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Alternative analytical method used in calculus of hyper static mechanical systems, in plotting the distribution of shear force, bending moment, displacements and rotations of section beam

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Abstract. The actual graphical methods used by engineers when plotting the stress distributions are based on integrating the differential equations of stresses for each beam segment. The resulting integration constants are obtained by imposing boundary conditions for each beam segment. This alternative proposed analytical method uses the *MATHCAD step function* $\Phi(x-a)$ which introduces a friendly analytical form of the shear force, bending moment, rotation (slope) cross-section and displacement expressions. The calculus of *hyper static mechanical systems* using the step function and matrix expression of equations and also the constructive optimization of structures is thus easier to be performed using this method. The traditional methods use integral expressions which are solved by means of numerical methods. The practical application presented in this chapter is representative for a large area of mechanical engineering.

Keywords: MATHCAD step function, hyper static mechanical systems

1. Expression of cross sectional and deflections functions

In this chapter we will present the expressions of the cross sectional functions for the shear force $T_z(x)$, bending moment $M_{iy}(x)$, transversal section rotations $\varphi_y(x)$ and displacement section $w(x)$, using the MATHCAD step function $\Phi(x-a)$ [1], [2], [3], [4]. The step function $\Phi(x-a)$ in MATHCAD have the well-known form:

$$\Phi(x-a) = \begin{cases} 0 & \text{if } x < a \\ 1 & \text{if } x \geq a \end{cases} \quad (1)$$

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Fig. 1. The Beam model loaded with 4 different loading types

One considers a beam subjected to bending. The beam is characterized by length L and constant bending stiffness El_y . There are 4 different load types [1], [2], [3], [4], presented in fig. 1:

- The bending moment N , placed at distance a from the left end of the beam;
- The concentrated force P , placed at distance b from the left end of the beam;
- The uniform distributed load q_0 which acts on a beam segment delimited by the distances e and f from the left end of the beam;
- The linear distributed load $q(x) = q_1 \frac{x-g}{h-g}$, $x \in [g, h]$ which acts on a beam segment delimited by the distances g and h from the left end of the beam.

The **analytical expressions for the cross sectional resultants** considering the one loading types from figure 1, written using the step function Φ are:

For the bending moment N :

$$M_i(x) = -N \cdot \Phi(x - a) \quad (2)$$

The bending cross sectional resultant diagram is represented using MATHCAD software in figure 2.

Fig. 2.

For the concentrated force P :

$$\begin{aligned} T(x) &= -P \cdot \Phi(x - b); \\ M_i(x) &= -P \cdot \Phi(x - b) \cdot (x - b) \end{aligned} \tag{3}$$

The bending and shear cross sectional resultants diagrams for this case is represented using MATHCAD software in figure 3.

Fig. 3.

For the uniform distributed load $q(x) = q_0, \quad x \in [e, f]$:

$$T(x) = -q_0 \cdot \Phi(x - e) \cdot (x - e) + q_0 \cdot \Phi(x - f) \cdot (x - f);$$
$$M_i(x) = -\frac{q_0}{2} \cdot \Phi(x - e) \cdot (x - e)^2 + \frac{q_0}{2} \cdot \Phi(x - f) \cdot (x - f)^2; \tag{4}$$

The bending and shear cross sectional resultants diagrams for this case is represented using MATHCAD software in figure 4.

Fig. 4.

For the linear distributed load $q(x) = q_1 \frac{x-g}{h-g}$, $x \in [g, h]$ (fig.5.)

Fig. 5.

The analytical expressions for the cross sectional resultant is obtained:

$$T(x) = -\frac{q_1}{2(h-g)} \cdot \Phi(x-g) \cdot (x-g)^2 + q_1 \cdot \Phi(x-h) \cdot (x-h) + \frac{q_1}{2(h-g)} \cdot \Phi(x-h) \cdot (x-h)^2;$$

$$M_i(x) = -\frac{q_1}{6(h-g)} \cdot \Phi(x-g) \cdot (x-g)^3 + \frac{q_1}{2} \cdot \Phi(x-h) \cdot (x-h)^2 + \frac{q_1}{6(h-g)} \cdot \Phi(x-h) \cdot (x-h)^3;$$
(5)

The bending and shear cross sectional resultants diagrams for this case is represented using MATHCAD software in figure 5.

If the beam is subjected simultaneous of the four types of load above, the analytical expressions of cross sectional are obtained using the principle of superposition effects:

■For shear force:

$$T_z(x) = -P \cdot \Phi(x-b) - q_0 \cdot (x-e) \cdot \Phi(x-e) + q_0 \cdot (x-f) \cdot \Phi(x-f) -$$

$$- q_1 \cdot \frac{(x-g)^2}{2(h-g)} \cdot \Phi(x-g) + q_1 \cdot (x-h) \cdot \Phi(x-h) + q_1 \cdot \frac{(x-h)^2}{2(h-g)} \cdot \Phi(x-h);$$
(6)

■For bending moment:

$$\begin{aligned} M_{iy}(x) = & -N \cdot \Phi(x-a) - P \cdot (x-b) \cdot \Phi(x-b) - \\ & - q_0 \cdot \frac{(x-e)^2}{2} \cdot \Phi(x-e) + q_0 \cdot \frac{(x-f)^2}{2} \cdot \Phi(x-f) - \\ & - q_1 \cdot \frac{(x-g)^3}{6(h-g)} \cdot \Phi(x-g) + q_1 \cdot \frac{(x-h)^2}{2} \cdot \Phi(x-h) + q_1 \cdot \frac{(x-h)^3}{6(h-g)} \cdot \Phi(x-h); \end{aligned} \quad (7)$$

The differential equation of displacements $w(x)$ corresponding to a cross-section is [1]:

$$\frac{d^2 w}{dx^2} = - \frac{M_{iy}(x)}{EI_y} \quad (8)$$

Integrating twice the differential equation (2), one obtains the function of cross-sectional rotations $\varphi_y(x)$ and displacement $w(x)$:

$$\begin{aligned} \varphi_y(x) &= \frac{dw}{dx} = \varphi_0 - \int_0^x \frac{M_{iy}(s)}{EI_y} ds \\ w(x) &= w_0 + \varphi_0 \cdot x - \int_0^x \int_0^t \left(\frac{M_{iy}(s)}{EI_y} ds \right) dt \end{aligned} \quad (9)$$

