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Performance Analysis of Multiple Antenna Systems with New Capacity Improvement Algorithm for MIMO Based 4G/5G Systems

Vinodh Kumar Minchula and Gottapu Sasibhushana Rao

Abstract

For a time varying channel, the channel capacity is determined by the Channel State Information (CSI) or its fading distribution at a transmitter or receiver. If CSI is perfectly known at both the transmitter and receiver, then the transmitter can adapt to its optimal transmission strategy (i.e., optimal antenna selection by power allocation scheme) relative to its instantaneous channel state for capacity enhancement. In the case where the channel information is not available at the transmitter (No CSIT), the transmitted power has to be distributed equally (i.e., uniform power allocation) between the transmitting antennas to improve the channel capacity. The IWFA (Iterative water filling allocation) strategy therefore allocates power to those spatial channels with positive non-zero singular values i.e. good quality channels and discards the lower eigenmodes channels resulting in maximum capacity in MIMO systems for perfect CSIT. In this chapter, the performance analysis of Multi Antenna systems under ICSIT/ICSIR, Perfect CSIT, No CSIT channel conditions have been implemented and a novel adaptive power allocation algorithm (SVD-based IWFAA) is considered to improve the spectral efficiency of next generation wireless MIMO communication (4G–5G). The algorithm considered is more efficient at high noise levels (low SNRs) under Perfect CSIT conditions because the strongest channel eigenmodes are allocated more power.

Keywords: MIMO, SVD, UPA, ICSIT, ICSIR, Perfect CSIT/CSIR, SNR, BER, Ergodic capacity

1. Introduction

Capacity is a maximal transmission rate that can be achieved by using higher order modulation schemes but these schemes are found to degrade the BER of wireless communication systems due to less separation between the constellation points. Another approach to increase the capacity is the use of multiple antennas at transmitter and receiver, which provides multiplexing gain. It also enhances the QoS by providing diversity gain, coverage and outage by array gain. Further increase in the capacity can be achieved with parallel decomposition of MIMO channels into r (r is rank of channel matrix) SISO channels by using SVD algorithm. MIMO system performance is closely related to allocation of optimal power with the help of CSI availability. Optimally allocating power to MIMO channels is

considered as an optimization problem to maximize the capacity. This chapter highlights the performance analysis of different multi antenna configurations based on its ICSIT/ICSIR, Perfect CSIT/CSIR, and No CSIT channel conditions. A new SVD-based IWFA algorithm is employed in case of Perfect CSIT and Uniform power allocation (UPA) algorithm is used when there is No CSIT as optimal schemes to enhance the MIMO channel capacity. The structure of this chapter is as follows. Section 2 discusses the pertinent system and various channel models for MIMO systems. The statistical properties of MIMO channel matrix are discussed in sections 3 and Section 4 explains the equivalent MIMO system model after decomposition. Capacity estimation algorithms of multiple antenna systems for Instantaneous CSI (ICSI), Perfect CSI at receiver (Perfect CSIR) and No CSIT channel conditions with UPA and IWFA algorithms are derived in Section 5. Section 6 presents results and discussion of the capacities of multiple antenna systems by means of simulations. Finally, Section 7 summarizes the chapter.

2. Multiple antenna system model

In this section the system model and channel matrix, ‘h’ of multiple antenna system are introduced representing the complex channel gain which incorporates the channel fading effect. In wireless mobile communications, multi antenna systems are four types: single input single output (SISO), SIMO, MISO, and MIMO systems. With one antenna on either side, SISO provides no diversity protection against fading. When compared to SISO, the use of multiple antenna configurations will improve the reliability and capacity of the system.

A Multiple antenna system with N_T transmitting and N_R receiving antennas is shown in **Figure 1**. It has multi element antenna arrays at both the transmitter and the receiver side of a radio link to drastically improve the capacity over more traditional SIMO system. SIMO channels can offer diversity gain, array gain, and interference reduction [1]. In addition to these advantages, MIMO links can offer a multiplexing gain by parallel spatial data channels within the same frequency band at no additional power expenditure [2–4]. It creates receiver and transmitter diversity, with beam forming on both sides of antennas, improves SINR, and provides greater spectral efficiency [5].

The MIMO channel can be expressed as

$$r_i^\alpha(t) = \sum_{j=1}^{N_T} h_{ij}(\tau, t) * s_j^\alpha(t) + n_i^\alpha(t) \tag{1}$$

Figure 1.
Schematic of $N_T \times N_R$ MIMO system.

