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A Comparison of the Undetermined Coefficient Method and the Adomian Decomposition Method for the Solutions of the Sasa-Satsuma Equation

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Abstract

This chapter will talk about the mathematical as well as numerical aspects of the Sasa-Satsuma equation that is the extended nontrivial version of nonlinear Schrödinger's equation. The exact solution will be found out by the undetermined coefficient method. After that, the Adomian decomposition method is secure numerical simulations of computed analytical solutions. The error plots are given to see the accuracy of the results.

Keywords: Sasa-Satsuma equation, solitons, Adomian decomposition method, undetermined coefficient method, telecommunication

1. Introduction

Solitons can be illustrated as special wave packets that are formed as a result of elegant balance among the fiber nonlinearity and dispersion. They have the ability to travel undistorted along trans-continental and trans-oceanic distances. Solitons are narrow pulses with immense peak power and exceptional properties. Mostly, pulses go through by spreading because of group velocity dispersion while propagating through optical fibers. Actually, solitons have the advantage of nonlinear effects that helps to overcome the broadening of pulse with the group velocity dispersion. When the corresponding dispersive effects and nonlinear effects are controlled and get the appropriate shape of the pulse. When these pulses balance compression and broadening and there is no change in the shape of the pulse or there are periodic changes in the shape of the pulse, this phenomenon is called soliton. Solitons are very advantageous for optical communication that they can overcome chromatic dispersion completely. In most Dense Wavelength Division Multiplexing (DWDM) systems, fiber losses are compensated periodically by using fiber amplifiers spaced 60-80 Km apart. Attenuation and higher powers are the indirect properties of solitons that are reimbursed by the optical amplifiers. When solitons and amplifiers are used together, they can assure the very high-bit rate, for the repeater-less data transmission for long distances. This combination can be responsible for the data transmission at a bit rate of 80 Gb/s for 10,000 km and it

was testified in the laboratory. Hasegawa and Tappert in 1973, have discussed the possibilities of propagation of solitons through optical fibers and showed their remarkable stability by numerical computation [1]. Seven years later, Mollenauer, Stolon, and Gordon succeeded in observing soliton propagation experimentally [2].

In this chapter, Sasa-Satsuma equation (SSE) is going to be studied for the sake of optical solitons. SSE is the expansion of nonlinear Schrödinger equation (NLSE). In 1991, Narimasa Sasa and Junkichi Satsuma reported significant results that have a great impact in the field of nonlinear optics and the telecommunication industry [3]. Initially, Sasa and Satsuma displayed a nonlinear complex wave equation that was composed with the aid of inverse scattering transformation [4].

The Sasa-Satsuma equation with the linear temporal evolution is [5]:

$$iq_t + a(|q|^n q)_{xx} + b|q|^2 q + i[\beta_3 q_{xxx} + \sigma(|q|^2)_x q + \theta|q|^2 q_x] = 0 \quad (1)$$

In (1), $q(x, t)$ is the dependent variable, x and t are independents variables and the subscripts serve as partial derivatives. The first term in (1) known as time evolution term, while a is the coefficient of nonlinear chromatic dispersion, b gives the self-phase modulation with kerr nonlinearity, β_3 is the coefficient of third-order dispersion, σ and θ are the coefficients of nonlinear dispersion. Finally, n gives the nonlinearity parameter of chromatic dispersion and $n > 0$.

2. Method of undetermined coefficients

Method of undetermined coefficients gives the spectrum of soliton solutions. In order to seek the soliton solution of SSE to (1) [6–10]:

$$q(x, t) = P(x, t)e^{i\phi(x, t)} \quad (2)$$

Where $P(x, t)$ is the amplitude segment of the soliton. The phase component is

$$\phi(x, t) = -\kappa x + \omega t + \theta_0 \quad (3)$$

Here, κ , ω and θ_0 are the soliton frequency, wave number and the phase constant respectively. By substituting (2) into (1) and equating real and imaginary parts leads to

