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Optimization of Multi-Tiered Supply Chain Networks with Equilibrium Flows

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1. Introduction

Consider a multi-tiered supply chain network which contains manufacturers, distributors and consumers. A manufacturer located at the top tier of this supply chain is supposed to be concerned with the production of products and shipments to the distributors for profit maximization. In turn, a distributor located in the middle tier of the supply chain is faced with handling and managing the products obtained from manufacturers as well as conducting transactions with consumers at demand markets. The consumer, who is the ultimate user for the product in the supply chain, located at the bottom tier of the supply chain agrees to the prices charged by distributors for the product if the associated business deal is done. The underlying behaviour of manufacturers, distributors and consumers is supposed to compete in a non-cooperative manner. Each decision maker individually wishes to find optimal shipments given the ones of other competitors. The problem of deciding optimal shipments in a supply chain equilibrium network was firstly noted by Nagurney et al. (2002). Dong et al. (2004) developed a supply chain network model where a finite-dimensional variational inequality was formulated for the behaviour of various decision makers. Zhang (2006), in turn, proposed a supply chain model that comprises heterogeneous supply chains involving multiple products and competing for multiple markets.

In this chapter we develop an optimal solution scheme for a multi-tiered supply chain network which contains manufacturers, distributors and consumers. In the multi-tiered supply chain network, there are two kinds of decision-making levels investigated: the management level and the operations level. For the management level, the decision maker wishes to find a set of optimal policies which aim to minimize total cost incurred by the whole supply chain network. For the operations level, assuming the underlying behaviour of the multi-tiered decision makers compete in a non-cooperative manner, each decision maker individually wishes to find optimal shipments given the ones of other competitors. Therefore a problem of deciding equilibrium productions and shipments in a multi-tiered supply chain network can be established. Nagurney et al. (2002) were the first ones to recognize the supply chain equilibrium behaviour, in this chapter, we enhance the modelling of supply chain equilibrium network by taking account of policy interventions at

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management level, which takes the responses of the decision makers at operations level to the changes made at management level for which a minimal cost of the supply chain can be achieved. A new solution scheme is also developed for optimizing a multi-tiered supply chain network with equilibrium flows.

Optimization for a multi-tiered supply chain network with equilibrium flows can be formulated as a mathematical program with equilibrium constraints (MPEC) where a two-level decision making process is considered. A MPEC program for a general network design problem is widely known as non-convex and non-differentiable. In this chapter, a non-smooth analysis is employed to optimize the policy interventions determined at the management level. The first order sensitivity analysis is carried out for supply chain equilibrium network flow which is determined at the operations level. The directional derivatives and associated generalized gradient of equilibrium product flows (shipments) with respect to the changes of policy interventions made at management level can be therefore obtained. Because the objective function of the multi-tiered supply chain network is non-smooth, a subgradient projection solution scheme (SPSS) is proposed to solve the multi-tiered supply chain network problem with global convergence. Numerical calculations are conducted using a medium-scale supply chain network. Computational results successfully demonstrate the potential of the SPSS approach in solving a multi-tiered supply chain equilibrium network problem with reasonable computational efforts.

The organization of this chapter is as follows. In next section, a MPEC formulation is addressed for a multi-tiered supply chain network with equilibrium flows where a two-level decision making process is considered. The first-order sensitivity analysis for equilibrium flows at operations level is carried out by solving an affine variational inequality. A subgradient projection solution scheme (SPSS), in Section 3, is proposed to globally solve the multi-tiered supply chain network problem with equilibrium flows. In Section 4 numerical calculations and comparisons with earlier methods in solving the supply chain network problem are conducted using a medium-scale network. Good results with far less computational efforts by the SPSS approach are also reported. Conclusions and further work associated are summarized in Section 5.

2. Problem formulation

In this section, a MPEC program is firstly given for a three-tiered supply chain network containing manufacturers, distributors and consumers where a two-level decision making process: the management level and the operations level, is considered. A first-order sensitivity analysis is conducted for which the generalized gradient and directional derivatives of variable of interests at operations level can be obtained. At the management level, suppose strong regularity condition (Robinson, 1980) holds at the variable of interests with respect to the policy interventions which are determined at management level, a one level MPEC program can be established. The directional derivatives for the three-tiered supply chain network can be also therefore found via the corresponding subgradients.

2.1 Notation

M : a set of manufacturers located at the top tier of the multi-tiered supply chain network.

R : a set of distributors located in the middle tier of the multi-tiered supply chain network.

