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A New BEM for Modeling and Optimization of 3T Fractional Nonlinear Generalized Magneto-Thermoelastic Multi-Material ISMFGA Structures Subjected to Moving Heat Source

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Abstract

The main purpose of this chapter, which represents one of the chapters of a fractal analysis book, is to propose a new boundary element method (BEM) formulation based on time fractional order theory of thermoelasticity for modeling and optimization of three temperature (3T) multi-material initially stressed multilayered functionally graded anisotropic (ISMFGA) structures subjected to moving heat source. Fractional order derivative considered in the current chapter has been found to be an accurate mathematical tool for solving the difficulty of our physical and numerical modeling. Furthermore, this chapter shed light on the practical application aspects of boundary element method analysis and topology optimization of fractional order thermoelastic ISMFGA structures. Numerical examples based on the multi-material topology optimization algorithm and bi-evolutionary structural optimization method (BESO) are presented to study the effects of fractional order parameter on the optimal design of thermoelastic ISMFGA structures. The numerical results are depicted graphically to show the effects of fractional order parameter on the sensitivities of total temperature, displacement components and thermal stress components. The numerical results also show the effects of fractional order parameter on the final topology of the ISMFGA structures and demonstrate the validity and accuracy of our proposed technique.

Keywords: boundary element method, modeling and optimization, time fractional order, three-temperature, nonlinear generalized thermoelasticity, initially stressed multilayered functionally graded anisotropic structures, moving heat source

1. Introduction

The fractional calculus has recently been widely used to study the theory and applications of derivatives and integrals of arbitrary non-integer order. This branch

of mathematical analysis has emerged in recent years as an effective and powerful tool for the mathematical modeling of various engineering, industrial, and materials science applications [1–3]. The fractional-order operators are useful in describing the memory and hereditary properties of various materials and processes, due to their nonlocal nature. It clearly reflects from the related literature produced by leading fractional calculus journals that the primary focus of the investigation had shifted from classical integer-order models to fractional order models [4, 5]. Fractional calculus has important applications in hereditary solid mechanics, fluid dynamics, viscoelasticity, heat conduction modeling and identification, biology, food engineering, econophysics, biophysics, biochemistry, robotics and control theory, signal and image processing, electronics, electric circuits, wave propagation, nanotechnology, etc. [6–8].

Numerous mathematicians have contributed to the history of fractional calculus, where Euler mentioned interpolating between integral orders of a derivative in 1730. Then, Laplace defined a fractional derivative by means of an integral in 1812.

Lacroix introduced the first fractional order derivative which appeared in a calculus in 1819, where he expressed the n th derivative of the function $y = x^m$ as follows:

$$\frac{d^n}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n} \quad (1)$$

Liouville assumed that $\frac{d^v}{dx^v}(e^{ax}) = a^v e^{ax}$ for $v > 0$ to obtain the following fractional order derivative:

$$\frac{d^v x^{-a}}{dx^v} = (-1)^v \frac{\Gamma(a+v)}{\Gamma(a)} x^{-a-v} \quad (2)$$

Laurent has been using the Cauchy's integral formula for complex valued analytical functions to define the integration of arbitrary order $v > 0$ as follows:

$${}_c D_x^v f(x) = {}_c D_x^{m-\rho} f(x) = \frac{d^m}{dx^m} \left[\frac{1}{\Gamma(\rho)} \int_c^x (x-t)^{\rho-1} f(t) dt \right], 0 < \rho \leq 1 \quad (3)$$

where ${}_c D_x^v$ denotes differentiation of order v of the function f along the x -axis. Cauchy introduced the following fractional order derivative:

$$f_+^{(\alpha)} = \int f(\tau) \frac{(t-\tau)^{\alpha-1}}{\Gamma(-\alpha)} d\tau \quad (4)$$

Caputo introduced his fractional derivative of order $\alpha < 0$ to be defined as follows:

$$D_*^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\alpha+1-m}} d\tau, m-1 < \alpha < m, \alpha > 0 \quad (5)$$

Recently, research on nonlinear generalized magneto-thermoelastic problems has received wide attention due to its practical applications in various fields such as geomechanics, geophysics, petroleum and mineral prospecting, earthquake engineering, astronautics, oceanology, aeronautics, materials science, fiber-optic communication, fluid mechanics, automobile industries, aircraft, space vehicles, plasma

physics, nuclear reactors, and other industrial applications. Due to computational difficulties in solving nonlinear generalized magneto-thermoelastic problems in general analytically, many numerical techniques have been developed and implemented for solving such problems [9–17]. The boundary element method (BEM) [18–31] has been recognized as an attractive alternative numerical method to domain methods [32–36] like finite difference method (FDM), finite element method (FEM), and finite volume method (FVM) in engineering applications. The superior feature of BEM over domain methods is that only the boundary of the domain needs to be discretized, which often leads to fewer elements and easier to use. This advantage of BEM over other domain methods has significant importance for modeling and optimization of thermoelastic problems which can be implemented using BEM with little cost and less input data. Nowadays, the BEM has emerged as an accurate and efficient computational technique for solving complicated inhomogeneous and non-linear problems in physical and engineering applications [37–69].

In the present chapter, we introduce a practical engineering application of fractal analysis in the field of thermoelasticity, where the thermal field is described by time fractional three-temperature radiative heat conduction equations. Fractional order derivative considered in the current chapter has high ability to remove the difficulty of our numerical modeling. A new boundary element method for modeling and optimization of 3T fractional order nonlinear generalized thermoelastic multi-material initially stressed multilayered functionally graded anisotropic (ISMFGA) structures subjected to moving heat source is investigated. Numerical results show that the fractional order parameter has a significant effect on the sensitivities of displacements, total three-temperature, and thermal stresses. Numerical examples show that the fractional order parameter has a significant effect on the final topology of ISMFGA structures. Numerical results of the proposed model confirm the validity and accuracy of the proposed technique, and numerical examples results demonstrate the validity of the BESO multi-material topology optimization method.

A brief summary of the chapter is as follows: Section 1 introduces the background and provides the readers with the necessary information to books and articles for a better understanding of fractional order problems and their applications. Section 2 describes the physical modeling of fractional order problems in three-temperature nonlinear generalized magneto-thermoelastic ISMFGA structures. Section 3 outlines the BEM implementation for modeling of 3T fractional nonlinear generalized magneto-thermoelastic problems of multi-material ISMFGA structures subjected to moving heat source. Section 4 introduces an illustration of the mechanisms of solving design sensitivities and optimization problem of the current chapter. Section 5 presents the new numerical results that describe the effects of fractional order parameter on the problem's field variations and on the final topology of multi-material ISMFGA structures.

2. Formulation of the problem

Consider a multilayered structure with n functionally graded layers in the xy -plane of a Cartesian coordinate. The x -axis is the common normal to all layers as shown in **Figure 1**. The thickness of the layer is denoted by h . The considered multilayered structure has been placed in a primary magnetic field H_0 acting in the direction of the y -axis.

Figure 1.
Geometry of the considered problem.

According to the three-temperature theory, the governing equations of nonlinear generalized magneto-thermoelasticity in an initially stressed multilayered functionally graded anisotropic (ISMFGA) structure for the i th layer can be written in the following form:

$$\sigma_{ab,b} + \tau_{ab,b} - \Gamma_{ab} = \rho^i(x+1)^m \ddot{u}_a^i \quad (6)$$

$$\sigma_{ab} = (x+1)^m \left[C_{abfg}^i u_{fg}^i - \beta_{ab}^i (T_\alpha^i - T_{\alpha 0}^i + \tau_1 \dot{T}_\alpha^i) \right] \quad (7)$$

$$\tau_{ab} = \mu^i(x+1)^m \left(\tilde{h}_a H_b + \tilde{h}_b H_a - \delta_{ba} (\tilde{h}_f H_f) \right) \quad (8)$$

$$\Gamma_{ab} = P^i(x+1)^m \left(\frac{\partial u_a^i}{\partial x_b} - \frac{\partial u_b^i}{\partial x_a} \right) \quad (9)$$

According to Fahmy [10], the time fractional order two-dimensional three-temperature (2D-3 T) radiative heat conduction equations in nondimensionless form can be expressed as follows:

$$D_\tau^\alpha T_\alpha^i(r, \tau) = \xi \nabla [\mathbb{K}_\alpha^i \nabla T_\alpha^i(r, \tau)] + \xi \overline{\overline{W}}(r, \tau), \quad \xi = \frac{1}{c_{sa}^i \rho^i \delta_1} \quad (10)$$

where

$$\overline{\overline{W}}(r, \tau) = \begin{cases} -\rho^i \mathbb{W}_{eI} (T_e^i - T_I^i) - \rho^i \mathbb{W}_{er} (T_e^i - T_p^i) + \overline{\overline{W}}, & \alpha = e, \delta_1 = 1 \\ \rho^i \mathbb{W}_{eI} (T_e^i - T_I^i) + \overline{\overline{W}}, & \alpha = I, \delta_1 = 1 \\ \rho^i \mathbb{W}_{er} (T_e^i - T_p^i) + \overline{\overline{W}}, & \alpha = p, \delta_1 = T_p^3 \end{cases} \quad (11)$$

in which

$$\overline{\overline{W}}(r, \tau) = -\mathbb{K}_\alpha^i \dot{T}_{\alpha,ab}^i + \beta_{ab}^i T_{a0}^i \tau_0 \ddot{u}_{a,b}^i + \rho^i c_{sa}^i \tau_0 \ddot{T}_\alpha^i - Q(x, \tau) \quad (12)$$

$$\mathbb{W}_{eI} = \rho^i \mathbb{A}_{eI} T_e^{i(-\frac{2}{3})}, \mathbb{W}_{er} = \rho^i \mathbb{A}_{er} T_e^{i(-\frac{1}{2})}, \mathbb{K}_\alpha = \mathbb{A}_\alpha T_\alpha^{i(\frac{2}{3})}, \alpha = e, I, \mathbb{K}_p = \mathbb{A}_p T_p^{i(3+\mathbb{B})} \quad (13)$$