can also be represented as,

$$r^\alpha = \sqrt{\frac{E_s}{N_T}} H s^\alpha + n^\alpha \quad (2)$$

where the received signal at instant α is $r^\alpha = [r_1^\alpha \ r_2^\alpha \ \dots \ r_{N_R}^\alpha]^T$, the transmitted signal is $s^\alpha = [s_1^\alpha \ s_2^\alpha \ \dots \ s_{N_T}^\alpha]^T$, the superscript 'T' stands for the matrix transpose, E_s is the signal power across the transmitting antennas N_T and n^α is Additive white Gaussian noise with zero mean and variance σ_n^2 . For a message from transmitter i , the j^{th} receiving antenna is weighted by AWGN and the channel coefficient H . The above Eq. (2) represents a MIMO model with or without perfect CSI. The $N_R \times N_T$ channel response is as follows

$$H = \begin{bmatrix} h_{1,1} & \dots & h_{1,N_T} \\ \dots & \dots & \dots \\ h_{N_R,1} & \dots & h_{N_R,N_T} \end{bmatrix}_{N_R \times N_T} \quad (3)$$

where $h_{j,i}$ is the complex channel coefficient between the j^{th} receiver and the i^{th} transmit antenna with zero mean complex Gaussian and circular symmetric channel at time instant α . With pilot symbols the receiver can estimate H matrix. The received signal at time α is given by

$$\begin{bmatrix} r_1^\alpha \\ r_2^\alpha \\ \vdots \\ r_{N_R}^\alpha \end{bmatrix}_{N_R \times 1} = \begin{bmatrix} h_{1,1} & \dots & h_{1,N_T} \\ \dots & \dots & \dots \\ h_{N_R,1} & \dots & h_{N_R,N_T} \end{bmatrix}_{N_R \times N_T} \begin{bmatrix} s_1^\alpha \\ s_2^\alpha \\ \vdots \\ s_{N_T}^\alpha \end{bmatrix}_{N_T \times 1} + \begin{bmatrix} n_1^\alpha \\ n_2^\alpha \\ \vdots \\ n_{N_R}^\alpha \end{bmatrix}_{N_R \times 1} \quad (4)$$

and the vector form of received signal $r(t)$ in MIMO systems is

$$[r(t)]_{N_R \times 1} = [h(t)]_{N_R \times N_T} [s(t)]_{N_T \times 1} + [n(t)]_{N_R \times 1} \quad (5)$$

3. Characterization of statistical properties of MIMO Channel matrix

In this section, the MIMO channel parallel decomposition i.e. the SVD of MIMO channel matrix H and Eigen value decomposition (EVD) of Hermitian matrix HH^H are explained.

3.1 Singular value decomposition (SVD)

Singular values of the channel matrix H gives highest number of data streams which can be sent at the same time. The channel matrix H can be decomposed into a product of three matrices as follows

$$[H]_{N_R \times N_T} = U \cdot \Sigma \cdot V^H \quad (6)$$

U & V are a unitary matrices with dimensions of $N_R \times N_R$ and $N_T \times N_T$ i.e.

$$UU^H = VV^H = I_r \quad (7)$$

The V^H is the transpose and complex conjugate of the matrix V , where rank of matrix H is expressed as,

$$r \leq \min(N_R, N_T) \quad (8)$$

and Σ is a $N_R \times N_T$ matrix with zero elements but the diagonal elements are non-zero

$$\Sigma = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_r\}, \sigma_i \geq 0 \text{ and } \sigma_i \geq \sigma_{i+1} \quad (9)$$

The diagonal elements are arranged in such a way that 1st element is greater than the 2nd element and so on the last element is lowest singular value and all of them are non-negative.

3.2 Eigen value decomposition (EVD)

The square of singular values, σ_i^2 , are the eigenvalues of the positive semi definite Hermitian matrix HH^H . So this can be decomposed into

$$HH^H = Q \cdot \Lambda \cdot Q^H \quad (10)$$

Where Q is an Orthonormal matrix with $N_R \times N_R$ dimension i.e.,

$$QQ^H = Q^H Q = I_{N_R} \quad (11)$$

and Λ is a matrix of dimension $N_R \times N_T$ with zero elements but the diagonal elements are non-zero

$$\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_{N_R}\}, \lambda_i \geq 0 \quad (12)$$

The order of eigenvalues are $\lambda_i \geq \lambda_{i+1}$, where

$$\lambda_i = \begin{cases} \sigma_i^2 & \text{if } i = 1, 2, \dots, r \\ 0 & \text{if } i = r + 1, r + 2, \dots, N_R \end{cases} \quad (13)$$

3.2.1. Importance of eigenvalues

Eigenvalue matrix of Q ($\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_r$) contains the strength information about the channel i.e. λ_i . The smallest eigenvalue λ_r is the minimum mode of the channel and it is exponentially distributed. This minimum eigenvalue plays an main role in the MIMO transceiver systems performance and useful in calculating the outage probability. It is used for antenna selection techniques in which antenna set with largest λ_r set is chosen [6], whereas the larger eigenvalues helps in choosing maximal SNR in maximal ratio transmission [7].

4. Equivalent decomposition model of MIMO system

In the previous section, the concepts of SVD, EVD and the significance of eigenvalues have been discussed. As aforementioned, the receiver and transmitter symbols relies on precoding and combining matrices obtained from channel

decomposition [1]. In this section, the basic structure of the channel and importance of channel state information are explained.

