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Delay Difference between the Linear Traffic Model and the Polynomial Traffic Model

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1. Introduction

In traffic-responsive signal control, a technique for interpreting the real-time detector data plays an important role in decision making process. The traditional Linear Traffic Model (LTM) is called a vertical queue traffic model and it is adopted widely in traffic signal control at signalized intersections. This model represents all vehicles have the same travel time before joining a queue. Thus, a queue is supposed to form vertically at the stop-line without occupying any space on the link. Due to the simplicity of this model, all vehicle motions are identical and unaffected by the signal display on their approach to the intersection. In the linear traffic model, the departure time of the leading vehicle is assumed to coincide with the start of the effective green time (Clayton, 1940; Webster & Cobbe, 1966; Allsop, 1970) and then departure times for the successive following vehicles are estimated in accordance with the saturation departure time at the stop-line. The delays and departure time estimates given by the vertical queue traffic models are not always realistic, because this model does not consider any braking motion until the stop-line. Namely, it is more focused on the queue delay rather than the geometric delay. The Robertson (1969) and TRANSYT (2004) uses vertical queue concepts in fixed-time signal optimization. Also, in traffic-responsive signal control, Miller (1963), Gartner (1983), and Heydecker (1990) use constant mean travel time from the detector to the stop-line. Traditional traffic theory focuses on modeling of queue discharge flow rates at signalized intersections with relatively little consideration of the corresponding queue discharge speed characteristics. This model uses a constant saturation flow rate and converts to an effective green time (Webster & Cobbe, 1966; Akcelick, 1981 & TRB, 2000).

In traffic-responsive signal control, the motion of each vehicle from the upstream detector to the downstream stop-line is needed for full interpretation of the detector output and for performance evaluation. It would be tedious and time consuming to describe the motion of each vehicle in detail at each time-step. Akcelick, et al (2002) describes the exponential queue discharge flow and speed models. In this paper, the concepts of kinematics in physics are applied to derive a Polynomial Traffic Model (PTM) at signalized intersections. The model developed in this paper requires one upstream detector, the position of detector is not an issue in this research. According to this model, arrival times and departure times of

all detected vehicles at the stop-line, and also their delays are estimated on the basis of the on-line detector data. The model is developed to represent the individual vehicle motion in relation to the general car-following concept. Thus, it can be applied in the dynamic signal optimization at a microscopic level (Ahn, 2004).

This paper explores a microscopic traffic model based on the one dimensional Kinematic equations in physics. The motion of vehicles from the upstream detector to the downstream stop-line is formulated analytically as a function of the start of green time. The formulae are derived for two different vehicle groups: a motion of the leading vehicle is determined by the signal indication and then successive following vehicles are estimated using the behavior of the vehicles in front of them. A comparative evaluation in delay and sensitivity of delay difference between the LTM and PTM is presented. The results show that the delay estimate in the LTM model is always greater than or equal to the PTM; however, the sensitivity of delay in the PTM is greater than the LTM.

2. Kinematic equations in physics

Kinematics is the study of motion irrespective of the forces; it deals with the mathematical description of motion in terms of position, speed, acceleration (or braking) and time. If any three of those variables are known, then the fourth variable can be calculated by using Kinematic equations. According to these concepts, we can describe the motion of vehicles in the vicinity of signalized intersections. If a vehicle is moving, the speed v is defined as the displacement of the vehicle divided by the time over which the displacement occurs. Furthermore, acceleration rate a refers to the rate of change of speed over time, which is defined as the change of speed divided by the change of time. The acceleration is equal to the second derivative x of with respect to time t :

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \quad (1)$$

The simplest case of translational motion assumes that the acceleration is constant, which means all vehicles are moving with maximum uniform acceleration rate. By using the Equation 1, the speed equation as a function of time, the position equation as a function of speed, and the position equation as a function of time are obtained as follows:

$$v_f = v_i + a(t_f - t_i) \quad (2)$$

$$x_f = x_i + \frac{v_f^2 - v_i^2}{2a} \quad (3)$$

$$x_f = x_i + v_i(t_f - t_i) + \frac{a}{2}(t_f - t_i)^2 \quad (4)$$

where v_i is the initial speed, v_f is the final speed, x_i is the initial position, and x_f is the final position.

