We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,300

Open access books available

172,000

International authors and editors

190M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

The Analysis and Application of Parallel Manipulator for Active Reflector of FAST

Xiao-qiang Tang and Peng Huang
Tsinghua University
People's Republic of China

1. Introduction

Since radio telescope is the main tool for human being to search the universe secret, the astronomer reached unanimity at the 24th URSI Conference in Kyoto, Japan, 1993, and proposed to construct the next generation of the large radio telescope (LT) (Nan & Peng, 2000). From then on, the astronomer of China began the project of Five-hundred meter Aperture Spherical radio Telescope (FAST) (Qiu, 1998; Li, 1998).

It is well known that Arecibo is the breakthrough of radio telescope. Its main mirror, 305m in diameter, is fixed on the karst base, and an elaborately designed feed system illuminates a part of the mirror which forms an effective aperture of the telescope with about 200m. The feed system is movable at a height of about 100 m for tracking the object to be observed. The enormous receiving area of the telescope will enable it to make many important astronomical discoveries inaccessible to lesser instruments, despite its small sky coverage (20° zenith scan angle), due to geometrical configuration, and narrow frequency bandwidth, originated from spherical aberration. An upgrade project has recently been carried out for the Arecibo telescope, in which a heavy and complex hence expensive Gregorian dual-reflector feed system is introduced for correcting the spherical aberration and a broad bandwidth is affected (Duan, 1999).

For the sake of satisfying the requirements of low cost and broad bandwidth, the project group of FAST decided to substitute the fixed spherical reflector with active reflector units. As shown in Fig. 1(a), the reflector consists of almost 2000 elementary reflector units. Fig. 1(b) shows some active reflector units and supporting mechanisms. The reflector unit is small part of spherical surface of regular hexagon and is driven by a supporting mechanism. The part of spherical reflector illuminated by the feed is continuously adjusted to fit a paraboloid of revolution in real-time, synchronous with the motion of the feed while tracking the object to be observed. As it is now free from spherical aberration, a simple, light, hence cheap feed system may be adopted to achieve broad bandwidth and full polarization.

In order to fit a paraboloid of revolution, it is necessary that every reflector units should be driven by a supporting mechanism with two rotational degrees of freedom and one translational degree of freedom (Luo et al., 2000). That means almost 6000 control nodes on the whole active reflector should be managed and controled at the same time. It is supposed to be very difficult, so a sharing strategy is derived to decrease the number of nodes, which

Source: Parallel Manipulators, Towards New Applications, Book edited by: Huapeng Wu, ISBN 978-3-902613-40-0, pp. 506, April 2008, I-Tech Education and Publishing, Vienna, Austria

requires three adjacent nodes combined together to share one driver. Basically, there are two types of mechanism which can fulfill the required movement for each reflector unit and fit for the sharing strategy, 3-PSS mechanism with constraint leg (Wang et al., 2006), shown in Fig. 2(a), 3-PSS+C for abbravation, and 3-PRS mechanism (Tang et al., 2007), shown in Fig. 2(b).

Fig. 1. The active reflector of spherical radio telescope

Fig. 2. The parallel supporting mechanism

These mechanisms will bring errors because of the control or dimensional factor. Moreover, the fitting surface of reflector will not match exactly with the nominal paraboloid, and the sharing strategy also brings accuracy problem. In order to guarantee the highest working frequency of large spherical radio telescope, 5GHz, the fitting accuracy of active reflector should be studied systematically. Based on the kinematics of 3-PSS+C mechanism, in this chapter, one-dimensional and two-dimensional fitting accuracy on the whole active reflector is analyzed considering control errors. However, about 2000 constraint legs increase almost one quarter of the cost. Thus 3-PRS mechanism is proposed and used as supporting

manipulator for reflector unit. Since 3-PRS mechanism has many problems such as parasitic motion, advanced research on kinematics with errors is necessary. Then three-dimensional fitting accuracy is analyzed based on error kinematics of 3-PRS mechanism.

2. The analysis of 3-PSS+C supporting mechanism

2.1 Supporting mechanism description

As shown in Fig. 2(a), the parallel supporting mechanism consists of a base plate, a movable platform, and four connecting legs, three of which have identical kinematic chains, PSS. Each of the three legs is composed of one fixed length link (3), and one union driven plate (5). The fixed length link (3) is connected to the movable platform (1) and the union driven plate (5) by two spherical joints (2) and (4), respectively. The union driven plate (5) is connected to the base plate (7) by a prismatic joint (6). The base plate and the movable platform are two regular triangles. The passive leg (8) connects the center points of the two regular triangles. One end of the passive leg has a 2-DOF universal joint (9), another end is fixed to the base plate (7) by a prismatic joint (10). The passive leg (8) can be extensible with the prismatic joint (10) along its axis line. Furthermore, when the supporting mechanism is assembled, the axis line of the prismatic joint (10) should pass the center of the spherical reflector. Since a supporting mechanism should be driven by three actuator legs, as shown in Fig. 2, the union driven plate (5) connects three fixed length links in order to reduce the actuator number. As a result, the number of actuators of the active reflector is equal to that of the reflector units.

From above description, one can see that the proposed mechanism is such a mechanism with n DOFs, which usually consists of n identical actuated legs with 6 DOFs and one passive leg with n DOFs connecting the movable platform and the base plate, i.e., the DOF of the mechanism is dependent on the passive leg's DOF. For the mechanism considered in this paper, the passive leg is with three DOFs, which means that n equals to 3. The three DOFs are one translation along z axis and two rotations about x and y axes.

2.2 Kinematics analysis

The mechanism kinematics deals with the study of the mechanism motion as constrained by the geometry of the links. Typically, the study of mechanism kinematics is divided into two parts, inverse kinematics and forward (or direct) kinematics (Wang & Tang, 2003). The inverse kinematics problem involves mapping a known pose (position and orientation) of the output platform of the mechanism to a set of input joint variables that will achieve that pose. The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the movable platform that results from those given inputs (Wang et al., 2001). Generally, as the number of closed kinematics loops in the parallel mechanism increases, the difficulty of solving the forward kinematics relationships increases while the difficulty of solving the inverse kinematics relationships decreases (Liu et al., 2001).

2.2.1 Inverse kinematics

A kinematics model of the mechanism is developed as shown in Fig. 3. The vertices of the movable platform are denoted as platform joints A_i ($i = 1, 2, 3$), and the vertices of the base plate are denoted as b_i ($i = 1, 2, 3$). A fixed global reference system $\mathcal{R}: o-xyz$ is located at the center of the regular triangles $b_1b_2b_3$ with the z axis normal to the base plate and the

y axis parallel to the side b_1b_2 . The circumcircle radius of triangles $b_1b_2b_3$ is denoted as R . Another reference frame, called the top frame $\mathcal{R}' : o' - x'y'z'$, is located at the center of regular triangles $A_1A_2A_3$. The z' axis is perpendicular to the movable platform and y' axis parallel to the side A_1A_2 . The circumcircle radius of triangles $A_1A_2A_3$ is denoted as r . Vector of fixed length links are denoted as L_i ($i = 1, 2, 3$), and the link length for each legs is denoted as l , where $A_iB_i = l$, ($i = 1, 2, 3$).