If the beam is subjected simultaneous of the four types of load above, the analytical expressions for the function of cross-sectional rotations $\varphi_y(x)$ and displacement $w(x)$ obtained using relation (9) are written:

$$\begin{aligned} EI_y \varphi_y(x) = & EI_y \varphi_0 + N \cdot (x-a) \cdot \Phi(x-a) + P \cdot \frac{(x-b)^2}{2} \cdot \Phi(x-b) + \\ & + q_0 \cdot \frac{(x-e)^3}{6} \cdot \Phi(x-e) - q_0 \cdot \frac{(x-f)^3}{6} \cdot \Phi(x-f) + \\ & + q_1 \cdot \frac{(x-g)^4}{24(h-g)} \cdot \Phi(x-g) - q_1 \cdot \frac{(x-h)^3}{6} \cdot \Phi(x-h) - q_1 \cdot \frac{(x-h)^4}{24(h-g)} \cdot \Phi(x-h); \end{aligned} \quad (10)$$

$$\begin{aligned} EI_y w(x) = & EI_y w_0 + EI_y \varphi_0 \cdot x + N \cdot \frac{(x-a)^2}{2} \cdot \Phi(x-a) + P \cdot \frac{(x-b)^3}{6} \cdot \Phi(x-b) + \\ & + q_0 \cdot \frac{(x-e)^4}{24} \cdot \Phi(x-e) - q_0 \cdot \frac{(x-f)^4}{24} \cdot \Phi(x-f) + \\ & + q_1 \cdot \frac{(x-g)^5}{120(h-g)} \cdot \Phi(x-g) - q_1 \cdot \frac{(x-h)^4}{24} \cdot \Phi(x-h) - q_1 \cdot \frac{(x-h)^5}{120(h-g)} \cdot \Phi(x-h); \end{aligned} \quad (11)$$

2. Numerical application for computing diagrams

Application 1. Beam supported by 2 rigid bearings (statically determined structure)

One considers a beam supported by two bearings. The length of the beam is $10a$ and the flexural stiffness EI constant along its entire length. The beam is loaded with two uniformly distributed loads q_0 , a linearly distributed load $0 - q_1$ and a force couple (moment) $N=qa^2$, as shown in figure 6. One should determine the analytical expressions of the shear force $T(x)$ and bending moment $M(x)$, cross-sectional rotations $\varphi(x)$ and displacements $w(x)$ and plot the variation diagrams along the beam's length using the step function $\Phi(x-a)$ from MATHCAD.

Particular values: $q_0=q$, $q_1=2q$; $q=1\text{ kN/m}$, $a=1\text{ m}$ and $EI=1000\text{ kNm}^2$.

Fig. 6.

For the statically determined system one will first determine the reaction forces V_1 and V_2 using the equilibrium equations from the Mechanics of Solids:

$$\begin{aligned}\sum M_{1y} = 0 &\Rightarrow V_2 \cdot 6a + q_0 \cdot 2a \cdot a + q_0 \cdot 2a \cdot a - \frac{1}{2} q_1 \cdot 3a \cdot 5a - N = 0 \Rightarrow V_2 = 2qa \\ \sum M_{2y} = 0 &\Rightarrow V_1 \cdot 6a - q_0 \cdot 2a \cdot 7a + q_0 \cdot 2a \cdot 5a - \frac{1}{2} q_1 \cdot 3a \cdot a + N = 0 \Rightarrow V_1 = qa\end{aligned}\quad (12)$$

The analytical expressions of the shear force $T(x)$ and bending moment $M(x)$ will be determined using the principle of effects superposition, considering the corresponding analytical expressions of the moment N , reaction forces V_1 , V_2 , uniformly distributed loads q_0 and linearly distributed loads $0-q_1$:

$$\begin{aligned}
 T(x) &= V_1 \cdot \Phi(x-2a) + V_2 \cdot \Phi(x-8a) - \\
 &\quad - q_0 \cdot \Phi(x) \cdot x + q_0 \cdot \Phi(x-2a) \cdot (x-2a) + q_0 \cdot \Phi(x-2a) \cdot (x-2a) - q_0 \cdot \Phi(x-4a) \cdot (x-4a) - \\
 &\quad - \frac{q_1}{2 \cdot 3a} \cdot \Phi(x-5a) \cdot (x-5a)^2 + q_1 \cdot \Phi(x-8a) \cdot (x-8a) + \frac{q_1}{2 \cdot 3a} \cdot \Phi(x-8a) \cdot (x-8a)^2. \\
 M_i(x) &= N \cdot \Phi(x-10a) + V_1 \cdot \Phi(x-2a)(x-2a) + V_2 \cdot \Phi(x-8a)(x-8a) - \\
 &\quad - \frac{q_0}{2} \cdot \Phi(x) \cdot x^2 + \frac{q_0}{2} \cdot \Phi(x-2a) \cdot (x-2a)^2 + \frac{q_0}{2} \cdot \Phi(x-2a) \cdot (x-2a)^2 - \frac{q_0}{2} \cdot \Phi(x-4a) \cdot (x-4a)^2 - \\
 &\quad - \frac{q_1}{6 \cdot 3a} \cdot \Phi(x-5a) \cdot (x-5a)^3 + \frac{q_1}{2} \cdot \Phi(x-8a) \cdot (x-8a)^2 + \frac{q_1}{6 \cdot 3a} \cdot \Phi(x-8a) \cdot (x-8a)^3
 \end{aligned} \tag{13}$$

The analytical expressions of the cross-sectional rotations $\varphi(x)$ will be determined using the principle of effects superposition, considering the general relations (10) for the 4 types of loads:

$$\begin{aligned}
 EI_y \varphi(x) &= EI_y \varphi_0 - N \cdot \Phi(x-10a) \cdot (x-10a) - V_1 \cdot \Phi(x-2a) \frac{(x-2a)^2}{2} - V_2 \cdot \Phi(x-8a) \frac{(x-8a)^2}{2} + \\
 &\quad + q_0 \cdot \Phi(x) \cdot \frac{x^3}{6} - q_0 \cdot \Phi(x-2a) \cdot \frac{(x-2a)^3}{6} - q_0 \cdot \Phi(x-2a) \cdot \frac{(x-2a)^3}{6} + q_0 \cdot \Phi(x-4a) \cdot \frac{(x-4a)^3}{6} + \\
 &\quad + \frac{q_1}{3a} \cdot \Phi(x-5a) \cdot \frac{(x-5a)^4}{24} - \frac{q_1}{3a} \cdot \Phi(x-8a) \cdot \frac{(x-8a)^4}{24} - q_1 \cdot \Phi(x-8a) \cdot \frac{(x-8a)^3}{6}
 \end{aligned} \tag{14}$$

The analytical expressions of the cross-sectional displacements $w(x)$ will be determined using the principle of effects superposition, considering the general relations (11) for the 4 types of loads:

$$\begin{aligned}
 EI_y w(x) &= EI_y w_0 + EI_y \varphi_0 \cdot x - N \cdot \Phi(x-10a) \frac{(x-10a)^2}{2} - V_1 \cdot \Phi(x-2a) \frac{(x-2a)^3}{6} - V_2 \cdot \Phi(x-8a) \frac{(x-8a)^3}{6} + \\
 &\quad + q_0 \cdot \Phi(x) \cdot \frac{x^4}{24} - q_0 \cdot \Phi(x-2a) \cdot \frac{(x-2a)^4}{24} - q_0 \cdot \Phi(x-2a) \cdot \frac{(x-2a)^4}{24} + q_0 \cdot \Phi(x-4a) \cdot \frac{(x-4a)^4}{24} + \\
 &\quad + \frac{q_1}{3a} \cdot \Phi(x-5a) \cdot \frac{(x-5a)^5}{120} - \frac{q_1}{3a} \cdot \Phi(x-8a) \cdot \frac{(x-8a)^5}{120} - q_1 \cdot \Phi(x-8a) \cdot \frac{(x-8a)^4}{24}
 \end{aligned} \tag{15}$$

