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A Probabilistic Approach in Fuselage Damage Analysis *via* Boundary Element Method

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Abstract

This chapter presents a new alternative approach to the analysis of the fatigue life of aircraft fuselage parts considering the compliance of internal elements to replace the classical model of critical crack size. In this case, from a global–local analysis using the boundary element method (BEM), induced stresses at a macro model, and their effects on micro models are evaluated. The BEM enables efficient simulations of the propagation of initial defects to assess the damage tolerance. For this purpose, computational techniques were developed that allowed evaluating these models, through a probabilistic treatment to assess damage tolerance and fatigue life. Finally, this technique is shown as an alternative to ensure the integrity and proper operation of fuselage panels avoiding reaching a Limit State during its projected lifespan.

Keywords: Fatigue, aircraft fuselage, compliance, global–local analysis, Boundary Element Method, damage tolerance

1. Introduction

Some documented studies to interpret the cause of aircraft accidents [1, 2]. One of the classical cases is the accidents with the Comet aircraft and Boeing 737–200 [2]. Regarding the Comet accident, the reports concluded fatigue failure as the main reason for the disintegration of the pressurized cabin. This study also reported that although designed for the operating conditions, its structure was unable to prevent the crack propagation, particularly unstable cracks that after reaching the critical length would continue to propagate until complete rupture of the structure.

As for the Boeing 737 Aloha Airlines, it stabilized at 7,000 meters altitude as planned, and a loud bang suddenly followed by the disintegration of the roof leaving a gap of six meters in the fuselage in flight, but it was still possible to land the aircraft with a damaged structure as shown in **Figure 1**. In this case, the investigations indicated that fatigue crack initiation in several areas (multiple damage site - MSD) greatly reduced the strength of the structure causing it to collapse.

Designers are always looking for fast and reliable simulation methods that produce accurate results to avoid damage processes and, consequently, the occurrence of accidents. Automation is then seen as a key point to evaluate several scenarios for parametric studies resulting in design optimization [3]. Thus, numerical methods as

Figure 1. Boeing 737-200 fuselage fatigue failure. NTSB. Aircraft Accident Report NTSB/AAR-89/03, Aloha Airlines, Flight 243, Boeing 737-200, N73711; Near Maui, Hawaii; April 28, 1988.

domain methods (Finite Element Method – FEM, Extended Finite Element Method – XFEM, Generalized Finite Element Method - GFEM), contour methods (Boundary Element Method – BEM, Dual Boundary Element Method – DBEM, Radial Integration Method - RIM) and mesh-free methods appear as an alternative for solving fracture problems. The DBEM shows further advantages simplifying the modeling of the crack area, direct SIF calculation, run times reduced, and accurate simulation of crack growth [4–6]. The need for discretization only of the solid contour allows, using the DBEM, the analysis of thousands of probabilistic and reliability simulations, such as flaws, initial defects, fatigue behavior prediction, multiple local damages, among others [7–11].

The literature analyzes the damage tolerance (number of cycles) based on the crack size. This chapter presents an innovative method for damage tolerance based on probabilistic global–local analysis. In this context, the method can find a relationship between the number of fatigue cycles and the respective compliance in local elements considering the statistical nature of the input parameters.

2. Literature review

2.1 Damage tolerance

Over the years several fatigue design methodologies have developed trying to combine structural safety and low cost in the aircraft manufacturing and operation process. The first methodology was called safe-life. This approach consists of designing and manufacturing a safe aeronautical structure throughout its useful life. For this, one must consider, in the prototype tests, the most extreme situations of fatigue stresses, foreseen during operation. Such methodology results in factors that oversize the structural elements to prevent the possibility of failure. This approach leads to high project costs and is not able to guarantee safety if an unforeseen design failure occurs during its useful life.

Rationally, a new methodology was developed based on the concept of damage tolerance. In this methodology, it is assumed that the structure is capable of withstanding the actions for which it was designed until the detection of a fatigue crack or other defects during its operation. The aircraft is then checked, repaired, and put back into service until the end of its useful life. The concept of damage tolerance began from statistical analysis to control the spread of fatigue cracking and considering inspection intervals to maintain a low probability of complete failure [12]. Later, the damage tolerance has been applied in the use of aluminum alloys for aircraft structural applications [13, 14]. Thus, the fatigue damage analysis

considered some aspects of the design, predictions, and experiments associated with tolerance to damage to aircraft structures [15–17]. From this, it was noticed that the load cycles have a direct linear relationship with the logarithm of the crack size and that the largest formed cracks grow in an approximately exponential way, known as the “main crack” methodology [18], from small discontinuities (flaws and microcracks) inherent to the material, as soon as an aircraft enters service [19]. Currently, the concept of damage tolerance is applied to aircraft with composite structures [20–23], in the analysis of multiple cracks [24], and shape optimization design [25, 26]. Studies on tolerance to probabilistic damage are based on manufacturing components [27] and the dispersion of fatigue life from the distribution of initial defects [28]. Other works relate damage tolerance through computational methods, using XFEM [29, 30], BEM [31], and DBEM [32].

