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The Optimal System for Complex Series-Parallel Systems with Cold Standby Units: A Comparative Analysis Approach

Ibrahim Yusuf and Ismail Muhammad Musa

Abstract

The purpose of this research is to propose three reliability models (configurations) with standby units and to study the optimum configuration between configurations analytically and numerically. The chapter considered the need for 60 MW generators in three different configurations. Configuration 1 has four 15 MW primary units, two 15 MW cold standby units and one 30 MW cold standby unit; Configuration 2 has three 20 MW primary units, three 20 cold standby units; Configuration 3 has two 30 MW primary units and three 30 MW cold standby units. Some reliability features of series-parallel systems under minor and complete failure were studied and contrasted by the current. Failure and repair time of all units is assumed to be exponentially distributed. Explanatory expressions for system characteristics such as system availability, mean time to failure (MTTF), profit function and cost benefits for all configurations have been obtained and validated by performing numerical experiments. Analysis of the effect of different system parameters on the function of profit and availability has been carried out. Analytical comparisons presented in terms of availability, mean time to failure, profit function and cost benefits have shown that configuration 3 is the optimal configuration. This is supported by numerical examples in contrast to some studies where the optimal configuration of the system is not uniform as it depends on some system parameters. Graphs and sensitivity analysis presented reveal the analytical results and accomplish that Configuration 3 is the optimal in terms of design, reliability physiognomies such as availability of the system, mean time to failure, profit and cost benefit. The study is beneficial to engineers, system designers, reliability personnel, maintenance managers, etc.

Keywords: optimality, availability, standby, partial, complete failure, MTTF

1. Introduction

Systems or configurations are designed with intention of meeting the optimal designed that has the reliability requirement at satisfaction of the buyers or customers usually studied with intention to the increase their reliability characteristics in terms such as mean time to failure (MTTF), busy period of repairman, availability, generated revenue as well as profit. Reliability models are vital in measuring the overall performance of system in ensuring quality of products. Achieving a high level of reliability through redundancy is often an essential requisite.

Literature on the reliability of comparative analysis of systems with standby units is numerous, and here we study previous papers on the issues of systems with standby units. Due to their significance in education, communication, military, industry and economics, many researchers have done excellent work in the field of reliability and performance analysis of serial systems by studying and constructing mathematical models to test their performance under different operating conditions. For instance; Singh et al. [1, 2] used copula to study the performance analysis of the complex system in the series configuration under different failure and repair discipline. Lado and Singh [3] recently discussed the cost assessment of complex repairable systems consisting two subsystems in series configuration using Gumbel Hougaard family copula. Yusuf [4] presented the availability modeling and evaluation of repairable system subject to minor deterioration under imperfect repairs. Singh and Ayagi [5] provided a frame work to analyze the performance of a complex system under preemptive resume repair policy using copula. Niwas and Garg [6] discussed the availability, reliability and profit of an industrial system based on cost free warranty policy. Monika et al. [7] provided a complex system having two subsystems in series configuration under k-out-of-n: G, policy. The k-out-of-n works if and only if at least k of the n components works. Gahlot et al. [8] analyzed the performance of repairable system in series configuration under different types of failure and repair policies using copula linguistics. Singh.,V.V and Singh, N. P [9] analyzed the performance of three-unit redundant system with switch and human failure. Saini and Kumar [10] discussed the performance evaluation of evaporation system in sugar industry using RAMD analysis. Malik and Tewari [11] presented performance modeling and maintenance priorities decision for water flow system of a coal based thermal power plant. Lado et al. [12] discussed the performance and cost assessment of repairable complex system with two subsystems connected in series configuration.

Researchers in the past have presented excellent works on reliability analysis of complex repairable systems and proclaimed better performance of the repairable system by their operations. Chen et al. [13] dealt with reliability analysis of a cold standby system with imperfect repair and under poisson shocks. Corvaro et al. [14] presented RAM analysis on reciprocating compressors. Garg [15] analyzed the reliability of industrial system using fuzzy kolmogrov's differential equations. Garg [16] presented an approach for analyzing the reliability of series-parallel system using credibility theory and different types of intuitionistic fuzzy numbers. Garg and Sharma [17] discussed two phase approach for reliability and maintainability analysis of an industrial system. Garg [18] presented RAM analysis of industrial systems using PSO and fuzzy methodology. Kakkar et al. [19] analyzed the reliability of two-unit parallel repairable industrial system. Kakkar [19] discussed the reliability of two dissimilar parallel unit repairable system with failure during preventive maintenance. Niwas and Kadyan [20] dealt with reliability modeling of a maintained system with warranty and degradation. Negi and Singh [21] analyzed the reliability of non-repairable complex system with weighted subsystems connected in series. Patil et al. [22] presented the reliability analysis of CNC turning center based on the assessment of trends in maintenance data. Tsarouhas [23] dealt with RAM analysis for wine packaging production line. Wang et al. [24] analyzed the reliability of two-dissimilar-unit warm standby repairable system with priority in use. Wu [25] analyzed the reliability of a cold standby system attacked by shocks. Wu and Wu [26] analyzed the reliability of two-unit cold standby repairable systems under Poisson shocks. Garg [27] analyzed the reliability of industrial system using fuzzy kolmogrov's differential equations. Kakkar et al. [28] analyzed the reliability of two dissimilar parallel unit repairable system with failure during preventive maintenance. Kumar and Malik [29, 30] dealt with reliability measures of a computer system with priority to PM over the H/W repair activities subject to MOT and MRT. Kumar and Lather [31] analyzed the reliability of a robotic system using hybridized technique.