3. Notation

The following notation will be used for the trajectory of the leading vehicle $n = 1$ and the following vehicles n ($2 \leq n \leq N$), where N is the serial number of the most recent detected vehicle at the position of detector. Let

a	be the acceleration rate ($a > 0$)
b	be the braking rate ($b > 0$)
v_0	be the free-flow speed
t_g	be the start of green time (including the reaction time)
v_g	be the speed of leading vehicle $n = 1$ at time t_g
X_g	be the position of leading vehicle $n = 1$ at time t_g
X_d	be the position of detector ($X_d \leq 0$)
X_s	be the position of stop-line ($X_s = 0$)
$\bar{X}_v$	be the position at which any delayed vehicles can regain the free-flow speed ($\bar{X}_v = v_0^2 / 2a$)
$\bar{X}_b$	be the maximum boundary of upstream ($\bar{X}_b = X_d - v_0^2 / 2b$)
t_d^n	be the time at which the vehicle n is detected at position X_d
v_d^n	be the speed of the vehicle n at time which it detected at position X_d
X_b^n	be the position at which the vehicle n starts to brake ($X_b^n = X_b^{n-1} - L$, where L is the safety margin)
t_b^n	be the time at which the vehicle n starts to brake
v_b^n	be the speed of the vehicle n at position X_b^n ($v_b^n = v_0$)
t_a^n	be the time at which the vehicle n starts to accelerate ($2 \leq n \leq N$)
v_a^n	be the speed of the vehicle n at time which the acceleration starts ($2 \leq n \leq N$)
X_a^n	be the position of vehicle n starts to accelerate ($2 \leq n \leq N$)
X_q^n	be the position at which the vehicle n stops completely after a full braking
t_q^n	be the time at which the vehicle n stops after a full braking at position X_q^n
X_v^n	be the position at which the vehicle n regains the free-flow speed
t_v^n	be the time at which the vehicle n regains the free-flow speed at position X_v^n
t_s^n	be the time at which the vehicle n crosses the stop-line X_s
v_s^n	be the speed of the vehicle n at time which it crosses the stop-line X_s

4. Polynomial Traffic Model (PTM) development

This model assumes a constant acceleration rate and braking rate, no overtaking is allowed and no vehicle exceeds the free-flow speed. Here, the start of green time t_g is defined as the beginning of the green time plus a reaction time τ , Gipps (1981) used $\tau = 2/3$ seconds

Fig. 1. Trajectory and delay for the leading vehicle in varying start of green time: Polynomial Traffic Model (PTM)

Fig. 2. Trajectory and delay for the leading vehicle in varying start of green time: Linear Traffic Model (LTM)

4.1 Trajectory of the leading vehicle

As can see in Figure 1, the motion of the leading vehicle is affected by the current signal indication. In respect of the start of green time t_g , the motion can be classified broadly into four regions: free-flow, braking (or braking and stopping), acceleration and free-flow. If current signal is green, the leading vehicle will cross the detector with free-flow speed and then reaches stop-line without experiencing any delays. However, if the signal is currently red, the vehicle from the detector can travel up to the braking position with free-flow speed, and on reaching that point the vehicle has to start to brake in order to stop safely at the stop-line. Meanwhile, if the signal changes to green, the vehicle will start to accelerate until it regains the free-flow speed; otherwise, the vehicle has to go through stopping until the next green starts. In this way, we can identify that the vehicle will pass the stop-line either at the free-flow speed or under. For the leading vehicle $n=1$, three different trajectories are considered as follows:

If $t_g \leq t_b^n$: maintain free-flow speed (see Figure 1a)

If $t_b^n < t_g \leq t_q^n$: free-flow $\rightarrow$ braking $\rightarrow$ acceleration $\rightarrow$ free-flow (see Figure 1b)

If $t_g > t_q^n$: free-flow $\rightarrow$ braking $\rightarrow$ stopping $\rightarrow$ acceleration $\rightarrow$ free-flow (see Figure 1c).