2.2 Fatigue

Fatigue is characterized by a cyclic loading process that causes progressive internal structural cumulative damage. In this case, the Paris Law [33], Eq. (2), relates the crack propagation rate (da/dN) with the variation of the Stress Intensity Factor (ΔK):

$$\frac{da}{dN} = C \cdot \Delta K^m \quad (1)$$

where a is the crack length, N is the number of load cycles, C and m are material dependent constants. According to classical theory, after a certain number of cycles, the cracks reach a critical length making the structure unstable and causing it to collapse. Thus, developing the Eq. (1), the condition for critical crack size a_c is determined by:

$$N = \frac{1}{C} \int_{a_0}^{a_c} \frac{da}{\Delta K^m} > N_{tot} \quad (2)$$

where N is the number of cycles required to increase the crack of the initial size a_0 up to a critical crack length a_c and N_{tot} is the number of cycles throughout life.

2.3 Boundary element method

The Finite Element Method (FEM) is probably the main technique used in engineering analysis, standing out due to its great versatility, quality of results, and relative ease to implement [34]. On the other hand, the Boundary Elements Method (BEM) has emerged as a complementary alternative to FEM, being indicated particularly in special cases that require better interpretation and data representation in problems with stress concentration or where the domain is infinite or semi-infinite [35].

Thus, the boundary elements technique began to be used in problems of incremental crack extension analysis [36]. The solution of crack problems, in general, cannot be achieved in an analysis of a single region with direct application of the conventional BEM, because the application of the same boundary integral equation at coincident source points on the opposing crack surfaces leads to degeneracy in the resulting system of algebraic equations. Among the techniques applied to work around this problem are the sub-regions, which models the structure in artificial contours connecting the cracks to the boundary in such a way that the domain is divided into sub-regions without cracks [5], as well as the Dual Boundary Elements

Method (DBEM), based on two distinct integral equations on each crack face (displacement and traction equations). Thus, degeneration of the equations system generated by the BEM is no longer present and the need for remeshing vicinity of the crack tip is not required, generating only new rows and columns to the existing matrix [6].

BemCracker2D is a program for elastostatic analysis of 2D problems to performing analyzes using the Boundary Element Method [37–41]. This software performs modeling of the standard BEM and the incremental analysis strategy for problems involving cracks [42]. For the analysis of the fracture mechanics problems, the BemCracker2D calculate elastic stresses using the conventional BEM and performs incremental analyzes of the crack extension through the DBEM.

3. Methodology

Initially, by submitting the macro model to the normal load P and shear load Q , the stress fields on the plate are obtained from the analysis of the continuum mechanics regarding the Eqs. (3), as shown in **Figure 2**.

Thus, analytically, the stress fields in a body in the elastic regime submitted to normal and shear stress, as shown in **Figure 2**, are represented in Eqs. (3):

$$\begin{aligned}\sigma_x &= -\frac{2z}{\pi} \int_{-a}^b \frac{p(s)(x-s)^2}{((x-s)^2 + y^2)^2} ds - \frac{2}{\pi} \int_{-a}^b \frac{q(s)(x-s)^3}{((x-s)^2 + y^2)^2} ds \\ \sigma_y &= -\frac{2z^3}{\pi} \int_{-a}^b \frac{p(s)}{((x-s)^2 + y^2)^2} ds - \frac{2y^2}{\pi} \int_{-a}^b \frac{q(s)(x-s)}{((x-s)^2 + y^2)^2} ds \\ \tau_{xy} &= -\frac{2z^2}{\pi} \int_{-a}^b \frac{p(s)(x-s)}{((x-s)^2 + y^2)^2} ds - \frac{2y}{\pi} \int_{-a}^b \frac{q(s)(x-s)^2}{((x-s)^2 + y^2)^2} ds\end{aligned}\quad (3)$$

With the stress field obtained in the macro model, analyzes are performed in the microelement. The microelement is represented by a square region of unitary side with a load on the right and top edges, and supported on the left and bottom edges, with a central hole and two 45° inclined cracks representing initial defects in the piece, as shown in **Figure 3**. Stresses σ_x , σ_y , and τ from the stress field at an internal point of the macro analysis, applied at the load ratio $R = 0.5$.