Kumar et al. [32] dealt with availability and cost analysis of an engineering system involving subsystems in series configuration. Suleiman et al. [33] dealt with comparative analysis between four dissimilar solar panel configurations.

Still, a further study on serial system of the new type of models with a justified and satisfactory assessment is required. For this reason, this chapter has three goals. The first goal is, to develop explicit expressions describing mean time to failure. The second is to compare the four configurations in terms of their mean time to failure. The third is to perform a parametric investigation of various system parameters with the mean time to failure, as well as to capture their effect on the mean time to failure. Analytical and numerical computations are presented to compare their mean time to failure (MTTF). Cost/benefit measure have been obtained for all configurations, where the benefit is mean time to failure.

The rest of the paper is organized as follows. Section 2 presents the notation used. Section 3 gives a description of the system. Section 4 deals with derivation of the models. Analytical comparison between configurations are presented in Section 4. The results of our numerical simulations are presented and discussed in Section 5. The paper is concluded in Section 6.

2. Notations

- α_0/β_0 : Unit failure/Repair rate.
- $p_i(t)$: Probability that Configuration 1/Configuration 2/Configuration 3 is in state i at time t .
- $P(t)$: Probability row vector.
- $Q_n/A_{Tn}/MTTF_n, n = 1, 2, 3$: Transition matrix/steady state Availability/Mean time to failure for the Configuration 1/Configuration 2/Configuration 3.
- $C_1/C_2/C_3$: cost for Configuration 1/Configuration 2/Configuration 3.
- $k_0/k_1/k_2$: Revenue generated/cost due to repair of partial failure/cost due to repair of complete failure.

3. Description of the systems

The present paper considered the requirement of 60 MW generators in following configurations: Configuration 1 has four 15 MW primary units, two 15 MW cold standby units and one 30 MW cold standby unit; Configuration 2 has three 20 MW primary units, three 20 cold standby units; Configuration 3 has two 30 MW primary units and three 30 MW cold standby units. It is assumed that units fail independent of the other (**Table 1**). It is also assumed that switching from standby to operation is automatic. Primary unit fails with exponential failure time distribution with parameter α_0 and immediately the cold standby is switch to operation. Also, unit fails independent of the other with exponential failure time distribution with parameter α_0 . Both units have exponential repair time distribution with parameter β_0 . The systems (Configurations) are depicted in **Figures 1–3** below.

Configuration	Number of Primary units	Number of standby units	Cost of Configuration
1	Four primary 15 MW	Two cold standby 15 MW units	$C_1 = 48,000,000$
2	Three primary 20 MW	Three cold standby 20 MW	$C_2 = 42,000,000$
3	Two primary 30 MW	Three cold standby 30 MW	$C_3 = 39,000,000$

Table 1.
Size of configurations and their corresponding cost.

Figure 1.
Reliability block diagram of configuration 1.

Figure 2.
Reliability block diagram of configuration 2.

Figure 3.
Reliability block diagram of configuration 3.

4. Reliability models formulation

4.1 Models formulation for configuration 1

The corresponding set of differential-difference equations for Configuration 1 as follows:

$$\left. \begin{aligned} \frac{d}{dt}p_0(t) &= -4\alpha_0p_0(t) + \beta_0p_1(t) + \beta_0p_2(t) \\ \frac{d}{dt}p_1(t) &= -(4\alpha_0 + \beta_0)p_1(t) + 2\alpha_0p_0(t) + \beta_0p_3(t) + \beta_0p_5(t) \\ \frac{d}{dt}p_2(t) &= -(4\alpha_0 + \beta_0)p_2(t) + 2\alpha_0p_0(t) + \beta_0p_3(t) + \beta_0p_4(t) \\ \frac{d}{dt}p_3(t) &= -(4\alpha_0 + \beta_0)p_3(t) + 2\alpha_0p_1(t) + 2\alpha_0p_2(t) + \beta_0p_6(t) + \beta_0p_7(t) \\ \frac{d}{dt}p_4(t) &= -\beta_0p_4(t) + 2\alpha_0p_2(t) \\ \frac{d}{dt}p_5(t) &= -\beta_0p_5(t) + 2\alpha_0p_1(t) \\ \frac{d}{dt}p_6(t) &= -\beta_0p_6(t) + 2\alpha_0p_3(t) \\ \frac{d}{dt}p_7(t) &= -\beta_0p_7(t) + 2\alpha_0p_3(t) \end{aligned} \right\} \quad (1)$$

With initial conditions

$$p_k(0) = \begin{cases} 1, & k = 0 \\ 0, & k = 1, 2, 3, \dots, 7 \end{cases} \quad (2)$$

Eq. (1) can be expressed in the form as:

$$\frac{d}{dt}p(t) = Q_1p(t) \quad (3)$$

With

$$Q_1 = \begin{pmatrix} -4\alpha_0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 \\ 2\alpha_0 & -(4\alpha_0 + \beta_0) & 0 & \beta_0 & 0 & \beta_0 & 0 & 0 \\ 2\alpha_0 & 0 & -(4\alpha_0 + \beta_0) & \beta_0 & \beta_0 & 0 & 0 & 0 \\ 0 & 2\alpha_0 & 2\alpha_0 & -(4\alpha_0 + 2\beta_0) & 0 & 0 & \beta_0 & \beta_0 \\ 0 & 0 & 2\alpha_0 & 0 & -\beta_0 & 0 & 0 & 0 \\ 0 & 2\alpha_0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & 0 & 2\alpha_0 & 0 & 0 & -\beta_0 & 0 \\ 0 & 0 & 0 & 2\alpha_0 & 0 & 0 & 0 & -\beta_0 \end{pmatrix}$$