The motion of the leading vehicle from the detector position up to the braking position is unaffected by the current signal indication, thus it maintains free-flow speed. For the leading vehicle $n=1$, the braking position X_b^n , the braking time t_b^n and the stopping time t_q^n are calculated as follows:

$$X_b^n = -\frac{v_0^2}{2b}, \quad t_b^n = t_d^n - \frac{v_0}{2b} - \frac{X_d}{v_0} \quad \text{and} \\ t_q^n = t_b^n + \Delta t_b^n = t_d^n + \frac{v_0}{2b} - \frac{X_d}{v_0} \quad (\text{where } \Delta t_b^n = v_0 / b) \quad (5)$$

When the vehicle has reached its braking position, its further motion is determined by the current signal indication. There are three different trajectories to be considered with respect to the start of green time t_g for the leading vehicle $n=1$:

If $t_g \leq t_b^n$ then

$$v_g = v_0, \quad X_g = X_d + v_0(t_g - t_d^n), \\ t_v^n = t_g \quad \text{and} \quad X_v^n = X_g \quad (6)$$

If $t_b^n < t_g \leq t_q^n$ then

$$v_g = v_0 - b(t_g - t_b^n), \quad X_g = X_b^n + v_0(t_g - t_b^n) - \frac{b}{2}(t_g - t_b^n)^2, \\ t_v^n = t_g + \frac{b}{a}(t_g - t_b^n) \quad \text{and} \quad X_v^n = X_b^n + v_0\left(\frac{a+b}{a}\right)(t_g - t_b^n) - \left(\frac{ab+b^2}{2a}\right)(t_g - t_b^n)^2 \quad (7)$$

If t_g starts after the vehicle has stopped ($t_g > t_q^n$) then

$$v_g = 0, \quad X_g = 0, \quad t_v^n = t_g + \frac{v_0}{a} \quad \text{and} \quad X_v^n = \frac{v_0^2}{2a} \quad (8)$$

Finally, for the leading vehicle $n=1$, the crossing time t_s^n and its speed v_s^n at the stop-line can be obtained by comparing the fixed position X_s and the varying position X_v^n :

If $X_v^n \leq X_s$, which means that the vehicle is crossing the stop-line with free-flow speed, then

$$t_s^n = t_v^n - \frac{X_v^n}{v_0} \text{ and } v_s^n = v_0 \quad (9)$$

If $X_v^n > X_s$, which means that the vehicle is crossing the stop-line during the acceleration, then

$$v_s^n = \sqrt{v_g^2 - 2aX_g} \text{ and } t_s^n = t_g + \frac{\sqrt{v_g^2 - 2aX_g} - v_g}{a} \quad (10)$$

Fig. 3. A standard motion test for the following vehicles

4.2 Trajectory of the following vehicles

The variables estimated for the leading vehicle are used as parameters for the following vehicles trajectory in the following section. The basic concepts we use in the following calculations are the following vehicles cannot depart the downstream free-flow

position $\bar{X}_v$ with less than the minimum headways. Once we have characterised the full trajectory of the leading vehicle $n=1$ as a function of the start of green time t_g , the trajectory of all successive following vehicles $n=2, 3, \dots, N$ can be calculated directly based on the motion in front.