The total energy of unit mass can be described by

$$P = P_e + P_I + p_p, P_e = c_e T_e^i, P_I = c_I T_I^i, P_p = \frac{1}{4} c_p T_p^{4i} \quad (14)$$

where σ_{ab} , τ_{ab} , and u_k^i are mechanical stress tensor, Maxwell's electromagnetic stress tensor, and displacement vector in the i th layer, respectively, c_α ($\alpha = c, I, p$) are constant $T_{\alpha 0}^i$, T_α^i , C_{abfg}^i , and β_{ab}^i are, respectively, reference temperature, temperature, constant elastic moduli, and stress-temperature coefficients in the i th layer: μ^i , $\tilde{h}$, P^i , ρ^i , and $c_{s\alpha}^i$ are, respectively, magnetic permeability, perturbed magnetic field, initial stress, density, isochore specific heat coefficients in the i th layer; τ is the time; τ_0 and τ_1 are the relaxation times; $i = 1, 2, \dots, n$ represents the parameters in multilayered structure; and m is a functionally graded parameter. Also, we considered in the current study that the medium is subjected to a moving heat source of constant strength moving along x -axis with a constant velocity v . This moving heat source is assumed to have the following form:

$$Q(x, \tau) = Q_0 \delta(x - v\tau) \quad (15)$$

where, Q_0 is the heat source strength and δ is the delta function.

3. BEM numerical implementation

By using Eqs. (7)–(9), we can write (6) as

$$L_{gb} u_f^i = \rho^i \ddot{u}_a^i - \left(D_a T_\alpha^i - P^i \left(\frac{\partial u_b^i}{\partial x_a} - \frac{\partial u_a^i}{\partial x_b} \right) \right) = f_{gb} \quad (16)$$

where inertia term, temperature gradient, and initial stress terms are treated as the body forces.

In this section, we are interested in using a boundary element method for modeling the two-dimensional three-temperature radiation heat conduction equations coupled with electron, ion, and phonon temperatures.

According to finite difference scheme of Caputo at times $(f + 1)\Delta t$ and $f\Delta\tau$, we obtain [1].

$$D_\tau^\alpha T_\alpha^{i(f+1)} + D_\tau^\alpha T_\alpha^{i(f)} \approx \sum_{j=0}^k W_{aj} \left(T_\alpha^{i(f+1-j)}(r) - T_\alpha^{i(f-j)}(r) \right) \quad (17)$$

where

$$W_{a,0} = \frac{(\Delta\tau)^{-a}}{\Gamma(2-a)}, W_{aj} = W_{a,0} \left((j+1)^{1-a} - (j-1)^{1-a} \right) \quad (18)$$

Based on Eq. (17), the fractional order heat Eq. (10) can be replaced by the following system:

$$\begin{aligned} W_{a,0} T_\alpha^{i(f+1)}(r) - \mathbb{K}_\alpha(x) T_{\alpha,II}^{i(f+1)}(r) - \mathbb{K}_{\alpha,I}(x) T_{\alpha,I}^{i(f+1)}(r) &= W_{a,0} T_\alpha^{i(f)}(r) - \mathbb{K}_\alpha(x) T_{\alpha,II}^{i(f)}(r) \\ - \mathbb{K}_{\alpha,I}(x) T_{\alpha,I}^{i(f)}(r) - \sum_{j=1}^f W_{aj} \left(T_\alpha^{i(f+1-j)}(r) - T_\alpha^{i(f-j)}(r) \right) &+ \overline{W}_m^{i(f+1)}(x, \tau) + \overline{W}_m^{i(f)}(x, \tau) \end{aligned} \quad (19)$$

where $j = 1, 2, \dots, F, f = 0, 1, 2, \dots, F$.

Now, according to Fahmy [10], and applying the fundamental solution which satisfies (19), the boundary integral equations corresponding to (10) without heat sources can be expressed as

$$T_\alpha^i(\xi) = \int_S [T_\alpha^i q^{i*} - T_\alpha^{i*} q^i] dC - \int_R f_{ab} T_\alpha^{i*} dR \quad (20)$$

Thus, the governing equations can be written in operator form as follows:

$$L_{gb} u_f^i = f_{gb}, \quad (21)$$

$$L_{ab} T_\alpha^i = f_{ab} \quad (22)$$

where the operators L_{gb} , f_{gb} , L_{ab} , and f_{ab} are as follows:

$$L_{gb} D_{abf} \frac{\partial}{\partial x_b} + D_{af} + \Lambda D_{a1f}, \quad L_{ab} = D_\tau^a \quad (23)$$

$$f_{gb} = \rho^i \ddot{u}_a^i - \left(D_a T_\alpha^i - P^i \left(\frac{\partial u_b^i}{\partial x_a} - \frac{\partial u_a^i}{\partial x_b} \right) \right) \quad (24)$$

$$f_{ab} = \frac{\mathbb{K}_\alpha}{D} T_\alpha^i \frac{\partial}{\partial \tau} \quad (25)$$

where

$$D_{abf} = C_{abfg} \varepsilon, \quad \varepsilon = \frac{\partial}{\partial x_g}, \quad D_{af} = \mu H_0^2 \left(\frac{\partial}{\partial x_a} + \delta_{a1} \Lambda \right) \frac{\partial}{\partial x_f},$$

$$D_a = -\beta_{ab}^i \left(\frac{\partial}{\partial x_b} + \delta_{b1} \Lambda + \tau_1 \left(\frac{\partial}{\partial x_b} + \Lambda \right) \frac{\partial}{\partial \tau} \right), \quad \Lambda = \frac{m}{x+1}.$$

The differential Eq. (21) can be solved using the weighted residual method (WRM) to obtain the following integral equation:

$$\int_R (L_{gb} u_f^i - f_{gb}) u_{da}^{i*} dR = 0 \quad (26)$$

Now, the fundamental solution u_{df}^{i*} and traction vectors t_{da}^{i*} and t_a^i can be written as follows:

$$L_{gb} u_{df}^{i*} = -\delta_{ad} \delta(x, \xi) \quad (27)$$

$$t_{da}^{i*} = C_{abfg} u_{df,g}^{i*} n_b \quad (28)$$

$$t_a^i = \frac{\bar{t}_a^i}{(x+1)^m} = \left(C_{abfg} u_{f,g}^i - \beta_{ab}^i (T_\alpha^i + \tau_1 T_\alpha^i) \right) n_b \quad (29)$$

Using integration by parts and sifting property of the Dirac distribution for (26), then using Eqs. (27) and (29), we can write the following elastic integral representation formula:

$$u_d^i(\xi) = \int_C (u_{da}^{i*} t_a^i - t_{da}^{i*} u_a^i + u_{da}^{i*} \beta_{ab}^i T_\alpha^i n_b) dC - \int_R f_{gb} u_{da}^{i*} dR \quad (30)$$

The fundamental solution T^{i*} can be defined as

$$L_{ab}T_{\alpha}^{i*} = -\delta(x, \xi) \quad (31)$$

By using WRM and integration by parts, we can write (23) as follows:

$$\int_R (L_{ab}T_{\alpha}^iT_{\alpha}^{i*} - L_{ab}T_{\alpha}^{i*}T_{\alpha}^i)dR = \int_C (q^{i*}T_{\alpha}^i - q^iT_{\alpha}^{i*})dC \quad (32)$$

where

$$q^i = -\mathbb{K}_{\alpha}^iT_{\alpha,b}^in_a \quad (33)$$

$$q^{i*} = -\mathbb{K}_{\alpha}^iT_{\alpha,b}^{i*}n_a \quad (34)$$

By the use of sifting property, we obtain from (32) the thermal integral representation formula

$$T_{\alpha}^i(\xi) = \int_C (q^{i*}T_{\alpha}^i - q^iT_{\alpha}^{i*})dC - \int_C f_{ab}T_{\alpha}^{i*}dR \quad (35)$$