Expression of availability, probability of partial and complete failure for configuration 1 are given by

$$A_{T1}(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) \quad (4)$$

$$B_{P1}(\infty) = p_1(\infty) + p_2(\infty) + p_3(\infty) \quad (5)$$

$$B_{P2}(\infty) = p_4(\infty) + p_5(\infty) + p_6(\infty) + p_7(\infty) \quad (6)$$

To obtained (4), the procedure is to compute the states probabilities $p_k(\infty), k = 0, 1, 2, \dots, 7$ by setting (3) to zero to give

$$Q_1P(t)^T = 0 \quad (7)$$

and using the following normalizing condition

$$p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty) + p_5(\infty) + p_6(\infty) + p_7(\infty) = 1 \quad (8)$$

to give

$$\begin{pmatrix} -4\alpha_0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 \\ 2\alpha_0 & -(4\alpha_0 + \beta_0) & 0 & \beta_0 & 0 & \beta_0 & 0 & 0 \\ 2\alpha_0 & 0 & -(4\alpha_0 + \beta_0) & \beta_0 & \beta_0 & 0 & 0 & 0 \\ 0 & 2\alpha_0 & 2\alpha_0 & -(4\alpha_0 + 2\beta_0) & 0 & 0 & \beta_0 & \beta_0 \\ 0 & 0 & 2\alpha_0 & 0 & -\beta_0 & 0 & 0 & 0 \\ 0 & 2\alpha_0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & 0 & 2\alpha_0 & 0 & 0 & -\beta_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (9)$$

By solving the system of equations in (9) using MATLAB package for the solution of $p_k(\infty)$ give in **Table 2** below.

(4), (5) and (6) are now expressed as:

$$A_{T1}(\infty) = \frac{\beta_0^3 + 4\alpha_0\beta_0^2 + 4\alpha_0^2\beta_0}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3} \quad (10)$$

$$B_{P1}(\infty) = \frac{4\alpha_0\beta_0(\alpha_0 + \beta_0)}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3} \quad (11)$$

$$B_{P2}(\infty) = \frac{8\alpha_0^2(2\alpha_0 + \beta_0)}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3} \quad (12)$$

Profit = total revenue generated – cost incurred by the repair man due to partial failure – cost incurred by the repair man due complete failure.

$$P_{F1} = k_0A_{T1}(\infty) - k_1B_{P1}(\infty) - k_2B_{P2}(\infty) \quad (13)$$

Using the method adopted in Wang and Kuo [34], Wang and Pearn [35], Wang et al. [36] and Yen, T,-S and Wang, K.-H [37], the mathematical model of mean time to failure for Configuration 1 is derived using the relation

$$MTTF_{.1} = P(0)(-M_1^{-1})[1, 1, 1, 1]^T = \frac{20\alpha_0^2 + 8\alpha_0\beta_0 + \beta_0^2}{8\alpha_0^2(4\alpha_0 + \beta_0)} \quad (14)$$

$p_0(\infty) = \frac{\beta_0^3}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$	$p_4(\infty) = \frac{4\alpha_0^2\beta_0}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$
$p_1(\infty) = \frac{2\alpha_0\beta_0^2}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$	$p_5(\infty) = \frac{4\alpha_0^2\beta_0}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$
$p_2(\infty) = \frac{2\alpha_0\beta_0^2}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$	$p_6(\infty) = \frac{8\alpha_0^3}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$
$p_3(\infty) = \frac{4\alpha_0^2\beta_0}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$	$p_7(\infty) = \frac{8\alpha_0^3}{16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3}$

Table 2.
Steady state probabilities of configuration 1.

Where $P(0) = [1, 0, 0, 0]$ and

$$M_1 = \begin{pmatrix} -4\alpha_0 & 2\alpha_0 & 2\alpha_0 & 0 \\ \beta_0 & -(4\alpha_0 + \beta_0) & 0 & 2\alpha_0 \\ \beta_0 & 0 & -(4\alpha_0 + \beta_0) & 2\alpha_0 \\ 0 & \beta_0 & \beta_0 & -(4\alpha_0 + 2\beta_0) \end{pmatrix}$$

obtained by transposing Q_1 and deleting rows and columns of failure states.

4.2 Models formulation for configuration 2

Applying similar description in 4.1 above, the differential-difference equations for Configuration 2 are expressed in the form:

$$\frac{d}{dt}p(t) = Q_2 p(t) \quad (15)$$

where

$$Q_2 = \begin{pmatrix} -3\alpha_0 & \beta_0 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & -\gamma_0 & \beta_0 & 0 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & -\gamma_0 & \beta_0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 & \beta_0 \\ \alpha_0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 \end{pmatrix}$$

and $\gamma_0 = (3\alpha_0 + \beta_0)$.