$$\begin{aligned} & -a\kappa^2(1+n)^2P^{n+1} + a\left(\frac{\partial}{\partial x}P(x, t)\right)^2 n(1+n)P^{n-1} + (a(1+n)(P(x, t))^n + 3\beta_3\kappa)\frac{\partial^2}{\partial x^2}P(x, t) \\ & P(x, t)\left((-2\sigma - \theta)\kappa - b\right)(P(x, t))^2 + \beta_3\kappa^3 + \omega = 0, \end{aligned} \quad (4)$$

and

$$\begin{aligned} & \left(\frac{\partial^3}{\partial x^3}P(x, t)\right)\beta_3 + \left(-2a\kappa(1+n)^2(P(x, t))^n + (2\sigma + \theta)(P(x, t))^2 - 3\beta_3\kappa^2\right)\frac{\partial}{\partial x}P(x, t) \\ & + \frac{\partial}{\partial t}P(x, t) = 0. \end{aligned} \quad (5)$$

2.1 Solution of bright soliton

For the solution of bright soliton, the starting hypothesis is [7–10]

$$P(x, t) = \frac{A}{(D + \cosh \tau)^p}, \quad (6)$$

Where

$$\tau = B(x - vt) \quad (7)$$

A is known as amplitude, B is the inverse width of the soliton, and D is the parameter that relates to A and B . The p is an unknown parameter that can be found with the aid of balancing principle. By putting (6) into (4) gives:

$$\begin{aligned} & \frac{A^3(\kappa\theta + b)}{E^{3p}} + \frac{A(a(np^2(n+2)B^2 - \kappa^2) + 3B^2\beta_3\kappa p^2 - \beta_3\kappa^3 - \omega) + AB^2p^2a}{E^p} \\ & + \frac{((n+1)(1+(n+1)p)a + 3\beta_3\kappa(p+1))(D+1)(D-1)ApB^2}{E^{p+2}} \\ & - 2 \frac{DAp((n+1)(1/2 + (n+1)p)a + 3\kappa(p+1/2)\beta_3)B^2}{E^{p+1}} = 0. \end{aligned} \quad (8)$$

where $E = D + (\cosh(\tau))$.

With the balancing principle, the exponents of $3p$ and $p+2$ gives $p=1$. By setting the coefficients of linearly independent function in (8) to zero that gives;

$$\omega = ((n+1)^2a + 3\beta_3\kappa)B^2 - \beta_3\kappa^3 - a\kappa^2 \quad (9)$$

$$A = \pm \frac{\sqrt{(\kappa\theta + b)(D-1)(D+1)((n^2 + 3n + 2)a + 6\beta_3\kappa)B}}{\kappa\theta + b}, \quad (10)$$

$$\kappa\theta + b > 0, \quad (11)$$

$$D = \frac{\sqrt{((n^2 + 3n + 2)a + 6\beta_3\kappa)(-B^2(n+2)(n+1)a - 6B^2\beta_3\kappa + (\kappa\theta + b)A^2)}}{((n^2 + 3n + 2)a + 6\beta_3\kappa)B}, \quad (12)$$

$$((n^2 + 3n + 2)a + 6\beta_3\kappa)B > 0. \quad (13)$$

The solution of bright soliton of (1) is

$$q(x, t) = \frac{A}{D + \cosh[B(x - vt)]} e^{i(-\kappa x + \omega t + \theta_0)}, \quad (14)$$

Figure 1 represents the bright optical soliton of SSE. The soliton solution appears with their corresponding constraints conditions.