In the **Figure 2**, $H = U\Sigma V^H$ is the channel matrix, the columns of U and V are the unitary matrices and diagonal matrix Σ is a $N_R \times N_T$ dimension with singular values σ_k , where σ_k is the k^{th} singular value of the Σ ordered decreasingly ($\sigma_1 > \sigma_2 > \dots > \sigma_r$) and rank 'r' of the matrix. For an input $s = V\tilde{s}$ using precoder at transmitter is sent in the channel and received at receiver as

$$r = \sqrt{\frac{E_s}{N_T}} Hs + n = \sqrt{\frac{E_s}{N_T}} U\Sigma V^H V\tilde{s} + \tilde{n}$$

also expressed as,

$$r = \sqrt{\frac{E_s}{N_T}} U\Sigma\tilde{s} + \tilde{n}$$

The output of channel is multiplied by U^H at receiver, resulting in

$$\tilde{r} = \sqrt{\frac{E_s}{N_T}} U^H U\Sigma\tilde{s} + U^H \tilde{n} = \sqrt{\frac{E_s}{N_T}} \Sigma\tilde{s} + \tilde{n} \quad (14)$$

further expressed as,

$$r_i = \sqrt{\frac{E_s}{N_T}} \sigma_i s_i + \tilde{n}_i = \sqrt{\frac{E_s}{N_T}} \sqrt{\lambda_i} s_i + \tilde{n}_i \quad (15)$$

where $\sigma_i = \sqrt{\lambda_i}$, λ_i is the eigenvalue.

$$\text{trace of } R_{ss} = \text{Tr}(R_{ss}) = N_T \text{ and } E(\tilde{n}\tilde{n}^H) = \sigma_n^2 I_r$$

From the above expression the number of non-zero elements in the diagonal matrix Σ corresponds to the number of independent channels. The processing of U matrix to the noise elements is not going to change the variance in the noise components $U^H \tilde{n} = \tilde{n}$. In order to find U and V matrices, the channel matrix H is needed i.e. to do transmit precoding CSI at transmitter (CSIT) and CSIR for receiver shaping is needed [8].

4.1 CSI at receiver (CSIR)

It has all the channel coefficients i.e. transmitted from transmitter to receiver. It is estimated by using pilot symbols insertion in the signal sent from the transmitter. It is assumed that the receiver is having perfect CSI to do receiver shaping. In open loop MIMO system the CSI is available only at the receiver but not at the transmitter.

Figure 2.
MIMO system decomposition model for perfect CSIT.

4.2 CSI at transmitter (CSIT)

The CSI at the transmitter mainly depends on duplexing mode i.e. TDD or FDD. CSIT can be acquired mainly on feedback from the channel estimate of the receiver using pilot symbols or on the channel reciprocity principle i.e. channel coefficients measured at the same time and frequency are identical. In TDD mode both feedback and channel reciprocity can be used, whereas in FDD only feedback mechanism is used because of different uplink and downlink frequencies.

5. Capacity of multiple antenna systems

This section discusses the determination of the channel capacities and MIMO performance improvement by spatial diversity gain with an increase in number of antennas for different configurations such as Uniform power allocation (UPA) with No CSIT. A new capacity improvement algorithm (SVD based IWFA algorithm) with Perfect CSIT channel conditions is also presented.

5.1 Capacity of multiple antenna channel for ICIR and ICSIT

It is assumed that channel is deterministic and noise is zero-mean complex Gaussian and circular symmetric random variable denoted as $n_k \sim N(0, \sigma_n^2 I_{N_R})$, instantaneous channel state information is available at the receiver (ICSIR) and transmitter knows instantaneous channel knowledge (ICSIT) [3]. So for this scenario, the MIMO channel capacities are build.

- MIMO channel capacity is obtained by maximization of mutual information over all possible source distribution for ICSIR.

$$C_{MIMO} = \max I(s; r), \text{ over all possible distribution of source signal } f(s)$$

$$\begin{aligned} I(s; r) &= h(r) - h(r/s) \\ &= h(r) - h(n) \end{aligned}$$

where $h(r)$ is the differential entropy of received signal vector r and since noise, received symbols are independent $h(r/s) = h(n)$.