Fig. 3. The geometric parameters of the parallel mechanism

The objective of the inverse kinematics solution is to define a mapping from the pose of the output platform in the Cartesian space to the set of actuated inputs that achieve that pose. For this analysis, the pose of the movable platform is considered known, and the position is given by the position vector $[o']_{\mathcal{R}}$ and the orientation is given by a matrix R_1 . Then there are

$$[o']_{\mathcal{R}} = (x \ y \ z)^T \quad (1)$$

where $x = y = 0$,

$$R_1 = \begin{bmatrix} c\beta & s\beta s\alpha & s\beta c\alpha \\ 0 & c\alpha & -s\alpha \\ -s\beta & c\beta s\alpha & c\beta c\alpha \end{bmatrix} \quad (2)$$

where c stands for cosine function, s stands for sine function, and α and β are the orientational DOFs of the movable platform with respect to x and y axes, respectively. The coordinate of point A_i in the frame $\mathcal{R}'$ can be described by the vector $[A_i]_{\mathcal{R}'}$ ($i = 1, 2, 3$), and

$$[A_1]_{\mathcal{R}'} = [r/2, -\sqrt{3}r/2, 0]^T, [A_2]_{\mathcal{R}'} = [r/2, \sqrt{3}r/2, 0]^T, [A_3]_{\mathcal{R}'} = [-r, 0, 0]^T \quad (3)$$

Vectors $[B_i]_{\mathcal{R}}$ ($i = 1, 2, 3$) will be defined as the position vectors of base joints B_i in frame $\mathcal{R}$, and

$$[\mathbf{B}_1]_{\mathfrak{R}} = [R/2, -\sqrt{3}R/2, z_1]^T, [\mathbf{B}_2]_{\mathfrak{R}} = [R/2, \sqrt{3}R/2, z_2]^T, [\mathbf{B}_3]_{\mathfrak{R}} = [-R, 0, z_3]^T \quad (4)$$

Vectors $[\mathbf{A}_i]_{\mathfrak{R}}$ ($i = 1, 2, 3$) in frame $o - xyz$ can be, therefore, written as

$$[\mathbf{A}_i]_{\mathfrak{R}} = \mathbf{R}_1[\mathbf{A}_i]_{\mathfrak{R}'} + [\mathbf{o}']_{\mathfrak{R}}, \quad (i = 1, 2, 3) \quad (5)$$

Then the inverse kinematics of the parallel mechanism can be solved by writing following constraint equation

$$\|[\mathbf{B}_i]_{\mathfrak{R}} - [\mathbf{A}_i]_{\mathfrak{R}}\| = \|\mathbf{L}_i\| = l, \quad (i = 1, 2, 3) \quad (6)$$

Hence, for a given mechanism and prescribed position and orientation of the movable platform, the required actuator inputs can be directly computed from Eq. (6), that is

$$\begin{cases} z_1 = \sqrt{l^2 - A_{11}^2 - A_{12}^2} + A_{13} \\ z_2 = \sqrt{l^2 - A_{21}^2 - A_{22}^2} + A_{23} \\ z_3 = \sqrt{l^2 - A_{31}^2 - A_{32}^2} + A_{33} \end{cases} \quad (7)$$

where

$$A_{11} = (R - r(c\beta - \sqrt{3}s\beta\alpha))/2$$

$$A_{12} = -\sqrt{3}(R - rc\alpha)/2$$

$$A_{13} = r(s\beta + \sqrt{3}c\beta\alpha)/2 + z$$

$$A_{21} = R - r(c\beta + \sqrt{3}s\beta\alpha)/2$$

$$A_{22} = \sqrt{3}(R - rc\alpha)/2$$

$$A_{23} = -r(s\beta - \sqrt{3}c\beta\alpha)/2 + z$$

$$A_{31} = -R + rc\beta$$

$$A_{32} = 0$$

$$A_{33} = rs\beta + z$$

2.2.2 Forward kinematics

The objective of the forward kinematics solution is to define a mapping from the known set of the actuated inputs to the unknown pose of the output platform. For the architecture with prismatic actuators, the inputs that are considered known are the lengths of the three actuator legs z_1 , z_2 and z_3 . The unknown pose of the output platform is described by the position vector $[\mathbf{o}']_{\mathfrak{R}}$ and angles α and β . Because it is very difficult to describe the direct kinematics in closed form for this type of parallel mechanism, the forward kinematics solution should be obtained by numerical methodology as following:

1. Decide the non-singularity workspace of the mechanism;
2. Give the initial value of direct kinematics solution;

3. Calculate the position coordinates of spherical joints, construct the nonlinear equations set by the geometry constraint relationship of fixed length links;
4. Solve the nonlinear equations set by Quasi-Newton method (Press et al., 1995).

From the Eq. (6), the nonlinear equations are

$$f_i(z, \alpha, \beta) = l^2 - A_{i1}^2 - A_{i2}^2 - (z_i - A_{i3})^2 = 0, \quad (i = 1, 2, 3) \quad (8)$$

where the direct kinematics solutions are z , α and β .

2.2.3 Velocity equation

Eq. (6) can be differentiated with respect to time to obtain the velocity equation. This leads to an equation of the form.

$$\mathbf{J}_p \dot{\mathbf{p}} = \mathbf{J}_q \dot{\mathbf{q}} \quad (9)$$

where $\dot{\mathbf{q}}$ is the vector of Cartesian velocities defined as

$$\dot{\mathbf{q}} = [\dot{z}, \dot{\alpha}, \dot{\beta}]^T \quad (10)$$

and $\dot{\mathbf{p}}$ is the vector of input velocities defined as

$$\dot{\mathbf{p}} = [\dot{z}_1, \dot{z}_2, \dot{z}_3]^T \quad (11)$$

Matrices $\mathbf{J}_p$ and $\mathbf{J}_q$ are the 3×3 forward and inverse Jacobian matrices of the mechanism and can be expressed as

$$\mathbf{J}_p = \begin{bmatrix} (z_1 - A_{13})/l & 0 & 0 \\ 0 & (z_2 - A_{23})/l & 0 \\ 0 & 0 & (z_3 - A_{33})/l \end{bmatrix} \quad (12)$$

$$\mathbf{J}_q = \begin{bmatrix} (\mathbf{w}_1)_z & (\mathbf{v}_1 \times \mathbf{w}_1)_x & (\mathbf{v}_1 \times \mathbf{w}_1)_y \\ (\mathbf{w}_2)_z & (\mathbf{v}_2 \times \mathbf{w}_2)_x & (\mathbf{v}_2 \times \mathbf{w}_2)_y \\ (\mathbf{w}_3)_z & (\mathbf{v}_3 \times \mathbf{w}_3)_x & (\mathbf{v}_3 \times \mathbf{w}_3)_y \end{bmatrix} \quad (13)$$

where $\mathbf{w}_i$ is the unit vector of $\mathbf{L}_i$, and $\mathbf{v}_i = \mathbf{R}_1[\mathbf{A}_i]_{\text{gr}}$. $(\mathbf{w}_i)_z$ is the element of vector $\mathbf{w}_i$ with respect to z axis coordinate, $(\mathbf{v}_i \times \mathbf{w}_i)_x$ and $(\mathbf{v}_i \times \mathbf{w}_i)_y$ are the elements of vector $\mathbf{v}_i \times \mathbf{w}_i$ with respect to x and y axis coordinates.

2.3 Mechanism accuracy analysis

When the large spherical radio telescope works, the feed system will illuminate a working area, which is the paraboloid reflector with a three-hundred-meter aperture. The part of spherical reflector illuminated by the feed is continuously adjusted to fit a paraboloid of revolution in real-time, synchronous with the motion of the feed while tracking the object to

be observed. For the fitting, the spherical surface reflector is divided into some small elementary units. When the mechanisms drive the reflector units to fit the paraboloid, the fitting surface of reflector will not match exactly with the nominal paraboloid. Moreover, the mechanism has error because of the control or dimensional factor. In this section, the mechanism accuracy is analyzed firstly.