2.2 Solution of dark solitons

For the solution of dark soliton, the assumption is [7–10]

$$P(x, t) = (A + B \tanh \tau)^p, \quad (15)$$

Figure 1.
Bright soliton.

where A and B are free parameters and

$$\tau = \mu(x - vt), \quad (16)$$

Here, p can be found with the balancing principle. By substituting (15) into (4), the real part gives:

$$\begin{aligned} & \mu^2(A(A+B) - B(A+B))^4(a(-1 + (n+1)p)(n+1) + 3\beta_3k(p-1))p E^{p-2} \\ & - 6(A-B)A(a(-1/2 + (n+1)p)(n+1) + 2\beta_3k(p-1/2))(A+B)\mu^2p E^{p-1} \\ & + ((1 + 2(n+1)p)a(n+1) + 3\beta_3k(2p+1))A\mu^2p E^{p+1} \\ & + ((1 + (n+1)p)a(n+1) + 3\beta_3k(1+p))\mu^2p E^{p+2} + B^2(k\theta + b)E^{3p} \\ & + 18 \left((A^2 - 1/3B^2)p^2 \left(1/3(n+1)^2a + \beta_3k \right) \mu^2 - 1/18B^2(\beta_3k^3 + ak^2 + w) \right) E^p = 0 \end{aligned} \quad (17)$$

where $E = (A + B \tanh(\tau))$

The value of p is similar to bright soliton and gives the value of coefficients of linearly independent function as zero that yields to the following relations of soliton parameters.

$$A = \pm B = \pm \frac{\sqrt{(k\theta + b)(an^2 + 3an + 6\beta_3k + 2a)}}{k\theta + b}, \quad (18)$$

$$(k\theta + b) > 0, \quad (19)$$

$$(an^2 + 3an + 6\beta_3k + 2) > 0, \quad (20)$$

$$\mu = \frac{\sqrt{(k\theta + b)(an^2 + 3an + 6\beta_3k + 2a)}}{an^2 + 3an + 6\beta_3k + 2} \quad (21)$$

$$\omega = -ak^2 + 4an^2 - k^3 + 8an + 4a + 12k \quad (22)$$

Figure 2.
 Dark soliton.

Hence, the solution of the dark soliton is given as:

$$q(x,t) = A\{1 \pm \tanh [\mu(x - vt)]\}e^{i(-\kappa x + \omega t + \theta_0)}, \tag{23}$$

Figure 2 represents the dark soliton of SSE. The soliton solution appears with their corresponding constraints conditions.

2.3 Solution of singular solitons

For the solution of singular soliton, the starting assumption is [7–10];

$$P(x,t) = \frac{A}{(D + \sinh \tau)^p}, \tag{24}$$

Here, A , B , and D are the free parameters with the unknown p . By putting (25) into (4) gives;

$$\begin{aligned} &\frac{((n + 1)(1 + (n + 1)p)a + 3\beta_3(p + 1)k)Ch^2ApB^2}{E^{p+2}} - \frac{AB^2Shp(an + 3\beta_3k + a)}{E^{p+1}} \\ &+ \frac{A^3(k\theta + b)}{E^{3p}} - \frac{A(\beta_3k^3 + ak^2 + w)}{E^p} = 0, \end{aligned} \tag{25}$$

where $E = (D + \sinh (\tau))$.
 By setting $3p = p + 2$, we get $p = 1$ and the free parameters are related as

$$A = \pm \sqrt{\frac{an^2 + 3an + 6\beta_3k + 2a}{k\theta + b}}B, \tag{26}$$

$$k\theta + b > 0, \tag{27}$$

$$\omega = \left(a(n+1)^2 + 3\beta_3 k\right)B^2 - \beta_3 k^3 - ak^2 \quad (28)$$

Hence the solution of singular soliton of (1) is as:

$$q(x, t) = \frac{A}{D + \sinh[B(x - vt)]} e^{i(-\kappa x + \omega t + \theta_0)}, \quad (29)$$

for designated parameters.