U : a set of demand markets located at the bottom tier of the multi-tiered supply chain network.

β : a set of policy settings determined at management level in the multi-tiered supply chain network.

x_{ij} : the product flow/shipment between agents at distinct tiers of the multi-tiered supply chain network.

$p_i(\cdot)$: the production cost function for a manufacturer i , $\forall i \in M$.

$h_j(\cdot)$: the handling cost function for a distributor j , $\forall j \in R$.

$t_{1ij}(\cdot)$: the transaction cost function on link (i, j) between manufacturer i and distributor j , $\forall i \in M$ and $\forall j \in R$.

$t_{2jk}(\cdot)$: the transaction cost function on link (j, k) between distributor j and consumers at demand market k ; $\forall j \in R$ and $\forall k \in U$.

d_k : the consumptions at the demand market k , $\forall k \in U$.

λ_{1ij} : the market price charged for distributor j by manufacturer i , $\forall i \in M$ and $\forall j \in R$.

λ_{2jk} : the market price charged for demand market k by distributor j , $\forall j \in R$ and $\forall k \in U$.

γ_j : the market clear price for distributor j , $\forall j \in R$.

μ_k : the price at demand market k , $\forall k \in U$.

2.2 Equilibrium conditions for a three-tiered supply chain network

According to Nagurney (1999), optimal production and shipments for manufacturers in a three-tiered supply chain network can be found by solving the following variational inequality formulation. Find the values $x_{ij} \in K_1$, $\forall i \in M, j \in R$ such that

$$\sum_{i \in M} \sum_{j \in R} (p_i(X_i) + t_{1ij}(x) - \lambda_{1ij}) (z - x_{ij}) \geq 0 \quad (1)$$

for all $z \in K_1 = \{x_{ij}, i \in M, j \in R\}$ where $X_i = \sum_{j \in R} x_{ij}$.

Akin to inequality (1), the optimal inbound shipments for distributor j , say x_{ij} , from the manufacturer i , and the outbound shipments, say x_{jk} , to the consumers at demand market

k , coincide with the solutions of the following variational inequality. Find values $x_{ij} \in K_1$ and $x_{jk} \in K_2$, $\forall i \in M, j \in R$ and $k \in U$ as well as the market clear price γ_j such that

$$\sum_{i \in M} \sum_{j \in R} (\lambda_{1ij} + h_j(X_j) - \gamma_j)(w - x_{ij}) + \sum_{j \in R} \sum_{k \in U} (t_{2jk}(x) + \gamma_j - \lambda_{2ij})(z - x_{jk}) + \sum_{j \in R} \left(\sum_{i \in M} x_{ij} - \sum_{k \in U} x_{jk} \right) (\gamma - \gamma_j) \geq 0 \quad (2)$$

for all $w \in K_1 = \{x_{ij}, i \in M, j \in R\}$, $z \in K_2 = \{x_{jk}, j \in R, k \in U\}$ and $X_j = \sum_{k \in U} x_{jk}$. The market clear price γ_j in a three-tiered supply chain network is associated with the product flow conservation which holds for each distributor j , $\forall j \in R$ as follows.

$$\sum_{i \in M} x_{ij} \geq \sum_{k \in U} x_{jk} \quad (3)$$

Assuming the underlying behavior of the consumers at demand market k , $\forall k \in U$ competing non-cooperatively with other consumers for the product provided by distributors, in the third tier supply chain network the governing equilibrium condition for the consumptions at demand market k can be, in a similar way to (1) and (2), coincide with the solutions of the following variational inequality in the following manner. Determine the consumptions d_k such that

$$\sum_{j \in R} \sum_{k \in U} (\lambda_{2jk} - \mu_k)(z - x_{jk}) \geq 0 \quad (4)$$

for all $z \in K_2 = \{x_{jk}, j \in R, k \in U\}$ and $d_k = \sum_{j \in R} x_{jk}$.

2.3 A three-tiered supply chain network equilibrium model

Consider the optimality conditions given in (1-2) and (4) respectively for manufacturers, distributors and consumers, a three-tiered supply chain network equilibrium model can be established in the following way.