MIMO channel capacity C_{MIMO} is expressed as.

$$C_{MIMO} = \max_{\text{Tr}(R_{ss})=N_T} \log_2 \left(\det \left(I_{N_R} + \frac{E_S}{N_T \sigma_n^2} H R_{ss} H^H \right) \right) \text{ (bit/s/Hz)} \quad (16)$$

Where, R_{ss} is the covariance matrix of transmitted signal and is represented as,

$$R_{ss} = E[s s^H]$$

after substituting 's' matrix, the expression is

$$R_{ss} = E \left(\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{N_T} \end{bmatrix} \begin{bmatrix} s_1^* & s_2^* & \cdots & s_{N_T}^* \end{bmatrix} \right)$$

the covariance matrix R_{ss} now becomes

$$R_{ss} = \begin{bmatrix} \frac{E_S}{N_T} & 0 & \cdots & 0 \\ 0 & \frac{E_S}{N_T} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{E_S}{N_T} \end{bmatrix} = \frac{E_S}{N_T} I_{N_T} \quad (17)$$

So the total signal power constraint is the trace of the covariance matrix R_{ss} and is equal to

$$E_S \text{Tr}(R_{ss}) = \sum_{i=1}^{N_T} E|s_i|^2 = E_S$$

5.2 MIMO Channel capacity with UPA algorithm for No CSIT with perfect CSIR

As discussed in Section 3, the decomposition of MIMO channel using SVD into r parallel channels helps in increasing the capacity of MIMO systems. Optimal Power allocation plays a significant function in the computation of enhanced MIMO capacity [1]. In this section the amount of power allocated to each transmitting antennas based on Perfect CSIR and No CSIT availability is presented.

The signals, $s = [s_1, s_2, \dots, s_{N_T}]^T$ is transmitted from the source and have no knowledge about what lying out there in the channel for the transmitter i.e. No CSIT. So vector 's' is statistically independent i.e.

$$R_{ss} = I_{N_T} \quad (18)$$

So substituting Eq. (18) in Eq. (16) the capacity expression is.

$$C_{MIMO}^{No\ CSIT} = \log_2 \left(\det \left(I_{N_R} + \frac{E_S}{N_T \sigma_n^2} HH^H \right) \right) \text{ (bit/s/Hz)} \quad (19)$$

From Section 3.2, it is observed that decomposition of Hermitian matrix HH^H is $Q \cdot \Lambda \cdot Q^H$, where Q is an Orthonormal matrix and Λ is a non-zero diagonal matrix. So the capacity expression becomes.

$$C_{MIMO}^{No\ CSIT} = \log_2 \left(\det \left(I_{N_R} + \frac{E_S}{N_T \sigma_n^2} Q \Lambda Q^H \right) \right) \text{ (bit/s/Hz)} \quad (20)$$

Now using matrix identity $\det(I_N + AB) = \det(I_N + BA)$, the above equation can be written as

$$C_{MIMO}^{No\ CSIT} = \log_2 \left(\det \left(I_{N_R} + \frac{E_S}{N_T \sigma_n^2} \Lambda Q Q^H \right) \right) \text{ (bit/s/Hz)}$$

also from Eq. (11) $Q Q^H = Q^H Q = I_{N_R}$,

$$C_{MIMO}^{No\ CSIT} = \log_2 \left(\det \left(I_{N_R} + \frac{E_S}{N_T \sigma_n^2} \Lambda \right) \right) \text{ (bit/s/Hz)} \quad (21)$$

where $\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_{N_R}\}$ and I_{N_R} are the diagonal matrix. In Eq. (21) the sum of two diagonal matrices is again a diagonal matrix with the elements as $\left(1 + \frac{E_S}{N_T \sigma_n^2} \lambda_i\right)$ diagonal entries [1]. The determinant of matrix is the product of diagonal values or eigenvalues i.e. $\det = \prod_{i=1}^r \lambda_i$ and log of product is sum of log of individuals. So the capacity expression of Eq. (21) is given as.

$$C_{MIMO}^{No\ CSIT} = \sum_{i=1}^r \log_2 \left(1 + \frac{E_S}{N_T \sigma_n^2} \lambda_i\right) \quad (\text{bit/s/Hz}) \quad (22)$$

Therefore, the capacity of a MIMO channel can be interpreted in terms of sum of 'r' number of SISO channels each having a signal strength of $\lambda_i, i = 1, 2, \dots, r$ corresponding to eigenvalue of that particular link and each of channel is excited by $\frac{E_S}{N_T}$ i.e. certain fraction of total transmitted power.

It is also observed that the maximum capacity is a function of channel coefficients and channel properties. The capacity of MIMO with No CSIT Eq. (22) and fixed total channel power Eq. (23) includes eigenvalues λ_i which are obtained from parallel decomposition of HH^H [7, 9, 10]. So here, MIMO systems with Perfect CSIT and No CSIT were considered for the development of a novel SVD-based IWFA algorithm (IWFAA) [1].