As can see in Fig 3, when the following vehicle $n=2$ crosses the detector at time t_d^n , the first order of task is finding the possible departure time $\bar{t}_v^n = \text{Max}[\hat{t}_v^n, \bar{t}_{d,v}^n]$ at position $\bar{X}_v$, here $\hat{t}_v^n = \bar{t}_v^{n-1} + 1/s$ (where s is a saturation flow) is the earliest departure time at position $\bar{X}_v$ and $\bar{t}_{d,v}^n = t_d^n + (\bar{X}_v - X_d)/v_0$ is the free-flow travel time from the detector to the position $\bar{X}_v$. By comparing variables $\hat{t}_v^n$ and $\bar{t}_{d,v}^n$, we can decide whether or not the following vehicle will be delayed. If $\bar{t}_{d,v}^n \geq \bar{t}_v^n$, the detected vehicle is identified as undelayed so it can reach the position $\bar{X}_v$ from the detector with free-flow speed. Thus, it is not necessary to find acceleration variables. However, if $\bar{t}_{d,v}^n < \bar{t}_v^n$, the vehicle is identified as delayed, then we need standard motion test to find acceleration variables, in which we can test whether or not any stopped lost time due to a queue is involved. Here, the standard arrival time is the longest approach time from the braking position X_b^n to the position $\bar{X}_v$ supposing that the vehicle has not stopped. The braking position is calculated by adding minimum safe spacing L . The final information we are seeking for each following vehicle n ($2 \leq n \leq N$) is its crossing time t_s^n , speed v_s^n at stop-line X_s and departure time $\bar{t}_v^n$ at position $\bar{X}_v$: the time t_s^n will be used to estimate the number of vehicles that can pass the stop-line if the green is extended by a certain control decision time, and $\bar{t}_v^n$ will be used to estimate its delays. In this analysis, the final information we are seeking for each following vehicle n ($2 \leq n \leq N$) is its crossing time t_s^n , speed v_s^n at stop-line X_s and departure time $\bar{t}_v^n$ at position $\bar{X}_v$: the time t_s^n will be used to estimate the number of vehicles that can pass the stop-line if the green is extended by a certain control decision time, and $\bar{t}_v^n$ will be used to estimate its delays. The calculation algorithm for the following vehicles trajectory is as follows:

Step 0: (Leading vehicle trajectory)

Identify the leading vehicle trajectory $n=1$ with respect to the current signal indication, and all identified variables are used as parameters in the following steps
 $\Rightarrow$ if $N=1$, stop processing (no following vehicles)

Step 1: (Departure time at $\bar{X}_v$)

Find the earliest departure time $\hat{t}_v^n$ and free-flow arrival time $\bar{t}_{d,v}^n$ at position $\bar{X}_v$, then the departure time will be $\bar{t}_v^n = \text{Max}[\hat{t}_v^n, \bar{t}_{d,v}^n]$, where $\hat{t}_v^n = \bar{t}_v^{n-1} + 1/s$ and $\bar{t}_{d,v}^n = t_d^n + (\bar{X}_v - X_d)/v_0$

Step 2: (Motion definition for undelayed case or delayed case)

Find the expected motion of the each following vehicle:

2-1) If $\bar{t}_{d,v}^n \geq \hat{t}_v^n$ which corresponds to undelayed motion, then standard motion test is not required

2-2) If $\bar{t}_{d,v}^n < \hat{t}_v^n$ which corresponds to delayed motion, then standard motion test is required

Step 3: (Braking position and time)

$X_b^n = X_b^{n-1} - L$ (t_b^n is only possible to find when $X_b^n \geq \bar{X}_b$; otherwise, out of range)

From Step 2, if the motion is identified as an undelayed case and:

If ($X_b^n \geq \bar{X}_b$), then $t_b^n = t_d^n + (X_b^n - X_d)/v_0$,

Else, t_b^n is unknown

⇒ go to Step 4 to find the variable of crossing time to the stop-line.

From Step 2, if the motion is identified as a delayed case:

If $X_d \leq X_b^n < X_s$, then $t_b^n = t_d^n + (X_b^n - X_d)/v_0$

If $\bar{X}_b \leq X_b^n < X_d$, then $t_b^n = t_d^n - (v_0 - \sqrt{v_0^2 + 2b(X_b^n - X_d)})/b$

⇒ go to Step 5 to find additional variables.

Step 4: (Crossing time to the stop-line for undelayed vehicle)

The crossing time t_s^n at stop-line X_s can be calculated by using a free-flow motion equation,

that is $t_s^n = t_d^n + (X_s - X_d)/v_0$

⇒ if $n < N$, go to Step 1; otherwise, stop processing

Step 5: (Standard motion test for delayed vehicle: halt case or not-halt case)

The standard motion test is only necessary if the vehicle is identified as delayed and $X_b^n \geq \bar{X}_b$

From the braking position standard motion test is needed to find whether or not it will come to a halt

5-1) The standard motion arrival time, that is $\hat{t}_{b,v}^n = t_b^n + v_0(a+b)/2ab + (\bar{X}_v - X_b^n)/v_0$

5-2) Motion definition: halt-case or not-halt case

If $\hat{t}_{b,v}^n < \bar{t}_v^n$, the motion is identified as a halt-case, then the stopping is involved