Figure 2.
Global–local analysis: Macro model with loadings and dimensions; and detail of the stress in the microelement (units in cm).

Figure 3.
Macro model and positions for initial defect analysis (microelements) and detail of the microelement for the microanalysis (units in cm).

Figure 4.
Points of the number of cycles versus compliance ratio at each increment. (a) Top edge. (b) Right edge.

The microanalysis was performed in three different positions of the macro element. Position 1 considers the microelement in the center of the plate with coordinates (0.10); position 2 considers this element at the origin of the system axis (0,0); and position 3, at the limit of the application of the external request (8.0), such positions are illustrated in **Figure 3**.

When performing the analysis on the microelement, BemCracker2D brings, as a result, the number of cycles and the displacement of the boundary mesh for each crack propagation increment. With these results, the compliance is calculated from the average of the displacements of each edge and the respective stress on the considered edge (right or top), thus obtaining the points to form the curve of the number of cycles *versus* compliance, as shown in **Figure 4**.

Each point (from left to right) represents an increase in crack size. Initial compliance is on the abscissa axis. As the cracks spread, the plate loses rigidity, exponentially increasing the compliance *versus* cycles ratio. From these points, a spline curve is fitted as seen in **Figure 5**. With the curves, the number of cycles is obtained, which corresponds to the compliance C (C_x and C_y) two times initial compliance $2C$ ($2C_x$ and $2C_y$), and three times initial compliance $3C$ ($3C_x$ and $3C_y$).

The results of the number of cycles were evaluated for the four crack tips of the microelement as shown in **Figure 6**, where C1 T1 means Crack 1 Tip 1; C2 T2, Crack 2 Tip 2, and so on, with the lowest number of cycles being considered to achieve the results for the $2C$ and $3C$.

Figure 5.
Number of cycles versus compliance curves. (a) Upper edge. (b) Right edge.

Figure 6.
Crack tips.

Structural aircraft fuselage commonly uses the 2024 aluminum alloy as a base material due to its high ability to withstand damage, good mechanical strength, and corrosion resistance [43, 44]. Thus, the material considered for the fuselage panel was aluminum alloy 2024-T3, yield and limit strengths were 338 MPa and 476 MPa, respectively, Young's modulus and Poisson's coefficient of 74 GPa and 0.33, respectively, and fracture toughness (K_{Ic}) of $34 \text{ MPa}\sqrt{\text{m}}$ [45]. The Paris Law's parameters $C = 7.20\text{e-}11$ and $m = 3.52$ [46] were considered with the fatigue load ratio being $R = 0.5$.

The proposed analysis considered the effects on fatigue life by changing the values of the following variables: external loadings P (normal) and Q (shear) the macro analysis; contained in the Paris Law C and m ; and initial defects R (hole radius), $L1$ (upper crack) and $L2$ (lower crack), from the microanalysis.

The following is an example of the technique, considering the values of the variables represented in **Table 1**:

P and Q are the normal and shear stresses, respectively, C and m are the Paris constants, r the radius of the central hole, $L1$ and $L2$ the size of the upper and lower cracks, respectively. For these values, we obtain the stress fields of the macro analysis illustrated in **Figure 7**.

Position 1.

Table 2 shows the resulting stresses considering the microelement at position 1 of **Figure 3**.

Figure 8(a) shows the 10 crack propagation increments, while **Figure 8(b)** shows the deformed microelement after all the increments, highlighting two details:

P (MPa)	360.47
Q (MPa)	92.78
C	7.20e-11
m	3.52
r (cm)	0.093
L1 (cm)	0.086
L2 (cm)	0.093

Table 1.
Values of case study 1 variables.

Figure 7.
Calculated stress field.

σ_x	42.90 MPa
σ_y	266.78 MPa
τ	11.04 MPa

Table 2.
Stress results at position 1.

Figure 8.
Analysis for the microelement in position 1.