5.2.1. Uniform power allocation (UPA) algorithm for No CSIT MIMO Channel capacity

Therefore, in MIMO system with No CSIT, the most optimal choice of power allocation is to use UPA algorithm i.e. to allocate power equally to all eigenvalues of HH^H channel. The fixed total channel power (i.e. the eigenvalues of channel are equal) is expressed as,

$$\|H\|_F^2 = \sum_{i=1}^r \lambda_i = \xi \quad (23)$$

If the full rank channel matrix H is $N_T = N_R = N$, then the capacity can be maximized when the eigenvalues, $\lambda_i = \lambda_j = \dots = \frac{\xi}{N}, i, j = 1, 2, \dots, N$. To achieve this, channel H should be orthogonal matrix

$$HH^H = H^H H = \frac{\xi}{N} I_N \quad (24)$$

then the maximized capacity for UPA No CSIT is.

$$\begin{aligned} C_{UPA\ MIMO}^{No\ CSIT} &= \sum_{i=1}^N \log_2 \left(1 + \frac{E_S}{N \sigma_n^2} \frac{\xi}{N}\right) \quad (\text{bit/s/Hz}) \\ &= N \log_2 \left(1 + \frac{E_S}{\sigma_n^2} \frac{\xi}{N^2}\right) \end{aligned}$$

now if the diagonal elements of H is $|H|^2 = 1$, then $\|H\|_F^2 = \xi = N^2$

$$C_{UPA\ MIMO}^{No\ CSIT} = N \log_2 \left(1 + \frac{E_S}{\sigma_n^2} \frac{N^2}{N^2}\right)$$

similarly the UPA capacity for No CSIT is

$$C_{UPA MIMO}^{No CSIT} = N \log_2 \left(1 + \frac{E_S}{\sigma_n^2} \right) \quad (25)$$

Therefore, the capacity of orthogonal MIMO channel is N times the capacity of SISO channel. If at transmitter CSI is not available, then the UPA is used and is given as,

$$C_{UPA MIMO}^{No CSIT} = \max_{Tr(R_{ss})=\bar{P}} \log_2 \left(\det \left(I_{N_R} + \frac{E_S}{N_T} HH^H \right) \right) \quad (26)$$

5.3 New capacity improvement algorithm for perfect CSIT

The system model after equivalent decomposition as described in Section 4 allow for the characterization of Perfect CSIT MIMO channel capacity. The MIMO capacity is

$$Tr(R_{SS}) = \sum_{k=1}^r E|s_k|^2 \leq \bar{P} \quad (27)$$

from Eq. (15), for the i^{th} received signal, r_i the Perfect CSIT MIMO capacity is.

$$C_{MIMO}^{Perfect CSIT} = \sum_{i=1}^r \log_2 \left(1 + \frac{E_S}{N_T} \frac{\lambda_i}{\sigma_n^2} \gamma_i \right) (\text{bit/s/Hz}) \quad (28)$$

where, γ_i is the transmit power at the i^{th} transmit antenna, $\gamma_i = E[|s_i|^2]$, $i = 1, 2, \dots, r$. Where in No CSIT case the transmit covariance is $R_{ss} = I_{N_T}$ i.e. each of them are having equal power allocated, but in case of Perfect CSIT system with transmit power γ_i indicates that there is certain amount of power to be given to each of transmitting signal with transmit constraint $\sum_{i=1}^r \gamma_i = N_T$. In the case of Perfect CSIT, the singular values of the sub channel matrix is calculated for the MIMO system and transmitter can allocate variable power levels to them (SVD based IWFA algorithm) for maximization of capacity or mutual information. As the transmitter has full information about the channel matrix H with given set of eigenvalues λ_i , it can allocate different energy across the sub channels to maximize the capacity. So the capacity maximization problem is

$$C = \sum_{\gamma_i=N_T}^{\max} \sum_{i=1}^r \log_2 \left(1 + \frac{E_S}{N_T} \frac{\lambda_i}{\sigma_n^2} \gamma_i \right), \text{ subject to constraint } \sum_{i=1}^r \gamma_i = N_T \quad (29)$$

Since the optimization function is concave and constraints is also defined. Using method of Lagrange multipliers [11], define a new variable, ζ , with objective function as

$$L(\gamma_i) = \sum_{i=1}^r \log_2 \left(1 + \frac{E_S}{N_T} \frac{\lambda_i}{\sigma_n^2} \gamma_i \right) + \zeta \left(N_T - \sum_{i=1}^r \gamma_i \right) \quad (30)$$

where, this function L is known as Lagrangian and the new variable, ' ζ ' is the Lagrange multiplier. The unknown transmit power is obtained by taking derivative of $L(\gamma_i)$ with respect to γ_i and equating it to zero.

$$\frac{\partial L(\gamma_i)}{\partial \gamma_i} = 0 \quad (31)$$

by substituting Eq. (30) in Eq. (31) results,

$$\frac{\partial}{\partial \gamma_i} \left[\log_2^e \sum_{i=1}^r \log \left(1 + \frac{E_S}{N_T} \frac{\lambda_i}{\sigma_n^2} \gamma_i \right) + \zeta \left(N_T - \sum_{i=1}^r \gamma_i \right) \right] = 0$$

by simplifying the above expression,

$$\frac{\log_2^e}{1 + \frac{E_S}{N_T} \frac{\lambda_i}{\sigma_n^2} \gamma_i} \frac{E_S}{N_T} \frac{\lambda_i}{\sigma_n^2} - \zeta = 0$$

Solving further the expression becomes

$$\begin{aligned} \log_2^e \frac{E_S \lambda_i}{N_T \sigma_n^2 + E_S \lambda_i \gamma_i} &= \zeta \\ (\log_e^2) \frac{N_T \sigma_n^2 + E_S \lambda_i \gamma_i}{E_S \lambda_i} &= \frac{1}{\zeta} \end{aligned}$$

and the expression for γ_i is

$$\gamma_i = \frac{1}{\zeta \log_e^2} - \frac{N_T \sigma_n^2}{E_S \lambda_i}$$

The power allocation expression is

$$\gamma_i = \mu - \frac{N_T \sigma_n^2}{E_S \lambda_i} \quad (32)$$

where, $\mu = \frac{1}{\zeta \log_e^2}$.