If $\hat{t}_{b,v}^n \geq \bar{t}_v^n$, the motion is identified as a not-halt case, then the stopping is not involved

Step 6: (Acceleration position, time and speed variables for delayed vehicle)

6-1) For a halt-case, acceleration position is equal to the stopped position, then

$$X_a^n = X_q^n \text{ (where } X_q^n = X_b^n + v_0^2/2b \text{), } v_a^n = 0 \text{ and } t_a^n = \bar{t}_v^n - \frac{v_0}{2a} - \frac{(\bar{X}_v - X_a^n)}{v_0}$$

6-2) For a not-halt case, acceleration starts meanwhile of braking, then

$$t_a^n = t_b^n + \sqrt{\frac{(X_b^n - \bar{X}_v) + v_0(\bar{t}_v^n - t_b^n)}{(ab + b^2)/2a}}, \quad X_a^n = X_b^n + v_0(t_a^n - t_b^n) - \frac{b}{2}(t_a^n - t_b^n)^2 \text{ and } v_a^n = v_0 - b(t_a^n - t_b^n)$$

Step 7: (Position and time of regaining the free-flow speed for delayed vehicle)

If $X_b^n \geq \bar{X}_b$, the time t_v^n and the position X_v^n can be calculated by using acceleration variables;

otherwise, they can be calculated corresponding to the departure time $\bar{t}_v^n$ at $\bar{X}_v$

7-1) If ($X_b^n \geq \bar{X}_b$), $t_v^n = t_a^n + (v_0 - v_a^n)/a$ and $X_v^n = X_a^n + v_a^n(t_v^n - t_a^n) + \frac{a}{2}(t_v^n - t_a^n)^2$

7-2) If ($X_b^n < \bar{X}_b$) which is defined as a out of boundary case, then

$$t_v^n = t_d^n + \sqrt{\frac{2[(X_d - \bar{X}_v) + v_0(\bar{t}_v^n - t_d^n)]}{a}} \quad \text{and} \quad X_v^n = X_d + v_d^n(t_v^n - t_d^n) + \frac{a}{2}(t_v^n - t_d^n)^2$$

where the estimated speed of $v_d^n = v_0 - a(t_v^n - t_d^n)$

Step 8: (Crossing time and speed to the stop-line for delayed vehicle)

The departure time t_s^n at the stop-line X_s can be calculated by comparing position variables

8-1) If $(X_b^n \geq \bar{X}_b)$ and

If $X_v^n \leq X_s$, then $t_s^n = t_v^n - (X_v^n / v_0)$

If $X_v^n > X_s$, then $t_s^n = t_a^n + (v_s^n - v_a^n) / a$,

where $v_s^n = \sqrt{(v_a^n)^2 - 2aX_a^n}$

8-2) If $(X_b^n < \bar{X}_b)$ and

If $X_v^n \leq X_s$, then $t_s^n = t_v^n - (X_v^n / v_0)$

If $X_v^n > X_s$, then $t_s^n = t_v^n + (v_0 - v_s^n) / a$,

where $v_s^n = \sqrt{(v_d^n)^2 - 2aX_d^n}$

$\Rightarrow$ if $n < N$, go to Step 1; otherwise, stop processing.

Fig. 4. Trajectory of the following vehicles (PTM vs LTM)

As can see in Figure 4a, the PTM can provide plenty of information around the detector and the stop-line. There are four different sets of motions have been identified at the position of the detector during one green period. In respect of the start of green time, any detected vehicles until the time A1 have crossed the detector with free-flow speed, the times between A1 and A2 vehicles have crossed it while braking, the times between A2 and A3 vehicles have crossed it while accelerating, and any following vehicles after the time A4 will cross the detector with free-flow speed. Presumably, the time A4 is the queue (or delay) dissipation time of this stage. More detailed analysis is suggested by Ahn (2004).

5. Delay and sensitivity of delay

In this section, delay and sensitivity of delay difference between the LTM and the PTM is discussed in relations to varying start of green time t_g . The delay estimates in the LTM and the PTM differs around the time of green starts; the LTM delay is always greater than or equal to the PTM, but so long as there is a queue the delay estimates are identical; however, the maximum sensitivity of delay in the PTM traffic model is higher than that in the LTM.