The mechanism accuracy involves the error caused by the actuator input error and the joint error of the mechanism. The actuator input error is denoted as $\delta \mathbf{p} = [\delta z_1, \delta z_2, \delta z_3]^T$ and the joint error is denoted as $\delta \mathbf{e} = [\delta \mathbf{A}_i^T \quad \delta \mathbf{B}_i^T]^T \in R^{18 \times 1} (i=1, \dots, 3)$, where $\delta \mathbf{B}_i^T \in R^{9 \times 1} (i=1, \dots, 3)$ includes the joint error on the base platform and the input error $\delta \mathbf{p} = [\delta z_1, \delta z_2, \delta z_3]^T$. The output error is denoted as $\delta \mathbf{q} = [\delta z, \delta \alpha, \delta \beta]^T$.

From Eq. (5) and (6), the inverse kinematics equation can be written as

$$\mathbf{R}_1 [\mathbf{A}_i]_{\mathfrak{R}'} + [\mathbf{o}']_{\mathfrak{R}} - [\mathbf{B}_i]_{\mathfrak{R}} = \mathbf{L}_i = \mathbf{w}_i l \quad (14)$$

Differentiating Eq. (14) leads to

$$\delta l = \mathbf{J}_q \delta \mathbf{q} + \mathbf{J}_e \delta \mathbf{e} \quad (15)$$

where

$$\mathbf{J}_e = \begin{bmatrix} \mathbf{w}_1^T \mathbf{R}_1 & -\mathbf{w}_1^T & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{w}_2^T \mathbf{R}_1 & -\mathbf{w}_2^T & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{w}_3^T \mathbf{R}_1 & -\mathbf{w}_3^T \end{bmatrix} \in R^{3 \times 18} \quad (16)$$

and $\delta l = [\delta l_1, \delta l_2, \delta l_3]^T$, $\delta l_i (i=1, 2, 3)$ is the manufacturing or measuring error of the i -th link. When $\mathbf{J}_q$ is nonsingular in the workspace, Eq. (15) can be rewritten as

$$\delta \mathbf{q} = \mathbf{J}_q^{-1} (\delta l - \mathbf{J}_e \delta \mathbf{e}) \quad (17)$$

3. Fitting accuracy analysis of active reflector

3.1 One dimensional fitting accuracy analysis

As shown in Fig. 4, the base active reflector of the radio telescope is a spherical surface with five-hundred-meter aperture, and the working reflector is a paraboloid with a three-hundred-meter aperture. When it works, the reflector units are driven by the parallel mechanism from the initial position to the fitting position to fit the paraboloid. Because the paraboloid is formed by the revolution of parabola, we can analyze the deviation about spherical surface and paraboloid in the reflector frame $\mathfrak{R}'' : o'' - y''z''$, which is built as shown in Fig. 4, where the spherical surface and the paraboloid in the frame $\mathfrak{R}''$ are circular arc and parabola, respectively.

Fig. 4. Configuration of the active reflector

Fig. 5. The i -th reflector unit

Fig. 5 shows one reflector unit which is in the initial position and fitting position, respectively. The initial position is located at the base spherical reflector surface. The deviation from the circular arc to the parabola is denoted as ΔK_{ij} , and symbol i represents the i -th reflector unit which corresponds to the i -th mechanism. The symbol j ($j=1,2,3$) represents the supporting point of the movable platform. The explanations of other symbols used in accuracy analysis are:

A_{ij} The supporting point while the reflector unit is in the initial position.

A'_{ij} The supporting point while the reflector unit is in the fitting position.

C_{ij} The intersecting points of line SA_{ij} and the parabola.

o'_i The reference center in the movable platform while the reflector unit is in the initial position.

o''_i The reference center in the movable platform while the reflector unit is in the fitting position.

S The center of spherical reflector.

K The radius of spherical reflector.

F The focal point of the paraboloid.

The absolute actuator input of the i -th mechanism is specified as ΔK_{ij} ($j=1,2,3$), while the i -th active reflector unit is driven to fit the paraboloid. Obviously, the driven reflector unit will not match exactly with the nominal paraboloid. In order to evaluate the fitting error, as shown in Fig. 5, Δe_i is defined as the center points deviation of the i -th reflector unit to the corresponding paraboloid and Δe_i is equal to $\|o''_i C_{i3}\|$, where the center points deviation Δe_i is called as one-dimensional fitting error.

3.1.1 Parabola equation and circle equation

According to the reference (Qiu 1998), the focal length of the parabola is specified as $0.476K$, then the parabola equation can be written as

$$z'' = \frac{1}{4 \times 0.467K} y''^2 \quad (18)$$

The base spherical surface in reflector coordinate system $\mathfrak{R}''$ is a circle. And the circle equation can be written as

$$z'' = K - \sqrt{K^2 - y''^2} \quad (19)$$

3.1.2 Actuator input range

The coordinate of the point A_{ij} in the frame $\mathfrak{R}''$ can be described by the vector $[A_{ij}]_{\mathfrak{R}''}$ ($j=1,2,3$), then

$$[A_{ij}]_{\mathfrak{R}''} = [y''_{ij}, z''_{ij}]^T, \quad (j=1,2,3) \quad (20)$$

The equation of straight line SA_{ij} can be written as

$$z'' = \frac{(z''_{ij} - K)y''}{y''_{ij}} + K, \quad (j=1,2,3) \quad (21)$$

According to Eqs. (19) and (21), the intersecting point C_{ij} between line SA_{ij} and the circle can be expressed by vector $[C_{ij}]_{\mathfrak{R}''}$, which is

$$[C_{ij}]_{\mathfrak{R}''} = [y''_{cij}, z''_{cij}]^T, \quad (j=1,2,3) \quad (22)$$

Actuator input value of the i -th reflector unit can be written as

$$\Delta K_{ij} = \|K - SC_{ij}\| = K - \sqrt{(y''_{cij})^2 + (K - z''_{cij})^2}, \quad (j=1,2,3) \quad (23)$$

3.1.3 One-dimensional fitting error

When actuator input ΔK_{ij} , ($j=1,2,3$) is specified, the fitting error Δe_i can be reached. The first step is to calculate the position coordinate $[o''_i]_{\mathfrak{R}} = [y, z]_{\mathfrak{R}}^T$ in the frame $\mathfrak{R}$ by the forward kinematics solution. The position vector of center point o''_i in the frame $\mathfrak{R}''$ is written as

$$[o''_i]_{\mathfrak{R}''} = [y''_{o'i}, z''_{o'i}]^T = R_2 [o''_i]_{\mathfrak{R}} + [y''_{i3}, z''_{i3}]^T \quad (24)$$

where R_2 is the rotation matrix about frame $\mathfrak{R} : o - yz$ to the frame $\mathfrak{R} : o'' - y''z''$, i.e.,

$$R_2 = \begin{bmatrix} c\alpha' & -s\alpha' \\ s\alpha' & c\alpha' \end{bmatrix} \quad (25)$$

where $\alpha' = \sin^{-1}\left(\frac{y''}{K}\right)$. Then the fitting error is expressed as

$$\Delta e_i = So''_i - SC_{i3} = \sqrt{(y''_{o'i})^2 + (z''_{o'i} - K)^2} - \sqrt{(y''_{ci3})^2 + (z''_{ci3} - K)^2} \quad (26)$$