2.4 Solution of W-shaped solitons

For the solution of w-shaped soliton, the starting assumption is [7–10];

$$P(x, t) = (\beta + \rho \operatorname{sech}(\tau))^p, \quad (30)$$

$$\tau = \mu(x - vt), \quad (31)$$

Substituting (30) into (4) gives

$$\begin{aligned} & 4\mu^2 p \beta (a(n+1)(1+2(n+1)p) + 3\beta_3 k(2p+1))E^{p+1} \\ & - p(\beta + \rho)\mu^2((-1+(n+1)p)a(n+1) + 3\beta_3 k(p-1))\beta^2(\rho - \beta)E^{p-2} \\ & - 2p((-1/2+(n+1)p)a(n+1) + 3\beta_3 k(p-1/2))(-2\beta^2 + \rho^2)\mu^2 \beta E^{p-1} \\ & + \rho^2(k\theta + b)E^{3p} + \left(p^2((n+1)^2 a + 3\beta_3 k)\mu^2 - \beta_3 k^3 - ak^2 - w\right)\rho^2 \\ & - 6\mu^2 \beta^2 p^2((n+1)^2 a + 3\beta_3 k)E^p \\ & - ((1+(n+1)p)a(n+1) + 3\beta_3 k(1+p))p\mu^2 E^{p+2} = 0, \end{aligned} \quad (32)$$

where $E = (\beta + \rho \operatorname{sech}(\tau))$

By the aid of balancing principle, the value of $p = 1$ that gives parameters as;

$$\beta = \pm 1/2 \sqrt{2}\rho, \quad (33)$$

$$\mu = \pm \sqrt{\frac{k\theta + b}{an^2 + 3an + 6k\beta_3 + 2a}}\rho, \quad (34)$$

$$\rho = \pm \sqrt{\frac{an^2 + 3an + 6k\beta_3 + 2a}{k\theta + b}}, \quad (35)$$

and

$$w = -\frac{6\beta^2 \mu^2 a n^2 - \mu^2 a n^2 \rho^2 + 12 a n \mu^2 \beta^2 - 2 a \mu^2 n \rho^2 + 18 \beta^2 \mu^2 k \beta_3 + \beta_3 k^3 \rho^2 - 3 \beta_3 k \mu^2 \rho^2 + 6 \mu^2 a \beta^2 + a k^2 \rho^2 - \mu^2 a \rho^2}{\rho^2}. \quad (36)$$

Therefore, W-shaped soliton solution is given by:

$$q(x, t) = (\beta + \rho \operatorname{sech}(\tau)) e^{i(-\kappa x + \omega t + \theta_0)}, \quad (37)$$

Figure 3 represents the W-shaped optical soliton of SSE. The soliton solution appears with their corresponding constraints conditions.

Figure 3.
W-shaped soliton.

3. Numerical investigation of soliton solutions

In this section, the Adomian decomposition method will be implemented. ADM has gained very much popularity in recent times in applied mathematics. This method is very robust, efficient, and effective to grasp a broad range of linear, nonlinear, ordinary or partial differential equations and linear or nonlinear integral equations. This method gives the fast convergence of the solution and has many symbolic advantages.

Georg Adomian has introduced and developed this method and very well treated it in the literature. A very appreciable amount of work has been explored for the wide range of linear, nonlinear, ordinary differential equations, partial differential equations as well as integral equation [11].

3.1 Recapitulation of Adomian decomposition method

In this section, ADM is used to handle SSE numerically that show the broad spectrum analytically results. This method tackles the problem in a direct way that shows the accuracy of the exact solution of soliton solutions.

$$q(x, t) = q_1(x, t) + iq_2(x, t) \quad (38)$$

By substituting (38) into (4) and breaking it down into real and imaginary parts, respectively

$$\begin{aligned} & -u_{2t} + a(u_1^2 + u_2^2)_{xx} u_{1xx} + u_1^3 + u_1 u_2^2 - \beta_3 u_{2xxx} - \sigma(u_{1x}^2 u_2 + u_{2x}^2 u_1) \\ & -\theta(u_1^2 u_{2x}^2 + u_2^2 u_{1x}^2) = 0, \end{aligned} \quad (39)$$

$$\begin{aligned} & u_{1t} + a(u_1^2 + u_2^2)_{xx} u_{2xx} + u_2^3 + u_1^2 u_2 + \beta_3 u_{1xxx} + \sigma(u_{1x}^2 u_1 + u_{2x}^2 u_1) \\ & +\theta(u_1^2 u_{1x}^2 + u_2^2 u_{1x}^2) = 0, \end{aligned} \quad (40)$$