With initial conditions

$$P(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \quad (16)$$

Expression for system availability, probability of partial and complete failure for Configuration 2 are given by

$$A_{T2}(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) \quad (17)$$

$$B_{P3}(\infty) = p_1(\infty) + p_2(\infty) + p_3(\infty) \quad (18)$$

$$B_{P4}(\infty) = p_4(\infty) + p_5(\infty) + p_6(\infty) + p_7(\infty) + \dots + p_{12}(\infty) \quad (19)$$

Setting (15) to zero to give

$$Q_2 p(\infty) = 0 \quad (20)$$

The normalizing condition for this analysis is

$$\sum_{j=0}^{12} p_j(\infty) = 1 \tag{21}$$

Combining (20) and (21) to give system of equations

$$\begin{pmatrix} -3\alpha_0 & \beta_0 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & -\gamma_0 & \beta_0 & 0 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & -\gamma_0 & \beta_0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\gamma_0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 & \beta_0 \\ \alpha_0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \\ p_{11}(\infty) \\ p_{12}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{22}$$

Solving the system of equations in (22) for the state probabilities $p_k(\infty), k = 0, 1, 2, \dots, 12$, using MATLAB package, to give states probabilities in **Table 3** below.

Expressions for the system availability, probability of partial and complete failure for configuration 2 in (17) to (19) as well as profit function are now

$$A_{T2}(\infty) = \frac{\beta_0^4 + 2\alpha_0\beta_0^3 + \alpha_0^2\beta_0^2 + \alpha_0^3\beta_0}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4} \tag{23}$$

$$B_{P3}(\infty) = \frac{\alpha_0\beta_0^3 + \alpha_0^3\beta_0 + \alpha_0^2\beta_0^2}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4} \tag{24}$$

$$B_{P4}(\infty) = \frac{3\alpha_0^4 + 2\alpha_0\beta_0^3 + 2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4} \tag{25}$$

$$P_{F2} = k_0A_{T2}(\infty) - k_1B_{P3}(\infty) - k_2B_{P4}(\infty) \tag{26}$$

$p_0(\infty) = \frac{\beta_0^4}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	$p_7(\infty) = \frac{\alpha_0^2\beta_0^2}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$
$p_1(\infty) = \frac{\alpha_0\beta_0^3}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	$p_8(\infty) = \frac{\alpha_0^3\beta_0}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$
$p_2(\infty) = \frac{\alpha_0^2\beta_0^2}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	$p_9(\infty) = \frac{\alpha_0^3\beta_0}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$
$p_3(\infty) = \frac{\alpha_0^3\beta_0}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	$p_{10}(\infty) = \frac{\alpha_0^4}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$
$p_4(\infty) = \frac{\alpha_0^3\beta_0}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	$p_{11}(\infty) = \frac{\alpha_0^4}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$
$p_5(\infty) = \frac{\alpha_0^3\beta_0}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	$p_{12}(\infty) = \frac{\alpha_0^4}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$
$p_6(\infty) = \frac{\alpha_0^2\beta_0^2}{3\alpha_0^4 + 3\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4}$	

Table 3.
Steady state probabilities of configuration 2.

Mathematical model of mean time to failure for Configuration 2 is derived using the relation

$$MTTF_2 = P(0)(-M_2^{-1})[1, 1, 1, 1]^T = \frac{40\alpha_0^3 + 27\alpha_0^2\beta_0 + 8\alpha_0\beta_0^2 + \beta_0^3}{\alpha_0(81\alpha_0^3 + 54\alpha_0^2\beta_0 + 16\alpha_0\beta_0^2 + 2\beta_0^3)} \quad (27)$$

Where $P(0) = [1, 0, 0, 0]$ and

$$M_2 = \begin{pmatrix} -3\alpha_0 & \alpha_0 & 0 & 0 \\ \beta_0 & -(3\alpha_0 + \beta_0) & \alpha_0 & 0 \\ 0 & \beta_0 & -(3\alpha_0 + \beta_0) & \alpha_0 \\ 0 & 0 & \beta_0 & -(3\alpha_0 + \beta_0) \end{pmatrix}$$

M_2 is obtained from Q_2 using similar argument above.

4.3 Models formulation for configuration 3

Following similar argument in 4.1 above, the differential-difference equations obtained for Configuration 3 are expressed in the form:

$$\frac{d}{dt}p(t) = Q_3p(t) \quad (28)$$

where

$$Q_3 = \begin{pmatrix} -2\alpha_0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & -\gamma_1 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 \\ 0 & \alpha_0 & 0 & -\gamma_1 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 & -\gamma_1 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 \end{pmatrix}$$

Where $\gamma_1 = (2\alpha_0 + \beta_0)$.

With initial conditions

$$P(0) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \quad (29)$$

Expression for system availability, probability of partial and complete failure for Configuration 3 are given by

$$A_{T3}(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty) \quad (30)$$

$$B_{P5}(\infty) = p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty) \quad (31)$$

$$B_{P6}(\infty) = p_5(\infty) + p_6(\infty) + p_7(\infty) + p_8(\infty) + p_9(\infty) + p_{10}(\infty) \quad (32)$$

Setting (28) to zero to give

$$Q_3p(\infty) = 0 \tag{33}$$

The normalizing condition for this analysis is

$$\sum_{j=0}^{10} p_j(\infty) = 1 \tag{34}$$

Combining (33) and (34) to give system of equations

$$\begin{pmatrix} -2\alpha_0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & -\gamma_1 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha_0 & 0 & -\gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_0 & \beta_0 \\ 0 & \alpha_0 & 0 & -\gamma_1 & 0 & \beta_0 & \beta_0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_0 & 0 & 0 & -\gamma_1 & 0 & 0 & \beta_0 & \beta_0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 & -\beta_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 & -\beta_0 & 0 & 0 \\ 0 & 0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{35}$$

Solving the system of equations in (35) for the state probabilities $p_k(\infty), k = 0, 1, 2, \dots, 10$, presented in **Table 4** below.