Definition 1. A three-tiered supply chain network equilibrium: The equilibrium state of the supply chain network is one where the product flows between the distinct tiers of the agents coincide and the product flows and prices satisfy the sum of the optimality conditions (1), (2) and (4). $\square$

Theorem 2. A variational inequality for the three-tiered supply chain network model: The equilibrium conditions governing the supply chain network model with competitions are equivalent to the solution of the following variational inequality. Find $(x_{ij}, x_{jk}) \in (K_1, K_2)$ such that

$$\begin{aligned} & \sum_{i \in M} \sum_{j \in R} (p_i(X_i) + h_j(X_j) + t_{1ij}(x) - \gamma_j)(u - x_{ij}) + \sum_{j \in R} \sum_{k \in U} (t_{2jk}(x) + \gamma_j - \mu_k)(v - x_{jk}) \\ & + \sum_{j \in R} \left(\sum_{i \in M} x_{ij} - \sum_{k \in U} x_{jk} \right) (\gamma - \gamma_j) \geq 0 \end{aligned} \quad (5)$$

for all $(u, v) \in (K_1, K_2)$, and γ_j is the market clear price for distributor j , $\forall j \in R$.

Proof. Following the Definition 1, the equilibrium conditions for a three-tiered supply chain network in determining optimal productions for manufacturers, optimal inbound and outbound shipments for distributors and optimal consumptions for consumers can be expressed as the following aggregated form of summing up the (1), (2) and (4). Find $(x_{ij}, x_{jk}) \in (K_1, K_2)$ such that

$$\begin{aligned} & \sum_{i \in M} \sum_{j \in R} (p_i(X_i) + h_j(X_j) + t_{1ij}(x) - \gamma_j)(u - x_{ij}) + \sum_{j \in R} \sum_{k \in U} (t_{2jk}(x) + \gamma_j - \mu_k)(v - x_{jk}) \\ & + \sum_{j \in R} \left(\sum_{i \in M} x_{ij} - \sum_{k \in U} x_{jk} \right) (\gamma - \gamma_j) \geq 0 \end{aligned}$$

for all $(u, v) \in (K_1, K_2)$, and γ_j is the market clear price for distributor j , $\forall j \in R$. $\square$

2.4 A generalized variational inequality

In the supply chain network equilibrium model (5), suppose $p_i(\cdot), h_j(\cdot), t_{1ij}(\cdot)$ and $t_{2jk}(\cdot)$, $\forall i \in M, j \in R$ and $k \in U$ are continuous and convex. Let

$$K = K_1 \cup K_2 \cup \left\{ (x_{ij}, x_{jk}) : \sum_{i \in M} x_{ij} \geq \sum_{k \in U} x_{jk}, \forall j \in R \right\} \quad (6)$$

And

$$F(\cdot) = (p_i, h_j, t_{1ij}, t_{2jk})_{i \in M, j \in R, k \in U} \quad (7)$$

a standard variational inequality for (5) can be expressed as follows. Determine $X \in K$ such that

$$F^t(X)(Z - X) \geq 0 \quad (8)$$

$\forall Z \in K$ where the superscript t denotes matrix transpose operation.

2.5 A link-based variational inequality

Regarding the inequality (8), a link-based variational inequality formulation for a three-tiered supply chain network equilibrium model can be expressed in the following way. Let s and d respectively denote total productions and demands for the supply chain. Let q denote the equilibrium link flow in the supply chain network, x denote the path flow between distinct tiers, Λ and Γ respectively denote the link-path and origin/destination-path incidence matrices. The set K in (6) can be re-expressed in the corresponding manner.

$$K = \{q : q = \Lambda x, \Gamma x = d, s = d, x \geq 0\} \quad (9)$$

Let f denote the corresponding cost for link flow q . A link-based variational inequality formulation for (8) can be expressed as follows. Determine values $q \in K$ such that

$$f^t(q)(z - q) \geq 0 \quad (10)$$

for all $z \in K$.

2.6 A MPEC programme

Optimal policy settings for a three-tiered supply chain equilibrium network (5) can be formulated as the following MEPC program.

$$\underset{\beta, q}{Min} \quad \Theta_0(\beta, q) \quad (11)$$

$$\text{subject to } \beta \in \Omega, q \in S(\beta)$$

where Ω denotes the domain set of the decision variables of the policy settings which are determined at management level, and $S(\cdot)$ denotes the solution set of equilibrium flows which is determined at operations level in a three-tiered supply chain network, which can be solved as follows.

$$f^t(\beta, q)(z - q) \geq 0 \quad (12)$$

for all $z \in K$.