By combining (30) and (35), we obtain

$$\begin{aligned} \begin{bmatrix} u_d^i(\xi) \\ T_{\alpha}^i(\xi) \end{bmatrix} &= \int_C \left\{ - \begin{bmatrix} t_{da}^{i*} & -u_{da}^{i*}\beta_{ab}n_b \\ 0 & -q^{i*} \end{bmatrix} \begin{bmatrix} u_a^i \\ T_{\alpha}^i \end{bmatrix} + \begin{bmatrix} u_{da}^{i*} & 0 \\ 0 & -T_{\alpha}^{i*} \end{bmatrix} \begin{bmatrix} t_a^i \\ q^i \end{bmatrix} \right\} dC \\ &\quad - \int_R \begin{bmatrix} u_{da}^{i*} & 0 \\ 0 & -T_{\alpha}^{i*} \end{bmatrix} \begin{bmatrix} f_{gb} \\ -f_{ab} \end{bmatrix} dR \end{aligned} \quad (36)$$

The nonlinear generalized magneto-thermoelastic vectors can be written in contracted notation form as

$$U_A^i = \begin{cases} u_a^i & a = A = 1, 2, 3 \\ T_{\alpha}^i & A = 4 \end{cases} \quad (37)$$

$$T_{\alpha A}^i = \begin{cases} t_a^i & a = A = 1, 2, 3 \\ q^i & A = 4 \end{cases} \quad (38)$$

$$U_{DA}^{i*} = \begin{cases} u_{da}^{i*} & d = D = 1, 2, 3; a = A = 1, 2, 3 \\ 0 & d = D = 1, 2, 3; A = 4 \\ 0 & D = 4; a = A = 1, 2, 3 \\ -T_{\alpha}^{i*} & D = 4; A = 4 \end{cases} \quad (39)$$

$$\tilde{T}_{\alpha DA}^{i*} = \begin{cases} t_{da}^{i*} & d = D = 1, 2, 3; a = A = 1, 2, 3 \\ -\tilde{u}_d^{i*} & d = D = 1, 2, 3; A = 4 \\ 0 & D = 4; a = A = 1, 2, 3 \\ -q^{i*} & D = 4; A = 4 \end{cases} \quad (40)$$

$$\tilde{u}_d^{i*} = u_{da}^{i*}\beta_{af}n_f \quad (41)$$

By using the above vectors, we can express (36) as

$$U_D^i(\xi) = \int_C \left(U_{DA}^{i*} T_{\alpha A}^i - \tilde{T}_{\alpha DA}^i U_A^i \right) dC - \int_R U_{DA}^{i*} S_A dR \quad (42)$$

The source vector S_A can be divided as

$$S_A = S_A^0 + S_A^T + S_A^u + S_A^{\dot{T}} + S_A^{\ddot{T}} + S_A^{\ddot{u}} \quad (43)$$

where

$$S_A^0 = \begin{cases} 0 & A = 1, 2, 3 \\ Q_0 \delta(x - v\tau) & A = 4 \end{cases} \quad (44)$$

$$S_A^T = \omega_{AF} U_F^i \text{ with } \omega_{AF} = \begin{cases} -D_a & A = 1, 2, 3; F = 4 \\ \xi \nabla [\mathbb{K}_\alpha^i] & \text{otherwise} \end{cases} \quad (45)$$

$$S_A^u = \psi U_F^i \text{ with } \psi = \begin{cases} P^i \left(\frac{\partial}{\partial x_b} - \frac{\partial}{\partial x_a} \right) & A = 1, 2, 3; F = 1, 2, 3, \\ 0 & A = 4; F = 4 \end{cases} \quad (46)$$

$$S_A^{\dot{T}} = \Gamma_{AF} \dot{U}_F^i \text{ with } \Gamma_{AF} = \begin{cases} -\beta_{ab}^i \tau_1 \left(\frac{\partial}{\partial x_b} + \Lambda \right) \frac{\partial}{\partial \tau} & A = 4; F = 4 \\ -\mathbb{K}_\alpha^i & \text{otherwise} \end{cases} \quad (47)$$

$$S_A^{\ddot{T}} = \delta_{AF} \ddot{U}_F^i \text{ with } \delta_{AF} = \begin{cases} 0 & A = 4; F = 4 \\ \rho^i c_{sa}^i \tau_0 & \text{otherwise} \end{cases} \quad (48)$$

$$S_A^{\ddot{u}} = \mathfrak{J} \ddot{U}_F^i \text{ with } \mathfrak{J} = \begin{cases} \rho^i & A = 1, 2, 3; F = 1, 2, 3, \\ \beta_{ab}^i T_{\alpha 0}^i \tau_0 & A = 4; F = 4 \end{cases} \quad (49)$$

The representation formula (36) can also be written in matrix form as follows:

$$[S_A] = - \begin{bmatrix} 0 \\ Q_0 \delta(x - v\tau) \end{bmatrix} + \begin{bmatrix} -D_a T_\alpha^i \\ \xi \nabla [\mathbb{K}_\alpha^i \nabla T_\alpha^i(r, \tau)] \end{bmatrix} + \begin{bmatrix} P^i (u_{b,a}^i - u_{a,b}^i) \\ 0 \end{bmatrix} \\ + \begin{bmatrix} -\beta_{ab}^i \tau_1 \left(\frac{\partial}{\partial x_b} + \Lambda \right) \dot{T}_\alpha^i \\ -\mathbb{K}_\alpha^i \dot{T}_\alpha^i \end{bmatrix} + \rho^i c_{sa}^i \tau_0 \begin{bmatrix} 0 \\ \ddot{T}_\alpha^i \end{bmatrix} + \begin{bmatrix} \rho^i \ddot{u}_a^i \\ \beta_{ab}^i T_{\alpha 0}^i \tau_0 \ddot{u}_{f,g}^i \end{bmatrix} \quad (50)$$

In order to convert the domain integral in (42) into the boundary, we approximate the source vector S_A by a series of known functions f_{AE}^q and unknown coefficients α_E^q as

$$S_A \approx \sum_{q=1}^E f_{AE}^q \alpha_E^q \quad (51)$$

Thus, the representation formula (42) can be written as follows:

$$U_D(\xi) = \int_C \left(U_{DA}^{i*} T_{\alpha A}^i - \tilde{T}_{\alpha DA}^{i*} U_A^i \right) dC - \sum_{q=1}^N \int_R U_{DA}^{i*} f_{AE}^q dR \alpha_E^q \quad (52)$$

By implementing the WRM to the following equations

$$L_{gb}u_{fe}^{iq} = f_{ae}^q \quad (53)$$

$$L_{ab}T_{\alpha}^{iq} = f_{pj}^q \quad (54)$$

Then the elastic and thermal representation formulae are given as follows (Fahmy [46]):

$$u_{de}^{iq}(\xi) = \int_C (u_{da}^{i*} t_{ae}^{iq} - t_{da}^{i*} u_{ae}^{iq}) dC - \int_R u_{da}^{i*} f_{ae}^q dR \quad (55)$$

$$T_{\alpha}^{iq}(\xi) = \int_C (q^{i*} T_{\alpha}^{iq} - q^{iq} T_{\alpha}^{i*}) dC - \int_R f^q T_{\alpha}^{i*} dR \quad (56)$$

The representation formulae (55) and (56) can be combined into the following single equation:

$$U_{DE}^{iq}(\xi) = \int_C (U_{DA}^{i*} T_{\alpha AE}^{iq} - T_{\alpha DA}^{i*} U_{AE}^{iq}) dC - \int_R U_{DA}^{i*} f_{AE}^q dR \quad (57)$$

By substituting from Eq. (57) into Eq. (52), we obtain the following BEM coupled thermoelasticity formula:

$$U_D^i(\xi) = \int_C (U_{DA}^{i*} T_{\alpha A}^i - \tilde{T}_{\alpha DA}^{i*} U_A^i) dC + \sum_{q=1}^E \left(U_{DE}^{iq}(\xi) + \int_C (T_{\alpha DA}^{i*} U_{AE}^{iq} - U_{DA}^{i*} T_{\alpha AE}^{iq}) dC \right) \alpha_E^q \quad (58)$$

In order to compute the displacement sensitivity, Eq. (58) is differentiated with respect to ξ_l as follows:

$$\frac{\partial U_D^i(\xi)}{\partial \xi_l} = - \int_C (U_{DA,l}^{i*} T_{\alpha A}^i - \tilde{T}_{\alpha DA,l}^{i*} U_A^i) dC + \sum_{q=1}^E \left(\frac{\partial U_{DE}^{iq}(\xi)}{\partial \xi_l} - \int_C (T_{\alpha DA,l}^{i*} U_{AE}^{iq} - U_{DA,l}^{i*} T_{\alpha AE}^{iq}) dC \right) \alpha_E^q \quad (59)$$

According to the procedure of Fahmy [44], we can write (58) in the following form:

$$\check{\zeta} U - \eta T_{\alpha} = (\check{\zeta} \check{U} - \eta \check{\phi}) \bar{\alpha} \quad (60)$$

The generalized displacements and velocities are approximated in terms of known tensor functions f_{FD}^q and unknown coefficients γ_D^q and $\tilde{\gamma}_D^q$:

$$U_F^i \approx \sum_{q=1}^N f_{FD}^q(x) \gamma_D^q \quad (61)$$

where

$$f_{FD}^q = \begin{cases} f_{fd}^q & f = F = 1, 2, 3; d = D = 1, 2, 3 \\ f^q & F = 4; D = 4 \\ 0 & \text{otherwise} \end{cases} \quad (62)$$

