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A Ball-Type Passive Tuned Mass Vibration Absorber for Response Control of Structures under Harmonic Loading

Jiří Náprstek and Cyril Fischer

Abstract

Ball-type tuned mass absorbers are growing in popularity. They combine a multi-directional effect with compact dimensions, properties that make them attractive for use at slender structures prone to wind excitation. Their main drawback lies in limited adjustability of damping level to a prescribed value. Insufficient damping makes ball-type absorbers more prone than pendula to objectionable effects stemming from the non-linear character of the system. Thus, the structure and design of the damping device have to be made so that the autoparametric resonance states, occurrence of which depends on system parameters and properties of possible excitation, are avoided for safety reasons. This chapter summarises available 3D mathematical models of a ball-pendulum and introduces the non-linear approach based on the Appell–Gibbs function. Efficiency of the models is then illustrated for the case of kinematic and random excitation. Interaction of the absorber and the harmonically forced simple linear structure is numerically analysed. Finally, the chapter provides examples of typical patterns of the autoparametric response and outlines possibilities of applications in practical engineering.

Keywords: passive vibration damping, non-linear dynamics, autoparametric systems, semi-trivial solution, dynamic stability

1. Introduction

Design of contemporary structures is often distinguished by their slenderness, which is either functionally, economically or aesthetically motivated. Consequently, the structure lacks required stiffness and vibration absorbers are thus required to be incorporated into the structure design. The generally accepted family of passive tuned mass vibration absorbers is well established in the engineering literature; see the exhaustive review paper [1], which reflects the situation until 2017. The history of tuned vibration absorbers dates back to the beginning of the twentieth century [2]. However, the history of the analysis of non-linear effects connected with these devices is much younger. A tuned mass absorber represents a non-linear system, which, in connection with the supporting structure, has an autoparametric character (for general description of the topic, see [3]). As such, the system is prone to an

autoparametric stability loss. This kind of problem was initially investigated in [4] and later elaborated in works by Tondl and others [5]. A new layer of complexity must be taken into account when both spatial components of the absorber are considered because the autoparametric interaction occurs between the two directions as well. This effect has been known for a long time (see the classical analysis of the chaotic behaviour of a spherical pendulum [6]), however, its relation to pendulum-based tuned mass absorbers was neglected until recently [3].

There are structures where installation of a classical pendulum-based absorber is not possible for spatial, aesthetic, or other reasons. Ball-type passive tuned mass absorbers, which are based on free movement of a heavy ball rolling in a spherical cavity, represent an alternative solution. They combine a multi-directional effect with compact dimensions and thus are convenient for use at towers, mast, foot-bridges, and other slender structures. For example, ball-type absorbers are increasingly popular in connection with wind turbines [7], namely, for offshore installations where the simultaneous influence of wind and wave loads makes the dynamic response of the turbines more complex. Moreover, such devices are almost maintenance-free; the importance of this property naturally increases with the number of installations [8]. Despite many advantages of ball-type absorbers, they have limited damping level adjustability. There are various techniques that implement additional damping into the absorber as a rubber coating or liquid introduced in the cavity. It also seems that the usage of several balls in one container may improve effectiveness of the absorber due to the impact effect and the rolling friction [7]. These modifications, however, entail significant maintenance costs with uncertain results. In any case, insufficient damping makes ball-type absorbers substantially more prone to objectionable effects stemming from the non-linear character of the system compared to pendula, namely, to an autoparametric-based energy transfer between individual components of the system. This effect can result in, for example, an increasing amplitude of the transverse motion of the absorber when only mild excitation takes place.

Mathematical modelling of the movement of a homogeneous sphere rolling on a perfectly rough surface has a long tradition in classical mechanics. The system is non-holonomic with linear constraints in the first derivatives with respect to time. The classical setting of several particular cases including rolling of a sphere on a spherical surface are considered in a classical monograph by Routh [9]. A similar Lagrangian approach was used in popular monographs [10] and is still used regularly [11]. As an alternative, the Appell–Gibbs approach, being based on an *energy acceleration function*, appears to be more effective in some cases, providing governing systems that are more transparent for further elaboration [12]. Abstract solutions using Lie group theoretical methods were derived recently [13]. This approach allows for generalisation of the cavity shape to non-symmetrical surfaces of the second order, however, it is less convenient for practical use.

The first papers dealing with theoretical, experimental and practical aspects of ball-type absorbers were published by Pirner [14]. His design procedure was based on a simplified planar approach. In a follow-up paper [15], the authors of this chapter modelled an absorber and a supporting structure as a non-linear planar structure. The detailed stability analysis of the complete system revealed the typical autoparametric behaviour exhibiting harmonic, chaotic or multi-valued response intervals, which can represent a dangerous state for the structure.

The spatial version of the absorber model is an autoparametric system where the longitudinal direction (parallel with the excitation movement) is supposed as the primary component and the lateral direction plays the role of a secondary component. If the system enters autoparametric resonance, vibration of the primary

coordinate acts as parametric excitation of the secondary coordinate due to mutual non-linear relations. If the secondary component remains in rest and the primary one vibrates, the so-called *semi-trivial state* occurs. The interaction of an absorber with a structure adds a new degree of complexity to the system. The structure, being driven by an external forcing, adopts the role of the primary component; however, both coordinates of the absorber maintain their mutual relations. The particular states of such an autoparametric system are characterised by the existence of bifurcation points that delimit stable and unstable solution branches.

The authors of this chapter put forth significant effort in describing the autoparametric character of pendulum- and ball-type absorbers. For the ball-pendulum, the 2D approach based on the Lagrangian formalism [15] offered a possibility of a detailed analytic description of the reduced problem, where the stable and unstable response domains were clearly identified. The numerical evaluation of the 3D model derived using the Appell–Gibbs function according to [16] revealed important physical properties of the system and many particular trajectory types in forced and free movement cases [17, 18]. It was also found that the resonance properties of the 3D model are similar to those obtained analytically and experimentally for the spherical pendulum [19, 20]. These results support validity of the mathematical model and numerical analyses presented and used in this chapter.