Since the three-hundred-meter aperture paraboloid is composed of a lot of reflector units, we should analysis all the error of reflector units. When the error is studied in the reflector frame $\mathfrak{R}'' : o'' - y''z''$ and the side length of reflector unit is specified, the one-dimensional root-mean-square (RMS) fitting error of the paraboloid reflector with three-hundred-meter aperture is defined as

$$Re = \sqrt{\frac{\sum_{i=1}^n \Delta e_i^2}{n}} \quad (27)$$

3.1.4 One-dimensional accuracy synthesis analysis

The accuracy synthesis analysis is defined as the composition RMS error that caused by the mechanism actuator input error and the fitting error. For the mechanism actuator input error has linear relationship with the value of ΔK_{ij} , Eq. (23) can be rewritten as

$$\Delta K_{ij} = \|K - SC_{ij}\| + \delta z_j = K - \sqrt{(y''_{cij})^2 + (z''_{cij} - K)^2} + \delta z_j, \quad (j=1,2,3) \quad (28)$$

where δz_j ($j=1,2,3$) is the actuator input error. Then Eqs. (24)- (27) can be used to calculate the one-dimensional composition RMS error Re .

3.1.5 Simulation example

Since the position of supporting point A_{ij} should be limited in the range of reflector unit, as shown in Fig. 5(b), the base plate parameter r of the parallel mechanism is written as

$$r = sl - M \quad (29)$$

where sl is the side length of reflector unit and M is the distance from the movable platform edge to reflector unit edge. In this work, $M = 0.5$ m and $R = r = 2l$.

Since the paraboloid reflector with the 300m aperture is symmetry, the error can be analyzed in the range of 150m. Fig. 6(a) shows the one-dimensional fitting error when the side length of reflector unit is specified.

Fig. 6. The fitting accuracy of active reflector

According to Eqs. (26) and (28), the one-dimensional RMS fitting error and composition RMS error can be drawn as shown in Fig. 6(b) for $[\delta z_1, \delta z_2, \delta z_3]_{\max} = [1, 1, 1]$ mm. In this work, we assume that the maximal input error of the mechanism is 1 mm. When the side length of reflector unit is equal to 7.5m, the one-dimensional RMS fitting error is 3.75 mm.

3.2 Two-dimensional fitting accuracy analysis

Fig. 7. Two-dimensional fitting error region of the i -th reflector unit

As shown in Fig. 5(a), the reflector unit fitting error is a closed region. And section 3.1 only considered the one-dimensional error. In this section, the area of the closed region will be used to analyze and evaluate the fitting error, which is called as two-dimensional fitting error. Obviously, the two-dimensional fitting error will provide more reliable index for us to analyzing the fitting accuracy of the large spherical radio telescope. Fig. 7 shows the two-dimensional fitting error of the i -th reflector unit, which is the sectional region.

3.2.1 Fitting position circle equation

When the reflector units are driven by mechanism, the circle arc equation of the i -th reflector unit will be changed in the frame $\mathfrak{R}''$. As shown in Fig. 7, the centre of circle arc $A'_{i1}A'_{i3}A'_{i2}$ is changed from S to S'_i . The coordinate of S'_i in the frame $\mathfrak{R}$ is written as

$$[S'_i]_{\mathfrak{R}} = [y_{S'_i}, z_{S'_i}]^T = R_1 [S]_{\mathfrak{R}'} + [y, z]^T \quad (30)$$

where

$$R_1 = \begin{bmatrix} c\alpha & s\alpha \\ s\alpha & c\alpha \end{bmatrix}, [S]_{\mathfrak{R}'} = [0, K] \quad (31)$$

The coordinate of S'_i in the frame $\mathfrak{R}''$ is written as

$$[S'_i]_{\mathfrak{R}''} = [y''_{S'_i}, z''_{S'_i}]^T = R_2 [S'_i]_{\mathfrak{R}} + [y''_{i3}, z''_{i3}]^T \quad (32)$$

The circle arc equation is changed to

$$z'' = K - \sqrt{K^2 - (y'' - y_{S'_i})^2} + z''_{S'_i} \quad (33)$$

3.2.2 Two-dimensional fitting error

According to the circle arc equation in the frame $\mathfrak{R}''$, the two-dimensional fitting error can be calculated. Firstly, as shown in Fig. 7, one point in parabola is denoted as $C(y''_C, z''_C)$, The equation of straight line SC can be written as

$$z'' = \frac{(z''_C - K)y''}{y''_C} + K \quad (34)$$

According to Eqs. (33) and (34), the intersecting point A between line SC and the circle arc $A'_{i1}A'_{i3}A'_{i2}$ can be expressed by $A(y''_A, z''_A)$. The fitting error of given point is expressed as

$$\Delta e_{y''} = SA - SC = \sqrt{(y''_C - y''_A)^2 + (z''_C - z''_A)^2} \quad (35)$$

The area of the closed region can be written as

$$S_{e_i} = \int_{y''_{C1}}^{y''_{C2}} |\Delta e_{y''}| dy'' \quad (36)$$

which is the two-dimensional fitting error of the i -th reflector unit. Then the average error of the two-dimensional fitting error is defined as

$$Qe_i = \frac{Se_i}{y''_{Ci2} - y''_{Ci1}} \quad (37)$$

In the end, the two-dimensional root-mean-square (RMS) fitting error of the paraboloid reflector with three-hundred-meter aperture is defined as

$$R_{Qe} = \sqrt{\frac{\sum_{i=1}^n Qe_i^2}{n}} \quad (38)$$

where n is the number of reflector units that consist of three-hundred meter aperture parabola.

3.2.3 Two-dimensional accuracy synthesis analysis

The two-dimensional accuracy synthesis analysis is defined as the composition RMS error that caused by the mechanism actuator input error and the two-dimensional fitting error, which is denoted as R'_{Qe} . The Eqs. (28), (37) and (38) can be used to calculate the two-dimensional composition RMS fitting error R'_{Qe} .

3.2.4 Simulation example

The two-dimensional RMS error and composition RMS fitting error are shown in Fig. 8, where all the dimensional design parameters are the same as the specified parameters in section 3.1.5. Comparing the Fig. 8 and Fig. 7, we can know although both the one-dimensional and two-dimensional RMS fitting error increase while the side length of reflector unit increases, the two-dimensional RMS error is larger than one-dimensional RMS error.

Fig. 8. The two-dimensional RMS fitting error of active reflector

According to the fitting error requirement given by reference (Qiu, 1998), when the highest working frequency of the radio telescope is 5GHz, the reflector RMS fitting error should be less than 3.75 mm. Now, we can decide the dimensional parameters and guarantee the implementation of the working frequency by the one-dimensional or two-dimensional RMS fitting error curves. For example, according to the Fig. 7, if the side length of reflector unit is specified as 7.5m, the specified dimension of reflector units can satisfy the requirement of 5GHz work frequency. However, as shown in Fig. 8, if the two-dimensional RMS fitting error is used to evaluate the fitting accuracy, the side length of reflector unit should be less than 7.0m for satisfying the requirement of 5GHz work frequency.

4. The error kinematics of 3-PRS mechanism

3-PRS mechanism has less chains which reduces its cost. The kinematics of 3-PRS mechanism has been fully analyzed (Carretero et al., 1997; Tsai & Shiau, 2003). Yet, when the mechanical manufacturing and assembling errors are brought into the model, kinematic analysis will become complicated. Therefore, analysis on parasitic motion and accuracy should be made to guarantee the application of 3-PRS mechanism as reflector unit supporting mechanism.