1. Since normal σ_y stress has a magnitude higher than the σ_x (**Table 2**), strains were much higher in y to x, deformation still occurs contrary to the direction of applied stress (tensile and σ_x deformation in the negative direction of the x-axis) due to Poisson effect. This implies disregarding this negative compliance for the analysis. Therefore, **Figure 9** considers only the analysis for the upper border.

Figure 9.
Number of cycles $\times$ compliance for the micro element in position 1.

σ_x	349.00 MPa
σ_y	360.47 MPa
τ	89.83 MPa

Table 3.
Stress results at position 2.

2. The analysis was able to detect the loss of local stiffness near the crack zones. At the upper edge, the microelement near the upper crack had a much greater displacement than at the other points; the same occurring on the lateral edge close to the lower crack.

The smallest number of cycles to reach 2Cy and 3Cy, we have $N(2C) = 1.3635e+04$ and $N(3C) = 1.3811e+04$, respectively.

Position 2.

Table 3 shows the resulting stress field considering the microelement at position 2 of **Figure 3**.

Figure 10(a) shows the crack increments. This analysis also resulted in ten propagation increments. **Figure 10(b)** shows the deformed microelement after all the increments. As a result, it can be seen that as the stresses have similar magnitudes ($\sigma_x = 349.00$ MPa and $\sigma_y = 360.47$ MPa), the deformations in the element have a certain symmetry. Thus, the two edges that have compliance, should be assessed the lowest number of cycles for each crack tip resulting in the first value of 2C and 3C. The result points $N(2C) = 1.8601e+03$ and $N(3C) = 1.8636e+03$, as shown in **Figure 11**.

Position 3.

Table 4 shows the resulting stress field considering the microelement at position 3 of **Figure 3**.

Figure 12(a) illustrates the crack increments. This analysis also resulted in 10 propagation increments. Then, **Figure 12(b)** represents the deformed microelement after all the increments. As a result, it is noticed that as σ_x has a magnitude

Figure 10.
Analysis for the microelement at position 2.

Figure 11.
Number of cycles x compliance for the microelement at position 3.

σ_x	406.67
σ_y	209.76
τ	160.38

Table 4.
Stress field (MPa) at position 3.

much greater than σ_y , the element has more prominent deformation on the right edge. In this analysis, the two edges also show compliance, and the lowest number of cycles for each crack tip that results in the first value of 2Cx and 3Cx must be evaluated. The result points $N(2C) = 1.2467e+03$ and $N(3C) = 1.2521e+03$, as shown in **Figure 13**.

Figure 12.
Analysis for the microelement at position 3.

Figure 13.
Number of cycles versus compliance for the microelement at position 3.

4. Probabilistic analysis

The values obtained previously refer to a deterministic analysis. A probabilistic analysis using Monte Carlo (MC) sampling performed a thousand simulations using BEM varying the values of P , Q , C , m , R , $L1$, and $L2$. The statistical parameters for those variables used for the MC simulation are listed in **Table 5**.

The results are presented from **Figures 14–16**. **Figure 14** refers to the results for microelement in Position 1. It can be seen that the instability of the microelement occurs for load cycles varying between 10^3 and 10^6 . **Figure 15** refers to the results

Variable	Mean	Standard deviation
C	7.0e-11	8.5e-12
m	3.2	0.4
P (MPa)	301.2	44.1
Q (MPa)	99.1	15.1
R (cm)	0.1	1.2e-02
L1 (cm)	0.1	9.8e-03
L2 (cm)	0.1	1.0e-02

Table 5.
Mean and standard deviation values for each variable.

Figure 14.
Monte Carlo simulations of compliance versus number of cycles in position 1.

for the microelement in Position 2. It can be seen that the instability of the microelement occurs for load cycles varying between 10^2 and 10^5 . **Figure 16** refers to the results for Position 3. It can be seen that the instability of the microelement occurs for small cycles of load ranging between 10^1 and 10^5 .

It can be noted that when the compliance reaches the value of three times the initial compliance ($3C$), the element already becomes unstable and tends to increase infinitely as can be seen in **Figures 16–18**. **Table 6** shows the minimum number of cycles N that leads the microelement to reach $3C$, thus occurring instability. The smallest N -value is the worst analysis case, which is the most conservative case. For Position 1 the element becomes unstable in $1.113\text{e}+03$ cycles. For Position 2, instability occurs in $1.12\text{e}+02$ cycles, in this case, there is a sudden reduction of load cycles regarding the previous case (ten times less than the element in Position 1). In position 3, the instability occurs in $4.7\text{e}+01$ cycles, being the worst situation in the case of occurrence of initial defects in the panel. **Figure 17** shows the superposition of the maximum and minimum limits of all curves in each position.