The idea of the ball-pendulum serving as a vibration absorber offers wide possible generalisations. Apart from the aforementioned usage of multiple balls in a cavity or multiple stacked devices for damping multiple frequencies, usage of non-homogeneous spheres, nested spheres, hemispheres or semielliptic spheres would allow the absorber to be fine-tuned for a precisely limited non-linear damping effect or multidirectional damping. For example, an analysis of a Chaplygin ball on a spherical surface is presented in [21]; the bidirectional damping based on a rolling-pendulum is introduced in [22]. A significant disadvantage of nonhomogeneous systems is their weaker stability when compared to traditional symmetric devices. Alternatively, usage of cavities with a general shape may represent a more convenient alternative; see, for example, a case with a semielliptic cavity analysed by Legeza [23]. It is worth noting that survey [1] does not mention any paper regarding dynamic stability analysis of the vibration absorber-equipped structures, although this topic is mentioned as one requiring significant attention. It seems that the research work being conducted on this topic is currently aimed at non-linear dynamic absorbers with different non-linearities in damping, as those involving friction elements [24] or different kinds of non-linear springs [25].

The chapter is organised as follows. After this introduction, the chapter describes the governing differential system based on the Appell–Gibbs approach. Next, the chapter discusses the autoparametric behaviour of the absorber itself for harmonic and random kinematical excitation. Then, the chapter presents results from numerical simulation of the simplified structure equipped with the absorber. The last section of the chapter concludes.

2. Mathematical model

2.1 Appell–Gibbs function of the system

The mathematical model of a simplified structure equipped with a ball-type vibration absorber (see **Figure 1**) consists of two main components: the simplified dynamical model of the supporting structure and the absorber (i.e., the cavity with a ball of mass m). The absorber is connected to the structure at point A , which is

Figure 1.
The structure modelled as two SDOF subsystems together with a 3D ball vibration absorber.

supposed to move horizontally with respect to coordinates x, y ; leaning of the structure and rotation around axis z are neglected.

Thus, the complete system includes eight degrees of freedom: two describing the movement of the top of the structure and six for the absorber, three of which are related by three non-holonomic constraints of the first order expressed in velocities. The detailed derivation of the model of the ball-type absorber was already published and thus we will only briefly summarise it here. For further information on the derivation, the reader is kindly referred to [26].

The system behaviour can be characterised by the Appell–Gibbs function where $\mathbf{u}_G = (u_{Gx}, u_{Gy}, u_{Gz})$ and $\mathbf{w} = (\omega_x, \omega_y, \omega_z)$ describes the translational and rotational movement of the ball in the cavity. Symbols $\mathbf{u}_A = (u_{Ax}, u_{Ay}, u_{Az})$ denote the displacement of the top of the structure, which is modelled as two SDOF linear damped oscillators representing movement of concentrated mass M independently in horizontal directions x, y :

$$S = \frac{1}{2}m(\ddot{u}_{Gx}^2 + \ddot{u}_{Gy}^2 + \ddot{u}_{Gz}^2) + \frac{1}{2}J(\dot{\omega}_x^2 + \dot{\omega}_y^2 + \dot{\omega}_z^2) + \frac{1}{2}M(\ddot{u}_{Ax}^2 + \ddot{u}_{Ay}^2), \quad (1)$$

$$\begin{aligned} M\ddot{u}_{Ax} + b_x\dot{u}_{Ax} + C_x u_{Ax} &= \Phi_x, \\ M\ddot{u}_{Ay} + b_y\dot{u}_{Ay} + C_y u_{Ay} &= \Phi_y, \end{aligned} \quad (2)$$

$u_{Az} = 0$, where forces Φ_x and Φ_y comprise effect of loading and interaction with the absorber.

In function S , the first and second parts (m, J) represent dynamics of the ball moving within the cavity of the absorber, while the third term (M) refers to the structure together with the case of the absorber (without the ball), as shown in **Figure 1**.

The following notation was adopted in Eq. (1):

R, r —radius of the cavity or the ball, respectively, [m],

M, m —mass representing the structure including mass of the static part of the absorber, mass of the ball moving inside the cavity, respectively, [kg],

J —central inertia moment of the ball with respect to point G ; parameter J allows to consider whatever type of spherical body with central symmetry, the mass of which is either concentrated in the centre ($J = 0$), uniformly distributed within the body ($J = 2/5mr^2$), or evenly dispersed mass over the outer envelope of the ball ($J = mr^2$), [kg m²],

ω —angular velocity vector of the ball with respect to its centre G , [rad s⁻¹],

A —moving origin related with the cavity in its bottom point,

$\mathbf{u}_A$ —displacement of the contact between the structure and the cavity,

$\mathbf{u}_G$ —displacement of the ball centre with respect to the moving origin A , [m],

C —contact point of the ball and cavity,

$\mathbf{u}_C = (u_{Cx}, u_{Cy}, u_{Cz})$ —displacement of the ball contact point with respect to the moving origin A , [m],

$\mathbf{x} = x, y, z$ —Cartesian coordinates with origin in the point O ,

C_x, b_x, C_y, b_y —structure stiffness and linear viscous damping in x, y horizontal directions, [N m⁻¹, Ns m⁻¹].

2.2 Ball movement inside the cavity

From the supposition of a non-sliding contact between the ball and cavity, the velocities of the ball centre with respect to origin can be deduced providing the respective non-holonomic constraints of “perfect” rolling. Thus, the conditions for displacement vectors $\mathbf{u}_C$ and $\mathbf{u}_G$ can be written as:

$$\begin{aligned} \dot{u}_{Cx} &= \omega_y(u_{Cz} - R) - \omega_z u_{Cy}, & \dot{u}_{Gx} &= \dot{u}_{Ax} + \rho(\omega_y(u_{Cz} - R) - \omega_z u_{Cy}), \\ \dot{u}_{Cy} &= \omega_z u_{Cx} - \omega_x(u_{Cz} - R), & \dot{u}_{Gy} &= \dot{u}_{Ay} + \rho(\omega_z u_{Cx} - \omega_x(u_{Cz} - R)), \\ \dot{u}_{Cz} &= \omega_x u_{Cy} - \omega_y u_{Cx}, & \dot{u}_{Gz} &= \dot{u}_{Az} + \rho(\omega_x u_{Cy} - \omega_y u_{Cx}), \end{aligned} \quad (3)$$

where : $\rho = 1 - r/R$ and $\dot{u}_{Az} = 0$.