2.7 Sensitivity analysis by directional derivatives at operations level

Following the technique employed (Qiu & Magnanti, 1989), the sensitivity analysis of (12) at operations level in a three-tiered supply chain network can be established in the following way. Let the changes in link or path flows with respect to the changes in the policy settings

made at management level be denoted by q' or x' , the corresponding change in path flow cost be denoted by F' , and let the demand market price be denoted by μ . Introduce

$$K' = \{q' : \exists x' \text{ such that } q' = \Lambda x', \Gamma x' = 0, \text{ and } x' \in K_0\} \quad (13)$$

where

$$K_0 = \left\{ x' : \begin{array}{ll} (i).x' \text{ free,} & \text{if } x > 0 \\ (ii).x' = 0, & \text{if } F > \mu \\ (iii).x' = 0, & \text{if } F = \mu, \text{ and } x = 0 \text{ with } F' > 0 \\ (iv).x' > 0, & \text{if } F = \mu, \text{ and } x = 0 \text{ with } F' \leq 0 \end{array} \right\} \quad (14)$$

Therefore the directional derivatives of (12) can be obtained by solving the following affine variational inequality. Find $q' \in K'$,

$$(\nabla_{\beta} f(\beta, q)\beta' + \nabla_q f(\beta, q)q')(z - q') \geq 0 \quad (15)$$

for all $z \in K'$ where $\nabla_{\beta} f$ and $\nabla_q f$ are gradients evaluated at (β, q) when the changes in the policy settings made at management level are specified. According to Rademacher's theorem (Clarke, 1980) in (11) the solution set $S(\cdot)$ is differentiable almost everywhere. Thus, the generalized gradient for $S(\cdot)$ can be denoted as follows.

$$\partial S(\beta^*) = \text{conv} \left\{ q'(\beta^*) = \lim_{k \rightarrow \infty} \nabla q(\beta^k) : \beta^k \rightarrow \beta^*, \nabla q(\beta^k) \text{ exists} \right\} \quad (16)$$

where conv denotes the convex hull.

2.8 A one level mathematical program

At the management level, suppose strong regularity condition (Robinson, 1980) holds at the variable of interests with respect to the policy interventions, due to inequality (15) a one level MPEC program can be established in the following way. Suppose the solution set $S(\cdot)$ is locally Lipschitz, a one level optimization problem of (11) is to

$$\underset{\beta}{Min} \quad \Theta(\beta) \quad (17)$$

subject to $\beta \in \Omega$

In problem (17), as it seen obviously from literature (Dempe, 2002; Luo et al., 1996), $\Theta(\cdot)$ function is a non-smooth and non-convex function with respect to the policy settings determined at management level in a three-tiered supply chain network because the solution set of equilibrium flow $S(\cdot)$ at operations level may not be explicitly expressed as a closed form.

3. A non-smooth optimization model

Due to non-differentiability of the solution set $S(\cdot)$ in (17), in this section, we propose an optimal solution scheme using a non-smooth approach for the three-tiered supply chain network problem (17). In the following we suppose that the objective function $\Theta(\cdot)$ is semi-smooth and locally Lipschitz. Therefore the directional derivatives of $\Theta(\cdot)$ can be characterized by the generalized gradient, which are also specified as follows.

Definition 3 <Semi-smoothness, adapted from Mifflin (1977)> We say that $\Theta(\cdot)$ is semismooth on set Ω if $\Theta(\cdot)$ is locally Lipschitz and the limit

$$\lim_{v \in \partial\Theta(\beta + t\bar{h}), \bar{h} \rightarrow h, t \downarrow 0} \{v\bar{h}\} \quad (18)$$

exists for all $\bar{h}$. $\square$

Theorem 4 <Directional derivatives for semismooth functions, adapted from Qi & Sun (1993)> Suppose that $\Theta(\cdot)$ is a locally Lipschitzian function and the directional derivative $\Theta'(\beta; h)$ exists for any direction h at β . Then

- (1). $\Theta'(\cdot; h)$ is Lipschitzian;
- (2). For any h , there exists a $v \in \partial\Theta(\beta)$ such that

$$\Theta'(\beta; h) = vh \quad (19)$$

$\square$

The generalized gradient of $\Theta(\cdot)$ can be expressed as follows.

$$\partial\Theta(\beta^*) = \text{conv} \left\{ \lim_{k \rightarrow \infty} \nabla\Theta(\beta^k) : \beta^k \rightarrow \beta^*, \nabla\Theta(\beta^k) \text{ exists} \right\} \quad (20)$$

According to Clarke (1980), the generalized gradient is a convex hull of all points of the form $\lim \nabla\Theta(\beta^k)$ where the subsequence $\{\beta^k\}$ converges to the limit value β^* . And the gradients in (20) evaluated at (β^k, q^k) can be expressed as follows.

$$\nabla\Theta(\beta^k) = \nabla_{\beta}\Theta_0(\beta^k, q^k) + \nabla_q\Theta_0(\beta^k, q^k)q'(\beta^k) \quad (21)$$

where the directional derivatives $q'(\beta^k)$ can be obtained from (15).