Figure 15.
Monte Carlo simulations of compliance versus number of cycles in position 2.

Figure 16.
Monte Carlo simulations of compliance versus number of cycles in position 3.

Figure 18 shows the Absolute Frequency (AF) of the incidences of compliance values corresponding to the initial compliance C , $2C$, and $3C$. **Figure 19(a)–(c)**, for the microelement, show the positions 1, 2, and 3, respectively. Note the increase in the dispersion of the value distribution due to higher standard deviation observed for increasing compliance levels.

Figure 17.
Compliance versus number of cycle limits in positions 1, 2, and 3.

Figure 18.
Absolute frequency of the incidences of compliances values corresponding to the initial compliance C , $2C$, and $3C$ for the three positions.

Position 1	Position 2	Position 3
1.113e+03	1.12e+02	4.7e+01

Table 6.
Minimum number of cycles that lead the microelement to instability.

Figure 19 shows the probability density function $f(N_{3c})$ to the micro element damage tolerance analysis for the N cycles at $3C$. It is noticed that the element in position 3 has a marked curvature and consequently a lesser variability of damage tolerance, that is, a selection range of N which is limited to $3C$ is limited to $0.4e+05$. The element in position 2 has a variation of N that reaches $3C$ up to $0.7e+05$.

Figure 19.
Estimated probability density functions for fatigue life for 3C criterion.

	Mean	Standard deviation	Coefficient of variation
Position 1	5.1e+04	4.5e+04	0.8823
Position 2	8.5e+03	9.0e+03	1.0544
Position 3	4.7e+03	5.1e+03	1.0894

Table 7.
Mean values and standard deviation of number of cycles to 3C.

Figure 20.
Characteristic value for fatigue life (N_k) considering for 3C criterion.

Position 1 presents a greater dispersion, which means a greater variation of the values of N_{3C} . This is confirmed in **Table 7** by the highest coefficient of variation.

Considering the determination of the characteristic fatigue life (N_k), related to an $\alpha\%$ probability that an N value would not be superior to N_k , **Figure 20** shows those characteristic values for that for α of 75% and 95%. Note the significantly higher values for position 1 rather than the other two positions for both α .

Figure 21.
Failure probability for N_{3C} for different λ values.

Regarding p_f as the failure probability of the structural element, $N_{G<0}$ is the number of simulations in which a performance function for the number of cycles to failure for the #C compliance $G_{N_{3C}} < 0$:

$$p_f = \frac{N_{G<0}}{N_{total}} \tag{4}$$

$$G_{N_{3C}} = \underbrace{\lambda \mu_{N_{3C}}}_{\substack{\text{resistant} \\ \text{fatigue} \\ \text{life}}} - \underbrace{N_{3C}}_{\substack{\text{loading} \\ \text{fatigue} \\ \text{life}}} \tag{5}$$

In which N_{total} the total number of simulations, λ is like an inverse safety factor that scales an allowable (resistant) fatigue life, and $\mu_{N_{3C}}$ the loading average life. **Figure 21** shows the probability of failure of the three positions and different values of λ . It is noticed that the failure probability is greater for Position 3, followed by Position 2 and later from Position 1.

5. Conclusions

Due to the flexibility of the Boundary Elements Method, it was possible to develop the damage tolerance technique by assessing the compliance of the micro-element edges on an aircraft fuselage metal plate. The automation of the thousands of Monte Carlo simulations using an in-house BEM computer program enabled the probabilistic analysis of the fatigue life of the aircraft fuselage in which the plate may have initial defects. The analysis carried out the influence of random input variables of material properties and loadings on the fatigue life predicted values. Based on this technique, the study showed that when the initial defects occur at the edge of the external loading application position for low numbers of cycles, the microelement has high compliance tending to sudden instability, being, therefore, the most unfavorable case. It was possible to observe that when compliance reaches the value of 3 times the initial compliance, it leads to local instability of the

microelement. With the data series, it was possible to perform a statistical treatment to define the damage tolerance to avoid the occurrence of a Limit State. In general, the use of this methodology is shown as an alternative to the analysis of damage tolerance regarding the process adopted by the literature in which the damage is considered from a critical crack size. In the method employed, critical size is disregarded and compliance is assessed as a variable that defines instability.

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