The ball centre acceleration $\ddot{\mathbf{u}}_G = (\ddot{u}_{Gx}, \ddot{u}_{Gy}, \ddot{u}_{Gz})$ consists of two parts: (i) acceleration of the moving origin A , denoted as $\ddot{\mathbf{u}}_A = (\ddot{u}_{Ax}, \ddot{u}_{Ay}, \ddot{u}_{Az})$, which represents kinematic excitation of the absorber by the movement of the structure, and (ii) acceleration of the ball centre G with respect to the point A being given by $\rho \ddot{\mathbf{u}}_C$. Components of acceleration $\ddot{\mathbf{u}}_C$ can be deduced when relations Eq. (3) are differentiated:

$$\begin{aligned} \ddot{u}_{Gx} &= \ddot{u}_{Ax} + \rho \frac{d}{dt} (\omega_y(u_{Cz} - R) - \omega_z u_{Cy}), \\ \ddot{u}_{Gy} &= \ddot{u}_{Ay} + \rho \frac{d}{dt} (\omega_z u_{Cx} - \omega_x(u_{Cz} - R)), \\ \ddot{u}_{Gz} &= \ddot{u}_{Az} + \rho \frac{d}{dt} (\omega_x u_{Cy} - \omega_y u_{Cx}), & \ddot{u}_{Az} &= 0. \end{aligned} \quad (4)$$

After substituting Eq. (4) into Eq. (1), the Appell–Gibbs function gets a form $S = S_2 + S_1 + S_0$, where S_2, S_1 and S_0 are polynomials of the second, first and zero degree of $\mathbf{w}$ and $\ddot{\mathbf{u}}_A$ components. The *reduced Appell–Gibbs function* is defined as $S^r = S_2 + S_1$. The term S_0 can be omitted because it disappears during differentiation with respect to $\mathbf{w}$ or $\ddot{\mathbf{u}}_A$ components.

The function S^r enables to formally write the Appell–Gibbs differential system:

$$\begin{aligned}\partial S^r / \partial \dot{\omega}_x &= F_{Gx}, & \partial S^r / \partial \ddot{u}_{Ax} &= F_{Ax}, \\ \partial S^r / \partial \dot{\omega}_y &= F_{Gy}, & \partial S^r / \partial \ddot{u}_{Ay} &= F_{Ay}, \\ \partial S^r / \partial \dot{\omega}_z &= F_{Gz},\end{aligned}\quad (5)$$

where $\mathbf{F}_G = (F_{Gx}, F_{Gy}, F_{Gz})$ and $\mathbf{F}_h = (F_{Ax}, F_{Ay}, F_{Az})$ are the external forces or moments acting in points G and A , respectively.

2.3 External forces

The right sides of Eq. (5) can be determined using the virtual displacements principle. In the discussed case, they include: (i) gravity forces acting in point G , (ii) external excitation in point A , (iii) influence of the lower part of the structure below point A , and (iv) dissipation forces in contact point C .

- i. *Gravity forces*: Forces $\mathbf{F}_{Gg}$ originate from the vector of gravity $(0, 0, -mg)$. The elementary work performed by force $\mathbf{F}_{Gg} = (F_{Ggx}, F_{Ggy}, F_{Ggz})$ along displacement $\delta \mathbf{u}_G = (\delta u_{Gx}, \delta u_{Gy}, \delta u_{Gz})$ can be expressed as

$$\delta W_{Gg} = 0 \cdot \delta u_{Gx} + 0 \cdot \delta u_{Gy} - mg \cdot \delta u_{Gz} \quad (6)$$

Virtual displacement δu_{Gz} can be determined using the third non-holonomic constraint in Eq. (3). Denoting by $\delta \varphi_-$ the virtual increments of individual components ω_- , it holds that

$$\delta u_{Gz} = \rho (u_{Cy} \delta \varphi_x - u_{Cx} \delta \varphi_y) \quad (7)$$

and therefore

$$\delta W_{Gg} = -mg\rho (u_{Cy} \delta \varphi_x - u_{Cx} \delta \varphi_y). \quad (8)$$

At the same time, the elementary work can be expressed in terms of virtual increments:

$$\delta W_{Gg} = F_{Ggx} \delta \varphi_x + F_{Ggy} \delta \varphi_y + F_{Ggz} \delta \varphi_z. \quad (9)$$

Comparison of coefficients at respective virtual components $\delta \varphi_-$ for x, y, z gives

$$F_{Ggx} = -\rho mg \cdot u_{Cy}, \quad F_{Ggy} = \rho mg \cdot u_{Cx}, \quad F_{Ggz} = 0. \quad (10)$$

- ii. *External excitation in the point A*: Excitation force Φ_A is considered in the horizontal direction. In the meaning of the virtual work, it acts along the displacement: $\delta \mathbf{u}_A = (\delta u_{Ax}, \delta u_{Ay}, \delta u_{Az})$. Elementary works performed by excitation forces acting in point A can be written as follows:

$$\delta W_{Ah} = \Phi_{Ax} \cdot \delta u_{Ax} + \Phi_{Ay} \cdot \delta u_{Ay} + 0 \cdot \delta u_{Az}, \quad (11)$$

where Φ_{Ax}, Φ_{Ay} are components of the horizontal excitation force. When comparing the relevant components of the elementary works the following relation arises:

$$F_{Ahx} = \Phi_{Ax}, \quad F_{Ahy} = \Phi_{Ay}, \quad F_{Ahz} = 0. \quad (12)$$

iii. *Influence of the lower part of the structure:* The force acting in point A consists of the stiffness and damping parts. Both are working on the identical virtual displacements $\delta \mathbf{u}_A$ as in the previous paragraph. Therefore, the relevant force components can be written as:

$$F_{Alx} = -C_x u_{Ax} - 2b_x \dot{u}_{Ax}, \quad F_{Aly} = -C_y u_{Ay} - 2b_y \dot{u}_{Ay}, \quad F_{Alz} = 0. \quad (13)$$

iv. *Dissipation forces in the contact point C:* The influence of damping in this case is rather complicated having a character between viscose force and dry friction. However, it can be modelled on a qualitative basis to prevent any non-pervious formulation of the model. With respect to real configuration of a structure, the damping effects are evidently sub-critical and, therefore, simplifications of its internal mechanism can be adopted. Supposing that no slipping arises in the contact, the dissipation process can be approximated as proportional to relevant components of the angular velocity vector $\boldsymbol{\omega}$ and the quality of the cavity/ball contact. The material coefficients characterising the rolling movement of the ball can be considered constant regardless of the direction in the tangential plane to the cavity in point C. The spin of the ball is related rather with a dry friction. Nevertheless, the influence of this damping force is even smaller than those acting in tangential directions and, therefore, such an approximation is acceptable.