The solution is decomposed into finite sums of components by decomposition method that is defined as;

$$u_i(z, t) = \sum_{n=0}^{\infty} u_{i,n}(x, t), \quad (41)$$

Here, $i \in \{1, 2\}$. The components $u_{i,n}$, $n \geq 0$ and $i = 1, 2$ will be found out recurrently. By using an operator form $L_t = \frac{\partial}{\partial t}$, Eqs. (39) and (40) becomes

$$-L_t u_2(x, t) + N_2(u_1, u_2) = 0 \quad (42)$$

$$L_t u_1(x, t) + N_1(u_1, u_2) = 0, \quad (43)$$

Where

$$N_2(u_1, u_2) = a(u_1^2 + u_2^2)_{xx}^n u_{1xx} + u_1^3 + u_1 u_2^2 - \beta_3 u_{2xxx} - \sigma(u_{1x}^2 u_2 + u_{2x}^2 u_1) - \theta(u_1^2 u_{2x}^2 + u_2^2 u_{2x}) = 0, \quad (44)$$

$$N_1(u_1, u_2) = a(u_1^2 + u_2^2)_{xx}^n u_{2xx} + u_2^3 + u_1^2 u_2 + \beta_3 u_{1xxx} + \sigma(u_{1x}^2 u_1 + u_{2x}^2 u_1) + \theta(u_1^2 u_{1x}^2 + u_2^2 u_{1x}) = 0, \quad (45)$$

By applying an inverse operator $L_t^{-1} = \int_0^t (\cdot) dt$ to Eqs. (42) and (43), we get

$$u_1(x, t) = u_1(x, 0) - L_t^{-1} N_2(u_1(x, t), u_2(x, t)) \quad (46)$$

$$u_2(x, t) = u_2(x, 0) + L_t^{-1} N_1(u_1(x, t), u_2(x, t)), \quad (47)$$

where $u_1(x, 0) = \text{Re}(u(x, 0))$ and $u_2(z, 0) = \text{Im}(q(z, 0))$.

$$\begin{aligned} u_{1,0}(x, t) &= u_1(x, 0) \\ u_{2,0}(x, t) &= u_2(x, 0) \\ u_{1,k+1}(x, t) &= -L_t^{-1}(N_{2,k}) \\ u_{2,k+1}(x, t) &= L_t^{-1}(N_{1,k}), \end{aligned} \quad (48)$$

$$A_n = \frac{1}{n!} \frac{d^n}{d\eta^n} N_j \left(\sum_{n=0}^{\infty} \eta^n u_{1,n}(x, t), \sum_{n=0}^{\infty} \eta^n u_{2,n}(x, t) \right). \quad (49)$$

4. Numerical simulations

4.1 Bright solitons

To depict the ability, reliability and the accuracy of the ADM for Sasa-Satsuma equation for bright solitons where, $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, and $\kappa = \frac{1}{2}$. The results and the profile of bright soliton shown in Table below. **Figures 4–14**, present the plots of exact and approximate solution with their error plots respectively by varying the values of t and x .

t	$ q_e - q_a $
0	$8. \times 10^{-17}$
0.1	$8. \times 10^{-17}$
0.2	8×10^{-17}
0.3	$8. \times 10^{-17}$
1	0.10
2	0.2
3	0.2
x	$ q_e - q_a $
0	$9. \times 10^{-8}$
0.1	$9. \times 10^{-8}$
0.2	$7. \times 10^{-8}$
0.3	$5. \times 10^{-8}$

Figure 4.
 The graph of analytical and numerical solution with absolute error at $t = 0$ and $a = \frac{1}{10}, b = \frac{4}{100}, \beta_3 = \frac{1}{100}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 5.
 The graph of analytical and numerical solution with absolute error at $t = 0.1$ and $a = \frac{1}{10}, b = \frac{4}{100}, \beta_3 = \frac{1}{100}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 6.