So an IWFA algorithm is used for finding optimal power γ_i^{opt} for achieving maximum capacity, as γ_i^{opt} corresponds to the power allocated to the i th branch and it should be always greater than or equal to zero its value is

$$\gamma_i^{opt} = \left(\mu - \frac{N_T \sigma_n^2}{E_s \lambda_i} \right)^+ \quad (33)$$

where, $(k)^+ = \begin{cases} k & \text{for } k > 0 \\ 0 & \text{for } k \leq 0 \end{cases}$

and to find the value of constant μ referring to the constraint $\sum_{i=1}^r \gamma_i^{opt} = N_T$

$$\sum_{i=1}^r \left(\mu - \frac{N_T \sigma_n^2}{E_s \lambda_i} \right) = N_T$$

Further solving the above expression

$$\left(r\mu - \frac{N_T \sigma_n^2}{E_s} \sum_{i=1}^r \frac{1}{\lambda_i} \right) = N_T$$

and can be simplified as

$$r\mu = N_T \left(1 + \frac{\sigma_n^2}{E_s} \sum_{i=1}^r \frac{1}{\lambda_i} \right)$$

the threshold value now for optimal power allocation γ_i^{opt} is

$$\mu = \frac{N_T}{r} \left(1 + \frac{\sigma_n^2}{E_s} \sum_{i=1}^r \frac{1}{\lambda_i} \right) \quad (34)$$

So the Perfect CSIT IWFA MIMO capacity is

$$C_{IWFA \text{ MIMO}}^{Perfect \text{ CSIT}} = \sum_{i=1}^r \log_2 \left(1 + \gamma_i \left(\mu - \frac{N_T \sigma_n^2}{E_s \lambda_i} \right)^+ \right) \quad (35)$$

The IWFA strategy therefore allocates power to those spatial channels with positive non-zero singular values i.e. good quality channels and discards the lower eigenmodes channels resulting in maximum capacity of MIMO systems when CSI is known at the transmitter. Usually the statistical fading channels are random in nature, so the expected value of capacity should be computed to obtain the average capacity or ergodic capacity. In the capacity analysis for MIMO channels, the ergodic capacity and outage capacity are obtained from the instantaneous capacity.

6. Results

The results obtained due to the implementation of the proposed IWFA algorithm under Perfect CSIT and uniform power allocation algorithm for No CSIT conditions. The concepts detailed in Section 5, along with Eqs. (18)–(35), were computed for the optimal power allocation and calculation of ergodic capacity. The random channel matrix for computation of ergodic and outage capacity are considered to be following the Rayleigh distribution. The ergodic capacity and outage capacity are calculated from the mean value of instantaneous capacity obtained by Monte-Carlo simulations. All simulations are done in MATLAB.

The spectral efficiencies of SISO, SIMO, MISO and MIMO systems under ICSIT and ICSIR conditions for different SNR value (0 to 20 dB) are illustrated in the **Figure 3**.

The capacities for different multi antenna systems under ICSIT and ICSIR conditions as discussed in section 5 are computed and tabulated in **Table 1**. The results indicate that for a Rayleigh fading channel, the spectral efficiency of MISO is lower than that of SIMO system. Due to spatial diversity antennas, MIMO system provides better spectral efficiency compared to other system. It is observed that for a given SNR value of 20 dB, in case of 4×4 MIMO performance (20.25 bit/s/Hz) is tetrad the Shannon limit of a SISO system performance (5.04 bit/s/Hz).

The cumulative distribution functions (CDFs) of the maximal achievable rate for different multi antenna systems are illustrated in **Figure 4**.

The CDFs are one of the ways to illustrate the benefits of MIMO system capacity and the capacity with 10% outage can easily be determined from the CDF.

The 10% outage capacity ($C_{out, 10}$) represents the information rate that is guaranteed for 90% of the channel realizations with probability

Figure 3. Spectral efficiency of SISO, SIMO, MISO, and MIMO systems under ICSIT and ICSIR conditions.