5.1 Delay and sensitivity of delay for the leading vehicle

As can see in Figure 2, the delay in the LTM is identified as the time difference between the start of effective green time $t_g + l_s$ (where start lag $l_s = v_0 / 2a$) and its free-flow arrival time to the stop-line $\bar{t}_{d,s}^n = t_{d,s}^n - X_d / v_0$. The LTM delay for the leading vehicle $n=1$ with respect to the time t_g is expressed as $D_V^n(t_g)$, which are calculated as follows:

If $(t_g + l_s \leq \bar{t}_{d,s}^n)$ then

$$D_V^n(t_g) = 0 \quad (11)$$

If $(t_g + l_s > \bar{t}_{d,s}^n)$ then

$$D_V^n(t_g) = (t_g + l_s) - \bar{t}_{d,s}^n \quad (12)$$

If t_g starts before the free-flow arrival time minus the start lag, no delay is incurred in LTM, and the resulting sensitivity is 0. However, from that time if t_g is increased by an amount ε , delay is increased by an identical amount ε , so that the resulting sensitivity is 1. By differentiating the delay Equation (11 and 12) with respect to the time t_g , we can get the sensitivity of delay as follows:

$$D_V^{1'}(t_g) = \begin{cases} 0 & t_g \leq \bar{t}_{d,s}^n - l_s \\ 1 & t_g > \bar{t}_{d,s}^n - l_s \end{cases} \quad (13)$$

As seen in Figure 1, the delay for the PTM is calculated on the basis of the time t_g in accordance with vehicular characteristic variables, such as, acceleration rate, braking rate and free-flow speed. The delay for the leading vehicle $n=1$ with respect to the time t_g is expressed as $D_K^n(t_g)$, which are calculated as follows:

If $t_g \leq t_b^n$ then

$$D_K^n(t_g) = 0 \quad (14)$$

If $t_b^n < t_g \leq t_q^n$ then

$$D_K^n(t_g) = (\bar{t}_b^n - \bar{t}_d^n) - \frac{(X_b^n - X_d)}{v_0} + \frac{(ab + b^2)}{2av_0}(t_g - \bar{t}_b^n)^2 \quad (15)$$

If $t_g > t_q^n$ then

$$D_K^n(t_g) = (t_g - \bar{t}_d^n) + \frac{v_0}{2a} + \frac{X_d}{v_0} \quad (16)$$

Fig. 5. Delay and sensitivity of delay comparison for the leading vehicle

As can see in Figure 5, if t_g starts before braking, no delay is incurred and the resulting sensitivity is 0. If t_g starts between braking and stopping, the delay increases quadratically, and the resulting sensitivity increases linearly. Once the vehicle has stopped, if t_g is increased by an amount ε , delay is increased by an identical amount ε , so that the resulting sensitivity is 1. By differentiating the delay Equations (14, 15 and 16) with respect to the varying time t_g , we can get the sensitivity of delay as follows:

If $t_g \leq t_b^n$ then

$$D_K^{n'}(t_g) = 0$$

(17)

If $t_b^n < t_g \leq t_q^n$ then

$$D_K^{n'}(t_g) = \frac{(ab + b^2)}{av_0}(t_g - t_b^n)$$

(18)

If $t_g > t_q^n$ then

$$D_K^{n'}(t_g) = 1$$

(19)

The maximum sensitivity of delay for the leading vehicle $n=1$ in the LTM is always 1. In contrast, the maximum sensitivity of delay in the PTM is $1+(b/a)$ that is obtained by differentiating the t_v^n in Equation (7) that obtained when $t_g = t_q^1$.