The integration constants $EI_y w_0$ and $EI_y \varphi_0$ will be determined for the displacements' conditions imposed in the supports:

$$\begin{cases} EI_y w(2a) = EI_y w_0 + EI_y \varphi_0 \cdot 2a + W(2a) = 0 \\ EI_y w(8a) = EI_y w_0 + EI_y \varphi_0 \cdot 8a + W(8a) = 0 \end{cases} \Rightarrow \begin{cases} EI_y \varphi_0 = \frac{1}{6a} [W(2a) - W(8a)] \\ EI_y w_0 = \frac{1}{3} [W(8a) - 4 \cdot W(2a)] \end{cases} \tag{16}$$

The shear forces $T(x)$ and bending moments $M_i(x)$ diagrams are shown in figure 7. In figure 8 one plotted the deflection function $EIW(x)$ and the rotation one $EIF(x)$, for the particular numerical data.

Fig. 7. Shear force T and bending moment M_i diagrams

Fig. 8. Cross-sectional deflection $EIW(x)$ and rotation $EIF(x)$ diagrams

Application 2: Continuous beam supported by 4 uneven rigid bearings (statically undetermined system)

One considers the continuous beam OA supported by 4 stiff bearings, having different levels with respect to the beam's axis: $w_1=0, w_2 \neq 0, w_3 \neq 0, w_4=0$. The beam has a constant bending stiffness EI , along its whole length. The beam's exterior loading is known: N, P, q_0 and q_1 . The support reactions V_1, V_2, V_3 and V_4 are unknown (fig.9). Particular values: $q_0=1\text{ kN/m}$; $a=1\text{ m}$; $EI=1000\text{ kNm}^3$; $w_2=0,001\text{ m}$; $w_3=0,003\text{ m}$; $r_1=2a$; $r_2=3a$; $r_3=7a$; $r_4=10a$; $w_1=w_4=0$

Fig. 9.

In order to compute the values of the unknown reaction forces V_1, V_2, V_3 și V_4 one uses the two equations of equilibrium from Mechanics:

$$\begin{aligned} \sum F_{zs} \downarrow &= V_1 + V_2 + V_3 + V_4 \\ \sum \tilde{M}_{4s} &= V_1(r_4 - r_1) + V_2(r_4 - r_2) + V_3(r_4 - r_3) \end{aligned} \quad (17)$$

where: $\sum F_{zs} \downarrow$ is the sum of exterior forces on Oz axis;

$\sum \tilde{M}_{4s}$ the sum of moments and exterior force couples with respect to the Oy axis through point 4.

The third and the fourth equations will result by replacing the origin parameters (w_0 and φ_0) in the displacement equations from the supports:

$$\begin{cases} w_0 + \varphi_0 \cdot r_1 + \frac{1}{EI} W(r_1) = w_1 \\ w_0 + \varphi_0 \cdot r_2 + \frac{1}{EI} W(r_2) = w_2 \end{cases} \quad \begin{cases} w_0 + \varphi_0 \cdot r_3 + \frac{1}{EI} W(r_3) = w_3 \\ w_0 + \varphi_0 \cdot r_4 + \frac{1}{EI} W(r_4) = w_4 \end{cases} \quad (18)$$

where $W(x)$ is the second integral of the bending efforts with changed sign for the four types of known efforts as well as for the unknown reaction forces V_1, V_2, V_3 and V_4 (fig.9):

$$\begin{aligned}
W(x) = & -\frac{N}{2} \cdot \Phi(x-11a) \cdot (x-11a)^2 - \frac{V_1}{6} \cdot \Phi(x-2a) \cdot (x-2a)^3 - \frac{V_2}{6} \cdot \Phi(x-3a) \cdot (x-3a)^3 - \\
& - \frac{V_3}{6} \cdot \Phi(x-7a) \cdot (x-7a)^3 - \frac{V_4}{6} \cdot \Phi(x-10a) \cdot (x-10a)^3 - \frac{q_0}{24} \cdot \Phi(x) \cdot x^4 - \frac{q_0}{24} \cdot \Phi(x-2a) \cdot (x-2a)^4 + \\
& + \frac{q_1}{120 \cdot 3a} \cdot \Phi(x-3a) \cdot (x-3a)^5 - \frac{q_1}{24} \cdot \Phi(x-6a) \cdot (x-6a)^4 - \frac{q_1}{120 \cdot 3a} \cdot \Phi(x-6a) \cdot (x-6a)^5 + \\
& + \frac{q_0}{24} \cdot \Phi(x-7a) \cdot (x-7a)^4 - \frac{q_0}{24} \cdot \Phi(x-10a) \cdot (x-10a)^4
\end{aligned} \tag{19}$$

Replacing w_0 and φ_0 in the first and last couple of equations (18) one obtains two equations:

$$\begin{aligned}
\frac{W(r_2) - W(r_1)}{r_2 - r_1} - EI \frac{w_2 - w_1}{r_2 - r_1} &= \frac{W(r_3) - W(r_2)}{r_3 - r_2} - EI \frac{w_3 - w_2}{r_3 - r_2} \\
\frac{W(r_3) - W(r_2)}{r_3 - r_2} - EI \frac{w_3 - w_2}{r_3 - r_2} &= \frac{W(r_4) - W(r_3)}{r_4 - r_3} - EI \frac{w_4 - w_3}{r_4 - r_3}
\end{aligned} \tag{20}$$

Denotes with $Ws(x)$ the second integral of the bending efforts with changed sign written *only for known exterior loads*:

$$\begin{aligned}
Ws(x) = & -N \cdot \Phi(x-11a) \cdot \frac{(x-11a)^2}{2} + q_0 \cdot \Phi(x) \cdot \frac{(x)^4}{24} - q_0 \cdot \Phi(x-2a) \cdot \frac{(x-2a)^4}{24} + \\
& + q_1 \cdot \Phi(x-3a) \cdot \frac{(x-3a)^5}{120(3a)} - q_1 \cdot \Phi(x-6a) \cdot \frac{(x-6a)^4}{24} - q_1 \cdot \Phi(x-6a) \cdot \frac{(x-6a)^5}{120(3a)} + \\
& + 2q_0 \cdot \Phi(x-7a) \cdot \frac{(x-7a)^4}{24} - 2q_0 \cdot \Phi(x-10a) \cdot \frac{(x-10a)^4}{24}
\end{aligned} \tag{21}$$

Notate: $Ws(r_1) = W_{s1}$; $Ws(r_2) = W_{s2}$; $Ws(r_3) = W_{s3}$, and so on.

$$\begin{aligned}
k_1(x) &= \Phi(x-r_1) \cdot \frac{(x-r_1)^3}{6}; \quad k_2(x) = \Phi(x-r_2) \cdot \frac{(x-r_2)^3}{6}; \\
k_3(x) &= \Phi(x-r_3) \cdot \frac{(x-r_3)^3}{6}; \quad k_4(x) = \Phi(x-r_4) \cdot \frac{(x-r_4)^3}{6};
\end{aligned} \tag{22}$$

Notate: $k_1(r_1) = k_{11}$; $k_1(r_2) = k_{12}$; $k_2(r_1) = k_{21}$, and so on.