4.1 Kinematic modeling with errors

The magnitude of the reflector driving machine is always at meter, so input error, length error of the legs and location error of the spherical joint have little influence on motion error of the moving platform. On the other hand, the location and angle error of the rotational joint, which will be extended by the legs, will mix with parasitic motion so as to greatly affect the motion. Therefore, we introduce angle error of the rotational axis and location error of the joint point in the rotational joint as the main error resources in order to analyze kinematics of 3-PRS error model.

Fig. 9. Kinematic error model of 3-PRS mechanism

In the error model representation of 3-PRS mechanism, as shown in Fig. 9(a), P_i is ideal axis vector of the rotational joint, and P'_i is actual axis vector with angle error. Similarly, B_i

is ideal vector of the rotational joint point, whereas $\mathbf{B}'_i$ is actual joint point vector with location error. Both $\mathbf{P}'_i$ and $\mathbf{B}'_i$ include three direction errors separately along x , y , and z axis, which means that there are six errors in each leg, as shown in Fig. 9(b), in which two-dot chain line represents ideal rotational joint and real line represents actual one.

The location error vector of the rotational joint is defined as

$$\Delta \mathbf{B}_i = [\Delta b_{ix} \quad \Delta b_{iy} \quad \Delta b_{iz}]^T$$

The angle error vector of the rotational joint is defined as

$$\Delta \mathbf{P}_i = [\Delta p_{ix} \quad \Delta p_{iy} \quad \Delta p_{iz}]^T$$

Then we can find

$$\mathbf{B}'_i = \mathbf{B}_i + \Delta \mathbf{B}_i \quad (39)$$

$$\mathbf{P}'_i = \mathbf{P}_i + \Delta \mathbf{P}_i \quad (40)$$

The three components of the vector $\Delta \mathbf{B}_i$ are independent while those of the vector $\Delta \mathbf{P}_i$ are not since the error on the direction can be given through two parameters only. So the relationship between the components of the vector $\Delta \mathbf{P}_i$ can be determined by $\|\mathbf{P}_i + \Delta \mathbf{P}_i\| = 1$, where $\|\mathbf{P}_i\| = \|\mathbf{P}'_i\| = 1$ are unit direction vectors. Thus, error resources are appropriately introduced and error modeling of 3-PRS mechanism is completed.

4.2 Inverse kinematics

The coordinate axes of the inertial frame fixed on the base platform are denoted by $\mathcal{R}: o-xyz$ while those of the moving frame fixed on the moving platform are denoted by $\mathcal{R}': o'-x'y'z'$ (see Fig. 9). In order to simplify the kinematic model, the origin of the inertial frame is located on the center of the base platform and x axis of the inertial frame points to one of the spherical joint on the base platform. The y axis is also on the plane of the base platform while z axis points upward forming a right-handed orthogonal frame. The coordinate axes of the moving frame are also located on the moving platform in the same way. The rotation matrix from the coordinate axes of the moving platform to those of the base platform is denoted by $\mathbf{R}$ which is expressed as

$$\mathbf{R} = \begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi s\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix}$$

where ψ , θ , and ϕ are variables which orderly specify the rotations around the x , y , and z axis, and s represents $\sin$, while c represents $\cos$.

$[\mathbf{H}]_{\mathcal{R}} = [x \quad y \quad z]^T$ is the vector from the origin of the inertial frame to the origin of the moving frame expressed in the inertial frame.

$[B_i]_{\mathfrak{R}}$ is the vector B_i expressed in the inertial frame.

$[B'_i]_{\mathfrak{R}}$ is the vector B'_i expressed in the inertial frame.

$[P_i]_{\mathfrak{R}} = [p_{i1} \ p_{i2} \ 0]^T$ is the vector P_i expressed in the inertial frame.

$[P'_i]_{\mathfrak{R}}$ is the vector P'_i expressed in the inertial frame.

$[A_i]_{\mathfrak{R}'} = [a_{i1} \ a_{i2} \ 0]^T$ is the vector from the origin of the moving frame to the i -th upper attachment point expressed in the moving frame.

$[L_i]_{\mathfrak{R}}$ is the vector from the rotational point to the upper attachment point of the i -th leg. It should be noted that $\| [L_i]_{\mathfrak{R}} \| = l_i$ is constant for each leg.

$S = (S_1, S_2, S_3)^T$ is the set of actuated joint variable of the 3-PRS mechanism which is the height of the rotational joint point. We can get

$$[B_i]_{\mathfrak{R}} = [b_{i1} \ b_{i2} \ S_i]^T$$

The inverse kinematic problem is supposed to determine the value of the actuated variables for a known position and orientation of the end-effector, that is: $S = f(x, y, z, \psi, \theta, \phi)$.

In those six variables, the known numbers are three desired motions which include z , ψ , and θ , while the unknown numbers are three parasitic motions which include x , y , and ϕ . The parasitic motions are determined by the target motions, that is: $(x, y, \phi) = g(z, \psi, \theta)$.

The structure of mechanical joint leads to two geometrical constraints which are rotation constraint and length limitation of the leg.

(a) The rotation constraint

Each attachment point of the moving platform should be restricted in the rotation plane formed by the wheeling leg. The constraint equations are

$$[P'_i]_{\mathfrak{R}}^T (R[A_i]_{\mathfrak{R}'} + [H]_{\mathfrak{R}}) + C_i = 0, i = 1, 2, 3 \quad (41)$$

where C_i is a constant of the rotation plane and determined by the following equation

$$[P'_i]_{\mathfrak{R}}^T [B'_i]_{\mathfrak{R}} + C_i = 0, i = 1, 2, 3 \quad (42)$$

Substituting Eq. (39) into Eq. (42)

$$([P_i]_{\mathfrak{R}} + [\Delta P_i]_{\mathfrak{R}})^T ([B_i]_{\mathfrak{R}} + [\Delta B_i]_{\mathfrak{R}}) + C_i = 0 \quad (43)$$

Expressing with the elements of those vectors, we get

$$m_i S_i + C_i + n_i = 0 \quad (44)$$

where

$$m_i = \Delta p_{iz}, \quad n_i = p_{i1}b_{i1} + p_{i2}b_{i2} + \Delta p_{ix}b_{i1} + \Delta p_{iy}b_{i2} + p_{i1}\Delta b_{ix} + p_{i2}\Delta b_{iy} + \Delta p_{ix}\Delta b_{ix} + \Delta p_{iy}\Delta b_{iy} + \Delta p_{iz}\Delta b_{iz}$$

Substituting Eq. (40) into Eq. (41)

$$([\mathbf{P}_i]_{\mathcal{R}} + [\Delta \mathbf{P}_i]_{\mathcal{R}})^T (\mathbf{R}[\mathbf{A}_i]_{\mathcal{R}'} + [\mathbf{H}]_{\mathcal{R}}) + C_i = 0 \quad (45)$$

Expressing with the elements of those vectors, we get

$$f_i c\phi + g_i s\phi + h_i x + k_i y + j_i + C_i = 0 \quad (46)$$

where

$$\begin{aligned} f_i &= a_{i1} p_{i1} c\theta + a_{i2} p_{i1} s\theta s\psi + a_{i2} p_{i2} c\psi + a_{i1} \Delta p_{ix} c\theta + a_{i2} \Delta p_{ix} s\theta s\psi + a_{i2} \Delta p_{iy} c\psi \\ g_i &= -a_{i2} p_{i1} c\psi + a_{i1} p_{i2} c\theta + a_{i2} p_{i2} s\theta s\psi - a_{i2} \Delta p_{ix} c\psi + a_{i1} \Delta p_{iy} c\theta + a_{i2} \Delta p_{iy} s\theta s\psi \\ h_i &= p_{i1} + \Delta p_{ix}, k_i = p_{i2} + \Delta p_{iy}, j_i = \Delta p_{iz} (-a_{i1} s\theta + a_{i2} c\theta s\psi + z) \end{aligned}$$