The graph of analytical and numerical solution with absolute error at $t = 0.2$ and $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 7.

The graph of analytical and numerical solution with absolute error at $t = 0.3$ and $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 8.

The graph of analytical and numerical solution with absolute error at $x = 0$ and $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 9.
 The graph of analytical and numerical solution with absolute error at $x = 0.1$ and $a = \frac{1}{10}, b = \frac{4}{100}, \beta_3 = \frac{1}{100}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 10.
 The graph of analytical and numerical solution with absolute error at $x = 0.2$ and $a = \frac{1}{10}, b = \frac{4}{100}, \beta_3 = \frac{1}{100}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 11.
 The graph of analytical and numerical solution with absolute error at $x = 0.3$ and $a = \frac{1}{10}, b = \frac{4}{100}, \beta_3 = \frac{1}{100}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 12.

The graph of analytical and numerical solution with absolute error at $t = 1$ and $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 13.

The graph of analytical and numerical solution with absolute error at $t = 2$ and $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 14.

The graph of analytical and numerical solution with absolute error at $t = 3$ and $a = \frac{1}{10}$, $b = \frac{4}{100}$, $\beta_3 = \frac{1}{100}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

4.2 Dark solitons

In order to depict ADM, we acknowledge the Sasa-Satsuma equation in investigated in detail where, $a = \frac{1}{10}$, $b = \frac{1}{2}$, $\beta_3 = \frac{1}{5}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0=0$, $\omega = \frac{311}{800}$, and $\kappa = \frac{1}{2}$. The results and the profile of dark soliton shown in Table below. **Figures 15–22**, present the plots of exact and approximate solution with their error plots respectively by varying the values of t and x .

t	$ q_e - q_a $
0	1.5×10^{-8}
0.1	$1. \times 10^{-16}$
0.2	$1. \times 10^{-16}$
0.3	$1. \times 10^{-16}$
1	$1. \times 10^{-16}$
2	$8. \times 10^{-17}$
4	$6. \times 10^{-17}$

x	$ q_e - q_a $
0.1	$4. \times 10^{-19}$
0.2	1.5×10^{-18}
0.3	$3. \times 10^{-18}$

4.3 W shaped solitons

In order to depict ADM, we acknowledge the Sasa-Satsuma equation in investigated in detail where, $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$, $\sigma = \frac{1}{10}$, $\rho = \frac{\sqrt{3}}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$,

Figure 15.
The graph of analytical and numerical solution with absolute error at $t = 0$ and $a = \frac{1}{10}$, $b = \frac{1}{2}$, $\beta_3 = \frac{1}{5}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 16.

The graph of analytical and numerical solution with absolute error at $t = 0.1$ and $a = \frac{1}{10}$, $b = \frac{1}{2}$, $\beta_3 = \frac{1}{5}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 17.

The graph of analytical and numerical solution with absolute error at $t = 0.2$ and $a = \frac{1}{10}$, $b = \frac{1}{2}$, $\beta_3 = \frac{1}{5}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 18.

The graph of analytical and numerical solution with absolute error at $t = 0.3$ and $a = \frac{1}{10}$, $b = \frac{1}{2}$, $\beta_3 = \frac{1}{5}$, $\sigma = \frac{1}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 19.
 The graph of analytical and numerical solution with absolute error at $x = 0.1$ and $a = \frac{1}{10}, b = \frac{1}{2}, \beta_3 = \frac{1}{5}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 20.
 The graph of analytical and numerical solution with absolute error at $x = 0.2$ and $a = \frac{1}{10}, b = \frac{1}{2}, \beta_3 = \frac{1}{5}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 21.
 The graph of analytical and numerical solution with absolute error at $x = 0.3$ and $a = \frac{1}{10}, b = \frac{1}{2}, \beta_3 = \frac{1}{5}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 22.
The graph of analytical and numerical solution with absolute error at $t = 1$ and $a = \frac{1}{10}, b = \frac{1}{2}, \beta_3 = \frac{1}{5}, \sigma = \frac{1}{10},$
 $\theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}.$

