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Numerical Integration Tools in Material Point Relative Motion Dynamics

Viviana Filip, Cornel Marin and Alexandru Marin

Abstract

The authors have proposed to show the advantages of different specialized software that can be used in solving the algebraic and transcendental differential equations applied in mechanics, as applications of material point relative motion dynamics.

Keywords: differential equations, motion dynamics, approximate numerical solution, specialized software.

1. INTRODUCTION

The relative motion of the material point can be described with relation [5]:

$$m \vec{a}_r = \vec{F} + \vec{F}_{Cor} + \vec{F}_t \quad (1)$$

where:

m is the material point mass

$\vec{a}_r$ is the relative acceleration

$\vec{F}$ is the exterior force resultant

$\vec{F}_{Cor}$ is the Coriolis force

$\vec{F}_t$ is the transport force

The equation (1) can be expressed by a differential equation at limit, like this:

$$\frac{d^2 \vec{y}}{d t^2} = f(\vec{y}, \frac{d \vec{y}}{d t}, t) \quad (2)$$

The initial conditions are:

$$\vec{y}(0) = \vec{y}_0 \text{ and } \frac{d \vec{y}(0)}{d t} = \vec{v}_0 \quad (3)$$

The relations (2) and (3) represent a differential equation of order two with initial conditions[1].

2. THE CALCULATION METHODS

The determination of numerical solution using the Euler method:

A differential equation of order n with initial conditions can be transformed in a system of n differential equations of first order by introducing $n-1$ auxiliary variables for the $n-1$ derivatives (in our example $n=2$).

This can be expressed in a matriceal form:

$$\vec{Y} = \begin{bmatrix} \vec{y} \\ \vec{y}' \end{bmatrix}, \quad \frac{d \vec{Y}}{d t} = \vec{F}(t, \vec{Y}), \quad \vec{Y}(t_0) = \begin{bmatrix} \vec{y}_0' \\ \vec{y}_0 \end{bmatrix} \quad (4)$$

where:

$$\vec{F}(t, \vec{Y}) = \begin{bmatrix} \vec{f}(t, \vec{y}, \vec{y}') \\ \vec{y}' \end{bmatrix} \quad (5)$$

The simplest method for solving these equations is the Euler method, where, to approximate the y variable at moment $t+dt$, it is used the Taylor series development until the first derivative:

$$\vec{y}(t + dt) \approx \vec{y}(t) + \frac{d \vec{y}(t)}{d t} = \vec{y}(t) + \vec{f}(t, \vec{y}(t)) \quad (6)$$

Any time there is this kind of problem, there must begin from an initial value of time t_0 , it is used a very small step dt , until it is obtained an approximate solution for equation with initial conditions.

The Euler method is very slow and it is necessary a very small time step dt to obtain good results, but it is typical for usual methods, like Runge-Kutta method.

The determination of numerical solution using the Runge-Kutta method:

The approximate solution y_{i+1} for equation (4) in next point $t_{i+1}=t_i+h$, is calculation with relation [4]:

$$y_{i+1} = y_i + \Delta y_i$$

$$\Delta y_i = \frac{1}{6} \left[K_1^{(i)} + 2K_2^{(i)} + 2K_3^{(i)} + K_4^{(i)} \right] \quad (7)$$

where:

$$K_1^{(i)} = h \cdot f(t_i, y_i)$$

$$K_2^{(i)} = h \cdot f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1^{(i)}\right)$$

$$K_3^{(i)} = h \cdot f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2^{(i)}\right) \quad (8)$$

$$K_4^{(i)} = h \cdot f(t_i + h, y_i + K_3^{(i)})$$

$$i = 1, 2, 3, \dots$$

3. THE SOLUTION OF DIFFERENTIAL EQUATION OF SECOND ORDER USING SPECIALIZED SOFTWARE

3.1. Application I:

It is considered a semicircular tube of R radius [8], which rotates with constant angular velocity $\dot{\theta}$ around the fixed axis presented in Figure 1. Inside the tube there is a material point of mass m that rotates with angular velocity $\dot{\varphi}$ around the center A of the semicircle. The authors proposed to determine the graphical representation of the relative motion $\varphi = \varphi(t)$ function, for t between t_i and t_f .

It is considered the following case: $R = 1$ m, $\theta(t) = 2t$, $\varphi(0) = 0$, $\varphi'(0) = 1$, $m = 0,01$ kg, $t_i = 0$, $t_f = 2$ s.

Fig. 1.

N_1 and N_2 are the reactions of the tube against the material point.