Isolating the unknowns V_1 , V_2 , V_3 and V_4 from equations (20) one obtains the third and the fourth equations:

$$\left\{ \begin{array}{l} V_1 \cdot \left(\frac{k_{11}-k_{12}}{d_{12}} - \frac{k_{12}-k_{13}}{d_{23}} \right) + V_2 \cdot \left(\frac{k_{21}-k_{22}}{d_{12}} - \frac{k_{21}-k_{23}}{d_{23}} \right) + V_3 \cdot \left(\frac{k_{31}-k_{32}}{d_{12}} - \frac{k_{32}-k_{33}}{d_{23}} \right) = \\ = \frac{W_{s1}-W_{s2}}{r_2-r_1} - \frac{W_{s2}-W_{s3}}{r_3-r_2} - EI \frac{w_1-w_2}{r_2-r_1} + EI \frac{w_2-w_3}{r_3-r_2} \\ V_1 \cdot \left(\frac{k_{12}-k_{13}}{d_{23}} - \frac{k_{12}-k_{13}}{d_{34}} \right) + V_2 \cdot \left(\frac{k_{22}-k_{23}}{d_{23}} - \frac{k_{23}-k_{24}}{d_{34}} \right) + V_3 \cdot \left(\frac{k_{32}-k_{33}}{d_{23}} - \frac{k_{33}-k_{34}}{d_{34}} \right) + \\ + V_4 \cdot \left(\frac{k_{42}-k_{43}}{d_{23}} - \frac{k_{42}-k_{43}}{d_{34}} \right) = \frac{W_{s2}-W_{s3}}{r_3-r_2} - \frac{W_{s3}-W_{s4}}{r_4-r_3} - EI \frac{w_2-w_3}{r_3-r_2} + EI \frac{w_3-w_4}{r_4-r_3} \end{array} \right. \quad (23)$$

The equations (17) and (22) are written in matrix form as it follows:

$$\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ \frac{r_4-r_1}{k_{11}-k_{12}} - \frac{k_{12}-k_{13}}{k_{12}-k_{13}} & \frac{r_4-r_2}{k_{21}-k_{22}} - \frac{k_{22}-k_{23}}{k_{22}-k_{23}} & \frac{r_4-r_3}{k_{31}-k_{32}} - \frac{k_{32}-k_{33}}{k_{32}-k_{33}} & 0 \\ \frac{r_2-r_1}{k_{12}-k_{13}} - \frac{k_{13}-k_{14}}{k_{13}-k_{14}} & \frac{r_3-r_2}{k_{22}-k_{23}} - \frac{k_{23}-k_{24}}{k_{23}-k_{24}} & \frac{r_2-r_1}{k_{32}-k_{33}} - \frac{k_{33}-k_{34}}{k_{33}-k_{34}} & 0 \\ \frac{r_3-r_2}{r_3-r_2} & \frac{r_4-r_3}{r_4-r_3} & \frac{r_3-r_2}{r_3-r_2} & \frac{r_4-r_3}{r_4-r_3} \end{array} \right] \left\{ \begin{array}{l} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \right\} = \left\{ \begin{array}{l} \sum \overleftarrow{F}_{zs} \downarrow \\ \sum \overleftarrow{M}_{4s} \\ \frac{W_{s1}-W_{s2}}{r_2-r_1} - \frac{W_{s2}-W_{s3}}{r_3-r_2} - EI \frac{w_1-w_2}{r_2-r_1} + EI \frac{w_2-w_3}{r_3-r_2} \\ \frac{W_{s2}-W_{s3}}{r_3-r_2} - \frac{W_{s3}-W_{s4}}{r_4-r_3} - EI \frac{w_2-w_3}{r_3-r_2} + EI \frac{w_3-w_4}{r_4-r_3} \end{array} \right\} \quad (24)$$

If one introduces the particular numerical data and the matrix equation (23) in MATHCAD, the following values of the reaction forces will yield:

$$\left(\begin{array}{c} V1 \\ V2 \\ V3 \\ V4 \end{array} \right) = \left(\begin{array}{c} 3.550 \\ -0.225 \\ 4.392 \\ 3.283 \end{array} \right) \quad (25)$$

The particular expressions of the shear force and bending moment are:

$$\begin{aligned}
 T(x) = & V_1 \cdot \Phi(x-2a) + V_2 \cdot \Phi(x-3a) + V_3 \cdot \Phi(x-7a) + V_4 \cdot \Phi(x-10a) - \\
 & - q_0 \cdot \Phi(x) \cdot x + q_0 \cdot \Phi(x-2a) \cdot (x-2a) - \\
 & - \frac{q_1}{2 \cdot 3a} \cdot \Phi(x-3a) \cdot (x-3a)^2 + q_1 \cdot \Phi(x-6a) \cdot (x-6a) + \frac{q_1}{2 \cdot 3a} \cdot \Phi(x-6a) \cdot (x-6a)^2 - \\
 & - q_0 \cdot \Phi(x-7a) \cdot (x-7a) + q_0 \cdot \Phi(x-10a) \cdot (x-10a)
 \end{aligned} \tag{26}$$

$$\begin{aligned}
 M(x) = & N \cdot \Phi(x-11a) + V_1 \cdot \Phi(x-2a) \cdot (x-2a) + V_2 \cdot \Phi(x-3a) \cdot (x-3a) + \\
 & + V_3 \cdot \Phi(x-7a) \cdot (x-7a) + V_4 \cdot \Phi(x-10a) \cdot (x-10a) - \\
 & - \frac{q_0}{2} \cdot \Phi(x) \cdot x^2 + \frac{q_0}{2} \cdot \Phi(x-2a) \cdot (x-2a)^2 + \\
 & - \frac{q_1}{6 \cdot 3a} \cdot \Phi(x-3a) \cdot (x-3a)^3 + \frac{q_1}{2} \cdot \Phi(x-6a) \cdot (x-6a)^2 + \frac{q_1}{6 \cdot 3a} \cdot \Phi(x-6a) \cdot (x-6a)^3 - \\
 & - \frac{q_0}{2} \cdot \Phi(x-7a) \cdot (x-7a)^2 + \frac{q_0}{2} \cdot \Phi(x-10a) \cdot (x-10a)^2
 \end{aligned} \tag{27}$$