5.2 Delay and sensitivity of delay for the following vehicles

In this section, the following results are based on the simulation. Two different cases of traffic are considered: a low density case and a high density case. In the present examples, the maximum number of vehicles that can be held in the queue with this given condition is 8-vehicle.

		vehicle $n =$							
		1	2	3	4	5	6	7	8
Low density $\alpha = 0.5$ $h_0 = 3.0$	u	-	0.613	0.321	0.824	0.569	0.851	0.312	0.67
	H	-	3.97	5.27	3.38	4.12	3.32	5.33	3.79
	t_d^n	0.0	4.0	9.3	12.7	16.8	20.1	25.4	29.2
High density $\alpha = 0.9$ $h_0 = 2.0$	u	-	0.613	0.321	0.824	0.569	0.851	0.311	0.671
	H	-	2.54	3.26	2.21	2.62	2.18	3.29	2.44
	t_d^n	0.0	2.5	5.8	8.0	10.6	12.8	16.1	18.5

Table 1. Vehicle generation by using shifted exponential distribution of headways

In this The detector is located 50m away from the stop-line, and the minimum spacing of each following vehicle is $L=7\text{m}$, saturation departure time is 1.8 sec/vehicle (2,000 veh/h) and two cases of vehicles are generated based on *shifted exponential distribution of headways*, which is given by

$$H = h_0 - \frac{1}{\alpha} \ln(u)$$

(20)

where

- u
- is the random value generation, in which variables are generated with equal probability between $[0, 1]$
- h_0
- is the minimum gap of following ($h_0 \geq \tau + L/v_0$, where τ is 0.67sec)
- α
- is the density parameter

(a) Low density case (b) High density case

Fig. 6. Delay and sensitivity of delay comparison for the following vehicles

Using Equation (20), we can generate the vehicles without causing any headway violation. The detection time t_d^n can be given by $t_d^n = t_d^{n-1} + H$ ($n \geq 2$). Based on Table 1 data, the sensitivity of delay for 8-vehicle is tested with respect to the variations in the start of green time: in the range of $t_g = 0 \sim 30$ sec and t_g is incremented by 0.1 sec. As can see in Figure 6, let T_E be the time at which the total delay of 8-vehicle for the PTM and the LTM become same, namely $T_E = t_q^1$, where T_L is the time at which total delay becomes increasing linearly.

If t_g starts before T_L , which means that all 8-vehicle have not delayed yet; in this range, we can suppose that some vehicles may go through the stop-line without experiencing any delay, thus the sensitivity fluctuates.

If t_g starts after T_L , which means that all 8-vehicle have been delayed, the sensitivity of delay in this range with respect to t_g is equal to the total number of delayed vehicles. The sensitivity of delay for the LTM is equal to the number of delayed vehicles in the time range. But it differs for the PTM, if t_g starts before T_E , in some time range it shows a sensitivity higher than the total number of delayed vehicles. For the high density case simulation (see Figure 6b), the LTM delay is greater than or equal to the PTM delay. $T_E = 5.7$ sec and $T_L = 7.1$ sec. At time $t_g = 5.5$ sec, the PTM shows a sensitivity of delay $D_K' = 17.28$, and the LTM shows $D_V' = 6$, which is equivalent to numbers of delayed vehicles. At this time, the sensitivity of delay for the PTM is about three times greater than for the LTM. At time $t_g = 7.1$ sec, the sensitivity of delay for both models is equal to 8. From that time, we can see that all vehicles will experience some delay.

6. Conclusions

As explored in this paper, the delay estimated in the Linear Traffic Model (LTM) and the Polynomial Traffic Model (PTM) differs around the time of the green start, but so long as there is a queue the delay estimates from these models are identical. With respect to variations in the start of green time and provided that the braking rate exceeds the acceleration rate, the delay estimated in the LTM is always greater than or equal to the PTM traffic model; however, the maximum sensitivity of delay in the PTM is greater than that in the LTM. The PTM proposed in this paper has some attractions compared to the simpler LTM. The delay estimate in the PTM is on the basis of the vehicular characteristics, such as vehicle length, acceleration rate, braking rate and free-flow speed; however, the LTM takes adjusted time parameter of the start of effective green time. Thus, the delay estimated from the LTM is not realistic and always greater than or equal to the PTM. From these results, it is clear that the leading vehicle trajectory is the most important determinant for estimating the motion of vehicles at signalized intersections. Namely, the delay can be minimized in signal operation, if the start of green time begins before the vehicle starts to brake. The work described so far in this study is only at an early stage, in the sense that more work should be done for estimating the motion of vehicles in various intersection configurations.

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