(b) The leg limitation

The distance between the attachment point of the moving platform and the rotational point of the rotational joint should be constant. The constraint equations are

$$\|\mathbf{R}[\mathbf{A}_i]_{\mathcal{R}'} + [\mathbf{H}]_{\mathcal{R}} - [\mathbf{B}'_i]_{\mathcal{R}}\| = \|\mathbf{L}_i\| = l_i, i = 1, 2, 3 \quad (47)$$

Substitute Eq. (39) into Eq. (47)

$$\|\mathbf{R}[\mathbf{A}_i]_{\mathcal{R}'} + [\mathbf{H}]_{\mathcal{R}} - ([\mathbf{B}_i]_{\mathcal{R}} + [\Delta \mathbf{B}_i]_{\mathcal{R}})\| = l_i \quad (48)$$

Expressing with the elements of those vectors, we get

$$S_i = \pm \sqrt{l_i^2 - u_i^2 - v_i^2} + w_i \quad (49)$$

where

$$\begin{aligned} u_i &= a_{i1} c\phi c\theta + a_{i2} c\phi s\theta s\psi - a_{i2} s\phi c\psi + x - b_{i1} - \Delta b_{ix} \\ v_i &= a_{i1} s\phi c\theta + a_{i2} s\phi s\theta s\psi + a_{i2} c\phi c\psi + y - b_{i2} - \Delta b_{iy} \\ w_i &= -a_{i1} s\theta + a_{i2} c\theta s\psi + z - \Delta b_{iz} \end{aligned}$$

In Eq. (49), there are two possible solutions for each leg, thereby yielding a total of 8 possible combinations of actuated height for a given position and orientation. In the present work, the negative square root is always selected to yield a solution where the legs are always beneath the moving platform.

Combining Eqs. (44) and (46), we get

$$f_i c\phi + g_i s\phi + h_i x + k_i y + j_i - m_i S_i - n_i = 0 \quad (50)$$

Combining Eqs. (49) and (50), we get equations set involved with variables S_i and three parasitic motions. We can find that if $m_i = \Delta p_{iz} = 0$, Eqs. (49) and (50) can be solved separately. But in ordinary condition, $m_i = \Delta p_{iz} \neq 0$, direct solution of the equations set will become impossible.

Fig. 10(b) is a top view of base sphere along radial direction. The x^g axis is perpendicular with the side of the regular hexagon reflector unit. Fig. 10(b) shows the location of the three attachment points and twenty four sampling points for calculation. It is noted that the upper surface of the reflector unit is spherical which means that the sampling points are all on a spherical surface whose radius is K .

As shown in Fig. 11, the coordinate axes of the inertial frame fixed on the platform in initial position are denoted by $\mathfrak{R} : o - xyz$, in which the location of the axes is similar with the location in the kinematic analyses introduced in section 4.1. The inertial frame is not moving with the reflector unit. Since the reflector unit in initial position is the tangent plane of the base sphere, z axis always points to the center of the sphere. It should be noticed that the directions of x axis and y axis in the coordinate frame $o - xyz$ is different from those in $o^g - x^g y^g z^g$. Similarly, the coordinate axes of the moving frame fixed on the platform in fitting position and orientation are denoted by $\mathfrak{R}' : o' - x' y' z'$. The coordinate frame $\mathfrak{R}$ and $\mathfrak{R}'$ are similar with the kinematic inertial and moving frame analyzed in section 4 so that the inverse and forward solution can be used.

Fig. 11. One reflector unit in initial and fitting position

We can divide the surface by several circular arcs. These arcs are intersections of the base sphere and the planes which are parallel with $x^g y^g z^g$ plane, and are all through the centers of the reflector units. So the fitting accuracy of all reflector units can be calculated along these arcs.

For analyses, the symbols used in the Fig. 10 and 11 are defined as

A_i is the i -th attachment on the reflector unit.

C is the center of the base sphere.

K is the radius of the base sphere.

F is the focal point of the ideal paraboloid of revolution.

sl is the side length of the reflector unit.

M is the distance from each attachment to the border of the reflector unit on radial direction.

$[A_{tji}]_{\mathfrak{R}}$ is the vector from the origin of $\mathfrak{R}$ to A_i on the (t, j) reflector unit in the initial position expressed in the inertial frame $\mathfrak{R}$ (the subscript t, j mean the j -th reflector unit on the t -th circular arc, the same as below).

$[A_{tji}]_{\mathfrak{R}^g}$ is the vector from the origin of $\mathfrak{R}^g$ to A_i in the initial position expressed in the global inertial frame $\mathfrak{R}^g$.

$[G_{tjk}]_{\mathfrak{R}'}$ is the vector from o' to the k -th sampling point in the fitting position expressed in the moving frame $\mathfrak{R}'$, so $[G_{tjk}]_{\mathfrak{R}'}$ is constant and known.

$[G_{tjk}]_{\mathfrak{R}}$ is the vector from o to the k -th sampling point in the fitting position expressed in the inertial frame $\mathfrak{R}$.

$[G_{tjk}]_{\mathfrak{R}^g}$ is the vector from o^g to the k -th sampling point in the fitting position expressed in the inertial frame $\mathfrak{R}^g$.

$[H_{tj}]_{\mathfrak{R}^g}$ is the vector from o^g to o expressed in the inertial frame $\mathfrak{R}^g$ which is determined by the position of current analyzing reflector unit.

$[H_{tj}]_{\mathfrak{R}} = (x, y, z)^T$ is the vector from o to o' expressed in the inertial frame $\mathfrak{R}$.

R_{tj} is the rotation matrix from coordinate frame $\mathfrak{R}$ to $\mathfrak{R}^g$.

R'_{tj} is the rotation matrix from coordinate frame $\mathfrak{R}'$ to $\mathfrak{R}$.

$S_{tj} = (S_{tj1}, S_{tj2}, S_{tj3})^T$ is the actuated joint variable.

ΔS_{tji} is the increment of the actuated variable of the i -th leg from the initial position to the fitting position.

Δl_{tji} is the distance between the i -th attachment in the initial position and the paraboloid of revolution on radial direction.

$f(\psi, \theta, z)$ is inverse solution which is analyzed in section 4 with output of $(S_1, S_2, S_3)^T$.

$f^{-1}(S_1, S_2, S_3)$ is forward solution with output of $(x, y, z, \psi, \theta, \phi)^T$.