S.no	SNR in dB	Capacity in bit/s/Hz											
		SISO		SIMO		MISO				MIMO			
		1 × 1	1 × 2	1 × 3	1 × 4	2 × 1	3 × 1	4 × 1	2 × 2	3 × 2	2 × 3	3 × 3	4 × 4
1	0	0.93	1.62	2.41	2.90	1.25	1.58	1.80	2.13	2.33	2.96	3.35	4.47
2	2	1.23	2.01	2.84	3.34	1.60	1.96	2.19	2.70	2.98	3.62	4.14	5.63
3	4	1.58	2.41	3.28	3.79	1.98	2.37	2.61	3.34	3.70	4.36	5.02	6.93
4	6	1.96	2.84	3.72	4.24	2.39	2.79	3.05	4.05	4.48	5.14	5.99	8.34
5	8	2.37	3.28	4.17	4.70	2.81	3.23	3.49	4.81	5.30	5.97	7.04	9.86
6	10	2.80	3.73	4.63	5.15	3.25	3.67	3.94	5.63	6.16	6.83	8.16	11.47
7	12	3.23	4.18	5.09	5.61	3.70	4.12	4.39	6.47	7.04	7.71	9.35	13.15
8	14	3.68	4.63	5.54	6.07	4.15	4.58	4.85	7.35	7.93	8.60	10.60	14.88
9	16	4.13	5.09	6.00	6.53	4.60	5.04	5.30	8.23	8.84	9.50	11.89	16.65
10	18	4.58	5.55	6.46	6.99	5.06	5.49	5.76	9.14	9.75	10.41	13.20	18.44
11	20	5.04	6.01	6.92	7.45	5.52	5.95	6.22	10.04	10.66	11.33	14.54	20.25

Table 1. Different multi antenna systems spectral efficiencies under ICSIT and ICSIR.

$\Pr_{\text{out}}(C_{\text{targeted}} \leq C_{\text{out}}, 10) = 10\%$. From the above figure it is observed that the 19.6, 14.1, and 8.71 bit/s/Hz obtained is the 10% outage capacity value for 4×4 , 3×3 , 2×2 MIMO systems respectively, whereas 6.8, 4.78 and 3.55 bit/s/Hz obtained is the 10% outage capacity value SIMO, MISO and SISO systems respectively. So it means that, 90% of time the MIMO system has achieved more capacity values than the capacity of SIMO, MISO and SISO systems. It is also observed that the outage probability decreases as the antenna size of MIMO channel increases.

The capacity with outage as a function of SNR can also be determined as shown in **Figure 5**, where a fixed outage probability of 10% is considered.

The outage capacities for different multi antenna systems as discussed under Section 5 are computed and tabulated in **Table 2**.

Figure 4.
10% outage capacity for different multi antenna systems.

Figure 5.
Comparison graph of Outage capacities for different multi antenna systems.

S.no	SNR in dB	10% Outage capacity					
		1 × 1	1 × 3	2 × 1	2 × 2	3 × 3	4 × 4
1	0	0.14	1.06	0.34	0.96	1.79	2.62
2	2	0.22	1.45	0.51	1.35	2.47	3.57
3	4	0.34	1.92	0.73	1.84	3.28	4.73
4	6	0.50	2.43	1.04	2.44	4.25	6.09
5	8	0.74	2.99	1.42	3.12	5.36	7.61
6	10	1.02	3.58	1.87	3.88	6.59	9.30
7	12	1.40	4.19	2.37	4.72	7.94	11.15
8	14	1.87	4.84	2.93	5.62	9.37	13.12
9	16	2.38	5.49	3.53	6.56	10.90	15.20
10	18	2.92	6.13	4.14	7.61	12.49	17.35
11	20	3.55	6.79	4.78	8.70	14.14	19.61

The capacity with 10% outage observed is 9.30, 1.87, and 3.58 bit/s/Hz at 10 dB of SNR for the 4 × 4 MIMO system, 2 × 1 MISO system, and 1 × 3 SIMO system.

Table 2.
Outage capacities for different multi antenna systems.

Similarly the ergodic capacities computed for different multi antenna systems are shown in **Figure 6**. The ergodic capacity values observed for SISO, SIMO and MISO are less than the values obtained for MIMO systems. From simulated results, the change in ergodic capacity due to SISO, SIMO and MISO systems to MIMO systems are

- i. At 2 dB of SNR the respective percentage reduction in ergodic capacities are 47.09% (SIMO), 71.8% (MISO) and 73.5% (SISO) systems.
- ii. At 18 dB of SNR the respective percentage reduction in ergodic capacities are 62.8% (SIMO), 71.37% (MISO) and 73.24% (SISO) systems.

In **Table 3** ergodic capacities for different SNR in dB and different antennas as discussed in Section 5 are computed.

Figure 6.
Comparison graph of Ergodic capacities for different multi antenna systems.