3.1 A subgradient projection solution scheme (SPSS)

Consider the non-smooth problem (17), a general solution by an iterative subgradient method can be expressed in the following manner. Let $\text{Pr}_{\Omega}(\beta) \in \Omega$ denote the projection of β on set Ω such that

$$\|x - \text{Pr}_{\Omega}(x)\| = \inf_{y \in \Omega} \|x - y\| \quad (22)$$

thus we have

$$\beta^{k+1} = \text{Pr}_{\Omega}(\beta^k - tv), v \in \partial\Theta(\beta^k) \quad (23)$$

and

$$t = \lambda \frac{\Theta(\beta^k) - \Theta(\beta^*)}{\|v\|^2}, v \in \partial\Theta(\beta^k), \quad 0 < a \leq \lambda \leq 2 - b, b > 0 \quad (24)$$

where the local minimum point β^* is supposed to be known and $\lambda = \frac{1}{k}$. Since the subgradient method is a non-descent method with slow convergence as commented and modified from literature, in this chapter, we are not going to investigate the details of these progress. On the other hand, a new globally convergent solution scheme for problem (17) is proposed via introducing a matrix in projecting the subgradient of the objective function onto a null space of active constraints in order to efficiently search for feasible points. In this proposed solution scheme, consecutive projections of the subgradient of the objective function help us dilate the direction provided by the negative of the subgradient which greatly improves the local solutions obtained. In the following, Rosen's gradient projection matrix is introduced first.

Definition 5. <Projection matrix> A $n \times n$ matrix G is called a projection matrix if $G = G^t$ and $GG = G$. $\square$

Thus the proposed Subgradient Projection Solution Scheme (SPSS) for the non-smooth problem (17) can be presented in the following way.

Theorem 6. <Subgradient Projection Solution Scheme> In problem (17), suppose $\Theta(\cdot)$ is lower semi-continuous on the domain set Ω . Given a β^1 such that $\Theta(\beta^1) = \alpha$, the level set $S_{\alpha}(\Omega) = \{\beta : \beta \in \Omega, \Theta(\beta) \leq \alpha\}$ is bounded and Θ is locally Lipschitzian and semi-smooth on the convex hull of S_{α} . A sequence of iterates $\{\beta^k\}$ can be generated in accordance with

$$\beta^{k+1} = \text{Pr}_{\Omega}(\beta^k - tG_k v^k), \quad v^k \in \partial\Theta(\beta^k) \quad (25)$$

where t is the step length which minimize Θ^k and the projection matrix G_k is of the following form.

$$G_k = I - M_k^t (M_k M_k^t)^{-1} M_k \quad (26)$$

In (26) M_k is the gradient of active constraints in (17) at β^k , where the active constraint gradients are linearly independent and thus M_k has full rank. The search direction h^k can be determined in the following form.

$$h^k = G_k v^k \quad (27)$$

Then the sequence of points $\{\beta^k\}$ generated by the SPSS approach is bounded whenever $G_k \nabla \Theta(\beta^k) \neq 0$.

Proof. For any x and y in the set Ω , by definition of the projection, we have

$$\|\text{Pr}_\Omega(x) - \text{Pr}_\Omega(y)\| \leq \|x - y\| \quad (28)$$

thus for β^{k+1} we have

$$\begin{aligned} \|\beta^{k+1} - \beta^*\|^2 &= \|\text{Pr}_\Omega(\beta^k - th^k) - \beta^*\|^2 \\ &\leq \|\beta^k - th^k - \beta^*\|^2 \\ &= \|\beta^k - \beta^*\|^2 + t^2 \|h^k\|^2 - 2t(\beta^k - \beta^*)^t h^k \end{aligned} \quad (29)$$

let

$$C = 2t(\beta^k - \beta^*)^t h^k - t^2 \|h^k\|^2 \quad (30)$$

then (29) can be rewritten as

$$\|\beta^{k+1} - \beta^*\|^2 \leq \|\beta^k - \beta^*\|^2 - C$$

Since Θ is locally Lipschitzian and semi-smooth on the convex hull of S_α , by convexity we have

$$(\beta^k - \beta^*)^t \nabla \Theta(\beta^k) \geq \Theta(\beta^k) - \Theta(\beta^*)$$

for any ε_1 and $\varepsilon_2 \in [0, 2]$ there exists λ such that $0 \leq \varepsilon_1 \leq \lambda \leq 2 - \varepsilon_2$, let

$$t = \lambda \frac{\Theta(\beta^k) - \Theta(\beta^*)}{G_k \|\nabla \Theta(\beta^k)\|^2} \quad (31)$$