The particular expression of the cross-sectional rotation is written:

$$\begin{aligned}
 EI_y \varphi(x) = & EI_y \varphi_0 + F(x); \\
 F(x) = & -N \cdot \Phi(x-11a) \cdot (x-11a) - \frac{V_1}{2} \cdot \Phi(x-2a) \cdot (x-2a)^2 - \frac{V_2}{2} \cdot \Phi(x-3a) \cdot (x-3a)^2 - \\
 & - \frac{V_3}{2} \cdot \Phi(x-7a) \cdot (x-7a)^2 - \frac{V_4}{2} \cdot \Phi(x-10a) \cdot (x-10a)^2 - \\
 & + \frac{q_0}{6} \cdot \Phi(x) \cdot x^3 - \frac{q_0}{6} \cdot \Phi(x-2a) \cdot (x-2a)^3 - \\
 & + \frac{2q}{24 \cdot 3a} \cdot \Phi(x-3a) \cdot (x-3a)^4 - \frac{2q_0}{6} \cdot \Phi(x-6a) \cdot (x-6a)^3 - \frac{2q_0}{24 \cdot 3a} \cdot \Phi(x-6a) \cdot (x-6a)^4 + \\
 & + \frac{q_0}{6} \cdot \Phi(x-7a) \cdot (x-7a)^3 - \frac{q_0}{6} \cdot \Phi(x-10a) \cdot (x-10a)^3
 \end{aligned} \tag{28}$$

The particular expression of the cross-sectional displacement is:

$$\begin{aligned}
 EI_y w(x) &= EI_y w_0 + EI_y \varphi_0 \cdot x + W(x) \\
 W(x) &= -\frac{N}{2} \cdot \Phi(x-11a) \cdot (x-11a)^2 - \frac{V_1}{6} \cdot \Phi(x-2a) \cdot (x-2a)^3 - \frac{V_2}{6} \cdot \Phi(x-3a) \cdot (x-3a)^3 - \\
 &\quad - \frac{V_3}{6} \cdot \Phi(x-7a) \cdot (x-7a)^3 - \frac{V_4}{6} \cdot \Phi(x-10a) \cdot (x-10a)^3 - \\
 &\quad + \frac{q_0}{24} \cdot \Phi(x) \cdot x^4 - \frac{q_0}{24} \cdot \Phi(x-2a) \cdot (x-2a)^4 + \\
 &\quad + \frac{q_1}{120 \cdot 3a} \cdot \Phi(x-3a) \cdot (x-3a)^5 - \frac{q_1}{24} \cdot \Phi(x-6a) \cdot (x-6a)^4 - \frac{q_1}{120 \cdot 3a} \cdot \Phi(x-6a) \cdot (x-6a)^5 + \\
 &\quad + \frac{q_0}{24} \cdot \Phi(x-7a) \cdot (x-7a)^4 - \frac{q_0}{24} \cdot \Phi(x-10a) \cdot (x-10a)^4
 \end{aligned} \tag{29}$$

$EI_y w_0$ and $EI_y \varphi_0$ integration constants, corresponding to the origin parameters will be determined for the zero displacement conditions from supports 1 and 4:

$$\begin{cases} EI_y w(2a) = EI_y w_0 + EI_y \varphi_0 \cdot 2a + W(2a) = 0 \\ EI_y w(10a) = EI_y w_0 + EI_y \varphi_0 \cdot 10a + W(10a) = 0 \end{cases} \quad \begin{cases} EI_y \varphi_0 = \frac{1}{8a} [W(2a) - W(10a)] \\ EI_y w_0 = \frac{1}{4} [W(10a) - 5 \cdot W(2a)] \end{cases} \tag{30}$$

The shear force $T(x)$ and bending moment $M_i(x)$ variation diagrams are shown in fig 10.

In figure 11 one plotted the deflection function $EIW(x)$ and the rotation one $EIF(x)$, for the particular numerical data

Fig. 10. Shear force T and bending moment M_i diagrams

Fig. 11. Cross-sectional deflection $Elw(x)$ and rotation $EI\phi(x)$ diagrams

In the particular case when all the supports are placed at the same level, the change will occur in $w_2=w_3=0$ in the MATHCAD sequence and the following results will be obtained for the reaction forces:

$$\begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \end{pmatrix} = \begin{pmatrix} 3.219 \\ 0.114 \\ 4.483 \\ 3.184 \end{pmatrix} \tag{31}$$

The shear force $T(x)$ and bending moment $M_i(x)$ variation diagrams are shown in fig 12. In figure 13 one plotted the deflection function $EIW(x)$ and the rotation one $EIF(x)$, for the particular numerical data.

Fig. 12. Shear force T and bending moment M_i diagrams

Fig. 13. Cross-sectional deflection $Elw(x)$ and rotation $El\phi(x)$ diagrams

Application 3: Continuous beam supported by 7 elastic bearings (statically undetermined system)

One considers a continuous beam OA supported by 7 level equidistant linear elastic bearings (having the same stiffness k). The beam has a constant flexural stiffness EI and it is subjected to the action of a load P as shown in figure 14. One should determine the reaction forces $V_1, V_2, \dots, V_7$ and plot the shear force $T(x)$ and bending moment $M(x)$, cross-sectional rotations $\phi(x)$ and displacements $w(x)$ variation diagrams, as a function of the stiffness k of the elastic elements. Numerical input data: $EI = 2 \text{ kN/m}^2$; $r_1 = 1 \text{ m}$; $r_2 = 2 \text{ m}$; $r_3 = 3 \text{ m}$; $r_4 = 4 \text{ m}$; $r_5 = 5 \text{ m}$; $r_6 = 6$; $r_7 = 7 \text{ m}$; $d = 4 \text{ m}$; $P = 10 \text{ kN}$; springs' stiffness particular cases: $k_1 = 100 \text{ kN/m}$; $k_2 = 10 \text{ kN/m}$; $k_3 = 1 \text{ kN/m}$.

In order to determine the reactions $V_1, V_2, \dots, V_7$ one will use the two equilibrium equations from Mechanics of Solids:

$$\begin{aligned} \sum F_{zs} \downarrow &= V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 \\ \sum \bar{M}_{7s} &= V_1 \cdot (r_7 - r_1) + V_2 \cdot (r_7 - r_2) + V_3 \cdot (r_7 - r_3) + V_4 \cdot (r_7 - r_4) + V_5 \cdot (r_7 - r_5) + V_6 \cdot (r_7 - r_6) \end{aligned} \tag{32}$$

where: $\sum F_{zs} \downarrow = P$ is the sum of exterior forces along the Oz axis;

$\sum \bar{M}_{7s} = P(r_7 - d)$ - is the sum of exterior moments around the Oy axis, passing through support 7.

Fig. 14.

