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Physical Only Modes Identification Using the Stochastic Modal Appropriation Algorithm

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Abstract

Many operational modal analysis (OMA) algorithms such as SSI, FDD, IV, ... are conceptually based on the separation of the signal subspace and the noise subspace of a certain data matrix. Although this is a trivial problem in theory, in the practice of OMA, this is a troublesome problem. Errors, such as truncation errors, measurement noise, modeling errors, estimation errors make the separation difficult if not impossible. This leads to the appearance of nonphysical modes, and their separation from physical modes is difficult. An engineering solution to this problem is based on the so-called stability diagram which shows alignments for physical modes. This still does not solve the problem since it is rare to find modes stable in the same order. Moreover, nonphysical modes may also stabilize. Recently, the stochastic modal appropriation (SMA) algorithm was introduced as a valid competitor for existing OMA algorithms. This algorithm is based on isolating the modes mode by mode with the advantage that the modal parameters are identified simultaneously in a single step for a given mode. This is conceptually similar to ground vibration testing (GVT). SMA is based on the data correlation sequence which enjoys a special physical structure making the identification of nonphysical modes impossible under the isolating conditions. After elaborating the theory behind SMA, we illustrate these advantages on a simulated system as well as on an experimental case.

Keywords: in-operation modal analysis, modal appropriation, spurious modes, SMA

1. Introduction

Operational modal analysis (OMA) is a good complement to classical modal analysis where the structure is installed in a laboratory and excited under well-controlled conditions. For structures under their operating conditions, the excitation cannot be measured, random, complex in nature, and can be nonstationary. Examples are offshore structures under swell, aircraft under turbulence, etc.

Several algorithms exist to extract the modal parameters from the output, only measurements. Most of these algorithms are stochastic realization algorithms, such as SSI, BR, CVA, FDD. These algorithms are based on the separation of two orthogonal subspaces, namely the signal subspace and the noise subspace. Although in theory, this is a trivial problem, in the practice of using them, strictly speaking, it is

impossible to separate them. Errors, such as finite sample length errors, estimation errors, modeling errors, noise, ... make the separation impossible leading to the problem of model order estimation. In order to solve this problem, the stability diagrams are used where the modal model is estimated at increased orders leading to alignments for physical modes. However, numerical modes, noise modes, spurious modes, harmonics, etc. appear and the challenge is how to reject these modes especially that the modal model has to be identified in unique model order.

The stochastic modal appropriation algorithm (SMA) is based on rotating and stretching the outputs correlation sequence which was derived based on physical background. We show that based on this idea, SMA rejects automatically nonphysical modes as well as harmonics. Harmonics are assumed to be modes with zero damping and we show that such a mode can never be appropriated because the phase angle between the input and output is always different from zero. On the other hand, the physical structure of the correlation sequence is respected if and only if the mode is physical. We illustrate this on a simulated example as well as experimentally.

2. The stochastic modal appropriation algorithm (SMA)

We describe here quickly the basics of the SMA algorithm. The considered system is a quarter car model excited with the unmeasured white noise of a certain variance. The impulse response of the system may be written as [1]:

$$h(t) = C_h e^{-\xi \omega_n t} \sin(\omega_d t) \quad (1)$$

where ξ is the system damping ratio, ω_n is the system natural frequency, and ω_d is the damped natural frequency.

Computing the correlation sequence of the system based on the above impulse response leads to the following expression [1]:

$$R(t) = C_r e^{-\xi \omega_n t} \sin(\omega_d t - \phi(\xi)) \quad (2)$$

where $\phi(\xi)$ is a known parametric function that depends on the system damping ratio. The impulse response, as well as the correlation sequence, may be considered as two rotating vectors in the complex plane but with decaying amplitudes (spirals).

In the INOPMA algorithm [2], it is assumed that the outputs correlation sequence is the system impulse response. As a consequence, it has been shown that the mode is appropriated at a frequency $\omega^* = \omega_n \sqrt{1 - 4\xi^2}$ and not the natural frequency ω_n . This is considered a limitation of INOPMA, especially that the natural frequency has to be estimated in two steps. In this work, we propose a different approach that allows us to overcome this limitation and we show that it is still possible to appropriate the mode at its natural frequency using a dynamic transformation on the correlation sequence.

Let $\bar{R}(t, \alpha)$ be the image of $R(t)$ by a linear anti-symmetric function that depends on a certain design parameter α and consider the following sequence:

$$H(t, \alpha) = R(t) + \bar{R}(t, \alpha) \quad (3)$$

$H(t, \alpha)$ may be interpreted as a combination of two transformations on the correlation sequence namely a rotation and stretching. By varying only α , one rotates and stretches the correlation sequence and hence it is possible to modify the

phase shift as well as the amplitude of the correlation sequence and consequently modify the damping ratio leading to a pure sinusoid. At this stage, the mode is appropriated and the modified correlation sequence is the system impulse response up to an unknown factor.