In (30), it can be rewritten as

$$\begin{aligned}
 C &= 2\lambda \frac{\Theta(\beta^k) - \Theta(\beta^*)}{G_k \|\nabla \Theta(\beta^k)\|^2} (\beta^k - \beta^*)^t G_k \nabla \Theta(\beta^k) - \lambda^2 \left(\frac{\Theta(\beta^k) - \Theta(\beta^*)}{G_k \|\nabla \Theta(\beta^k)\|^2} \right)^2 \|G_k \nabla \Theta(\beta^k)\|^2 \\
 &\geq 2\lambda \frac{(\Theta(\beta^k) - \Theta(\beta^*))^2}{\|\nabla \Theta(\beta^k)\|^2} - \lambda^2 \frac{(\Theta(\beta^k) - \Theta(\beta^*))^2}{\|\nabla \Theta(\beta^k)\|^2} \quad (\text{due to convexity}) \\
 &= \left(\frac{\Theta(\beta^k) - \Theta(\beta^*)}{\|\nabla \Theta(\beta^k)\|} \right)^2 (2\lambda - \lambda^2) \geq 0
 \end{aligned}$$

thus we have $\|\beta^{k+1} - \beta^*\|^2 \leq \|\beta^k - \beta^*\|^2$ for $k = 1, 2, 3, \dots$. It implies $\|\beta^k - \beta^*\|$ is monotonically decreasing and $\|\beta^k - \beta^*\| \leq \|\beta^1 - \beta^*\|$. $\square$

Theorem 7. Following Theorem 6, when $G_k \nabla \Theta(\beta^k) = 0$, if all the Lagrange multipliers corresponding to the active constraint gradients in (17) are positive or zeros, it implies the current point is a Karush-Kuhn-Tucker (KKT) point. Otherwise choose one negative Lagrange multiplier, say η_j , and construct a new $\hat{M}_k$ of the active constraint gradients by deleting the j th row of $\hat{M}_k$, which corresponds to the negative component η_j , and make the projection matrix of the following form

$$\hat{G}_k = I - \hat{M}_k^t (\hat{M}_k \hat{M}_k^t)^{-1} \hat{M}_k \quad (32)$$

The search direction then can be determined by (27) and the results of Theorem 6 hold. $\square$

Theorem 8 <Convergence of SPSS> In problem (17) assuming that $\Theta(\cdot)$ is lower semi-continuous on the domain set Ω , given a β^1 such that $\Theta(\beta^1) = \alpha$, the level set $S_\alpha(\Omega) = \{\beta : \beta \in \Omega, \Theta(\beta) \leq \alpha\}$ is bounded and Θ is locally Lipschitzian and semi-smooth on the convex hull of S_α . Let $\{\beta^k\}$ be the sequence of points generated by the SPSS approach as described above. Then every accumulation point β^* satisfies

$$0 \in \partial \Theta(\beta^*) \quad (33)$$

Proof. We proof this theorem by contradiction. Supposing $0 \notin \partial \Theta(\beta^*)$, by definition there is no subgradient $\nabla \Theta(\beta^k) = 0$ in the convex hull of S_α , whose accumulation point is β^* . Then there is a $t^* > 0$ minimizing $\Theta(\beta^* - th^*)$ and a $\delta > 0$ such that

$\Theta(\beta^*) = \Theta(\beta^* - t^* h^*) + \delta$ and $\beta^* - t^* h^*$ is an interior point of S_α . By the mean value theorem, for any β^k we have

$$\Theta(\beta^k - t^* h^k) = \Theta(\beta^* - t^* h^*) + \nabla \Theta(\xi^k)^t (\beta^k - \beta^* - t^* (h^k - h^*)) \quad (34)$$

where $\xi^k = \beta^* - t^* h^* + \varepsilon (\beta^* - \beta^k - t^* (h^* - h^k))$ for some $0 < \varepsilon < 1$. Following the Bozano-Weierstrass theorem that there is a subsequence $\{\beta^{kn}\}$ of $\{\beta^k\}$ that converges to β^* , then $\{\nabla \Theta(\xi^{kn})\}$ converges to $\nabla \Theta(\beta^* - t^* h^*)$ and $\{\beta^{kn} - \beta^* - t^* (h^{kn} - h^*)\}$ converges to zero. For sufficiently large kn , the vector ξ^{kn} belongs to the convex hull of S_α and

$$\Theta(\beta^{kn} - t^* h^{kn}) \leq \Theta(\beta^* - t^* h^*) + \frac{\delta}{2} = \Theta(\beta^*) - \frac{\delta}{2} \quad (35)$$