Expressions for the system availability, probability of partial and complete failure for configuration 3 in (30) to (32) as well as profit function are now

$$A_{T3}(\infty) = \frac{\beta_0^3 + 2\alpha_0\beta_0^2 + 2\alpha_0^2\beta_0}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3} \tag{36}$$

$$B_{P5}(\infty) = \frac{2\alpha_0\beta_0(\alpha_0 + \beta_0)}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3} \tag{37}$$

$$B_{P6}(\infty) = \frac{2\alpha_0^2(2\alpha_0 + \beta_0)}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3} \tag{38}$$

$p_0(\infty) = \frac{\beta_0^3}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$	$p_6(\infty) = \frac{\alpha_0^3}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$
$p_1(\infty) = \frac{\alpha_0\beta_0^2}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$	$p_7(\infty) = \frac{\alpha_0^3}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$
$p_2(\infty) = \frac{\alpha_0\beta_0^2}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$	$p_8(\infty) = \frac{\alpha_0^3}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$
$p_3(\infty) = \frac{\alpha_0^2\beta_0}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$	$p_9(\infty) = \frac{\alpha_0^2\beta_0}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$
$p_4(\infty) = \frac{\alpha_0^2\beta_0}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$	$p_{10}(\infty) = \frac{\alpha_0^2\beta_0}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$
$p_5(\infty) = \frac{\alpha_0^3}{4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3}$	

Table 4.
Steady state probabilities of configuration 3.

$$P_{F3} = k_0 A_{T3}(\infty) - k_1 B_{P5}(\infty) - k_2 B_{P6}(\infty) \quad (39)$$

Mathematical model of mean time to failure for Configuration 3 is derived using the relation

$$MTTF_3 = P(0)(-M_3^{-1})[1, 1, 1, 1, 1]^T = \frac{20\alpha_0^3 + 16\alpha_0^2\beta_0 + 6\alpha_0\beta_0^2 + \beta_0^3}{2\alpha_0^2(8\alpha_0^2 + 4\alpha_0\beta_0 + \beta_0^2)} \quad (40)$$

Where $P(0) = [1, 0, 0, 0, 0]$

$$M_3 = \begin{pmatrix} -2\alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 \\ \beta_0 & -(2\alpha_0 + \beta_0) & 0 & \alpha_0 & \alpha_0 \\ \beta_0 & 0 & -(2\alpha_0 + \beta_0) & 0 & 0 \\ 0 & \beta_0 & 0 & -(2\alpha_0 + \beta_0) & 0 \\ 0 & \beta_0 & 0 & 0 & -(2\alpha_0 + \beta_0) \end{pmatrix}$$

M_3 is obtained from Q_3 using similar argument above.

5. Results and discussion

5.1 Analytical comparison

In this section, the configurations are compared analytically in terms of their availability and mean time to failure to determine the optimal configuration by taking the difference between mean time to failure (MTTF) and availability for the configurations using MAPLE software package.

$$MTTF_3 - MTTF_1 = \frac{16\alpha_0^4 + 192\alpha_0^3\beta_0 + 100\alpha_0^2\beta_0^2 + 28\alpha_0\beta_0^3 + 3\beta_0^4}{8\alpha_0^2(4\alpha_0 + \beta_0)(8\alpha_0^2 + 4\alpha_0\beta_0 + \beta_0^2)} \quad (41)$$

$$\Rightarrow MTTF_3 > MTTF_1 \quad \forall \alpha_0, \beta_0 > 0$$

$$MTTF_3 - MTTF_2 = \frac{980\alpha_0^6 + 1624\alpha_0^5\beta_0 + 1246\alpha_0^4\beta_0^2 + 567\alpha_0^3\beta_0^3 + 158\alpha_0^2\beta_0^4 + 26\alpha_0\beta_0^5 + 2\beta_0^6}{2\alpha_0^2(8\alpha_0^2 + 4\alpha_0\beta_0 + \beta_0^2)(81\alpha_0^3 + 54\alpha_0^2\beta_0 + 16\alpha_0\beta_0^2 + 2\beta_0^3)} \quad (42)$$

$$\Rightarrow MTTF_3 > MTTF_2 \quad \forall \alpha_0, \beta_0 > 0$$

$$MTTF_1 - MTTF_2 = \frac{340\alpha_0^5 + 544\alpha_0^4\beta_0 + 361\alpha_0^3\beta_0^2 + 126\alpha_0^2\beta_0^3 + 24\alpha_0\beta_0^4 + 2\beta_0^5}{8\alpha_0^2(4\alpha_0 + \beta_0)(81\alpha_0^3 + 54\alpha_0^2\beta_0 + 16\alpha_0\beta_0^2 + 2\beta_0^3)} \quad (43)$$

$$\Rightarrow MTTF_1 > MTTF_2 \quad \forall \alpha_0, \beta_0 > 0$$

Using mean time to failure models of configurations, it is clear from (41)–(43) that