Consequently, with reference to [26], the resistance force can be assumed proportional to components of the respective angular speeds. Thus, the damping forces in directions x, y, z can be defined as

$$(D_{Gx}, D_{Gy}, D_{Gz})^T = \mathbf{T}_c \cdot \mathbf{A} \cdot \mathbf{T}_c^T \cdot (\omega_x, \omega_y, \omega_z)^T, \quad (14)$$

where $\mathbf{T}_c$ is the transformation matrix from the local coordinate system of the ball to the moving coordinates and matrix $\mathbf{A}$ reflects the damping coefficients for rolling (α) and spinning (β):

$$\mathbf{T}_c = \begin{pmatrix} \frac{u_{Cx}(R - u_{Cz})}{R\nu}, & \frac{u_{Cy}(R - u_{Cz})}{R\nu}, & \frac{\nu}{R} \\ \frac{-u_{Cy}}{\nu}, & \frac{u_{Cx}}{\nu}, & 0 \\ \frac{-u_{Cx}}{R}, & \frac{-u_{Cy}}{R}, & \frac{R - u_{Cz}}{R} \end{pmatrix}, \quad \mathbf{A} = \text{diag}(\alpha, \alpha, \beta), \quad \nu^2 = u_{Cx}^2 + u_{Cy}^2. \quad (15)$$

Finally, the external forces can be summarised as follows:

$$\begin{aligned} F_{Gx} &= -\rho mg \cdot u_{Cy} + D_{Gx} \\ F_{Gy} &= \rho mg \cdot u_{Cx} + D_{Gy} \\ F_{Gz} &= D_{Gz} \end{aligned} \quad (16)$$

$$F_{Ax} = F_{Ahx} + F_{Alx} = \Phi_{Ax} - C_x u_{Ax} - 2b_x \dot{u}_{Ax}$$

$$F_{Ay} = F_{Ahy} + F_{Aly} = \Phi_{Ay} - C_y u_{Ay} - 2b_y \dot{u}_{Ay}$$

2.4 Governing differential system

When the derived quantities are introduced into Eq. (5), the resulting system, together with the left part of Eq. (3), includes eight differential equations for eight unknowns $u_{Cx}, u_{Cy}, u_{Cz}, \omega_x, \omega_y, \omega_z, u_{Ax}, u_{Ay}$. Using the geometric relations

$$u_{Cx}\dot{\omega}_x + u_{Cy}\dot{\omega}_y + (u_{Cz} - R)\dot{\omega}_z = 0 \quad \text{and} \quad u_{Cx}^2 + u_{Cy}^2 + (R - u_{Cz})^2 = R^2, \quad (17)$$

which reflect the orthogonality of vectors $\mathbf{u}_C$ and $\dot{\boldsymbol{\omega}}$ and geometric properties of the cavity, the final system reads:

$$\begin{aligned} J_s \dot{\omega}_x &= \rho^2 \left((-u_{Cy}\omega_z - \omega_y(R - u_{Cz}))\Omega_1 - \frac{1}{\rho} \left((R - u_{Cz})\ddot{u}_{Ay} + g u_{Cy} \right) - \frac{D_{Gx}}{m} \right), \\ J_s \dot{\omega}_y &= \rho^2 \left((u_{Cx}\omega_z + \omega_x(R - u_{Cz}))\Omega_1 + \frac{1}{\rho} \left((R - u_{Cz})\ddot{u}_{Ax} + g u_{Cx} \right) - \frac{D_{Gy}}{m} \right), \\ J_s \dot{\omega}_z &= \rho^2 \left((u_{Cy}\omega_x - u_{Cx}\omega_y)\Omega_1 + \frac{1}{\rho} (u_{Cy}\ddot{u}_{Ax} - u_{Cx}\ddot{u}_{Ay}) - \frac{D_{Gz}}{m} \right), \end{aligned} \quad (18)$$

$$\begin{aligned} m_s \ddot{u}_{Ax} &= \Phi_{Ax} - 2b_x \dot{u}_{Ax} - C_x u_{Ax} + m\rho \frac{d}{dt} (\omega_y(R - u_{Cz}) + u_{Cy}\omega_z), \\ m_s \ddot{u}_{Ay} &= \Phi_{Ay} - 2b_y \dot{u}_{Ay} - C_y u_{Ay} - m\rho \frac{d}{dt} (\omega_x(R - u_{Cz}) + u_{Cx}\omega_z), \end{aligned} \quad (19)$$

where it has been denoted:

$$\Omega_1 = u_{Cx}\omega_x + u_{Cy}\omega_y - (R - u_{Cz})\omega_z, \quad J_s = J + m\rho^2 R^2, \quad m_s = m\rho + M. \quad (20)$$

The damping forces enable to be simplified in the following way

$$\begin{aligned} D_{Gx} &= \alpha\omega_x + (\beta - \alpha)u_{Cx}\Omega_1/R^2, \\ D_{Gy} &= \alpha\omega_y + (\beta - \alpha)u_{Cy}\Omega_1/R^2, \\ D_{Gz} &= \alpha\omega_z - (\beta - \alpha)(R - u_{Cz})\Omega_1/R^2. \end{aligned} \quad (21)$$

The quantities given by a solution to system Eqs. (3) and (18) describe behaviour of the structure with the absorber. Vector $\mathbf{u}_C$ depicts displacements of the contact point C of the ball and can be used to study its trajectories within the cavity. Vector $\mathbf{u}_A$ characterises horizontal movement of the point A , where the absorber is fixed to the structure. The detailed behaviour of the ball as a rotating body is given by angular velocities $\boldsymbol{\omega}$. The time history of the ball rotation can be enumerated, if necessary, by means of the Euler angles.