Now, the gradients of the generalized displacements and velocities can also be approximated in terms of the tensor function derivatives as

$$U_{F,g}^i \approx \sum_{q=1}^N f_{FD,g}^q(x) \gamma_K^q \quad (63)$$

By substituting (63) into Eq. (45), we get

$$S_A^T = \sum_{q=1}^N S_{AF} f_{FD,g}^q \gamma_D^q \quad (64)$$

By applying the point collocation procedure of Gaul et al. [43] to Eqs. (51) and (61), we obtain

$$\check{S} = J\bar{\alpha}, \quad U^i = J'\gamma, \quad (65)$$

Similarly, applying the same point collocation procedure to Eqs. (64), (46), (47), (48), and (49) yields

$$\check{S}^{Ti} = \mathcal{B}^T \gamma \quad (66)$$

$$S_A^u = \psi U^i \quad (67)$$

$$\check{S}^{Ti} = \bar{\Gamma}_{AF} \dot{U}^i \quad (68)$$

$$\check{S}^{Ti} = \bar{\delta}_{AF} \ddot{U}^i \quad (69)$$

$$\check{S}^{\ddot{u}} = \bar{\Xi} \ddot{U}^i \quad (70)$$

where $\bar{\psi}$, $\bar{\Gamma}_{AF}$, $\bar{\delta}_{AF}$, and $\bar{\Xi}$ are assembled using the submatrices $[\psi]$, $[\Gamma_{AF}]$, $[\delta_{AF}]$, and $[\Xi]$, respectively.

Solving the system (65) for $\bar{\alpha}$ and γ yields

$$\bar{\alpha} = J^{-1} \check{S}, \quad \gamma = J'^{-1} U^i \quad (71)$$

Now, the coefficient $\bar{\alpha}$ can be written in terms of the unknown displacements U^i , velocities $\dot{U}^i$, and accelerations $\ddot{U}^i$ as

$$\bar{\alpha} = J^{-1} \left(\check{S}^0 + \left(\mathcal{B}^T J'^{-1} + \bar{\psi} \right) U^i + \bar{\Gamma}_{AF} \dot{U}^i + (\bar{\Xi} + \bar{\delta}_{AF}) \ddot{U}^i \right) \quad (72)$$

An implicit-implicit staggered algorithm has been implemented for use with the BEM to solve the governing equations which can now be written in a suitable form after substitution of Eq. (72) into Eq. (60) as

$$\widehat{M} \ddot{U}^i + \widehat{\Gamma} \dot{U}^i + \widehat{K} U^i = \widehat{Q}^i \quad (73)$$

$$\widehat{X} \ddot{T}_\alpha^i + \widehat{A} \dot{T}_\alpha^i + \widehat{B} T_\alpha^i = \widehat{Z} \ddot{U}^i + \widehat{R} \quad (74)$$

where $V = (\eta \check{\phi} - \zeta \check{U})J^{-1}$, $\widehat{M} = V(\bar{\Xi} + \bar{\delta}_{AF})$, $\widehat{\Gamma} = V\bar{\Gamma}_{AF}$,
 $\widehat{K} = -\check{\zeta} + V(\mathcal{B}^T J'^{-1} + \bar{\psi})$, $\widehat{Q}^i = -\eta \check{T} + V\check{S}^0$, $\widehat{X} = -\rho^i c_{s\alpha}^i \tau_0$, $\widehat{A} = -\mathbb{K}_\alpha^i$,
 $\widehat{B} = \xi \nabla [\mathbb{K}_\alpha^i \nabla]$, $\widehat{Z} = \beta_{ab}^i T_{\alpha 0}^i \tau_0$, $\widehat{R} = -Q_0 \delta(x - v\tau)$.
 where $\ddot{U}^i, \dot{U}^i, U^i, T^i$ and $\widehat{Q}^i$ are, respectively, acceleration, velocity, displacement, temperature, and external force vectors, and $V, \widehat{M}, \widehat{\Gamma}, \widehat{K}, \widehat{A}$, and $\widehat{B}$ are, respectively, volume, mass, damping, stiffness, capacity, and conductivity matrices.

In many applications, the coupling term $\widehat{Z} \ddot{U}_{n+1}^i$ that appear in the heat conduction equation is negligible. Therefore, it is easier to predict the temperature than the displacement.

Hence Eqs. (73) and (74) lead to the following coupled system of differential-algebraic equations (DAEs):

$$\widehat{M} \ddot{U}_{n+1}^i + \widehat{\Gamma} \dot{U}_{n+1}^i + \widehat{K} U_{n+1}^i = \widehat{Q}_{n+1}^{ip} \quad (75)$$

$$\widehat{X} \ddot{T}_{\alpha(n+1)}^i + \widehat{A} \dot{T}_{\alpha(n+1)}^i + \widehat{B} T_{\alpha(n+1)}^i = \widehat{Z} \ddot{U}_{n+1}^i + \widehat{R} \quad (76)$$

where $\widehat{Q}_{n+1}^{ip} = \eta T_{\alpha(n+1)}^{ip} + V\check{S}^0$ and $T_{\alpha(n+1)}^{ip}$ is the predicted temperature.
 By integrating Eq. (73) and using Eq. (75), we get

$$\dot{U}_{n+1}^i = \dot{U}_n^i + \frac{\Delta\tau}{2} (\ddot{U}_{n+1}^i + \ddot{U}_n^i) \quad (77)$$

$$= \dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \widehat{M}^{-1} \left(\widehat{Q}_{n+1}^{ip} - \widehat{\Gamma} \dot{U}_{n+1}^i - \widehat{K} U_{n+1}^i \right) \right]$$

$$U_{n+1}^i = U_n^i + \frac{\Delta\tau}{2} (\dot{U}_{n+1}^i + \dot{U}_n^i)$$

$$= U_n^i + \Delta\tau \dot{U}_n^i + \frac{\Delta\tau^2}{4} \left[\ddot{U}_n^i + \widehat{M}^{-1} \left(\widehat{Q}_{n+1}^{ip} - \widehat{\Gamma} \dot{U}_{n+1}^i - \widehat{K} U_{n+1}^i \right) \right] \quad (78)$$

From Eq. (77) we have

$$\dot{U}_{n+1}^i = \bar{\gamma}^{-1} \left[\dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \widehat{M}^{-1} \left(\widehat{Q}_{n+1}^{ip} - \widehat{K} U_{n+1}^i \right) \right] \right] \quad (79)$$

where $\bar{\gamma} = \left(I + \frac{\Delta\tau}{2} \widehat{M}^{-1} \widehat{\Gamma} \right)$.

Substituting Eq. (79) into Eq. (78), we derive

$$U_{n+1}^i = U_n^i + \Delta\tau \dot{U}_n^i + \frac{\Delta\tau^2}{4} \left[\ddot{U}_n^i + \widehat{M}^{-1} \left(\widehat{\mathbb{Q}}_{n+1}^{ip} - \widehat{\Gamma} \bar{\gamma}^{-1} \left[\dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \widehat{M}^{-1} \left(\widehat{\mathbb{Q}}_{n+1}^{ip} - \widehat{K} U_{n+1}^i \right) \right] \right] - \widehat{K} U_{n+1}^i \right) \right] \quad (80)$$

Substituting $\dot{U}_{n+1}^i$ from Eq. (79) into Eq. (75), we obtain

$$\ddot{U}_{n+1}^i = \widehat{M}^{-1} \left[\widehat{\mathbb{Q}}_{n+1}^{ip} - \widehat{\Gamma} \left[\bar{\gamma}^{-1} \left[\dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \widehat{M}^{-1} \left(\widehat{\mathbb{Q}}_{n+1}^{ip} - \widehat{K} U_{n+1}^i \right) \right] \right] \right] - \widehat{K} U_{n+1}^i \right] \quad (81)$$

By integrating the heat Eq. (74) and using Eq. (76), we obtain

$$\begin{aligned} \dot{T}_{\alpha(n+1)}^i &= \dot{T}_n^i + \frac{\Delta\tau}{2} \left(\ddot{T}_{\alpha(n+1)}^i + \ddot{T}_{an}^i \right) \\ &= \dot{T}_{an}^i + \frac{\Delta\tau}{2} \left(\widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{A} \dot{T}_{\alpha(n+1)}^i - \widehat{B} T_{\alpha(n+1)}^i \right] + \ddot{T}_{an}^i \right) \end{aligned} \quad (82)$$

$$\begin{aligned} T_{\alpha(n+1)}^i &= T_{an}^i + \frac{\Delta\tau}{2} \left(\dot{T}_{\alpha(n+1)}^i + \dot{T}_{an}^i \right) \\ &= T_{an}^i + \Delta\tau \dot{T}_{an}^i + \frac{\Delta\tau^2}{4} \left(\ddot{T}_{an}^i + \widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{A} \dot{T}_{\alpha(n+1)}^i - \widehat{B} T_{\alpha(n+1)}^i \right] \right) \end{aligned} \quad (83)$$

From Eq. (82) we get

$$\dot{T}_{\alpha(n+1)}^i = \gamma^{-1} \left[\dot{T}_{an}^i + \frac{\Delta\tau}{2} \left(\widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{B} T_{\alpha(n+1)}^i \right] + \ddot{T}_{an}^i \right) \right] \quad (84)$$

where $\gamma = \left(I + \frac{1}{2} \widehat{A} \Delta\tau \widehat{X}^{-1} \right)$.

Substituting Eq. (84) into Eq. (83), we obtain

$$\begin{aligned} T_{\alpha(n+1)}^i &= T_{an}^i + \Delta\tau \dot{T}_{an}^i + \frac{\Delta\tau^2}{4} \left(\ddot{T}_{an}^i + \widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{A} \left(\gamma^{-1} \left[\dot{T}_{an}^i + \frac{\Delta\tau}{2} \left(\widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{B} T_{\alpha(n+1)}^i \right] + \ddot{T}_{an}^i \right) \right] \right) - \widehat{B} T_{\alpha(n+1)}^i \right] \right) \end{aligned} \quad (85)$$

Substituting $\dot{T}_{n+1}^i$ from Eq. (84) into Eq. (76), we get

$$\begin{aligned} \ddot{T}_{\alpha(n+1)}^i &= \widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{A} \left(\gamma^{-1} \left[\dot{T}_{an}^i + \frac{\Delta\tau}{2} \left(\widehat{X}^{-1} \left[\widehat{\mathbb{Z}} \ddot{U}_{n+1}^i + \widehat{\mathbb{R}} - \widehat{B} T_{\alpha(n+1)}^i \right] + \ddot{T}_{an}^i \right) \right] \right) - \widehat{B} T_{\alpha(n+1)}^i \right] \end{aligned} \quad (86)$$

Now, a displacement predicted staggered procedure for the solution of (80) and (85) is as follows:

The first step is to predict the propagation of the displacement wave field: $U_{n+1}^{ip} = U_n^i$. The second step is to substitute for $\dot{U}_{n+1}^i$ and $\ddot{U}_{n+1}^i$ from Eqs. (77) and (75), respectively, in Eq. (85) and solve the resulted equation for the three-temperature fields. The third step is to correct the displacement using the computed three-temperature fields for the Eq. (80). The fourth step is to compute $\dot{U}_{n+1}^i$, $\ddot{U}_{n+1}^i$, $\dot{T}_{\alpha(n+1)}^i$, and $\ddot{T}_{\alpha(n+1)}^i$ from Eqs. (79), (81), (82), and (86), respectively.

The continuity conditions for temperature, heat flux, displacement, and traction that have been considered in the current chapter can be expressed as

$$T_{\alpha}^i(x, z, \tau)|_{x=h^i} = T_{\alpha}^{(i+1)}(x, z, \tau)|_{x=h^i} \quad (87)$$

$$q^i(x, z, \tau)|_{x=h^i} = q^{(i+1)}(x, z, \tau)|_{x=h^i} \quad (88)$$

$$u_f^i(x, z, \tau)|_{x=h^i} = u_f^{(i+1)}(x, z, \tau)|_{x=h^i} \quad (89)$$

$$\bar{t}_a^i(x, z, \tau)|_{x=h^i} = \bar{t}_a^{(i+1)}(x, z, \tau)|_{x=h^i} \quad (90)$$

where n is the total number of layers, $\bar{t}_a$ are the tractions which is defined by $\bar{t}_a = \sigma_{ab}n_b$, and $i = 1, 2, \dots, n - 1$.

The initial and boundary conditions of the present study are

$$u_f^i(x, z, 0) = \dot{u}_f^i(x, z, 0) = 0 \quad \text{for } (x, z) \in R \cup C \quad (91)$$

$$u_f^i(x, z, \tau) = \Psi_f(x, z, \tau) \quad \text{for } (x, z) \in C_3 \quad (92)$$

$$\bar{t}_a^i(x, z, \tau) = \Phi_f(x, z, \tau) \quad \text{for } (x, z) \in C_4, \tau > 0 \quad (93)$$

$$T_{\alpha}^i(x, z, 0) = \dot{T}_{\alpha}^i(x, z, 0) = 0 \quad \text{for } (x, z) \in R \cup C \quad (94)$$

$$T_{\alpha}^i(x, z, \tau) = \bar{f}(x, z, \tau) \quad \text{for } (x, z) \in C_1, \tau > 0 \quad (95)$$

$$q^i(x, z, \tau) = \bar{h}(x, z, \tau) \quad \text{for } (x, z) \in C_2, \tau > 0 \quad (96)$$

where Ψ_f , Φ_f , f , and $\bar{h}$ are prescribed functions, $C = C_1 \cup C_2 = C_3 \cup C_4$, and $C_1 \cap C_2 = C_3 \cap C_4 = \emptyset$.

4. Design sensitivity and optimization

According to Fahmy [58, 60], the design sensitivities of displacements components and total 3T can be performed by implicit differentiation of (75) and (76), respectively, which describe the structural response with respect to the design variables, and then we can compute thermal stresses sensitivities.

The bi-directional evolutionary structural optimization (BESO) is the evolutionary topology optimization method that allows modification of the structure by either adding or removing material to or from the structure design. This addition or removal depends on the sensitivity analysis. Sensitivity analysis is the estimation of the response of the structure to the modification of design variables and is dependent on the calculation of derivatives [70–80].

The homogenized vector of thermal expansion coefficients α^H can be written in terms of the homogenized elasticity matrix D^H and the homogenized vector of stress-temperature coefficients β^H as follows:

$$\alpha^H = (D^H)^{-1} \beta^H \quad (97)$$

For the material design, the derivative of the homogenized vector of thermal expansion coefficients can be written as

$$\frac{\partial \alpha^H}{\partial X_{kl}^m} = (D^H)^{-1} \left(\frac{\partial \beta^H}{\partial X_{kl}^m} - \frac{\partial D^H}{\partial X_{kl}^m} \alpha^H \right) \quad (98)$$

where $\frac{\partial D^H}{\partial X_{kl}^m}$ and $\frac{\partial \beta^H}{\partial X_{kl}^m}$ for any l th material phase can be calculated using the adjoint variable method [73] as

$$\frac{\partial D^H}{\partial X_{kl}^m} = \frac{1}{|\Omega|} \int_Y (I - B^m U^m)^T \frac{\partial D^m}{\partial X_{kl}^m} (I - B^m U^m) dy \quad (99)$$

and

$$\frac{\partial \beta^H}{\partial X_{kl}^m} = \frac{1}{|Y|} \int_Y (I - B^m U^m)^T \frac{\partial D^m}{\partial X_{kl}^m} (\alpha^m - B^m \varphi^m) dy + \frac{1}{|\Omega|} \int_Y (I - B^m U^m)^T \frac{\partial \alpha^m}{\partial X_{kl}^m} dy \quad (100)$$

where $|\Omega|$ is the volume of the base cell.

5. Numerical examples, results, and discussion

In order to show the numerical results of this study, we consider a monoclinic graphite-epoxy as an anisotropic thermoelastic material which has the following physical constants [57].

The elasticity tensor is expressed as

$$C_{ijkl} = \begin{bmatrix} 430.1 & 130.4 & 18.2 & 0 & 0 & 201.3 \\ 130.4 & 116.7 & 21.0 & 0 & 0 & 70.1 \\ 18.2 & 21.0 & 73.6 & 0 & 0 & 2.4 \\ 0 & 0 & 0 & 19.8 & -8.0 & 0 \\ 0 & 0 & 0 & -8.0 & 29.1 & 0 \\ 201.3 & 70.1 & 2.4 & 0 & 0 & 147.3 \end{bmatrix} \text{ GPa} \quad (101)$$

The mechanical temperature coefficient is

$$\beta_{pj} = \begin{bmatrix} 1.01 & 2.00 & 0 \\ 2.00 & 1.48 & 0 \\ 0 & 0 & 7.52 \end{bmatrix} \cdot 10^6 \frac{\text{N}}{\text{Km}^2} \quad (102)$$

The thermal conductivity tensor is

$$k_{pj} = \begin{bmatrix} 5.2 & 0 & 0 \\ 0 & 7.6 & 0 \\ 0 & 0 & 38.3 \end{bmatrix} \text{ W/Km} \tag{103}$$

Mass density $\rho = 7820\text{kg/m}^3$ and heat capacity $c = 461 \text{ J/kg K}$.

The proposed technique that has been utilized in the present chapter can be applicable to a wide range of three-temperature nonlinear generalized thermoelastic problems of ISMFGA structures. The main aim of this chapter was to assess the impact of fractional order parameter on the sensitivities of total three-temperature, displacement components, and thermal stress components.

Figure 2 shows the variation of the total temperature sensitivity along the x -axis. It was shown from this figure that the fraction order parameter has great effects on the total three-temperature sensitivity.

Figures 3 and 4 show the variation of the displacement components u_1 and u_2 along the x -axis for different values of fractional order parameter. It was noticed from these figures that the fractional order parameter has great effects on the displacement sensitivities.

Figures 5–7 show the variation of the thermal stress components σ_{11} , σ_{12} , and σ_{22} , respectively, along the x -axis for different values of fractional order parameter. It was noted from these figures that the fractional order parameter has great influences on the thermal stress sensitivities.

Since there are no available results for the three-temperature thermoelastic problems, except for Fahmy’s research [10–14]. For comparison purposes with the special cases of other methods treated by other authors, we only considered one-dimensional numerical results of the considered problem. In the special case under consideration,

Figure 2.
Variation of the total 3T sensitivity along x-axis.

Figure 3.
Variation of the displacement u_1 sensitivity along x-axis.

Figure 4.
Variation of the displacement u_2 sensitivity along x-axis.

the displacement u_1 and thermal stress σ_{11} results are plotted in **Figures 8** and **9**. The validity and accuracy of our proposed BEM technique were demonstrated by comparing our BEM results with the FEM results of Xiong and Tian [81], it can be noticed that the BEM results are found to agree very well with the FEM results.

Example 1. Short cantilever beam.

The mean compliance has been minimized, to obtain the maximum stiffness, when the structure is subjected to moving heat source. In this example, we consider a short cantilever beam shown in **Figure 10**, where the BESO final topology of considered short cantilever beam shown in **Figure 11a** for $\alpha = 0.5$ and shown in **Figure 11b** for $\alpha = 1.0$. It is noticed from this figure that the fractional order parameter has a significant effect on the final topology of the multi-material ISMFGA structure.

Figure 5.
Variation of the thermal stress σ_{11} sensitivity along x-axis.

Figure 6.
Variation of the thermal stress σ_{12} sensitivity along x-axis.

Example 2. MBB beam.

It is known that extraordinary thermo-mechanical properties can be accomplished by combining more than two materials phases with conventional materials [75]. For this reason, it is essential that the topology optimization strategy permits more than two materials phases within the design domain. In this example, we consider a MBB beam shown in **Figure 12**, where the BESO final topology of MBB beam has been shown in **Figure 13a** for $\alpha = 0.5$ and shown in **Figure 13b** for $\alpha = 1.0$ to show the effect of fractional order parameter on the final topology of the multi-material ISMFGA structure.

Example 3. Roller-supported beam.

Figure 7.
Variation of the thermal stress σ_{22} sensitivity along x-axis.

Figure 8.
Variation of the displacement u_1 sensitivity along x-axis.

In this example, we consider a roller-supported beam shown in **Figure 14**, where the BESO final topology of a roller-supported beam shown in **Figure 15a** for $\alpha = 0.5$ and shown in **Figure 15b** for $\alpha = 1.0$.

Example 4. Cantilever beam (validation example).

In order to demonstrate the validity of our implemented BESO topology optimization technique, we consider isotropic case of a cantilever beam shown in **Figure 16** as a special case of our anisotropic study to interpolate the elasticity matrix and the stress-temperature coefficients using the design variables X^M , then we compare our

Figure 9.
Variation of the thermal stress σ_{11} waves along x-axis.

Figure 10.
Design domain of a short cantilever beam.

Figure 11.
The final topology of a short cantilever beam: (a) $\alpha = 0.5$ and (b) $\alpha = 1.0$.

Figure 12.
Design domain of a MBB beam.

Figure 13.
The final topology of MBB beam: (a) $\alpha = 0.5$ and (b) $\alpha = 1.0$.

Figure 14.
Design domain of a roller-supported beam.

Figure 15.
The final topology of a multi-material roller-supported beam: (a) $\alpha = 0.5$ and (b) $\alpha = 1.0$.

Figure 16.
Design domain of a cantilever beam.

BESO final topology shown in **Figure 17a** with the material interpolation scheme of the solid isotropic material with penalization (SIMP) shown in **Figure 17b**.
The BESO topology optimization problem implemented in Examples 1 and 4, to find the distribution of the M material phases, with the volume constraint can be stated as

Figure 17.
The final topology of a cantilever beam: (a) MMA and (b) BESO.

Find X^M
That minimize $C^M = \frac{1}{2} (P^M)^T u^M = \frac{1}{2} \left(f^{M,ter} + f^{M,mec} \right)^T u^M$
Subject to $V^{M,*} - \sum_{i=1}^N V_i^M X_i^M = 0$

$$K^M u^M = P^M$$
$$X_i^M = x_{min} V1$$

where X^M is the design variable; C^M is the mean compliance; P is the total load on the structure, which is the sum of mechanical and thermal loads; u^M is the displacement vector; $V^{M,*}$ is the volume of the solid material; N is the total number of elements; K^M is the global stiffness matrix; x_{min} is a small value (e.g., 0.0001), which it guarantee that none of the elements will be removed completely from design domain; $f^{M,mec}$ is the mechanical load vector; and $f^{M,ter}$ is the thermal load vector. Also, the BESO parameters considered in Examples 1–4 can be seen in **Tables 1–4**, respectively.

The BESO topology optimization problem implemented in Examples 2 and 3, to find the distribution of the two materials in the design domain, which minimize the compliance of the structure, subject to a volume constraint in both phases can be stated as.

Find X^M
That minimize $C^M = \frac{1}{2} (P^M)^T u^M = \frac{1}{2} \left(f^{M,ter} + f^{M,mec} \right)^T u^M$
Subject to $V_j^{M,*} - \sum_{i=1}^N V_i^M X_{ij}^M - \sum_{i=1}^{j-1} V_i^{M,*} = 0; j = 1, 2$

$$K^M u^M = P^M$$
$$X_i^M = x_{min} V1; j = 1, 2$$

Variable name	Variable description	Variable value
V_f^M	Final volume fraction	0.5
ER^M	Evolutionary ratio	1%
AR_{max}^M	Volume addition ratio	5%
r_{min}^M	Filter ratio	3 mm
τ	Convergence tolerance	0.1%
N	Convergence parameter	5

Table 1.
BESO parameters for minimization of a short cantilever beam.

Variable name	Variable description	Variable value
V_{f1}^M	Final volume fraction of the material 1 for both interpolations	0.10
V_{f2}^M	Final volume fraction of the material 2 for both interpolations	0.20
ER^M	Evolutionary ratio for interpolation 1	2%
ER^M	Evolutionary ratio for interpolation 2	3%
AR_{max}^M	Volume addition ratio for interpolation 1	3%
AR_{max}^M	Volume addition ratio for interpolation 2	2%
r_{min}^M	Filter ratio for interpolation 1	4 mm
r_{min}^M	Filter ratio for interpolation 2	3 mm
τ	Convergence tolerance for both interpolations	0.01%
N	Convergence parameter for both interpolations	5

Table 2.
Multi-material BESO parameters for minimization of a MBB beam.

Variable name	Variable description	Variable value
V_{f1}^M	Final volume fraction of the material 1 for both interpolations	0.25
V_{f1}^M	Final volume fraction of the material 2 for both interpolations	0.25
ER^M	Evolutionary ratio for interpolation 1	3%
ER^M	Evolutionary ratio for interpolation 2	3%
AR_{max}^M	Volume addition ratio for interpolation 1	1%
AR_{max}^M	Volume addition ratio for interpolation 2	1%
r_{min}^M	Filter ratio for interpolation 1	4 mm
r_{min}^M	Filter ratio for interpolation 2	4 mm
τ	Convergence tolerance for both interpolations	0.5 %
N	Convergence parameter for both interpolations	5

Table 3.
Multi-material BESO parameters for minimization of a roller-supported beam.

Variable name	Variable description	Variable value
V_f^M	Final volume fraction	0.4
ER^M	Evolutionary ratio	1.2%
AR_{max}^M	Volume addition ratio	3%
r_{min}^M	Filter ratio	0.19 mm
τ	Convergence tolerance	0.1%
N	Convergence parameter	5

Table 4.
BESO parameters for minimization of a cantilever beam.

where $V_j^{M^*}$ is the volume of j th material phase and i and j denote the element i th which is made of j th material.

6. Conclusion

The main purpose of this chapter is to describe a new boundary element formulation for modeling and optimization of 3T time fractional order nonlinear generalized thermoelastic multi-material ISMFGA structures subjected to moving heat source, where we used the three-temperature nonlinear radiative heat conduction equations combined with electron, ion, and phonon temperatures.

Numerical results show the influence of fractional order parameter on the sensitivities of the study's fields. The validity of the present method is examined and demonstrated by comparing the obtained outcomes with those known in the literature. Because there are no available data to confirm the validity and accuracy of our proposed technique, we replace the three-temperature radiative heat conduction with one-temperature heat conduction as a special case from our current general study of three-temperature nonlinear generalized thermoelasticity. In the considered special case of 3T time fractional order nonlinear generalized thermoelastic multi-material ISMFGA structures, the BEM results have been compared graphically with the FEM results; it can be noticed that the BEM results are in excellent agreement with the FEM results. These results thus demonstrate the validity and accuracy of our proposed technique. Numerical examples are solved using the multi-material topology optimization algorithm based on the bi-evolutionary structural optimization method (BESO). Numerical results of these examples show that the fractional order parameter affects the final result of optimization. The implemented optimization algorithm has proven to be an appropriate computational tool for material design.

Nowadays, the knowledge of 3T fractional order optimization of multi-material ISMFGA structures, can be utilized by mechanical engineers for designing heat exchangers, semiconductor nano materials, thermoelastic actuators, shape memory actuators, bimetallic valves and boiler tubes. As well as for chemists to observe the chemical processes such as bond breaking and bond forming.

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References

- [1] Cattaneo C. Sur une forme de l'équation de la chaleur éliminant le paradoxe d'une propagation instantanée. *Comptes Rendus de l'Académie des Sciences*. 1958;**247**:431-433
- [2] Oldham KB, Spanier J. *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*. Mineola: Dover Publication; 2006
- [3] Podlubny I. *Fractional Differential Equations*. San Diego, California, USA: Academic Press; 1999
- [4] Ezzat MA, El Karamany AS, Fayik MA. Fractional order theory in thermoelastic solid with three-phase lag heat transfer. *Archive of Applied Mechanics*. 2012;**82**:557-572
- [5] Kilbas AA, Srivastava HM, Trujillo JJ. *Theory and Applications of Fractional Differential Equations*, vol. 204 of North-Holland Mathematics Studies. Amsterdam, The Netherlands: Elsevier Science; 2006
- [6] Sabatier J, Agrawal OP, Machado JAT. *Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering*. Dordrecht, The Netherlands: Springer; 2007
- [7] Ezzat MA, El-Bary AA. Application of fractional order theory of magneto-thermoelasticity to an infinite perfect conducting body with a cylindrical cavity. *Microsystem Technologies*. 2017; **23**:2447-2458
- [8] Soukkou A, Belhour MC, Leulmi S. Review, design, optimization and stability analysis of fractional-order PID controller. *International Journal of Intelligent Systems Technologies and Applications*. 2016;**8**:73-96
- [9] El-Naggar AM, Abd-Alla AM, Fahmy MA. The propagation of thermal stresses in an infinite elastic slab. *Applied Mathematics and Computation*. 2003;**12**:220-226
- [10] Fahmy MA. A new boundary element strategy for modeling and simulation of three temperatures nonlinear generalized micropolar-magneto-thermoelastic wave propagation problems in FGA structures. *Engineering Analysis with Boundary Elements*. 2019;**108**:192-200
- [11] Fahmy MA. A new computerized boundary element model for three-temperature nonlinear generalized thermoelastic stresses in anisotropic circular cylindrical plate structures. In: Awrejcewicz J, Grzelczyk D, editors. *Dynamical Systems Theory*. London, UK: IntechOpen; 2019. pp. 1-17
- [12] Fahmy MA. Boundary element model for nonlinear fractional-order heat transfer in magneto-thermoelastic FGA structures involving three temperatures. In: Ebrahimi F, editor. *Mechanics of Functionally Graded Materials and Structures*. London, UK: IntechOpen; 2019. pp. 1-22
- [13] Fahmy MA. Boundary element mathematical modelling and boundary element numerical techniques for optimization of micropolar thermoviscoelastic problems in solid deformable bodies. In: Sivasankaran S, Nayak PK, Günay E, editors. *Mechanics of Solid Deformable Bodies*. London, UK: IntechOpen; 2020. pp. 1-21
- [14] Fahmy MA. Boundary element modeling and optimization based on fractional-order derivative for nonlinear generalized photo-thermoelastic stress wave propagation in three-temperature anisotropic semiconductor structures. In: Sadollah A, Sinha TS, editors. *Recent Trends in Computational Intelligence*. London, UK: IntechOpen; 2020. pp. 1-16

- [15] Abd-Alla AM, El-Naggar AM, Fahmy MA. Magneto-thermoelastic problem in non-homogeneous isotropic cylinder. *Heat and Mass Transfer*. 2003; **39**:625-629
- [16] Hu Q, Zhao L. Domain decomposition preconditioners for the system generated by discontinuous Galerkin discretization of 2D-3T heat conduction equations. *Communications in Computational Physics*. 2017; **22**: 1069-1100
- [17] Sharma N, Mahapatra TR, Panda SK. Thermoacoustic behavior of laminated composite curved panels using higher-order finite-boundary element model. *International Journal of Applied Mechanics*. 2018; **10**:1850017
- [18] Fahmy MA. A time-stepping DRBEM for magneto-thermo-viscoelastic interactions in a rotating nonhomogeneous anisotropic solid. *International Journal of Applied Mechanics*. 2011; **3**:1-24
- [19] Fahmy MA. A time-stepping DRBEM for the transient magneto-thermo-visco-elastic stresses in a rotating non-homogeneous anisotropic solid. *Engineering Analysis with Boundary Elements*. 2012; **36**:335-345
- [20] Fahmy MA. Numerical modeling of transient magneto-thermo-viscoelastic waves in a rotating nonhomogeneous anisotropic solid under initial stress. *International Journal of Modeling, Simulation and Scientific Computing*. 2012; **3**:1250002
- [21] Fahmy MA. Transient magneto-thermo-viscoelastic stresses in a rotating nonhomogeneous anisotropic solid with and without a moving heat source. *Journal of Engineering Physics and Thermophysics*. 2012; **85**:950-958
- [22] Fahmy MA. Transient magneto-thermo-elastic stresses in an anisotropic viscoelastic solid with and without moving heat source. *Numerical Heat Transfer, Part A: Applications*. 2012; **61**: 547-564
- [23] Fahmy MA. Transient magneto-thermoviscoelastic plane waves in a non-homogeneous anisotropic thick strip subjected to a moving heat source. *Applied Mathematical Modelling*. 2012; **36**:4565-4578
- [24] Fahmy MA. The effect of rotation and inhomogeneity on the transient magneto-thermoviscoelastic stresses in an anisotropic solid. *ASME Journal of Applied Mechanics*. 2012; **79**:1015
- [25] Fahmy MA. Computerized Boundary Element Solutions for Thermoelastic Problems: Applications to Functionally Graded Anisotropic Structures. Saarbrücken: LAP Lambert Academic Publishing; 2017
- [26] Fahmy MA. Boundary Element Computation of Shape Sensitivity and Optimization: Applications to Functionally Graded Anisotropic Structures. Saarbrücken: LAP Lambert Academic Publishing; 2017
- [27] Fahmy MA. A time-stepping DRBEM for 3D anisotropic functionally graded piezoelectric structures under the influence of gravitational waves. In: *Proceedings of the 1st GeoMEast International Congress and Exhibition (GeoMEast 2017)*; 15–19 July 2017; Sharm El Sheikh, Egypt. Facing the Challenges in Structural Engineering, Sustainable Civil Infrastructures. 2017. pp. 350-365
- [28] Fahmy MA. 3D DRBEM modeling for rotating initially stressed anisotropic functionally graded piezoelectric plates. In: *Proceedings of the 7th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2016)*; 5–10 June 2016; Crete Island, Greece. 2016. pp. 7640-7658
- [29] Fahmy MA. Boundary element solution of 2D coupled problem in

- anisotropic piezoelectric FGM plates. In: Proceedings of the 6th International Conference on Computational Methods for Coupled Problems in Science and Engineering (Coupled Problems 2015); 18–20 May 2015; Venice, Italy. 2015. pp. 382-391
- [30] Fahmy MA. The DRBEM solution of the generalized magneto-thermo-viscoelastic problems in 3D anisotropic functionally graded solids. In: Proceedings of the 5th International Conference on Coupled Problems in Science and Engineering (Coupled Problems 2013); 17–19 June 2013; Ibiza, Spain. 2013. pp. 862-872
- [31] Fahmy MA. A computerized boundary element model for simulation and optimization of fractional-order three temperatures nonlinear generalized piezothermoelastic problems based on genetic algorithm. In: AIP Conference Proceedings 2138 of Innovation and Analytics Conference and Exhibition (IACE 2019); 25-28 March 2019; Sintok, Malaysia. 2019. p. 030015
- [32] Soliman AH, Fahmy MA. Range of applying the boundary condition at fluid/porous Interface and evaluation of beavers and Joseph's slip coefficient using finite element method. *Computation*. 2020;8:14
- [33] Eskandari AH, Baghani M, Sohrabpour S. A time-dependent finite element formulation for thick shape memory polymer beams considering shear effects. *International Journal of Applied Mechanics*. 2019;10:1850043
- [34] Huang R, Zheng SJ, Liu ZS, Ng TY. Recent advances of the constitutive models of smart materials—Hydrogels and shape memory polymers. *International Journal of Applied Mechanics*. 2020;12:2050014
- [35] Othman MIA, Khan A, Jahangir R, Jahangir A. Analysis on plane waves through magneto-thermoelastic microstretch rotating medium with temperature dependent elastic properties. *Applied Mathematical Modelling*. 2019;65:535-548
- [36] El-Naggar AM, Abd-Alla AM, Fahmy MA, Ahmed SM. Thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder. *Heat and Mass Transfer*. 2002;39:41-46
- [37] Fahmy MAA. New computerized boundary element algorithm for cancer modeling of cardiac anisotropy on the ECG simulation. *Asian Journal of Research in Computer Science*. 2018;2: 1-10
- [38] Brebbia CA, Telles JCF, Wrobel L. *Boundary Element Techniques in Engineering*. New York: Springer-Verlag; 1984
- [39] Wrobel LC, Brebbia CA. The dual reciprocity boundary element formulation for nonlinear diffusion problems. *Computer Methods in Applied Mechanics and Engineering*. 1987;65:147-164
- [40] Partridge PW, Wrobel LC. The dual reciprocity boundary element method for spontaneous ignition. *International Journal for Numerical Methods in Engineering*. 1990;30:953-963
- [41] Partridge PW, Brebbia CA. Computer implementation of the BEM dual reciprocity method for the solution of general field equations. *Communications in Applied Numerical Methods*. 1990;6:83-92
- [42] Partridge PW, Brebbia CA, Wrobel LC. *The Dual Reciprocity Boundary Element Method*. Southampton: Computational Mechanics Publications; 1992
- [43] Gaul L, Kögl M, Wagner M. *Boundary Element Methods for*

Engineers and Scientists. Berlin:
 Springer-Verlag; 2003

- [44] Fahmy MA. Implicit-explicit time integration DRBEM for generalized magneto-thermoelasticity problems of rotating anisotropic viscoelastic functionally graded solids. *Engineering Analysis with Boundary Elements*. 2013; **37**:107-115
- [45] Fahmy MA. Generalized magneto-thermo-viscoelastic problems of rotating functionally graded anisotropic plates by the dual reciprocity boundary element method. *Journal of Thermal Stresses*. 2013; **36**:1-20
- [46] Fahmy MA. A three-dimensional generalized magneto-thermo-viscoelastic problem of a rotating functionally graded anisotropic solids with and without energy dissipation. *Numerical Heat Transfer, Part A: Applications*. 2013; **63**:713-733
- [47] Fahmy MA. A 2-D DRBEM for generalized magneto-thermo-viscoelastic transient response of rotating functionally graded anisotropic thick strip. *International Journal of Engineering and Technology Innovation*. 2013; **3**:70-85
- [48] Fahmy MA, Salem AM, Metwally MS, Rashid MM. Computer implementation of the DRBEM for studying the generalized Thermoelastic responses of functionally graded anisotropic rotating plates with one relaxation time. *International Journal of Applied Science and Technology*. 2013; **3**:130-140
- [49] Fahmy MA, Salem AM, Metwally MS, Rashid MM. Computer implementation of the DRBEM for studying the classical uncoupled theory of thermoelasticity of functionally graded anisotropic rotating plates. *International Journal of Engineering Research and Applications*. 2013; **3**: 1146-1154
- [50] Fahmy MA. A computerized DRBEM model for generalized magneto-thermo-visco-elastic stress waves in functionally graded anisotropic thin film/substrate structures. *Latin American Journal of Solids and Structures*. 2014; **11**:386-409
- [51] Fahmy MA, Salem AM, Metwally MS, Rashid MM. Computer implementation of the DRBEM for studying the classical coupled thermoelastic responses of functionally graded anisotropic plates. *Physical Science International Journal*. 2014; **4**: 674-685
- [52] Fahmy MA, Salem AM, Metwally MS, Rashid MM. Computer implementation of the DRBEM for studying the generalized thermo elastic responses of functionally graded anisotropic rotating plates with two relaxation times. *British Journal of Mathematics & Computer Science*. 2014; **4**:1010-1026
- [53] Fahmy MA. A 2D time domain DRBEM computer model for magneto-thermoelastic coupled wave propagation problems. *International Journal of Engineering and Technology Innovation*. 2014; **4**:138-151
- [54] Fahmy MA, Al-Harbi SM, Al-Harbi BH. Implicit time-stepping DRBEM for design sensitivity analysis of magneto-thermo-elastic FGA structure under initial stress. *American Journal of Mathematical and Computational Sciences*. 2017; **2**:55-62
- [55] Fahmy MA. The effect of anisotropy on the structure optimization using golden-section search algorithm based on BEM. *Journal of Advances in Mathematics and Computer Science*. 2017; **25**:1-18
- [56] Fahmy MA. DRBEM sensitivity analysis and shape optimization of rotating magneto-thermo-viscoelastic FGA structures using golden-section

search algorithm based on uniform bicubic B-splines. *Journal of Advances in Mathematics and Computer Science*. 2017;**25**:1-20

[57] Fahmy MA. A predictor-corrector time-stepping DRBEM for shape design sensitivity and optimization of multilayer FGA structures. *Transylvanian Review*. 2017;**XXV**:5369-5382

[58] Fahmy MA. Shape design sensitivity and optimization for two-temperature generalized magneto-thermoelastic problems using time-domain DRBEM. *Journal of Thermal Stresses*. 2018;**41**: 119-138

[59] Fahmy MA. Boundary element algorithm for modeling and simulation of dual-phase lag bioheat transfer and biomechanics of anisotropic soft tissues. *International Journal of Applied Mechanics*. 2018;**10**:1850108

[60] Fahmy MA. Shape design sensitivity and optimization of anisotropic functionally graded smart structures using bicubic B-splines DRBEM. *Engineering Analysis with Boundary Elements*. 2018;**87**:27-35

[61] Fahmy MA. Modeling and optimization of anisotropic viscoelastic porous structures using CQBEM and moving asymptotes algorithm. *Arabian Journal for Science and Engineering*. 2019;**44**:1671-1684

[62] Fahmy MA. Boundary element modeling and simulation of biothermomechanical behavior in anisotropic laser-induced tissue hyperthermia. *Engineering Analysis with Boundary Elements*. 2019;**101**: 156-164

[63] Fahmy MA, Al-Harbi SM, Al-Harbi BH, Sibih AM. A computerized boundary element algorithm for modeling and optimization of complex magneto-thermoelastic problems in

MFGA structures. *Journal of Engineering Research and Reports*. 2019;**3**:1-13

[64] Fahmy MA. A new LRBFCM-GBEM modeling algorithm for general solution of time fractional order dual phase lag bioheat transfer problems in functionally graded tissues. *Numerical Heat Transfer, Part A: Applications*. 2019;**75**:616-626

[65] Fahmy MA. Design optimization for a simulation of rotating anisotropic viscoelastic porous structures using time-domain OQBEM. *Mathematics and Computers in Simulation*. 2019;**66**: 193-205

[66] Fahmy MA. A new convolution variational boundary element technique for design sensitivity analysis and topology optimization of anisotropic thermo-poroelastic structures. *Arab Journal of Basic and Applied Sciences*. 2020;**27**:1-12

[67] Fahmy MA. Thermoelastic stresses in a rotating non-homogeneous anisotropic body. *Numerical Heat Transfer, Part A: Applications*. 2008;**53**: 1001-1011

[68] Abd-Alla AM, Fahmy MA, El-Shahat TM. Magneto-thermo-elastic problem of a rotating non-homogeneous anisotropic solid cylinder. *Archive of Applied Mechanics*. 2008;**78**:135-148

[69] Fahmy MA, El-Shahat TM. The effect of initial stress and inhomogeneity on the thermoelastic stresses in a rotating anisotropic solid. *Archive of Applied Mechanics*. 2008, 2008;**78**:431-442

[70] Huang X, Xie Y. Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method. *Finite Elements in Analysis and Design*. 2007;**43**(14):1039-1049

- [71] Huang X, Xie Y. Evolutionary Topology Optimization of Continuum Structures. U.S.A: John Wiley & Sons Ltd.; 2010
- [72] Huang X, Xie YM. Bi-directional evolutionary topology optimization of continuum structures with one or multiple materials. *Computational Mechanics*. 2008;**43**(3):393
- [73] Huang X, Zhou S, Xie Y, Li Q. Topology optimization of microstructures of cellular materials and composites for macrostructures. *Computational Materials Science*. 2013; **67**:397-407
- [74] Sigmund O. Design of multiphysics actuators using topology optimization—Part i: One-material structures. *Computer Methods in Applied Mechanics and Engineering*. 2001; **190**(49):6577-6604
- [75] Sigmund O, Torquato S. Composites with extremal thermal expansion coefficients. *Applied Physics Letters*. 1996;**69**(21):3203-3205
- [76] Sigmund O, Torquato S. Design of materials with extreme thermal expansion using a three-phase topology optimization method. *Journal of the Mechanics and Physics of Solids*. 1997; **45**(6):1037-1067
- [77] Sigmund O, Torquato S. Design of smart composite materials using topology optimization. *Smart Materials and Structures*. 1999;**8**:365-379
- [78] Yang XY, Xie YM, Steven GP, Querin OM. Bidirectional evolutionary method for stiffness optimization. *AIAA Journal*. 1999;**37**(11):1483-1488
- [79] Wang Y, Luo Z, Zhang N, Wu T. Topological design for mechanical metamaterials using a multiphase level set method. *Structural and Multidisciplinary Optimization*. 2016b; **54**:937-954
- [80] Xu B, Huang X, Zhou S, XIE Y. Concurrent topological design of composite thermoelastic macrostructure and microstructure with multi-phase material for maximum stiffness. *Composite Structures*. 2016;**150**:84-102
- [81] Xiong QL, Tian XG. Generalized magneto-thermo-microstretch response during thermal shock. *Latin American Journal of Solids and Structures*. 2015; **12**:2562-2580