Figure 23.
The graph of analytical and numerical solution with absolute error at $t = 2$ and $a = \frac{1}{10}, b = \frac{1}{2}, \beta_3 = \frac{1}{5}, \sigma = \frac{1}{10},$
 $\theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}.$

and $\kappa = \frac{1}{2}$. The results and the profile of W-shaped soliton shown in Table below.
Figures 23–34, present the plots of exact and approximate solution with their error plots respectively by varying the values of t and x .

t	$ q_e - q_a $
0	1.6×10^{-11}
0.1	1.6×10^{-11}
0.2	1.6×10^{-11}
0.3	1.6×10^{-11}
2	$3. \times 10^{-18}$

t	$ q_e - q_a $
3	$2. \times 10^{-18}$
5	$3. \times 10^{-18}$

x	$ q_e - q_a $
0.1	$8. \times 10^{-19}$
0.2	$8. \times 10^{-19}$
0.3	$8. \times 10^{-19}$

Figure 24.
 The graph of analytical and numerical solution with absolute error at $t = 4$ and $a = \frac{1}{10}, b = \frac{1}{2}, \beta_3 = \frac{1}{5}, \sigma = \frac{1}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 25.
 The graph of analytical and numerical solution with absolute error at $t = 0$ and $a = \frac{4}{100}, b = \frac{1}{2}, \beta_3 = \frac{4}{100}, \sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$.

Figure 26.

The graph of analytical and numerical solution with absolute error at $t = 0.1$ and $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$

$$\sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$$

Figure 27.

The graph of analytical and numerical solution with absolute error at $t = 0.2$ and $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$

$$\sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$$

Figure 28.

The graph of analytical and numerical solution with absolute error at $t = 0.3$ and $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$

$$\sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}$$

Figure 29.
 The graph of analytical and numerical solution with absolute error at $x = 0.1$ and $a = \frac{4}{100}, b = \frac{1}{2}, \beta_3 = \frac{4}{100},$
 $\sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}.$

Figure 30.
 The graph of analytical and numerical solution with absolute error at $x = 0.2$ and $a = \frac{4}{100}, b = \frac{1}{2}, \beta_3 = \frac{4}{100},$
 $\sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}.$

Figure 31.
 The graph of analytical and numerical solution with absolute error at $x = 0.3$ and $a = \frac{4}{100}, b = \frac{1}{2}, \beta_3 = \frac{4}{100},$
 $\sigma = \frac{1}{10}, \rho = \frac{\sqrt{3}}{10}, \theta = -\frac{173}{200}, \theta_0 = 0, \omega = \frac{311}{800}, \kappa = \frac{1}{2}.$

Figure 32.

The graph of analytical and numerical solution with absolute error at $t = 2$ and $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$, $\sigma = \frac{1}{10}$, $\rho = \frac{\sqrt{3}}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 33.

The graph of analytical and numerical solution with absolute error at $t = 3$ and $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$, $\sigma = \frac{1}{10}$, $\rho = \frac{\sqrt{3}}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

Figure 34.

The graph of analytical and numerical solution with absolute error at $t = 5$ and $a = \frac{4}{100}$, $b = \frac{1}{2}$, $\beta_3 = \frac{4}{100}$, $\sigma = \frac{1}{10}$, $\rho = \frac{\sqrt{3}}{10}$, $\theta = -\frac{173}{200}$, $\theta_0 = 0$, $\omega = \frac{311}{800}$, $\kappa = \frac{1}{2}$.

5. Conclusion

In this chapter, SSE has been discussed. First, the undetermined coefficient method has been used. This method secured bright, dark, singular, and W-shaped solitons solutions. The method has given a spectrum of solitons. After that, the Adomian decomposition method has been used for the numerical simulations. This is a very powerful method that has given rapid convergence. Along with, error plots have also been given to witness the accuracy of the exact solution. The graphs have also shown the comparison of exact and absolute solution.

Conflict of interest

“The authors declare no conflict of interest”.

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