5.2 Paraboloid equation and circle equation

The focal length of the paraboloid is specified as $0.467K$, then the paraboloid equation can be written as

$$z^g = \frac{1}{4 \times 0.467K} (x^{g2} + y^{g2})$$

The distance from one point $(x_0^g, y_0^g, z_0^g)^T$ to the paraboloid on radial direction is

$$\Delta l = \left\| (x_0^g, y_0^g, z_0^g)^T - (x^g, y^g, z^g)^T \right\|$$

where

$$z^g = K + \frac{2(z_0^g - K)^2 \times 0.467K}{x_0^{g2} + y_0^{g2}} - \sqrt{\left(K + \frac{2(z_0^g - K)^2 \times 0.467K}{x_0^{g2} + y_0^{g2}}\right)^2 - K^2}$$

$$x^g = \frac{z^g - K}{z_0^g - K} x_0^g$$

$$y^g = \frac{z^g - K}{z_0^g - K} y_0^g$$

By combining the equations above, we define the function to calculate distance between one point $(x_0^g, y_0^g, z_0^g)^T$ and the paraboloid on radial direction as

$$dis((x_0^g, y_0^g, z_0^g)^T) = \Delta l$$

5.3 Driving strategy

Driving strategy determines the method to drive the reflector unit to fit for paraboloid. In order to simplify calculation, the actuated variable S_{ij} is the value of actuated variable in initial position plus Δl_{iji} .

5.4 Fitting accuracy calculation

Calculating the number of the arcs in the half base sphere of positive y^g axis surface, we get:

$$m = \text{ceil}\left(\frac{K\pi}{9sl}\right) \quad (51)$$

where $\text{ceil}(x)$ means the least integer which is no less than x .

The radius of the t -th arc is written as

$$K_t = K \cos(3t \cdot sl/2K) \quad (52)$$

The meeting point of the t -th arc to the border of the base sphere is written as

$$x_t^g = \sqrt{(K/2)^2 - (K \sin(3t \cdot sl/2K))^2} \quad (53)$$

Thus the length of the t -th positive half circular arc is written as

$$al_t = K_t \arcsin(x_t^g / K_t) \quad (54)$$

So the number of reflector units on the positive half t -th arc can be obtained as

$$n_t = \text{ceil}(al_t / \sqrt{3}sl) \quad (55)$$

First, we focus on the coordinate frame $\mathfrak{R}$ and $\mathfrak{R}^g$. The equation of the base sphere is

$$x^{g2} + y^{g2} + (z^g - K)^2 = K^2$$

or

$$z^g = K - \sqrt{K^2 - x^{g2} - y^{g2}}$$

So that along the arc of even number, the position of the j -th reflector unit on the t -th arc can be written as

$$x_{ij}^g = K_t \sin(\sqrt{3}j \cdot sl / K_t), \quad y_{ij}^g = K \sin(3t \cdot sl / 2K), \quad z_{ij}^g = K - \sqrt{K^2 - x_{ij}^{g2} - y_{ij}^{g2}}$$

While along the arc of odd number, the x_{ij}^g is

$$x_{ij}^g = K_t \sin(\sqrt{3}(j + 0.5)sl / K_t)$$

Thus position of the coordinate frame $\mathfrak{R}$ which is fixed on the reflector unit in the initial position is determined by

$$[\mathbf{H}_{ij}]_{\mathfrak{R}^g} = (x_{ij}^g, y_{ij}^g, z_{ij}^g)^T \quad (56)$$

According to the orientation of the coordinate frame $\mathfrak{R}$, the rotation matrix to $\mathfrak{R}^g$ can be written as

$$\mathbf{R}_{ij} = \text{Rot}(\mathbf{p}_{ij}, \arcsin(\frac{\sqrt{x_{ij}^{g2} + y_{ij}^{g2}}}{K})) \text{Rot}(z, 90) \quad (57)$$

where $\text{Rot}(a, b)$ is rotation matrix of rotating angle of b degree around vector a , and $\mathbf{p}_{ij}$ is rotating vector in $o^g x^g y^g$ plane which is expressed as

$$\mathbf{p}_{ij} = (y_{ij}^g / \sqrt{x_{ij}^{g2} + y_{ij}^{g2}}, -x_{ij}^g / \sqrt{x_{ij}^{g2} + y_{ij}^{g2}}, 0)^T$$

The vector of sampling point can be written as

$$[\mathbf{G}_{ijk}]_{\mathfrak{R}^g} = \mathbf{R}_{ij} [\mathbf{G}_{ijk}]_{\mathfrak{R}} + [\mathbf{H}_{ij}]_{\mathfrak{R}^g} \quad (58)$$

while the vector of the upper attachment in the initial position can be written as

$$[\mathbf{A}_{tji}]_{\mathfrak{R}^g} = \mathbf{R}_{ij} [\mathbf{A}_{tji}]_{\mathfrak{R}} + [\mathbf{H}_{ij}]_{\mathfrak{R}^g} \quad (59)$$

Then, we focus on the coordinate frame $\mathfrak{R}'$ and $\mathfrak{R}$. The value of actuated variable should be determined for the (t, j) unit. According to the driving strategy, the actuated variable of the reflector unit in fitting position can be obtained as

$$\mathbf{S}_{ij} = \mathbf{S}_0 + (\Delta l_{ij1}, \Delta l_{ij2}, \Delta l_{ij3})^T \quad (60)$$

where $\mathbf{S}_0 = \mathbf{f}(0, 0, 0)$ is the initial value of the actuated variable and $\Delta l_{ji} = \text{dis}(\mathbf{A}_{ji})_{\mathfrak{R}^s}$.

According to forward solution, the Cartesian variables which specify the position and orientation from the coordinate frame $\mathfrak{R}'$ to the inertial frame $\mathfrak{R}$ can be obtained as

$$(x, y, z, \psi, \theta, \phi)^T = \mathbf{f}^{-1}(\mathbf{S}_{ij}) \quad (61)$$

$$[\mathbf{H}_{ij}]_{\mathfrak{R}} = (x, y, z)^T \quad (62)$$

$$\mathbf{R}'_{ij} = \begin{bmatrix} c\phi c\theta & c\phi s\theta s\psi - s\phi c\psi & c\phi s\theta c\psi + s\phi c\psi \\ s\phi c\theta & s\phi s\theta s\psi + c\phi c\psi & s\phi s\theta c\psi - c\phi s\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} \quad (63)$$

The vector of the k -th sampling point expressed in inertial frame $\mathfrak{R}$ can be obtained by

$$[\mathbf{G}_{ijk}]_{\mathfrak{R}} = \mathbf{R}'_{ij} [\mathbf{G}_{ijk}]_{\mathfrak{R}'} + [\mathbf{H}_{ij}]_{\mathfrak{R}} \quad (64)$$

Finally, we calculate the fitting accuracy based on the results obtained above. The fitting accuracy of one sampling point on the j -th reflector unit in the t -th arc can be written as

$$e_{ijk} = \text{dis}(\mathbf{G}_{ijk}^g) \quad (65)$$

Synthetically, substituting Eqs. (56)-(64) into (65), we can calculate e_{ijk} with specified t, j and k as

$$e_{ijk} = \text{dis}(\text{Rot}(\mathbf{p}_{ij}, \arcsin(\frac{\sqrt{x_{ij}^{g2} + y_{ij}^{g2}}}{K})) \text{Rot}(z, 90)(\mathbf{R}'_{ij} [\mathbf{G}_{ijk}]_{\mathfrak{R}'} + [\mathbf{H}_{ij}]_{\mathfrak{R}}) + (x_{ij}^g, y_{ij}^g, z_{ij}^g)^T)$$

The RMS fitting accuracy on the whole fitting surface can be written as

$$E_{RMS} = \sqrt{\left(\sum_{t=-m}^m \sum_{j=-n_t}^{n_t} \sum_{k=1}^{24} e_{ijk}^2 \right) / \left(24 \sum_{t=-m}^m (2n_t + 1) \right)}$$

5.5 Driving strategy optimization

In order to reduce the RMS fitting accuracy of the whole reflector, the current driving strategy should be optimized. Considering the real-time compensation, the optimization algorithm should be no more difficult than the current strategy.