It is worth emphasising that the system Eqs. (3) and (18) have a significantly expressed autoparametric character. Hence, the existence of semi-trivial solutions (STS) should be expected outside the resonance zone. However, it emerged that the STS can have a more general character than that defined, for example, in [3]. In other words, for values of bifurcation parameter ω , which produce the STS either in the sub- or super-resonance zone, other solutions can also exist. It depends on a character of related bifurcation points, if the newly emerging solution branch reaches outside the autoparametric resonance zone, possibly involving more or all response components. These solutions, however, are generally not accessible from homogeneous initial conditions and should be looked for from relevant bifurcation points. We provide some details in the next section.

3. Autoparametric behaviour of the absorber

Behaviour of the ball absorber, when it is excited in one direction only, has a strong autoparametric character. It is characterised by a non-linear interaction between both components (longitudinal and lateral), when an enforced movement in one (longitudinal) direction destabilises the resting state of the other component. Depending on parameters of the system and excitation, the response may attain a periodic, quasi-periodic or chaotic character, which generally prevents the device from working properly. Alternatively, a similar autoparametric effect is used for the sake of a structure when an autoparametric absorber is installed. Designers often overlook the former effect because it involves non-linear relations between individual components. In the case of pendulum-type absorbers, this unwanted effect can be mitigated efficiently when a sufficiently large damping is applied [20]. However, due to small damping, ball-type absorbers are much more prone to this type of response.

The authors of this chapter thoroughly studied the effect of the autoparametric resonance of the ball-type vibration absorber. A number of distinctive solutions of the homogeneous system (no external excitation, various settings of the non-homogeneous initial conditions) were presented in [18]. Despite visually attractive shapes of certain solutions, the most important ones were used as limits separating solution groups of a certain character. Particular effects of a harmonic external excitation were studied in [17], namely different regimes of periodic or aperiodic responses and their stability, together with the effect of different values of damping. The most relevant results are summarised in this section. **Table 1** lists the numerical parameters used in figures and simulations.

The design procedure of a ball-type vibration absorber generally involves an assumption of small horizontal amplitudes of the ball [14]. Depending on the moment of inertia of the ball, the rotation inertia of the rolling sphere reduces the natural frequency of the ball-type vibration absorber as compared with the pendulum-type absorber of an equivalent length $(R - r)$. Similarly, the rolling motion of the spherical absorber reduces the efficiency of the device according to the value of the moment of inertia of the ball. For example, if a homogeneous sphere in the ball-type absorber should have the same effect as the pendulum absorber, its mass would have to be increased by a factor of 7/5 with respect to the mass of the pendulum [14].

It appears, however, that an assumption of small horizontal vibrations can be violated easily in the resonance. Due to a limited damping there exists a significant probability that a movement of the ball within the cavity exceeds “small” values. It also appears that small damping enables various limit cycles to exist—at least for a limited time (see **Figure 2**). Such regular limit cycles are, of course, very sensitive to carefully selected initial conditions. Their existence, however even theoretical, emphasises the importance of sufficient damping in tuned mass absorbers. In conjunction with a spatial resonance movement, which can be induced by

Parameter	Absorber				Structure			
	R	r	m	α, β	M	C_x, C_y	b_x, b_y	g
	[m]	[m]	[kg]	[Ns m ⁻¹]	[kg]	[N m ⁻¹]	[Ns m ⁻¹]	[m s ⁻²]
Value	1	3/4 ^a	1	0.1	10	90	0.1	9.81

^aValue 1/4 is used in **Figure 2**.

Table 1.
Model parameters used in figures and simulations.

Figure 2.

Free movement of the ball for prescribed initial conditions $u_{Cx} = 0.75R$, $u_{Cy} = 0$, $\omega_x = -298.942$, $\omega_y = 0$, $\omega_z = 265.757$, $r = 1/4R$, no damping assumed. Left: Time history of three displacement components for two periods $T = 2.6$. Right: Trajectory of the centre of the ball in the xy plane.

uni-directional excitation, the movement of the ball in a limit cycle negatively affects the structure. It seems that the difficulty in introducing the appropriate damping is the main weakness of the ball-based absorbers and, therefore, geometrical measures should be adopted to tune the absorber.

3.1 Harmonic excitation of the cavity

In this section, we present the numerically evaluated frequency response curves for a ball-type absorber. The ball moves in a vertical plane that passes the cavity centre if an in-plane non-zero initial condition is prescribed and/or an uni-directional excitation component is applied. However, due to the non-linear character of the mathematical model, the in-plane movement is susceptible to a loss of the stability of the semi-trivial planar state for certain parameters of excitation.

Harmonic kinematic excitation (i.e., the sinusoidal form of a prescribed movement of the cavity), represents an easily understandable case, which is very convenient for both analytical and numerical treatment. It is also very popular for an assessment of dynamical properties of linear engineering structures or systems because a simple composition of the response components for individual excitation frequencies gives a realistic image of the complex response. In a non-linear case, however, this approach is not generally feasible and the frequency response curves have to be interpreted with sufficient care. Nevertheless, harmonic excitation in x direction is assumed in the following text:

$$\ddot{u}_{Ax} = u_0 \omega^2 \sin(\omega t), \quad \ddot{u}_{Ay} = 0. \quad (22)$$

The initial conditions are prescribed as very small, but non-zero values in both components and the excitation amplitude are assumed as $u_0 = 0.025$. For certain excitation frequencies and amplitudes the planar response movement loses stability and lateral movement emerges. **Figure 3** shows the corresponding plots for an undamped case. The graph on the left shows the resonance curves for the longitudinal (top plot, solid curve) and lateral (bottom plot, dashed) components, obtained using a bunch of mutually independent simulation runs. The figure shows that for the selected excitation amplitude in the resonance (i) the response in the longitudinal direction increases dramatically and (ii) the zero position of the lateral component loses stability. The response in y direction attains values comparable to the longitudinal component, which represents spatial movement of the ball.

The non-linear resonance curves given by **Figure 3** have only an illustrative meaning, especially in the resonance interval $\omega \in (2.8, 3.1)$. Due to lack of damping,

Figure 3. Left: The frequency response curve for longitudinal u_{Cx} (top plot, solid) and lateral u_{Cy} (bottom plot, dashed) components. Right: Projection of the contact point to the horizontal plane xy for a stabilised motion. The four plots correspond to excitation frequencies indicated by verticals in the left-hand plot. The absorber parameters are given in Table 1; $u_0 = 0.05R$, no damping assumed, $\alpha = 0, \beta = 0$.