It is projected the motion equation (1) on the x axis:

$$m \ddot{\varphi} R = 2 m \dot{\theta}^2 R \sin \frac{\varphi}{2} \cos \frac{\varphi}{2} \quad (9)$$

It is projected the motion equation (1) on the y axis:

$$m \dot{\varphi}^2 R = N_1 - m \dot{\theta}^2 2 R \sin^2 \frac{\varphi}{2} - m 2 \dot{\theta} \dot{\varphi} R \quad (10)$$

It is projected the motion equation (1) on the z axis:

$$N_2 = m g \quad (11)$$

3.1.1. The solution obtained using Mathematica

In order to determine the graphical representation for $\varphi=\varphi(t)$ it will be solved the differential equation of second order, using the Mathematica software.

It was run this program for equation (9):

`a = NDSolve[{fi''[t] + 1 == 2^2 * Sin[fi[t]], fi[0] == 1, fi'[0] == 0}, fi[t], {t, 0, 2}]` (12)

and it was obtained the following results:

`{(fi[t] -> InterpolatingFunction[{{0., 2.}}, <>][t])}`

After the graphical construction command

`Plot[fi[t] /. a, {t, 0, 2}]` (13)

it was obtained the curve presented in Figure 2:

Fig. 2.

3.1.2. The solution obtained using Mathcad

The differential equation (2) that must be solved is:

$$\frac{d^2}{dt^2} \phi(t) = 4 \sin(\phi(t))$$
 (14)

with the initial conditions:

$$\begin{bmatrix} \phi_0(0) \\ \phi_1(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
 (15)

It was defined the initial conditions vector:

$$\text{ic} := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(16)

It was defined the time period against it will be represented the $\varphi = \varphi(t)$ function, and the number of iteration (N):

$$t_0 := 0 \quad t_1 := 2$$

(17)

$$N := 10$$

The Mathcad software needs to transform the differential equation of second order (2), in a two differential equations of first ordered system, as following:

$$\begin{aligned} \phi_0(t) &= \phi(t) & \phi_1(t) &= \frac{d}{dt} \phi_0(t) \\ D(t, \Phi) &:= \begin{bmatrix} \Phi_1 \\ 4 \sin(\Phi_0) \end{bmatrix} \end{aligned}$$

(18)

The command:

$$S := \text{rkfixed}(\text{ic}, t_0, t_1, N, D)$$

(19)

will solve the differential equations system.
In order to construct the graphical representation, it was assigned:

$$t := S^{<0>} \quad \phi(t) := S^{<1>}$$

(20)

After the running of the program it was obtained the curve presented in Figure 3:

Fig. 3.

3.2. Application II:

One considers the following problem [3]:

Inside a tube OA, a material point M of mass m slides with friction (Fig.4). In the same time, the tube is rotating with the angular velocity ω , around a vertical axis which intersects its ends.

In this example, the absolute and relative motion of the material point will be studied considering the following cases:

Fig. 4.

Case I: $F_r = k f(\dot{x})$

a) $\dot{x} < 1 \text{ m/s}$, $F_r = k \dot{x}$, $k=12.5 \text{ kg/s}$

b) $1 \text{ m/s} < \dot{x} < 250 \text{ m/s}$, $F_r = k \dot{x}^2$, $k=12.5 \text{ kg/m}$

c) $250 \text{ m/s} < \dot{x} < 300 \text{ m/s}$, $F_r = k \dot{x}^3$, $k=0.05 \text{ kg}\cdot\text{s}/\text{m}^2$

Case II: $F_r = \mu \sqrt{N_1^2 + N_2^2}$

where

- F_r is the resistant force which acts on the material point;
- k is a constant which depends on the domains of material point velocity values;
- μ is the friction coefficient between the material point and the tube's wall;
- N_1, N_2 are the normal reactions of the tube's wall on the material point.

In order to be able to compare the motion of the point in the two cases, using numerical methods, the following particular situations will be considered:

$m = 0,2 \text{ kg}$, $OA = 250 \text{ m}$, $\omega = 8 \text{ rad/s}$.