In this work, we propose to use the following anti-symmetric transformation:

$$F(R(t), \alpha) = j\alpha R(t) \quad (4)$$

The key idea is similar to the INOPMA algorithm in the sense that one takes the convolution of a driving harmonic force with the modified correlation sequence; notice, however, that with SMA one varies two parameters namely the driving frequency and the parameter alpha.

In the frequency domain, this means that the system transfer function (the Laplace transform of the correlation sequence) is multiplied by a complex factor $(1 + j\alpha)$. It can easily be shown that the transfer function phase angle is zero exactly at the following condition:

$$\begin{cases} \omega = \omega_n \\ \alpha = 2\xi \end{cases} \quad (5)$$

Geometrically, one interpretation is that when the mode is appropriated the correlation sequence vector describes a circle in the complex plane meaning that the conservative part of the system is isolated. The nonconservative part follows immediately. Consequently, the system modal parameters are identified simultaneously at the same step. This is one advantage of the SMA algorithm.

3. Harmonics rejection

Harmonics are assumed here to be modes with zero damping. We show that the algorithm SMA automatically rejects these modes. This avoids hand-based removal of these harmonics as done in practice.

Let us consider the correlation sequence of an SDOF system excited with unmeasured white noise. We propose to show in the sequel that if the damping ratio is zero then the mode cannot be appropriated (the phase angle is never zero) hence rejected.

The SMA algorithm starts by considering the following modified parametric correlation sequence:

$$H(t, \alpha) = (1 + j\alpha)R(t)$$

The Laplace transform of this function can be shown to write as:

$$G(s) = (1 + j\alpha) \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The imaginary part of the frequency response is:

$$I = (2\xi\alpha\omega_n + \omega)(\omega_n^2 - \omega^2) - (2\xi\omega_n - \alpha\omega)(2\xi\omega\omega_n)$$

While the real part is:

$$\text{Re} = (\omega_n^2 - \omega^2)(2\xi\omega_n - \alpha\omega) + 2\omega^2\xi\omega_n$$

When the damping ratio is zero the tangent of the phase angle of the frequency response reduces to:

$$tg = -\frac{\omega(\omega_n^2 - \omega^2)}{\alpha\omega(\omega_n^2 - \omega^2)} = -1/\alpha$$

Which is always different from zero. Consequently, for a harmonic, the angle between the input and the output is never zero meaning that the harmonic is never identified (no zero-crossing).

4. Spurious modes rejection

Spurious/numerical modes appear in an OMA procedure due to many reasons, such as finite sample length effects, truncation orders, measurement noise, ... These modes appear because they are fitted to the system characteristic equation and rejecting them is a challenge. This leads to a spurious frequency and damping that we still denote in the sequel as ω_n and ξ . The correlation sequence of the system output is given by [3, 4]:

$$R(t) = e^{-\xi\omega_n t} \left[\cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}} \sin(\omega_d t) \right] \quad (6)$$

The phase shift in this correlation sequence is given by:

$$tg(\theta) = \frac{\xi}{\sqrt{1-\xi^2}} \quad (7)$$

This particular expression of the phase shift is valid for physical modes only [3]. We propose to show in the sequel that if the phase shift of a correlation sequence enjoys this particular expression, then the mode is necessarily physical.

Consider the following correlation sequence:

$$R_x(t) = e^{-\xi\omega_n t} [\cos(\omega_d t) + x \sin(\omega_d t)]$$

The Laplace transform of $(1+j\alpha)R_x(t)$ is:

$$G_x(s) = (1+j\alpha) \frac{s + \omega_n \left(\xi + x \sqrt{1-\xi^2} \right)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The numerator writes as:

$$(1+j\alpha) \left(j\omega + \omega_n \left(\xi + x \sqrt{1-\xi^2} \right) \right) (-\omega^2 + 2j\omega\xi\omega_n + \omega_n^2)$$

And the imaginary part writes as:

$$\begin{aligned} \text{Im} = & -\omega^3 + \omega\omega_n^2 - \alpha\omega^2\omega_n \left(\xi + x \sqrt{1-\xi^2} \right) + \alpha\omega_n^3 \left(\xi + x \sqrt{1-\xi^2} \right) \\ & + 2\xi\omega_n^2\omega \left(\xi + x \sqrt{1-\xi^2} \right) - 2\xi\omega^2\omega_n\alpha \end{aligned}$$

Under the appropriation conditions

$$\begin{cases} \omega = \omega_n \\ \alpha = 2\xi \end{cases}$$
$$\omega_n^3 \left(2\xi \left(\xi + x\sqrt{1-\xi^2} \right) - 4\xi^2 \right) = 0$$

Leading to:

$$x = \frac{\xi}{\sqrt{1-\xi^2}}$$

This proves that under the SMA isolating conditions, the mode is necessarily physical.