Figure 4. Left: Maximum ($\textcircled{U}$, blue) and minimum ($\textcircled{L}$, yellow) amplitudes of horizontal displacements. Top: Longitudinal u_{Cx} component—Solid curves; bottom: Lateral u_{Cy} component—Dashed curves. Right: Projections of the contact point of the ball to the horizontal plane xy for a stabilised motion; pictures a–d represent trajectories for frequencies indicated on the left. The absorber parameters are given in Table 1, $u_0 = 0.05R$.

the largest values represent only the covered integration interval. Increasing the integration time could cause a physically meaningless response.

Introduction of moderate damping between the ball and the cavity changes the resonance plots, as shown in **Figure 4**. The recorded maximal amplitudes of the stabilised response for the same excitation amplitude $u_0 = 0.025R$ are slightly lower and the resonance interval is narrower, however, the most significant change is the emergence of the stable circular or elliptic limit cycle in $\omega \in (2.94, 3.03)$.

The response remains planar outside the resonance zone in both damped and undamped cases; the lateral component vanishes. The STS is stable in the sense that for general initial conditions the trajectory stabilises in the planar state. For excitation frequencies $\omega \in (2.82, 3.03)$ the semi-trivial behaviour is unstable and the lateral movement suddenly emerges. General initial conditions for an excitation frequency in this resonance area produce a spatial response when transitional effects subside. The shape of the spatial trajectory is visible from both parts of **Figure 4**. On the left, the upper (blue, $\textcircled{U}$) and lower (yellow $\textcircled{L}$) curves in each plot correspond to maximal and minimal amplitudes of the settled response, obtained for the respective frequency ω . If both curves coincide, the response is harmonic and stationary (planar or spatial). For the non-stationary response, both curves differ, and their vertical difference indicates a width of a strip where the response takes place. If the lower curve approaches the zero value in one or both coordinates, the response at least temporarily vanishes in that coordinate (see point (a) in **Figure 4**). A positive value of the lower amplitude indicates movement in a circular strip around the vertical axis (see points (b) and (d) in **Figure 4**).

The corresponding vertical projections into the plane (u_{Cx}, u_{Cy}) of the ball trajectories are shown in the right-hand part of **Figure 4** for frequencies marked on the left by verticals (a–d). The plots (a–b) show a multi-harmonic or quasi-periodic response, where the length of the quasi-period decreases for an increasing excitation frequency. The relevant Lyapunov exponent is positive but small in this area. The very stable periodic trajectory shows plot (d) in $\omega \in (2.94, 3.03)$, with a narrow exceptional interval, plot (c).

It appears that from a certain threshold value of the excitation amplitude, the overall face of the resonance plot remains the same with the non-stationary area in the left-hand part of the resonance interval and increasing ramp representing periodic response on the right. Naturally, an increased excitation amplitude or decreased value of damping causes broadening of the resonance interval and enlargement of the response amplitude or vice versa. Changes in other parameters (m, r) influence the face of the plot more significantly, yet the overall character of the graph remains the same.

Similarly to multiple settled solutions in the resonance interval, there exist multiple solution branches also outside the resonance. They differ in stability. The best approach to their identification is an analytical way, if possible (see [15] for the case of the 2D model or [19] for a spherical pendulum). Although the unstable solutions are usually difficult to identify numerically, there are certain exceptions. **Figure 5** shows the resonance curves ① and ② from **Figure 4** enriched by two additional branches, which were obtained when the numerical simulation followed the frequency sweeping from low to high and vice versa. The sweeping process means that in every new step performed for $\omega \pm \Delta\omega$ the simulation starts from initial conditions corresponding to the final state of the previous run. This way, in fact, a small change in the driving frequency can be accommodated by the stable solution, which would be otherwise hardly accessible from random initial conditions.

The result is demonstrated in **Figure 5**. The planar response branch ①,② was obtained when the sweeping was performed from high to low, starting above the resonance interval with small but non-zero initial conditions. During continuing on the stable part of this branch above $\omega = 3.3$, curve ②, the lateral component value decreased below the machine epsilon before the resonance interval was entered and the numerical round-off then ensured continuance also on the unstable part of planar branch below $\omega = 3.3$. Here the movement remains stable with respect to perturbations in the u_{Cx} variable within the resonance interval and even further for $\omega < 2.82$, curve ①. The response is formed by a planar movement with large amplitudes. Any perturbation in the lateral direction causes a switch from ① to a generally stable planar solution ③ in $\omega \in (2.44, 2.82)$ or to a non-stationary behaviour

Figure 5. Amplitudes of u_{Cx} (upper plot, solid) and u_{Cy} (bottom plot, dashed) component. Sweeping from high to low in green, ②,①; from low to high in red ③,④. The branches ①–③ exhibit the planar response; the branch ④ is spatial. The absorber parameters are as in **Figure 4**.

represented by curves ⑨ and ⑩ in the resonance interval $\omega \in (2.82, 3.03)$. Although the existence of this unstable type solution sounds theoretically, it was actually measured during the experimental examination of the spherical pendulum, see [27].

The spatial response branch ④ in **Figure 5** can be identified when starting simulation in the resonance interval, for example at $\omega = 2.85$, from small but non-zero initial conditions; sweeping the excitation frequency upwards enables the response to continue with the circular type response outside the resonance interval. The stability of the periodic trajectory gradually decreases in term of a sensitivity to perturbations, cf. [17] for details, however, the sweeping process itself is able to continue up to physically meaningless frequency values. This effect is indicated by an arrow on the right in **Figure 5**. The maximal approach of the circular trajectory to the equator of the cavity occurred for $\omega = 9.7$; for ω increased further the amplitudes start to very slowly decrease.

It is worth noting that such a periodic high-energy trajectory may represent a serious danger to the structure. Although this regime is not accessible easily, the numerical experiments show that it is unfavourably stable against perturbations in the excitation frequency and amplitude—at least for lower excitation frequencies. The spatial response in the resonance area attains also large amplitudes, however, they are not synchronised and so this case could even help to dissipate the vibrational energy to modes that are not excited by the primary loading. The planar periodic motion exhibiting high in-plane amplitudes synchronised with excitation in the sub-resonance zone may also represent a possibly dangerous state, but this effect quickly attenuates whenever the lateral component gains a non-zero value.