One considers the fixed reference frame $x_1y_1z_1$ and the moving reference frame, bonded to the tube. The differential equation of the material point motion is:

$$m\vec{a}_r = \vec{F}_r + \vec{G} + \vec{N}_1 + \vec{N}_2 + \vec{F}_t + \vec{F}_C \quad (21)$$

where G is the weight of the material point and F_t respectively F_C are the transport and Coriolis forces which act on the material point (Fig. 5). The terms of the equation (21) have the following expressions (22) and (23).

$$\begin{aligned} m\vec{a}_r &= m\ddot{x}\vec{i}, \quad \vec{F}_r = -k\dot{x}\vec{i}, \\ \vec{G} &= -mg\vec{k}, \quad \vec{N}_1 = N_1\vec{j} \end{aligned} \quad (22)$$

$$\begin{aligned} \vec{N}_2 &= N_2\vec{k} \\ \vec{F}_C &= -2m\omega\dot{x}\vec{j}, \quad \vec{F}_t = m\omega^2 x\vec{i} \end{aligned} \quad (23)$$

Fig. 5.

Case I a)

In the case of velocity values in the domain $[0,1]$ m/s, the resistant force F_r is proportional to the velocity: $F_r = k\dot{x}$, where $k = 12,5$ kg/s.

If one projects the equation (21) on the x axis of the moving reference system, one obtains:

$$m\ddot{x} = -k\dot{x} + m x \omega^2 \quad (24)$$

The solution of this equation is given by relation (25).

$$x[t] = C_1 e^{\frac{t(-k_1 - \sqrt{k_1^2 + 4m^2\omega^2})}{2m}} + C_2 e^{\frac{t(-k_1 + \sqrt{k_1^2 + 4m^2\omega^2})}{2m}} \quad (25)$$

where C_1 and C_2 are the integration constants.

Considering the particular numerical data and the initial conditions $x[0] = 0,25 \text{ m}$, $\dot{x}[0] = 0 \text{ m/s}$, one obtains the solution (26), plotted in Fig.6 .

$$x[t] = 0,00390507 e^{-63,5078 t} + 0,46095 e^{1,00775 t} \quad (26)$$

Fig. 6.

Considering that the tube's law of the motion is $\theta(t) = 8 t$, the parametric equations of the material point with respect to the fixed reference system are:

$$x_1[t] = x[t] \cos 8t = (0.00390507 e^{-63,5078 t} + 0.246095 e^{1,00775 t}) \cos 8t \quad (27)$$

$$y_1[t] = x[t] \sin 8t = (0.00390507 e^{-63,5078 t} + 0.246095 e^{1,00775 t}) \sin 8t$$

The trajectory of the material point for the time $[0, 1.38]s$ can be seen in Fig. 7.

At moment $t = 1,38 \text{ s}$, the velocity value reaches $\dot{x} = 1 \text{ m/s}$, so the material point enters the velocity domain where the resistant force is proportional to the squared velocity.

At instant $t = 1,38 \text{ s}$, the material point has the coordinate $x = 0,98 \text{ m}$. Due to the fact that the point is now in the velocity domain $[1, 250] \text{ m/s}$, the resistant force has the expression $F_r = k \dot{x}^2$, where $k = 12,5 \text{ kg/m}$.

Fig. 7.

Case I b)

The projection of the motion equation (21) on the x axis of the mobile reference system is:

$$m \ddot{x} + k \dot{x}^2 - m x \omega^2 = 0 \tag{28}$$

This equation is solved for the following initial conditions: $x[1,38] = 0,98 \text{ m}$, $\dot{x}[1,38] = 1 \text{ m/s}$.

Fig. 8.

Due to the non-linearity of this differential equation, the solution may be obtained only by advanced numerical methods. An analytical solution is not available, just a numerical one can be obtained. This solution's graph is plotted in Fig. 8. This numerical solution may be approximated using interpolation methods on the polynomial function shown by relation (29).

$$\begin{aligned} x = & 0.98 + (1.14516 + (0.263249 + (-0.00200546 + \\ & 0.000765443 + (-0.000579773 + 0.000311021(-0.0001123 + \\ & (0.0000299515 + (6.30748 \times 10^{-6} + 1.10011 \times 10^{-6} \\ & (-9 + t))(-8 + t))(-7 + t))(-6 + t))(-5 + t)) \\ & (-4 + t))(-3 + t))(-2.5 + t))(-2 + t))(-1.38 + t)) \end{aligned} \tag{29}$$

At instant $t = 12.5218 \text{ s}$, the material point's velocity reaches the value $\dot{x} = 250 \text{ m/s}$ which is in another domain of values. In this domain the resistant force is proportional with the velocity's power of 3. Considering the parametric equations of the material point with respect to the fixed reference frame, the trajectory of the material point in the time span $[1.38, 12.5218] \text{ s}$ can be plotted in Fig.9.

Case I c)

At instant $t = 12.5218 \text{ s}$, the material point ahs the coordinate $x = 182.713 \text{ m}$. The resistant force has now the following expression $F_r = k \dot{x}^3$, where $k = 0,05 \text{ kg s/m}^2$.