In order to obtain the **3 Displacements Equation** one will write the displacements corresponding to 3 beam sections denoted by *i*, *j* and *k* as in figure 15 ($x_j = x_i + L_i$; $x_k = x_i + L_i + L_j$), according to the general relations (11):

$$\begin{cases} EI \cdot w_i = EI \cdot w_0 + EI\varphi_0 \cdot x_i + W(x_i) & \left| \begin{array}{l} L_j \\ -(L_i + L_j) \\ L_i \end{array} \right. \end{cases} \quad (33)$$

Fig. 15.

If one multiplies the first equation with L_j , the second one with $(-L_i - L_j)$, the third one with L_i and if one adds up the obtained expressions, by eliminating the origin parameters one will obtain the **3 Displacements Equation**:

$$EI[w_i L_j - w_j (L_j + L_i) + w_k L_i] = W(x_i) L_j - W(x_j) (L_j + L_i) + W(x_k) L_i \quad (34)$$

The other five equations required to solve the statically undetermined system form fig. 14 can be obtained using the **3 Displacements Equation** for 5 sets of 3 consecutive supports, as it follows:

$$\begin{aligned}
 1-2-3: \quad & EI[w_1(r_3-r_2)-w_2(r_3-r_1)+w_3(r_2-r_1)] = [W(r_1) \cdot (r_3-r_2) - W(r_2) \cdot (r_3-r_1) + W(r_3) \cdot (r_2-r_1)] \\
 2-3-4: \quad & EI[w_2(r_4-r_3)-w_3(r_4-r_2)+w_4(r_3-r_2)] = [W(r_2) \cdot (r_4-r_3) - W(r_3) \cdot (r_4-r_2) + W(r_4) \cdot (r_3-r_2)] \\
 3-4-5: \quad & EI[w_3(r_5-r_4)-w_4(r_5-r_3)+w_5(r_4-r_3)] = [W(r_3) \cdot (r_5-r_4) - W(r_4) \cdot (r_5-r_3) + W(r_5) \cdot (r_4-r_3)] \\
 4-5-6: \quad & EI[w_4(r_6-r_5)-w_5(r_6-r_4)+w_6(r_5-r_4)] = [W(r_4) \cdot (r_6-r_5) - W(r_5) \cdot (r_6-r_4) + W(r_6) \cdot (r_5-r_4)] \\
 5-6-7: \quad & EI[w_5(r_7-r_6)-w_6(r_7-r_5)+w_7(r_6-r_5)] = [W(r_5) \cdot (r_7-r_6) - W(r_6) \cdot (r_7-r_5) + W(r_7) \cdot (r_6-r_5)]
 \end{aligned}
 \tag{35}$$

where $W(x)$ has the particular shape:

$$\begin{aligned}
 W(x) = & P \cdot \Phi(x-d) \cdot \frac{(x-d)^3}{6} - V_1 \cdot \Phi(x-r_1) \cdot \frac{(x-r_1)^3}{6} - V_2 \cdot \Phi(x-r_2) \cdot \frac{(x-r_2)^3}{6} - V_3 \cdot \Phi(x-r_3) \cdot \frac{(x-r_3)^3}{6} - \\
 & - V_4 \cdot \Phi(x-r_4) \cdot \frac{(x-r_4)^3}{6} - V_5 \cdot \Phi(x-r_5) \cdot \frac{(x-r_5)^3}{6} - V_6 \cdot \Phi(x-r_6) \cdot \frac{(x-r_6)^3}{6} - V_7 \cdot \Phi(x-r_7) \cdot \frac{(x-r_7)^3}{6}
 \end{aligned}
 \tag{36}$$

The displacements $w_1, w_2, \dots, w_5$ are linear functions of $V_1, V_2, \dots, V_7$, as it follows:

$$w_1 = \frac{V_1}{k}; \quad w_2 = \frac{V_2}{k}; \quad w_3 = \frac{V_3}{k}; \quad w_4 = \frac{V_4}{k}; \quad w_5 = \frac{V_5}{k}; \quad w_6 = \frac{V_6}{k}; \quad w_7 = \frac{V_7}{k}$$

The step functions in $W(x)$ will be denoted like:

$$\begin{aligned}
 k_P(x) &= \Phi(x-d) \cdot \frac{(x-d)^3}{6}; \quad k_1(x) = \Phi(x-r_1) \cdot \frac{(x-r_1)^3}{6}; \\
 k_2(x) &= \Phi(x-r_2) \cdot \frac{(x-r_2)^3}{6}; \quad k_3(x) = \Phi(x-r_3) \cdot \frac{(x-r_3)^3}{6}; \\
 k_4(x) &= \Phi(x-r_4) \cdot \frac{(x-r_4)^3}{6}; \quad k_5(x) = \Phi(x-r_5) \cdot \frac{(x-r_5)^3}{6}; \\
 k_6(x) &= \Phi(x-r_6) \cdot \frac{(x-r_6)^3}{6}; \quad k_7(x) = \Phi(x-r_7) \cdot \frac{(x-r_7)^3}{6}
 \end{aligned}
 \tag{37}$$

One will denote the values of $k_i(x)$ functions given by (34) in the beam's supports as it follows:

$$\begin{aligned} k_P(r_1) &= k_{P1}; \quad k_1(r_1) = k_{11}; \quad k_2(r_1) = k_{21}; \quad k_3(r_1) = k_{31}; \quad k_4(r_1) = k_{41}; \quad \dots \\ k_P(r_2) &= k_{P2}; \quad k_1(r_2) = k_{12}; \quad k_2(r_2) = k_{22}; \quad k_3(r_2) = k_{32}; \quad k_4(r_2) = k_{42}; \quad \dots \end{aligned} \quad (38)$$

One will denote: $EI / k = \beta$

After introducing the functions $W(r_i)$ in equations (32) and separating the unknown values $V_1, V_2, \dots, V_7$, one obtains:

$$\begin{aligned} &V_1 \cdot (k_{11}d_{32} - k_{12}d_{31} + k_{13}d_{21} + \beta \cdot d_{32}) + V_2 \cdot (k_{21}d_{32} - k_{22}d_{31} + k_{23}d_{21} - \beta \cdot d_{31}) + \\ &+ V_3 \cdot (k_{31}d_{32} - k_{32}d_{31} + k_{33}d_{21} + \beta \cdot d_{21}) + V_4 \cdot (k_{41}d_{32} - k_{42}d_{31} + k_{43}d_{21}) + V_5 \cdot (k_{51}d_{32} - k_{52}d_{31} + k_{53}d_{21}) + \\ &+ V_6 \cdot (k_{61}d_{32} - k_{62}d_{31} + k_{63}d_{21}) + V_7 \cdot (k_{71}d_{32} - k_{72}d_{31} + k_{73}d_{21}) = P \cdot (k_{P1}d_{32} - k_{P2}d_{31} + k_{P3}d_{21}) \\ &V_1 \cdot (k_{12}d_{43} - k_{13}d_{42} + k_{14}d_{32}) + V_2 \cdot (k_{22}d_{43} - k_{23}d_{42} + k_{24}d_{32} + \beta \cdot d_{43}) + V_3 \cdot (k_{32}d_{43} - k_{33}d_{42} + k_{34}d_{32} - \beta \cdot d_{42}) + \\ &+ V_4 \cdot (k_{42}d_{43} - k_{43}d_{42} + k_{44}d_{32} + \beta \cdot d_{32}) + V_5 \cdot (k_{52}d_{43} - k_{53}d_{42} + k_{54}d_{32}) + \\ &+ V_6 \cdot (k_{62}d_{43} - k_{63}d_{42} + k_{64}d_{32}) + V_7 \cdot (k_{72}d_{43} - k_{73}d_{42} + k_{74}d_{32}) = P \cdot (k_{P2}d_{43} - k_{P3}d_{42} + k_{P4}d_{32}) \\ &V_1 \cdot (k_{13}d_{54} - k_{14}d_{53} + k_{15}d_{43}) + V_2 \cdot (k_{23}d_{54} - k_{24}d_{53} + k_{25}d_{43}) + V_3 \cdot (k_{33}d_{54} - k_{34}d_{53} + k_{35}d_{43} + \beta \cdot d_{54}) + \\ &+ V_4 \cdot (k_{43}d_{54} - k_{44}d_{53} + k_{45}d_{43} - \beta \cdot d_{53}) + V_5 \cdot (k_{53}d_{54} - k_{54}d_{53} + k_{55}d_{43} + \beta \cdot d_{43}) + \\ &+ V_6 \cdot (k_{63}d_{54} - k_{64}d_{53} + k_{65}d_{43}) + V_7 \cdot (k_{73}d_{54} - k_{74}d_{53} + k_{75}d_{43}) = P \cdot (k_{P3}d_{54} - k_{P4}d_{53} + k_{P5}d_{43}) \end{aligned} \quad (39)$$

$$\begin{aligned} &V_1 \cdot (k_{14}d_{65} - k_{15}d_{64} + k_{16}d_{54}) + V_2 \cdot (k_{24}d_{65} - k_{25}d_{64} + k_{26}d_{54}) + V_3 \cdot (k_{34}d_{65} - k_{35}d_{64} + k_{36}d_{54}) + \\ &+ V_4 \cdot (k_{44}d_{65} - k_{45}d_{64} + k_{46}d_{54} + \beta \cdot d_{65}) + V_5 \cdot (k_{54}d_{65} - k_{55}d_{64} + k_{56}d_{54} - \beta \cdot d_{64}) + \\ &+ V_6 \cdot (k_{64}d_{65} - k_{65}d_{64} + k_{66}d_{54} + \beta \cdot d_{65}) + V_7 \cdot (k_{74}d_{65} - k_{75}d_{64} + k_{76}d_{54}) = P \cdot (k_{P4}d_{65} - k_{P5}d_{64} + k_{P6}d_{54}) \\ &V_1 \cdot (k_{15}d_{76} - k_{16}d_{75} + k_{17}d_{65}) + V_2 \cdot (k_{25}d_{76} - k_{26}d_{75} + k_{27}d_{65}) + V_3 \cdot (k_{35}d_{76} - k_{36}d_{75} + k_{37}d_{65}) + \\ &+ V_4 \cdot (k_{45}d_{76} - k_{46}d_{75} + k_{47}d_{65}) + V_5 \cdot (k_{55}d_{76} - k_{56}d_{75} + k_{57}d_{65} + \beta \cdot d_{76}) + \\ &+ V_6 \cdot (k_{65}d_{76} - k_{66}d_{75} + k_{67}d_{65} - \beta \cdot d_{75}) + V_7 \cdot (k_{75}d_{76} - k_{76}d_{75} + k_{77}d_{65} + \beta \cdot d_{65}) = P \cdot (k_{P5}d_{76} - k_{P6}d_{75} + k_{P7}d_{65}) \end{aligned}$$

Equations (32) and (39) will have the matrix shape:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{X} \quad (40)$$

Case a. for $k_1=1 \text{ kN/m}$, the matrices $\mathbf{A}$ and $\mathbf{B}$ have the particular shape:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 3 & -3.833 & 2 & 0 & 0 & 0 & 0 \\ 2 & 3 & -3.833 & 2 & 0 & 0 & 0 \\ 3 & 2 & 3 & -3.833 & 2 & 0 & 0 \\ 4 & 3 & 2 & 3 & -3.833 & 2 & 0 \\ 5 & 4 & 3 & 2 & 3 & -3.833 & 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 10 \\ 30 \\ 0 \\ 0 \\ 1.667 \\ 10 \\ 20 \end{pmatrix}$$

Solving this matrix equation in MATHCAD, one obtains the reaction forces for this particular case:

$$X := \text{Isolve}(A, B)$$

$$X = \begin{pmatrix} -0.194 \\ 1.131 \\ 2.458 \\ 3.21 \\ 2.458 \\ 1.131 \\ -0.194 \end{pmatrix} \quad \begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \end{pmatrix} := X$$

The analytical expressions of the shear force $T_z(x)$ and bending moment $M_{iy}(x)$ determined using the step function Φ are:

$$T_z(x) = V_1 \cdot \Phi(x-r_1) + V_2 \cdot \Phi(x-r_2) + V_3 \cdot \Phi(x-r_3) + V_4 \cdot \Phi(x-r_4) + \\ + V_5 \cdot \Phi(x-r_5) + V_6 \cdot \Phi(x-r_6) + V_7 \cdot \Phi(x-r_7) - P \cdot \Phi(x-d)$$

$$M_{iy}(x) = V_1 \cdot (x-r_1) \cdot \Phi(x-r_1) + V_2 \cdot (x-r_2) \cdot \Phi(x-r_2) + V_3 \cdot (x-r_3) \cdot \Phi(x-r_3) + V_4 \cdot (x-r_4) \cdot \Phi(x-r_4) + \\ + V_5 \cdot (x-r_5) \cdot \Phi(x-r_5) + V_6 \cdot (x-r_6) \cdot \Phi(x-r_6) + V_7 \cdot (x-r_7) \cdot \Phi(x-r_7) - P \cdot (x-d) \cdot \Phi(x-d)$$

(43)

The shear force $T(x)$ and bending moment $M_i(x)$ diagrams for this case are shown in figure 16.

Fig. 16. $T_z(x)$ and $M_{iy}(x)$ diagrams for $k=1 \text{ kN/m}$ $T_{max}=3,395 \text{ kN}$; $M_{max}=4,138 \text{ kNm}$

Case b. for $k_2=10\text{ kN/m}$ the matrices **A** and **B** have the particular shape:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1.2 & -0.233 & -0.2 & 0 & 0 & 0 & 0 \\ 2 & 1.2 & -0.233 & -0.2 & 0 & 0 & 0 \\ 3 & 2 & 1.2 & -0.233 & 0.2 & 0 & 0 \\ 4 & 3 & 2 & 1.2 & -0.233 & 0.2 & 0 \\ 5 & 4 & 3 & 2 & 1.2 & -0.233 & 0.2 \end{pmatrix}$$

$$B = \begin{pmatrix} 10 \\ 30 \\ 0 \\ 0 \\ 1.667 \\ 10 \\ 20 \end{pmatrix} \tag{44}$$

Solving this matrix equation in MATHCAD one obtains the reaction forces for this particular case:

$$X := \text{Isolve}(A, B)$$

$$X = \begin{pmatrix} -0.373 \\ 0.24 \\ 2.519 \\ 5.229 \\ 2.519 \\ 0.24 \\ -0.373 \end{pmatrix}$$

$$\begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \end{pmatrix} := X \tag{45}$$

The shear force $T(x)$ and bending moment $M_i(x)$ diagrams for this case are shown in figure 17.