$$MTTF_3 > MTTF_1 > MTTF_2$$

$$A_{T3}(\infty) - A_{T1}(\infty) = \frac{2\alpha_0^2\beta_0(4\alpha_0^2 + 4\alpha_0\beta_0 + 3\beta_0^2)}{(16\alpha_0^3 + 12\alpha_0^2\beta_0 + 4\alpha_0\beta_0^2 + \beta_0^3)(4\alpha_0^3 + 4\alpha_0^2\beta_0 + 2\alpha_0\beta_0^2 + \beta_0^3)} \quad (44)$$

$$\Rightarrow A_{T3} > A_{T1} \quad \forall \alpha_0, \beta_0 > 0$$
$$A_{T3}(\infty) - A_{T2}(\infty) = \frac{\alpha_0 \beta_0 (2\alpha_0^5 + 4\alpha_0^4 \beta_0 + \alpha_0^3 \beta_0^2 + 2\alpha_0 \beta_0^4 + \beta_0^5)}{(4\alpha_0^3 + 4\alpha_0^2 \beta_0 + 2\alpha_0 \beta_0^2 + \beta_0^3)(3\alpha_0^4 + 3\alpha_0^3 \beta_0 + 3\alpha_0^2 \beta_0^2 + 3\alpha_0 \beta_0^3 + \beta_0^4)}$$

(45)

$$\Rightarrow A_{T3} > A_{T2} \quad \forall \alpha_0, \beta_0 > 0$$
$$A_{T2}(\infty) - A_{T1}(\infty) = \frac{\alpha_0 \beta_0 (2\alpha_0^4 + \alpha_0^3 \beta_0 + 10\alpha_0^2 \beta_0^2 + 4\alpha_0 \beta_0^3 - \beta_0^4)}{(3\alpha_0^4 + 3\alpha_0^3 \beta_0 + 3\alpha_0^2 \beta_0^2 + 3\alpha_0 \beta_0^3 + \beta_0^4)(16\alpha_0^3 + 12\alpha_0^2 \beta_0 + 4\alpha_0 \beta_0^2 + \beta_0^3)}$$

(46)

Using availability models of configurations, it is clear from (44)–(46) that

$$A_{T3}(\infty) > A_{T2}(\infty) > A_{T1}(\infty)$$

5.2 Comparison based on ranking of the configurations

Tables 5 and 6 depict the ranking of configuration base on their availability and mean time to failure. It clear from Table 5 that configuration 3 is the optimal configuration whenever $0 \leq \beta_0 \leq 1$. Thus, verifying the above analytical claim that $A_{T3}(\infty) > A_{T2}(\infty) > A_{T1}(\infty)$ and $MTTF_3 > MTTF_1 > MTTF_2$ $\alpha_0, \beta_0 > 0$.

5.3 Comparison based on availability, profit and mean time to failure

In this section, $\beta_0 = 0.8, k_0 = 50,000,000, k_1 = 1250$ and $k_2 = 2150$ are fixed and vary α_0 from 0.1 to 1 in Figures 4–6 and $\alpha_0 = 0.018, k_0 = 50,000,000,$

Case	Parameter Range	Result		Constant parameters
1	$0 < \alpha_0 < 0.2$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	$\beta_0 = 0.6$
	$0.2 < \alpha_0 < 0.4$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	
	$0.4 < \alpha_0 < 0.6$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	
	$0.6 < \alpha_0 < 0.8$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	
	$0.8 < \alpha_0 < 1.0$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	

Table 5.
Ranking of configurations based on their availability and MTTF for $\alpha_0 \in [0, 1]$.

Case	Parameter Range	Result		Constant parameters
2	$0 < \beta_0 < 0.2$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	$\alpha_0 = 0.2$
	$0.2 < \beta_0 < 0.4$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	
	$0.4 < \beta_0 < 0.6$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	
	$0.6 < \beta_0 < 0.8$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	
	$0.8 < \beta_0 < 1.0$	$A_{V3}(\infty) > A_{V2}(\infty) > A_{V1}(\infty)$	$MTTF_3 > MTTF_1 > MTTF_2$	

Table 6.
Ranking of configurations based on their availability and MTTF for $\beta_0 \in [0, 1]$.

Figure 4.
Availability against α_0 .

Figure 5.
Profit against α_0 .

Figure 6.
MTTF against α_0 .

$k_1 = 1250$ and $k_2 = 2150$ are fixed and vary β_0 from 0.1 to 1 in **Figures 7–9** and obtained the following results.

Simulations in **Figures 4–6** compare the steady state availability, profit and MTTF with respect to α_0 for all the three configurations considered. From these figures, availability, profit and MTTF decreases as α_0 increases for any configuration. Furthermore, Configuration 3 seems to be most effective and reliable configuration among all the three configurations. From these figures, it is clear that Configuration 3 produces more availability, profit and MTTF than the other configurations. Thus, Configuration 3 is the optimal configuration in this study. On the other hand, simulations in **Figures 7–9** compare the steady state availability, profit and MTTF with respect to β_0 for all the three configurations. It is evident from these figures that availability, profit and MTTF increases as β_0 increases for any

Figure 7.
Availability against β_0 .

Figure 8.
Profit against β_0 .

Figure 9.
MTTF against β_0 .

Figure 10.
 C_k/A_{vk} against α_0 .

configuration. Similar to **Figures 4–6**, Configuration 3 seems to be most effective and reliable configuration among all the three configurations and hence is the optimal configuration.

5.4 Comparison based on cost benefit

In this section, the configurations are compared based on their C_k/A_{vk} and $C_k/MTTF_k$ using MATLAB software. The following parameters values are used for the purpose of analysis:

Figure 11.
 $C_k/MTTF_k$ against α_o .