Fig. 9.

The projection of the motion equation (21) on the x axis of the moving reference frame will be:

$$m \ddot{x} + k_3 \dot{x}^3 - m x \omega^2 = 0 \quad (30)$$

This equation is solved considering the initial conditions $x[12.5218]=182.713m$, $\dot{x}[12.5218]=250m/s$

The obtained numerical solution is plotted in Figure 10.

Fig. 10.

The numerical solution can be approximated with the polynomial relation (31):

$$\begin{aligned} x = & 182.713 + (36.7001 + (1.06405 + (0.0082531 + \\ & (-0.00769165 + (0.00186645 + (-0.00034831 + \\ & (0.0000550529 - 7.7067 \times 10^{-6} (-19 + t)) \\ & (-18 + t)) (-17 + t)) (-16 + t)) (-15 + t)) \\ & (-14 + t)) (-13 + t)) (-12.5 + t)) \end{aligned} \quad (31)$$

At instant $t = 26.46$ s, the material point leaves the tube.

Taking into account the parametric equations of the material point with respect to the fixed reference frame, the trajectory of the point in the time span $[12.5218, 26.46]$ will be Figure 11.

Fig. 11.

Case II

One considers the case when the resistant force which acts on the material point is a

Coulombian force $F_r = \mu \sqrt{N_1^2 + N_2^2}$.

The projections of the motion equation (21) on the axis of the moving reference system are:

$$\begin{cases} m \ddot{x} = -\mu \sqrt{N_1^2 + N_2^2} + m x \omega^2 \\ N_1 = 2m \omega \dot{x} \\ N_2 = G \end{cases} \quad (32)$$

This leads to the differential equation of motion on the x axis:

$$m \ddot{x} + \mu \sqrt{(2m \omega \dot{x})^2 + (m g)^2} - m x \omega^2 = 0 \quad (33)$$

Solving this equation with the initial conditions $x[0] = 0.25 \text{ m}$, $\dot{x}[0] = 0 \text{ m/s}$ one obtains the numerical solution plotted by Figure 12.

Fig. 12.

The material point exists the tube at instant $t = 0,955\text{ s}$ and up to this point the numerical solution may be approximated by the polynomial relation (34).

$$\begin{aligned} x = & 0.25 + (1.945 + (24.6484 + (392.125 + \\ & (-462.338 + 5874.51 (-0.8 + t))(-0.6 + t)) \\ & (-0.4 + t)) (-0.2 + t)) t \end{aligned}$$

(34)

Considering the parametric equations of the material point with respect to the fixed reference frame the trajectory of the point for the time span $[0, 0.955]\text{ s}$, has the shape shown by Figure 13:

Fig. 13.

4. CONCLUSIONS

4.1. Conclusions about Application I:

In Mathematica and Mathcad, it is not necessary that user ask the program to display the φ values at different time moments between 0 and 2 seconds, in order to construct the graphical representation.

In Mathematica it is not necessary to specify the time step for interpolation, while in Mathcad, the step must be specified.

The NDSolve function from Mathematica has implemented the algorithm for solving the differential equations of first and superior order, when it is known the initial values. Mathematica is an algebraic instrument that can automatically convert any differential equation of superior order with initial conditions to a first ordered differential equation and can determine the numerical solution.

When using Mathcad, the user must make this conversion from superior to first order equation.

The time necessary for solving the equation is smaller when using Mathematica, due to the great number of input data required by the Mathcad.

The disadvantage of Mathematica and Mathcad is the necessity for user to learn the syntax for the specific function.

The advantage for Mathcad and Mathematica is the greater number of mathematical problems that can be solved.

Mathcad has developed a graphical user interface that makes this program very user friendly, despite the great level of complexity of the problems that can be solved using it.

The authors recommend Mathematica for solvers that use frequently the numerical methods due to the reduced time required for solving.

For these who are not interested in learning the syntax of specific functions, the authors recommend Mathcad due to the user friendly interface, which makes it easy to use.

4.2. Conclusions about Application II:

In the given conditions from case I a), the differential equation is linear and admits an analytical solution. The other cases though exhibit non-linear equations with numerical solutions that can be reached only by means of advanced mathematical software. They can be approximated by polynomial functions using interpolation methods.

The contribution of this paper is the study of the material point relative motion dynamics by using the advanced mathematical techniques. This advances techniques allow the trajectory determination, the numerical solutions determination, the motion significant moments calculation.

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