3.2 Random excitation of the cavity

If the harmonic excitation can be regraded as the most simple excitation case, the opposite extreme is a completely random case. For this purpose, a stationary random process is generally used, which is described by a spectral density matrix and an underlining—preferably Gaussian—probability distribution. For the sake of simplicity, only the white noise excitation will be assumed in this section. For details and more complex examples, see [28]. This simplified case of random excitation was used to assess the possibility of emergence of the high-energy spatial response due to an ambient broadband noise.

When dealing with non-linear models, the results of simulation are generally not Gaussian even for normally distributed inputs. This applies also to this case and, consequently, the results have to be represented by an estimate of a (time-dependent) probability distribution. Histograms are used for this purpose in this work.

The spatial response of the upper part of branch ④ in **Figure 5** for deterministic harmonic excitation is periodic and the relevant trajectory intersects the coordinate axes always in the same points. When random excitation is assumed, solution trajectories deviate from an ideal ellipse depending on a variance of the random process. Positions of intersections of trajectories with the coordinate axes then represent a random variable, distribution of which characterises the stochastic response. For deterministic excitation, the histograms would be concentrated in values corresponding to intersection points of the elliptic trajectory and both axes. When random perturbation of a harmonic input increases, the centre of gravity of the histogram becomes blurred. A further increase in the random perturbation intensity may cause a change of the type of the response and a switch to the lower solution branch, which is characterised by a negligible value of the lateral component and a non-zero value of the longitudinal component that reflects the relevant amplitude.

Figure 6.

The probability density estimates for components $u_{Cx}|_{y=0}$ and $u_{Cy}|_{x=0}$ for $t \in (400, 600)$ and increasing white noise intensity σ . The absorber parameters are given in **Table 1**.

From the simulation, it can be concluded that the spatial response may emerge depending on the variance of the input random process. This result follows from **Figure 6**, which shows probability density estimates for components $u_{Cx}|_{y=0}$ and $u_{Cy}|_{x=0}$ for an increasing white noise intensity; each simulation begins from “small” initial conditions $u_{Cx}(0) = u_{Cy}(0) = 0.01$ and counts axes crossings for the both components. In order to neglect the transient effects, the initial part of each simulation is not taken into account and only the time interval $t \in (400, 600)$ is considered. **Figure 6** shows that starting from the white noise intensity $\sigma = 0.15$, the lateral component becomes positive and for $\sigma \geq 0.35$ is the random response almost symmetric in the both components. However, the elliptic periodic response, which is typical for the spatial branch for $\omega > 2.94$, does not appear dominant in any histogram.

The random simulation was performed using the Itô version of the modified stochastic Euler method, [29], with $\Delta t = 2^{-6}$. The computation was restarted 240 times. Approximately 100 axes crossings were counted in each simulation for $t \in (400, 600)$, which number gives in total ca. 2.4×10^5 samples for each histogram.

4. Interaction of the structure and the absorber

Frequency response curves serve as a main evaluation tool when regards an efficiency estimate of the absorber. It was already shown using the analytical tools—which are available for the 2D simplified case—that the shape of such non-linear frequency response curves may be fairly complicated, see [15]. The illustrative simulation results regarding the complete equation system Eqs. (3) and (18) will be presented in this section.

The non-linear frequency response curves are shown in **Figure 7** for multiple settings of the absorber and excitation frequency. The reference data of the sample

structure and the absorber used during simulation are given in **Table 1**, which setting correspond the natural frequency of the structure $\omega_0 = 3$ and an appropriate choice of the ball size $r = 3/4$. For simplicity, the damping coefficients are set equal for the absorber, $\alpha = 0.1, \beta = 0.1$, and also for the structure, $b_x = 0.1, b_y = 0.1$. The harmonic forcing is supposed in the form

$$\Phi_{Ax} = F_0 \sin \omega t, \quad \Phi_{Ay} = 0, \tag{23}$$

where the forcing amplitude F_0 varies between 0.1 and 0.7.

The resonance curve of the linear model of the supporting structure without an absorber is shown in each plot in **Figure 7** as the black dashed curve. For cases with the absorber, the blue solid line indicates the amplitude of the structure response in

Figure 7. Frequency response curves of the structure equipped with the ball-type absorber. In columns—Left: $r = 0.7R$; middle: $r = 0.75R$ —The optimal value; right: $r = 0.8R$. In rows: From top to bottom, the excitation amplitude $F_0 = 0.1, \dots, 0.7$. Black dashed: Frequency response of the linear structure without an absorber, blue solid and red dotted curves denote frequency response of the structure with the absorber in longitudinal and lateral components, respectively. Model parameter of the absorber and structure are given in **Table 1**.

the longitudinal direction, u_{Ax} , the red dotted curve corresponds to the lateral direction, u_{Ay} . Three columns show the response properties of three radii of the ball: $r = 0.7R$, $0.75R$, $0.8R$ for the left, the middle and the right column, respectively. Finally, each row shows the response for a particular value of the excitation amplitude: $F_0 = 0.1, \dots, 0.7$.

The plots in **Figure 7** show that in the depicted case the non-zero amplitude of the lateral component arises even for the lowest forcing amplitude, namely for the case of a maximal efficiency of the absorber ($r = 0.75R$). This effect is naturally dependent on the physical properties of the structure, namely on its rigidity. In most cases are the maximal amplitudes of the both components comparable and the originally unidirectional vibration transforms into a spatial movement of the structure. See [18] for details. As the excitation amplitude increases, an additional peak emerges in the resonance frequency of the structure besides the both side extremes. This peak is significantly lower than that originating in the linear resonance, however, it appears in the both directions. Comparison of all three columns illustrates the fact that the efficiency as a function of the tuning of the absorber in terms of the radius of the ball deteriorates for $r > 0.75R$ faster than for $r < 0.75R$. Although this effect becomes less noticeable when the ratio r/R is getting smaller, in real cases it would be safer to underestimate the radius of the ball than the opposite.