5. Simulation validation

We propose in this section to study the performance of the SMA algorithm on a simple simulated example. A SDOF system is taken as an example. The considered system parameters are taken as $m = 2$ kg, $k = 10,000$ N/m, and $c = 8$ Ns/m. The excitation is a white noise with unit variance. This leads to the following modal parameters; $\omega_n = 11.254$ Hz and $\xi = 2.83\%$. The output is then simulated using a sampling frequency of $F_s = 64$ Hz and 2% measurement noise is added to the output. **Figure 1** shows the identification results of this data set.

5.1 Harmonics rejection

We propose to study in the section the ability of SMA to reject harmonics. Let us consider an SDOF system excited with unmeasured white noise. We add a harmonic component with frequency 5 Hz and amplitude 0.1 N. **Figure 2** shows the phase angle corresponding to the identification results and we notice that the harmonic component is rejected and only the system frequency is identified.

Figure 1.
Phase angle as a function of frequency and alpha.

Figure 2.
Phase angle for harmonics rejection.

5.2 Spurious modes rejection

Spurious modes may arise from different sources such as noise, measurements, errors, ... In order to simulate spurious modes, we consider adding noise to the system as well as introducing colored noise. We drive a unit of white noise through an AR [5] process whose output serves as the excitation to the system. **Figure 3** shows that SMA is robust against spurious modes and only the physical mode is identified.

Figure 3.
Spurious modes rejection.

6. Experimental validation

The considered test object is a standard B&K demo plate (WA0846), which is a rectangular aluminum plate with dimensions $290 \times 250 \times 8 \text{ mm}^3$; for the test, the plate was placed on soft foam. A B&K demo motor WB 1471 with an unbalanced rotor was attached to the plate; the motor was set to operate at 374 rps (**Figure 4**). For the experiment, the plate was excited by tapping its surface by the tip of a plastic pen. About 16 monoaxial accelerometers B&K Type 4507 were mounted equidistantly on the plate on the grid points of a 4×4 grid, oriented to measure in the direction perpendicular to the plate surface. The data acquisition was performed by B&K LAN-Xi DAQ, the sampling frequency was set to 4096 Hz, and 60 seconds of the acceleration data were recorded, which was used as an input to both SMA and SSI algorithms.

Figure 4.
The test setup.

Figure 5.
The stability diagram.

Frequency [Hz]	Damping [%]	Comment
353.4	0.57	1st torsional mode (along Y-axis)
371.9	0.05	Harmonic, automatically identified as a noise mode
491.2	0.62	1st bending mode (along Y-axis)
720.3	0.98	1st bending mode (along X-axis)
866.7	0.47	2nd torsional mode (along Y-axis)
971.7	0.60	2nd torsional mode (along X-axis)
1424	1.1	2nd bending mode (along Y-axis)
1663	0.69	2nd membrane mode
1706	0.67	3rd torsional mode (along Y-axis)

Table 1.
Identification results.

Frequency [Hz]	Damping [%]	Comment
354	0.57	1st torsional mode (along Y-axis)
372.5	0.05	Harmonic, automatically identified as a noise mode
492	0.62	1st bending mode (along Y-axis)
721.4	0.98	1st bending mode (along X-axis)
871	0.47	2nd torsional mode (along Y-axis)
972	0.60	2nd torsional mode (along X-axis)
1422	1.1	2nd bending mode (along Y-axis)
1661	0.69	2nd membrane mode
1705	0.67	

Table 2.
Identification results.

To validate the results of the SMA algorithm, we used the commercial OMA software package “PULSE Operational Modal Analysis 5.1.0.4—x64”; the software was used in automatic identification mode, that is, all default settings were applied; OMA-SSI-UPC method was employed. The stabilization diagram is shown in **Figure 5**, and the modal identification results are presented in **Table 1**.

The SMA algorithm was used with 256 correlation lags. Sensors 1 and 5 were used for the identification. The modes were identified as the angle crossings with zero. The results are reported in **Table 2**.

Notice that the harmonics as well as spurious/numerical modes were not identified and were automatically rejected.

7. Conclusion

Nonphysical modes, as well as harmonics, present a challenge in OMA. Although stability diagrams help in solving this problem, rejecting these modes is not trivial. Although stability diagrams help to solve this problem, the results will remain user-dependent.

The SMA algorithm seems to present an advantage. Not only the correlation sequence has a physical meaning, but also the simultaneity in the identification of the modal parameters makes a constraint on the modes to be exclusively physical. This was illustrated on a simulated example as well as experimentally.

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