Let t_{kn}^* be the minimizing point of $\Theta(\beta^{kn} - t_{kn} h^{kn})$. Since $\{\Theta(\beta^{kn})\}$ is monotone decreasing and converges to $\Theta(\beta^*)$, we have

$$\Theta(\beta^*) < \Theta(\beta^{kn} - t_{kn}^* h^{kn}) \leq \Theta(\beta^{kn} - t^* h^{kn}) \leq \Theta(\beta^*) - \frac{\delta}{2} \quad (36)$$

a contradiction. Therefore every accumulation point β^* satisfies $0 \in \partial \Theta(\beta^*)$. $\square$

Corollary 9 <Stopping condition> If β^k is a KKT point for problem (17) satisfying Theorem 8 then the search process may stop; otherwise a new search direction at β^k can be generated according to Theorem 6. $\square$

3.2 Implementation Steps

In this subsection, ways in solving the non-smooth problem (17) for a three-tiered supply chain network involving the management level and the operations level are conducted by steps in the following manner.

Step 1. At the management level, start with the initial policy setting β^k , and set index $k = 1$.

Step 2. At the operations level, solve a three-tiered supply chain equilibrium problem by means of (5) when the decision variables of policy β^k are specified at management level. Find the subgradients for equilibrium products and shipments by means of (15), and obtain the generalized gradient for the objective function of the supply chain network via (21).

Step 3. Use the SPSS approach to determine a search direction.

Step 4. If $G_k \nabla \Theta(\beta^k) \neq 0$, find a new β^{k+1} by means of (25) and let $k \leftarrow k + 1$. Go to Step 2.

If $G_k \nabla \Theta(\beta^k) = 0$ and all the Lagrange multipliers corresponding to the active constraint

gradients are positive or zeros, β^k is a KKT point and stop. Otherwise, follow the results of Theorem 6 and find a new projection matrix and go to Step 3.

4. Numerical calculations

In this section, we used a 9-node network from literature (Bergendorff et al., 1997) as an illustration for a three-tiered supply chain network problem with equilibrium flows. In Fig. 1, a three-tiered supply chain is considered in which there are two pairs of manufacturers and consumers, and 4 product-mix pairs: [1,3], [1,4], [2,3] and [2,4], can be accordingly specified. In Fig. 1, manufacturers are denoted by nodes 1 and 2, distributors are denoted by nodes 7 and 8, and the consumers are denoted by nodes 3 and 4. The corresponding demand functions can be determined in the following manner: $d_{1,3} = 10 - 0.5\mu_{1,3}$, $d_{1,4} = 20 - 0.5\mu_{1,4}$, $d_{2,3} = 30 - 0.5\mu_{2,3}$ and $d_{2,4} = 40 - 0.5\mu_{2,4}$. In this numerical illustration a new set of link tolls at the management level is to be determined optimally such that traffic congestion on the connected links between various distinct tiers can be consistently reduced. In Fig. 1 let A_a and k_a be given parameters and specified as a pair (A_a, k_a) near each link. The transaction costs on links are assumed in the following way.

$$t_a(q_a) = A_a(1 + 0.15(\frac{q_a}{k_a})^4) \quad (37)$$

Computational results are summarized in Table 1 for a comparative analysis at two distinct initial tolls. Three earlier well-known methods in solving the network design problem are also considered: the sensitivity analysis method (SAB) proposed by Yang & Yagar (1995), the Genetic Algorithm (GA) proposed by Ceylan & Bell (2004), and recently proposed Generalized Projected Subgradient (GPS) method by Chiou (2007). As it seen in Table 1, the SPSS approach improved the minimal toll revenue at two distinct initial tolls nearly by 18% and 16% while the SAB method only did by 8% and 6%. The SPSS approach successfully outperformed the GA method and newly proposed GPS method by 4% and 2% on average in reduction of minimal toll revenue. For two sets of initial tolls the relative difference of the minimal toll revenue did the SPSS is within 0.07 % while that did the SAB method is within nearly 0.3%. Regarding the efficiency of the SPSS approach in solving the three-tiered supply chain network with equilibrium flows when the toll settings are considered at management level, the SPSS approach required the least CPU time in all cases. Furthermore, as it obviously seen in Table 1, various sets of resulting tolls can be found due to the non-convexity of the MPEC problem. Computational efforts on all methods mentioned in this chapter were conducted on SUN SPARC SUNW, 900 MHZ processor with 4Gb RAM under operating system Unix SunOS 5.8 using C++ compiler gnu g++ 2.8.1.