Fig. 17. $T_z(x)$ and $M_{iy}(x)$ diagrams for $k=10\text{ kN/m}$ $T_{\max}=2,385\text{ kN}$; $M_{\max}=1,879\text{ kNm}$

Case c. For $k_2=100\text{ kN/m}$ the matrices **A** and **B** have the particular shape:

$$A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ 1.02 & 0.127 & 0.02 & 0 & 0 & 0 & 0 \\ 2 & 1.02 & 0.127 & 0.02 & 0 & 0 & 0 \\ 3 & 2 & 1.02 & 0.127 & 0.02 & 0 & 0 \\ 4 & 3 & 2 & 1.02 & 0.127 & 0.02 & 0 \\ 5 & 4 & 3 & 2 & 1.02 & 0.127 & 0.02 \end{pmatrix}$$

$$B = \begin{pmatrix} 10 \\ 30 \\ 0 \\ 0 \\ 1.667 \\ 10 \\ 20 \end{pmatrix} \tag{46}$$

Solving this matrix equation in MATHCAD one obtains the reaction forces for this particular case:

$$X := \text{lsolve}(A, B)$$

$$X = \begin{pmatrix} 0.02 \\ -0.351 \\ 1.22 \\ 8.224 \\ 1.22 \\ -0.351 \\ 0.02 \end{pmatrix} \begin{pmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \end{pmatrix} := X \tag{47}$$

The shear force $T(x)$ and bending moment $M_i(x)$ diagrams for this case are shown in figure 18.

Fig. 18. $T_z(x)$ and $M_{iy}(x)$ diagrams for $k=100 \text{ kN/m}$ $T_{\max}=0,888 \text{ kN}$; $M_{\max}=0,576 \text{ kNm}$

3. Conclusions

- This is a method of expressing the variation of the shear force $T_z(x)$, bending moment $M_{iy}(x)$, transversal cross-section rotations $\varphi_y(x)$ and displacements $w(x)$ using the MATHCAD step function $\Phi(x-a)$ in a more compact form than the traditional methods;
- The above shown method has a high degree of generality. One can use it to determine the reactions in the N (rigid or elastic) supports (equidistant or randomly placed) of a continuous beam. Thus one can better assimilate the results of the Winkler Method for a continuous perfect elastic space with the results obtained for a discrete elastic space.
- Figures 16, 17 and 18 show that increasing the stiffness of the elastic elements leads to an increase of the elastic supports load in the central part of the beam ($3,21 \text{ kN} \uparrow 8.224 \text{ kN}$) and a decrease of the maximum bending moments ($4,138 \text{ kNm} \downarrow 0,576 \text{ kNm}$); on the other hand, when decreasing the elastic elements' stiffness, a decrease of the central supports load and maximum bending moments occurs.
- Considering the fact that the step function method uses simple operation expressions, the numerical applications are being solved fast with a minimum number of computational cycles. The traditional methods use integral expressions which are solved by means of numerical methods. These methods imply a large number of computational cycles, which will cause slower obtained results;

4. References

- Cornel MARIN, Alexandru MARIN – *Metoda analitica pentru trasarea diagramelor de eforturi în barele drepte*. Sesiunea de comunicări științifice SIMEC 2006, UTCB Bucuresti, Facultatea de Utilaj Tehnologic, 31 martie 2006, pp.81-86
- Cornel MARIN, Alexandru MARIN – *Metoda analitica pentru calculul deplasărilor și rotiriilor barelor drepte supuse la încovoiere*. Sesiunea de comunicări științifice SIMEC 2006, UTCB Bucuresti, Facultatea de Utilaj Tehnologic, 31 martie 2006, pp.87-92
- Cornel MARIN, Alexandru MARIN – *O metodă analitica pentru rezolvarea sistemelor static nedeterminate de tipul grinzilor continue pe mai multe reazeme punctuale rigide la același nivel* Sesiunea de comunicări științifice a UTCB, Facultatea de Utilaj Tehnologic – SIMEC 2007.
- Cornel MARIN, Alexandru MARIN – *Calculul grinzilor continue situate pe reazeme elastice la același nivel, considerând terenul de sub reazeme ca un mediu perfect elastic* Sesiunea de comunicări științifice a UTCB, Facultatea de Utilaj Tehnologic – SIMEC 2007.
- Cornel Marin, Viviana Filip, Alexandru Marin - *Analytical Method Used for Plotting the Shear Force and Bending Moment Diagrams, Translations and Rotations Distributions on Beams Subjected to Bending*, International MultiConference of Engineers and Computer Scientists ICMES 2008, Hong Kong, 19-21 March 2008. Lecture Notes IMECS 2008, Volume II, ISBN 978-988-17012-1-3, pp 1629-1633
- Marin Cornel, Hadar Anton, Marin Alexandru, *Step Function Method Used in Calculating Continuous Footing Foundation Placed of Elastic and Discrete Soil* - Annals of DAAAM for 2008, ISSN 1726-9679, ISBN 978-3-901 509-68-1, pp.0793-0794.
- Cornel MARIN, Nicolae POPA, Elena DINU, *Grinzi continue situate pe 7 reazeme elastice la același nivel pe un mediu discret perfect elastic*, Sesiunea de comunicări științifice a studenților FIMMR Târgoviște, iunie 2008.
- Cornel MARIN - *Simularea încărcării unui pod elastic situat pe șapte reazeme elastice proporționale*. Al VI lea Simpozion Național de Mecatronica, Microtehnologii și Materiale Noi, 7 noiembrie 2008, Târgoviște.
- Cornel MARIN - *Rezistența materialelor și elemente de Teoria elasticității*, Editura Bibliotheca, Târgoviște 2006, ISBN (10) 973-721-198-8, ISBN (13) 973-973-721-198-1;
- Cornel MARIN - *Aplicații ale Teoriei elasticității și plasticității în inginerie*, Editura Bibliotheca, Târgoviște 2007, ISBN 978-973-712-297-1.
- Cornel Marin, Viviana Filip, Alexandru Marin, *IAENG TRANSACTIONS ON ENGINEERING TECHNOLOGIES VOLUME I -Ch. ICINDE_23: Alternative analytical method used in plotting the shear force and bending moment diagrams, displacements and rotations distributions for beams subjected to bending*, America Institute of Physics, USA 2009, ISBN: 978-0-7354-0622-3.

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