$\beta_0 = 0.8$, $C_1 = 48,000,000$, $C_2 = 42,000,000$, $C_3 = 39,000,000$ (Yen, T.-S and Wang, K.-H [37]) are fixed and vary α_0 between 0.1 and 1 in **Figures 10 and 11**.

$\alpha_0 = 0.018$, $C_1 = 42,000,000$, $C_1 = 39,000,000$ and vary β_0 between 0.1 to 1 in **Figures 12 and 13** and obtained the following results:

Figure 12.
 C_k/A_{vk} against β_o .

Figure 13.
 $C_k/MTTF_k$ against β_0 .

Figures 10 and 11 depict the results of C_k/A_{vk} and $C_k/MTTF_k$ for each configuration i ($i = 1, 2, 3$) with respect to α_0 . From these figures, it is evident that C_k/A_{vk} and $C_k/MTTF_k$ increase as α_0 increases for each configuration. It is evident from these Figures that the optimal configuration using both C_k/A_{vk} and $C_k/MTTF_k$ is Configuration 3.

From **Figures 12 and 13**, it is clear that C_k/A_{vk} and $C_k/MTTF_k$ decrease as β_0 increases. It is clear from these Figures that the optimal configuration using both C_k/A_{vk} and $C_k/MTTF_k$ is again Configuration 3. Configurations 1 and 2 tend to have more C_k/A_{vk} and $C_k/MTTF_k$ than Configuration 3. From the result presented in this study, it is clear that the survival of manufacturing and industrial systems depends upon its design and reliability characteristics. Through the system design and reliability characteristics, management can tend to realize whether such systems operate at minimum cost of maintenance, quality of the product, production output as well as revenue mobilization.

5.5 Sensitivity analysis

Sensitivity analysis presented in **Tables 7 and 8** depict the change in availability, MTTF and profit of the three configurations with respect to failure rate α_0 for different values of β_0 . It is clear from these tables that availability, MTTF and profit of the three configurations decreases as α_0 increase. Availability, MTTF and profit tend to be higher for the three configurations for whenever $\beta_0 = 0.9$. This sensitivity analysis implies that maintenance action such as inspection, preventive maintenance, etc. should be invoke to reduce the occurrence of failure in order to attain maximum value of availability, MTTF and profit. From these tables it is evident that Configuration 3 has higher values of availability, MTTF and profit than configurations 1 and 2 for different values of β_0 . On the other hand, Sensitivity analysis in depicted in **Tables 9 and 10**, displayed the variation of availability, MTTF and

	α_0	$A_{V1}(\infty)$	$A_{V2}(\infty)$	$A_{V3}(\infty)$	$MTTF_1$	$MTTF_2$	$MTTF_3$
$\beta_0 = 0.3$	0.1	0.6522	0.7313	0.8361	9.4643	4.9887	25.6897
	0.2	0.3962	0.5287	0.6084	3.8920	2.4878	9.3654
	0.3	0.2727	0.3846	0.4545	2.4167	1.6558	5.5128
	0.4	0.2050	0.2902	0.3565	1.7475	1.2405	3.8699
	0.5	0.1632	0.2286	0.2912	1.3674	0.9916	2.9717
$\beta_0 = 0.6$	0.1	0.8571	0.8538	0.9494	13.0000	4.9965	40.2941
	0.2	0.6522	0.7313	0.8361	4.7321	2.4943	12.8448
	0.3	0.5000	0.6230	0.7143	2.7778	1.6605	7.0000
	0.4	0.3962	0.5287	0.6084	1.9460	1.2439	4.6827
	0.5	0.3243	0.4494	0.5227	1.4923	0.9942	3.4809
$\beta_0 = 0.9$	0.1	0.9252	0.8989	0.9764	16.6346	4.9985	55.1600
	0.2	0.7852	0.8112	0.9154	5.6066	2.4970	16.4662
	0.3	0.6522	0.7313	0.8361	3.1548	1.6629	8.5632
	0.4	0.5445	0.6576	0.7537	2.1531	1.2459	5.5391
	0.5	0.4609	0.5899	0.6768	1.6224	0.9959	4.0169

Table 7.
Availability and MTTF sensitivity as function of α_0 for different values of β_0 .

α_0	Profit * 10^7								
	$\beta_0 = 0.3$			$\beta_0 = 0.6$			$\beta_0 = 0.9$		
	P_{F1}	P_{F2}	P_{F3}	P_{F1}	P_{F2}	P_{F3}	P_{F1}	P_{F2}	P_{F3}
0	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
0.1	3.0666	3.5304	4.0420	4.1659	4.1968	4.6934	4.5546	4.4432	4.8555
0.2	1.8056	2.4578	2.8393	3.0666	3.5304	4.0420	3.7692	3.9643	4.4934
0.3	1.2298	1.7423	2.0850	2.3044	2.9493	3.3837	3.0666	3.5304	4.0420
0.4	0.9205	1.2982	1.6218	1.8056	2.4578	2.8393	2.5232	3.1335	3.5928
0.5	0.7316	1.0172	1.3190	1.4682	2.0586	2.4143	2.1147	2.7751	3.1881
0.6	0.6054	0.8304	1.1085	1.2298	1.7423	2.0850	1.8056	2.4578	2.8393
0.7	0.5156	0.6995	0.9547	1.0542	1.4939	1.8270	1.5673	2.1818	2.5441
0.8	0.4486	0.6035	0.8378	0.9205	1.2982	1.6218	1.3800	1.9446	2.2952
0.9	0.3968	0.5305	0.7461	0.8157	1.1426	1.4556	1.2298	1.7423	2.0850

Table 8.
Profit sensitivity as function of α_0 for different values of β_0 .

profit with respect to β_0 for different values of α_0 . It is evident from the tables that availability, MTTF and profit increases as β_0 increases for different values of α_0 . Increase in the values of α_0 decrease the availability, MTTF and profit as shown in the tables. This sensitivity analysis suggest that perfect repair, preventive maintenance, inspection should be invoke at early failure to restore the system to its position as good as new. It is also clear from these tables that Configuration III has higher values of availability, MTTF and profit than configurations 1 and 2.