Figure 1. 9-node supply chain network

5. Conclusions and discussions

This chapter addresses a new solution scheme for a three-tiered supply chain equilibrium network problem involving two-level kinds of decision makers. A MPEC program for the three-tiered supply chain network problem was established. In this chapter, from a non-smooth approach, firstly, we proposed a globally convergent SPSS approach to optimally solve the MPEC program. The first order sensitivity analysis for the three-tiered supply chain equilibrium network was conducted. Numerical computations using a 9-node supply chain network from literature were performed. Computational comparative analysis was also carried out at two sets of distinct initial data in comparison with earlier and recent proposed methods in solving the multi-tiered supply chain network problem. As it shown, the proposed SPSS approach consistently made significant improvements over other alternatives with far less computational efforts. Regarding near future work associated, a multi-tiered supply chain network optimization problem with multi-level decision makers is being investigated as well as implementations on large-scale supply chain networks.

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	1 st set initials				2 nd set initials			
	SAB	GA	GPS	SPSS	SAB	GA	GPS	SPSS
Initial toll (in \$)	2.0	2.0	2.0	2.0	5.5	5.5	5.5	5.5
Initial revenue (in \$)	1223	1223	1223	1223	1198	1198	1198	1198
$\beta_{(1,5)}$	0.5	2.9	0	0	2.5	3.2	0	0
$\beta_{(1,6)}$	0.3	1.2	0.5	1.2	3.2	3.0	0.4	2.1
$\beta_{(2,5)}$	1.2	0	1.6	1.2	1.2	1.2	1.8	1.5
$\beta_{(2,6)}$	0.4	0	1.4	0.4	0	0	1.5	0.9
$\beta_{(5,6)}$	0	2.5	0	2.1	0	0	0.4	1.8
$\beta_{(5,7)}$	8.6	7.6	5.6	7.8	6.3	1.6	5.2	7.5
$\beta_{(5,9)}$	0.7	1.4	3.7	0.7	2.7	0.3	3.5	0.5
$\beta_{(6,5)}$	0	2.8	0	0	0	0	0.2	0
$\beta_{(6,8)}$	1.3	0	3.2	1.3	0	0	3.1	1.1
$\beta_{(6,9)}$	0	0	0	0	1.5	2.2	0.9	0.5
$\beta_{(7,3)}$	0.6	1.7	0.6	0.6	1.2	1.2	0	0
$\beta_{(7,4)}$	0.4	1.8	0.2	0.4	0.4	3.4	0	0
$\beta_{(7,8)}$	0	0	1.2	0	1.2	1.1	1.1	0.8
$\beta_{(8,3)}$	0	0	1.1	0	1.1	0	1.1	0.7
$\beta_{(8,4)}$	0.6	1.2	0.6	1.8	0.6	0.6	0	1.8
$\beta_{(8,7)}$	0	0	2.2	0.6	0.8	0	2.2	0.6
$\beta_{(9,7)}$	0.9	1.5	0	1.9	0.9	2.7	1.2	2.4
$\beta_{(9,8)}$	0	0	3.2	0	1.3	1.8	3.5	0.8
Revenue (in \$)	1120.5	1048.9	1026.7	1005.5	1123.5	1050.7	1027.1	1004.8
cpu time (in sec)	132	258	84	7	154	263	85	6

Table 1. Computational results for 9-node supply chain network

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Traditionally supply chain management has meant factories, assembly lines, warehouses, transportation vehicles, and time sheets. Modern supply chain management is a highly complex, multidimensional problem set with virtually endless number of variables for optimization. An Internet enabled supply chain may have just-in-time delivery, precise inventory visibility, and up-to-the-minute distribution-tracking capabilities. Technology advances have enabled supply chains to become strategic weapons that can help avoid disasters, lower costs, and make money. From internal enterprise processes to external business transactions with suppliers, transporters, channels and end-users marks the wide range of challenges researchers have to handle. The aim of this book is at revealing and illustrating this diversity in terms of scientific and theoretical fundamentals, prevailing concepts as well as current practical applications.

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