S.no	SNR (dB)	Ergodic Capacity (bit/s/Hz)					
		1 × 1	1 × 3	2 × 1	2 × 2	3 × 3	4 × 4
1	0	0.87	1.87	0.93	1.67	2.52	3.35
2	2	1.17	2.37	1.26	2.24	3.37	4.48
3	4	1.53	2.91	1.65	2.91	4.37	5.81
4	6	1.95	3.49	2.11	3.69	5.52	7.34
5	8	2.41	4.10	2.62	4.57	6.82	9.06
6	10	2.93	4.72	3.18	5.53	8.24	10.94
7	12	3.47	5.36	3.76	6.57	9.77	12.96
8	14	4.05	6.01	4.37	7.67	11.40	15.11
9	16	4.65	6.67	5.00	8.83	13.11	17.37
10	18	5.27	7.32	5.64	10.03	14.88	19.72
11	20	5.90	7.98	6.29	11.27	16.71	22.17

Table 3.
Ergodic capacities for different multi antenna systems.

In **Figure 7** ergodic capacities plotted for different configurations of MIMO systems under UPA and IWFA are shown. It indicates that the proposed algorithms is having better ergodic capacity than the UPA. The percentage of improvement in ergodic capacity is given below.

- i. At 2 dB of SNR, the respective ergodic capacity improvement in percentage is 25.67% for 10×10 MIMO system.
 - ii. At 18 dB of SNR, the respective ergodic capacity improvement in percentage is 3.07%, for 10×10 MIMO system.
- so it can infer that, at different SNR values an percentage improvement in ergodic capacity is obtained.

In **Figure 8**, different Eigenmode capacities are illustrated. It is observed that the maximum eigenmode is having more capacity for proposed IWFA algorithm then

Figure 7.
Ergodic capacity of different configurations of MIMO systems under UPA and IWFA.

Figure 8.
Capacities for Eigenmodes for 4×4 MIMO system under perfect CSIT and No CSIT.

UPA algorithm i.e. at high SNR IWFA tends to be like UPA and at low SNR the stronger streams are assigned more power with IWFA algorithm to increase the MIMO system capacity.

7. Conclusions

In this chapter, the MIMO system ergodic capacities for different power allocation methods under PCSIT and No CSIT are estimated. For a Rayleigh fading channel, the spectral efficiency of MISO is lower than that of SIMO system. Because of considering spatial diversity at both transmitter and receiver, MIMO system has improved performance. It is found that for a given SNR value of 20 dB, in case of 4×4 MIMO performance (20.25 bit/s/Hz) is tetrad the Shannon limit of a SISO system performance (5.04 bit/s/Hz). The proposed MIMO IWFA algorithm for different MIMO configurations performs better with improved percentages in their channel capacities.

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References

- [1] Minchula VK, Sasibhushana Rao G. SVD-based IWFA for next generation wireless MIMO communication. Elsevier Information & Communication Technology (ICT) Express, ISSN 2405–9595. 2018;4(3):171-174. DOI: 10.1016/j.icte.2018.01.011
- [2] Rao GS. Mobile Cellular Communication. New Delhi: Pearson Education; 2013
- [3] Vinodh Kumar Minchula, G. Sasibhushana Rao, “Multi-Antenna system Performance under ICSIT and ICSIR channel conditions”, Springer Advances in Intelligent Systems and Computing, Vol. 695, pp. 505-511, 2018. DOI:10.1007/978-981-10-7566-7_50
- [4] Blum RS, Winters JH, Sollenberger NR. On the capacity of cellular systems with MIMO. IEEE Communications Letters. Jun. 2002; 6(6):242-244. DOI: 10.1109/LCOMM.2002.1010868
- [5] Catreux S, Driessen PF, Greenstein LJ. Simulation results for an interference-limited multiple-input multipleoutput cellular system. IEEE Communications Letters. Nov. 2000; 4(11):334-336. DOI: 10.1109/4234.892193
- [6] Heath RW, Sandhu S, Paulraj A. Antenna selection for spatial multiplexing systems with linear receivers. IEEE Commun. Lett. Apr. 2001;5:142-144. DOI: 10.1109/4234.917094
- [7] Dighe PA, Mallik RK, Jamuar SS. Analysis of transmit receive diversity in Rayleigh fading. IEEE Com. Apr. 2003; 51:694-703. DOI: 10.1109/TCOMM.2003.810871
- [8] Minchula VK, Rao GS. Performance analysis of LTE systems in T-F dispersive channels. In: Pp. 296–302, ICRTEST-2016, Hyderabad, 25th–27th. October 2016
- [9] Vinodh Kumar Minchula, G. Sasibhushana Rao, “SAC channel effects on MIMO wireless system Capacity”, Springer Microelectronics, Electromagnetics and Telecommunications, ISSN: 1876–1100, Vol. 521, pp 759–765, 2018. DOI: 10.1007/978-981-13-1906-8_77
- [10] Minchula VK, Sasibhushana Rao G. Multi antenna correlation effects on closed form MIMO system capacity. In: ICRTEST-2018, Visakhapatnam, Feb 20th & 21st. 2018
- [11] Arfken GB, Weber HJ. Mathematical Methods for Physicists. New Delhi: Academic Press; 2005