	α_0	$A_{V1(\infty)}$	$A_{V2(\infty)}$	$A_{V3(\infty)}$	$MTTF_1$	$MTTF_2$	$MTTF_3$
$\alpha_0 = 0.012$	0.1	0.9150	0.8915	0.9726	131.8506	41.6518	432.21
	0.2	0.9749	0.9432	0.9929	217.2939	41.6638	778.30
	0.3	0.9883	0.9615	0.9968	303.5201	41.6657	1125.2
	0.4	0.9933	0.9708	0.9982	390.0050	41.6662	1472.4
	0.5	0.9956	0.9765	0.9989	476.6069	41.6664	1819.5
$\alpha_0 = 0.015$	0.1	0.8784	0.8671	0.9583	92.0139	33.3142	290.60
	0.2	0.9623	0.9299	0.9890	146.3675	33.3294	511.70
	0.3	0.9821	0.9523	0.9950	201.3889	33.3319	733.60
	0.4	0.9896	0.9638	0.9972	256.6425	33.3327	955.70
	0.5	0.9933	0.9708	0.9982	312.0040	33.3330	1177.90
$\alpha_0 = 0.018$	0.1	0.8399	0.8434	0.9418	69.2650	27.7550	211.6955
	0.2	0.9479	0.9168	0.9843	106.7765	27.7726	364.8292
	0.3	0.9749	0.9432	0.9929	144.8626	27.7759	518.8338
	0.4	0.9854	0.9569	0.9960	183.1581	27.7769	673.0276
	0.5	0.9904	0.9652	0.9974	221.2852	27.7773	827.2852

Table 9.
 Availability and MTTF sensitivity as function of β_o for different values of α_o .

β_0	Profit * 10^7								
	$\alpha_0 = 0.012$			$\alpha_0 = 0.015$			$\alpha_0 = 0.018$		
	P_{F1}	P_{F2}	P_{F3}	P_{F1}	P_{F2}	P_{F3}	P_{F1}	P_{F2}	P_{F3}
0	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002	-0.0002
0.1	4.6436	4.5075	4.8881	4.4850	4.3959	4.8289	4.3156	4.2875	4.7597
0.2	4.8969	4.7431	4.9711	4.8444	4.6825	4.9552	4.7839	4.6230	4.9359
0.3	4.9521	4.8260	4.9871	4.9268	4.7842	4.9799	4.8969	4.7431	4.9711
0.4	4.9725	4.8684	4.9927	4.9577	4.8366	4.9887	4.9400	4.8051	4.9837
0.5	4.9822	4.8942	4.9953	4.9725	4.8684	4.9927	4.9609	4.8429	4.9895
0.6	4.9875	4.9116	4.9968	4.9807	4.8899	4.9949	4.9725	4.8684	4.9927
0.7	4.9908	4.9240	4.9976	4.9857	4.9054	4.9963	4.9796	4.8868	4.9946
0.8	4.9929	4.9334	4.9982	4.9890	4.9170	4.9972	4.9843	4.9007	4.9959
0.9	4.9944	4.9407	4.9986	4.9913	4.9261	4.9977	4.9875	4.9116	4.9968

Table 10.
 Profit sensitivity as function of β_o for different values of α_o .

6. Conclusion

In this paper, three different standby serial systems each supplying 60 MW are considered. The expressions for the reliability characteristics such as system availability, busy period of repairman due to partial and complete failure as well as profit functions for all the configurations have been obtained and validated by performing analytical and numerical experiments. Analysis of the effect of various system parameters on mean time to failure, profit function and availability was performed.

These are the main contributions of this study. On the basis of the numerical results obtained in Figures and Tables for a particular case, it is evident that the optimal system configuration is configuration 3. This is supported from analytical comparison presented in terms of the availability and mean time to failure models obtained in which configuration III is the optimal configuration for all $\alpha_0, \beta_0 > 0$ contrary to some studies where the optimality among the system configuration is not uniform as it depends on some system parameters. The contributions of this paper are as follows.

- i. Failure is categorized into partial and complete failure
- ii. Analytical comparison between the configuration in terms of their availability and mean time to failure is performed
- iii. Optimal configuration in analytical comparison agrees with that of numerical comparison
- iv. Optimal configuration is unique for all parameter values

Conflict of interest

There are no conflicts of interest to this chapter.

Additional classification

AMS (2010) subject classification: 90B25

Author details

Ibrahim Yusuf^{1*} and Ismail Muhammad Musa²

1 Department of Mathematical Sciences, Bayero University, Kano, Nigeria

2 Department of Mathematics and Computer Science, Alqalam University, Katsina, Nigeria

*Address all correspondence to: iyusuf.mth@buk.edu.ng

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