Character of the responses of both the ball and the structure in the auto-parametric-resonance regions is mostly quasi-periodic or chaotic. Some basic properties are evident from **Figure 8**. For a single forcing amplitude $F_0 = 0.5$ are shown the frequency response curves of components u_{Cx} , u_{Cy} , u_{Ax} , u_{Ay} (four rows) in three columns for three radii of the ball: $r = 0.70R$, $0.75R$, $0.80R$. There are two curves in each plot which indicate (non-)stationarity of the response; the upper (blue) shows maximal amplitudes for a given forcing frequency, the lower (yellow) corresponds to minimal ones, cf. description of **Figure 4**. The plots are grouped to vertically stacked pairs. The response of the structure is shown in the second row, i.e., variables u_{Ax} , u_{Ay} and, for the sake of comparison, the response of the ball, u_{Cx} , u_{Cy} , is in

Figure 8. Detailed frequency analysis of the response of the ball (top row) and the structure (bottom row) for three radii $r = 0.7R$, $0.75R$, $0.8R$ in three columns. Plots for lateral and longitudinal components are vertically stacked. Model parameters as in **Figure 7**.

the first row. Part of the relevant linear resonance curve is shown in the row for u_{Ax} as the dashed black curve.

It can be seen that the spatial response is mostly non-stationary. The most noticeable exception is a hardly visible interval $\omega \in (2.78, 2.80)$ for $r = 0.75R$, where the minimal and maximal response curves are non-zero and coincide for all four variables; it means that the ball and the structure move in elliptic curves. It is, however, interesting that whereas in the ball movement is dominant the lateral direction ($u_{Cy} > u_{Cx}$), for the structure is the dominant component the longitudinal one ($u_{Ay} < u_{Ax}$). Another example of such a behaviour is for $\omega \in (3.22, 3.24)$. There is one such interval for $r = 0.70R$ in frequencies above resonance $\omega \in (3.44, 3.48)$ and for $r = 0.80R$ in frequencies below resonance: $\omega \in (2.55, 2.58)$.

It is also worth noting that the movement of the ball for the depicted case $F_0 = 0.5$ reach the equator of the cavity when the radius of the ball is not optimal ($r = 0.7, 0.8$). This case should be considered as unacceptable in a real device. However, it appears that even in this case the absorber is able to work for the sake of the structure.

The colour map plots in **Figure 9** show the sensitivity of the maximal response of the structure on the radius of the ball (vertical axis) and the loading frequency (horizontal axis). The coloured spots in both plots correspond to positions of extremes of frequency response curves in **Figure 8** for different values of the radius r . The value $r = 0.75R$, which corresponds to cases shown in the middle column of **Figure 8**, is indicated by the horizontal dashed line. Two observations are worth mentioning. The first regards position of one or both extremes when the tuning of the absorber is changing (variable r). Whereas the upper (right) extreme of the longitudinal variable decreases in magnitude and moves to higher frequencies for r decreasing from $0.75R$, the position of the lower one remains stable and its value increases. For r increasing from $0.75R$, the lower (left) extreme vanishes and the position of the upper one increasingly coincides with the resonance frequency of the structure. This behaviour is natural because for $r \rightarrow R$ the absorber ceases to work. The amplitude of the structure is maximal. Similar behaviour is visible also for the lateral component in the right-hand plot.

The other observation supports the previously mentioned remark regarding sensitivity of the absorber efficiency to the radius of the ball. The gradient of the response amplitudes is significantly steeper when moving up from the level $r = 0.75R$.

Figure 9.
 Dependence of the maximal response of the structure on the radius of the ball r and the loading frequency. Left: Longitudinal component u_{Ax} . Right: Lateral component u_{Ay} . Model parameters as in **Figure 7**, $F_0 = 0.5$.

Figure 10.

Dependence of the maximal response of the structure on the natural frequency of the structure and the loading frequency. Left: Longitudinal component u_{Ax} . Right: Lateral component u_{Ay} . Model parameters as in Figure 7, $F_0 = 0.5$.

Similar information is provided by Figure 10. The ball radius (i.e., the natural frequency of the absorber) is fixed in this case to $r = 0.75R$ and the natural frequency of the structure is changing in the interval $\omega_0 \in (2.3, 3.8)$. The frequency $\omega_0 = 3$ used in Figures 7–9 is indicated by the horizontal dashed line. It passes both extreme areas in places where the amplitudes are relatively small, a situation that corresponds to the setting shown in the middle column of plots in Figure 7, row for $F_0 = 0.5$.

5. Conclusions

The tuned mass absorbers are supposed to work in semi-trivial mode, avoiding any type of the autoparametric resonance effects described in this chapter. They are traditionally designed using a simplified linear, or non-linear but planar, approach, which is adequate to such an expected behaviour. However, the lack of sufficient damping makes the ball-type vibration absorbers prone to unwanted autoparametric effects, which stem from the non-linear character of the system. Thus correct and safe design has to consider possible occurrence of the autoparametric resonance. To facilitate this procedure, the non-linear mathematical model of the ball-type absorber was presented and analysed in connection to a linear model of an elastic supporting structure. The model of the absorber consists of six degrees of freedom constrained by three non-holonomic relations. The complete system with the structure comprises ten first-order ordinary differential equations.

It was shown that in systems with small damping, the desired planar STS is prone to loss of stability even for small excitation amplitudes. This danger increases with increasing excitation amplitude. Although the resonance interval is relatively narrow, the spatial response of the absorber can emerge also due to a broadband random excitation, provided that the intensity of the random noise exceeds a certain limit. The spatial movement of the ball within the absorber is unfavourably stable with respect to random perturbations that correlate with the resonance frequency of the structure.

The efficiency of the absorber is obviously dependent on a proper tuning. It was shown that the absorber efficiency deteriorates faster if the ratio between radii of the ball and the cavity is greater than the optimal one, rather than in the opposite

case. Although this effect becomes less noticeable when the ratio r/R is getting smaller, in real cases it would be safer to underestimate the radius of the ball with respect to the cavity, rather than the opposite.

Although the described resonance state should be preferably avoided, it appears, however, that a limited induced lateral movement of the ball may help to dissipate the harmonic loading energy and stabilise the structure. However, this mechanism should not be relied upon in the design procedure as it can set off movement in dangerous non-linear high-energy limit cycles.

Both cases of harmonic and random excitation indicate a need for further investigation of the topic. A more thorough parametric study should comprise different system parameters and structure types in the case of harmonic loading. A deeper stochastic analysis is also necessary, which should comprise the effect of a supporting structure. Nevertheless, it can be concluded that autoparametric resonance effects may be encountered in practice more often than expected. This can be dangerous for structures if adequate countermeasures are not applied.

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