According to the analyses above, we put forward one modified driving strategy as

$$\mathbf{S}_{ij} = \mathbf{S}_0 + (\Delta l_{ij1}, \Delta l_{ij2}, \Delta l_{ij3})^T + (3\text{dis}([\mathbf{H}_{ij}]_{\mathfrak{R}^s}) - \Delta l_{ij1} - \Delta l_{ij2} - \Delta l_{ij3})/6 \quad (66)$$

The strategy will compensate the actuated variable of each reflector unit with the algebraic average among three supporting points and the center of the reflector so as to get less RMS fitting accuracy on each unit spherical surface.

5.6 Simulation example

Fig. 12(a) shows the fitting accuracy on the whole reflector range with and without optimized driving strategy when the side length is changeable, which are respectively expressed as E_{RMS}^O and E_{RMS} . All the dimensional design parameters are the same as the specified parameters in section 3.1.5.

Fig. 12. The three-dimensional fitting accuracy of active reflector

As shown in Fig. 12(a), the fitting accuracy is reduced approximately by 40% when the driving strategy is optimized. Fig. 12(b) shows the fitting accuracy on the whole reflector range with and without errors when the optimized driving strategy is used, which are respectively expressed as $E_{RMS}^{O'}$ and E_{RMS}^O . In this work, we assume that the axis angle tolerance of the rotational joint is $[0,0.2^\circ]$ and the position tolerance is $[-10mm,10mm]$, and $E_{RMS}^{O'}$ is the worst situation with these tolerances. In order to guarantee the working frequency of the large radio telescope, the side length of the reflector unit should be less than 7.3mm.

6. Conclusion and future works

In order to guarantee the usage of active reflector and achieve the highest working frequency requirement, 5GHz, in FAST, fitting accuracy of the active reflector is supposed to be analyzed. In this chapter, a novel 3 DOFs parallel mechanism, 3-PSS with constraint leg, is proposed. This mechanism can fulfill the required movement to fit a paraboloid of revolution for the active reflector. The kinematics of 3-PSS+C mechanism is studied. Based on that, the one and two-dimensional fitting accuracy are calculated and the side length

limit of reflector units is evaluated as 7.0m. However, due to more expensive cost of the extra constraint chain, 3-PSS+C is not very appropriate as reflector supporting mechanism. So 3-PRS mechanism becomes more attractive and deserves to pay more attention. Then error kinematics with rotational joint tolerance is analyzed for actual application. Based on that, three-dimensional fitting accuracy is calculated with optimized driving strategy, and the side length limit turns out to be 7.3m.

The future work will still focus on the fitting accuracy not only on kinematics, but also on synthetical design, stiffness and control, as well as sharing strategy study. Further more, experiment research will be taken into account as certification for the theoretical analysis.

7. Acknowledgment

This research is sponsored by National Natural Science Foundation of China (No.50605035, 10778625), the State High-Technology Development Program of China (No.2006AA04Z133), and the National 973 Program of China (No. 2007CB714007).

8. References

- Carretero, J. A.; Nahon, M.; Buckham, B. & Gosselin, C. M. (1997). Kinematic analysis of a three-dof parallel mechanism for telescope applications, *Proceedings of The 1997 ASME Design Engineering Technical Conference and Computers in Engineering Conference*, pp. 1-8, California, Sep. 1997, ASME, Sacramento
- Duan, B.-Y. (1999). A new design project of the line feed structure for large spherical radio telescope and its nonlinear dynamic analysis. *Mechatronics*, Vol. 9, pp. 53-64
- Li, H. (1998). China hopes to move FAST on largest telescope. *Science*, Vol. 281, pp. 771-773
- Liu, X.-J; Wang, J.-S.; Gao, F. & Wang, L.-P. (2001). On the analysis of a new spatial three degrees of freedom parallel manipulator *IEEE Transactions on Robotics and Automation*, *IEEE Transactions on Robotics and Automation*, Vol. 17, pp. 959-968
- Luo, Y.-F.; Deng, C.-G. & Li, G.-Q. (2000). Structural Analysis of the Reflector Supporting System of the 500 m Active Spherical Telescope. *Journal of Tongji University*, Vol. 28, No. 4, pp. 497-500
- Nan, R.-D. & Peng, B. (2000). Chinese concept for the 1km² radio telescope. *Acta Astronautica*, Vol. 46, pp. 667-675
- Press, W. H.; Teukolsky, S. A.; Vetterling, W. T. & Flannery, B. F. (1995). *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University, U.K.
- Qiu, Y.-H. (1998). A novel design for a giant Arecibo-type spherical radio telescope with an active main reflector. *Mon. Not. R. Astron. Soc*, Vol. 301, pp. 222-228
- Tang, X.-Q.; Wang, J.-S.; Huang P. et al. (2007). Patent: 3-PRS Based Co-driving Array Parallel Mechanism for Large Radio Telescope, 200710177308.3
- Tsai, M.-S. & Shiau, T.-N. (2003). Direct kinematic analysis of a 3-PRS parallel mechanism. *Mechanism and Machine Theory*, Vol. 38, No. 1, pp. 71-83
- Wang, J.-S.; Tang, X.-Q.; Duan, G.-H. & Li, J.-F. (2001). Design methodology for a novel planar three degrees of freedom parallel machine tool. *Proceedings of the 2001 IEEE*

- International Conference on Robotics & Automation*, pp. 2448-2453, Korea, May. 2001, IEEE, Seoul
- Wang, J.-S. & Tang, X.-Q. (2003). Analysis and dimensional design of a novel hybrid machine tool. *International Journal of Machine Tools and Manufacture*, Vol. 43, No. 7, pp. 647-655
- Wang, Q.-M.; Tang, X.-Q.; Zhu, L.-C. et al. (2006). Patent: Co-driving Array Parallel Mechanism for Large Radio Telescope, 200610011941.0

Parallel Manipulators, towards New Applications

Edited by Huapeng Wu

ISBN 978-3-902613-40-0

Hard cover, 506 pages

Publisher I-Tech Education and Publishing

Published online 01, April, 2008

Published in print edition April, 2008

In recent years, parallel kinematics mechanisms have attracted a lot of attention from the academic and industrial communities due to potential applications not only as robot manipulators but also as machine tools. Generally, the criteria used to compare the performance of traditional serial robots and parallel robots are the workspace, the ratio between the payload and the robot mass, accuracy, and dynamic behaviour. In addition to the reduced coupling effect between joints, parallel robots bring the benefits of much higher payload-robot mass ratios, superior accuracy and greater stiffness; qualities which lead to better dynamic performance. The main drawback with parallel robots is the relatively small workspace. A great deal of research on parallel robots has been carried out worldwide, and a large number of parallel mechanism systems have been built for various applications, such as remote handling, machine tools, medical robots, simulators, micro-robots, and humanoid robots. This book opens a window to exceptional research and development work on parallel mechanisms contributed by authors from around the world. Through this window the reader can get a good view of current parallel robot research and applications.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Xiao-qiang Tang and Peng Huang (2008). The Analysis and Application of Parallel Manipulator for Active Reflector of FAST, *Parallel Manipulators, towards New Applications*, Huapeng Wu (Ed.), ISBN: 978-3-902613-40-0, InTech, Available from:

http://www.intechopen.com/books/parallel_manipulators_towards_new_applications/the_analysis_and_application_of_parallel_manipulator_for_active_reflector_of_fast

INTECH
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2008 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen