We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,300

Open access books available

172,000

International authors and editors

190M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Scaling Investigation of Low Prandtl Number Flow and Double Diffusive Heat and Mass Transfer over Inclined Walls

*Mubashir O. Quadri, Matthew N. Ottah,
Olayinka Omowunmi Adewumi and Ayowole A. Oyediran*

Abstract

This paper presents an essential study of scale analysis and double diffusive free convection boundary layer laminar flow of low Prandtl fluids over an inclined wall kept at uniform surface temperature. Buoyancy effect (N) was considered for an assisting flow when $N \geq 0$, which implies that the thermal and solutal forces are consolidating each other to help drive the fluid flow in the same direction. Scale analysis and similarity transformation methods are used to obtain the governing equations, and the resulting system of coupled ordinary differential equations (ODEs) is solved with the differential transform method (DTM). Results for the distributions of velocity, temperature, and concentration boundary layer of the fluid adjacent to the wall are presented. The study includes the effects of the ratio of solutal buoyancy to thermal buoyancy and important dimensionless parameters used in this work with varying angles of inclination of the wall on fluid flow and heat transfer.

Keywords: scale analysis, free convection, double diffusion, low Prandtl flow, boundary layer

1. Introduction

The impact of temperature and species concentration distribution on heat and mass transfer of fluid flow has received renewed interest to researchers and the academic community due to its multiple application areas notably in physical and chemical processes, food and manufacturing industries, geophysics, oceanography, and photosynthesis. These occurrences of thermo-solutal convection not only involve temperature variation but also concentration variation. Free convection problems driven only by temperature difference have been studied extensively by many investigators notably Akter et al. [1], Schlichting [2], Venkateswara [3], Gebhart and Pera [4], Khair and Bejan [5], and Mongruel et al. [6]. Species concentration variation sometimes plays a major role in creating the buoyancy needed in driving flow and influencing rate of heat transfer. Double

diffusion has been studied by few investigators over the years. The pioneer of this area of research is the work of Gebhart and Pera in 1971 [4] where they investigated the combined buoyancy effects of thermal and mass diffusion on natural convection flow. Also, Bejan and Khair [7] carried out some analysis on heat and mass transfer by natural convection in porous media. Furthermore, the Schmidt number is the appropriate number in the concentration equation for $Pr < 1$ regime, while in the $Pr > 1$, regime Lewis number is the appropriate dimensionless number for vertical walls and this extends to inclined walls. This important criterion is sometimes omitted from heat and mass transfer studies. Allain et al. [8] also considered the problem of combined heat and mass transfer convection flows over a vertical isothermal plate. These contributors used a combination of integral and scaling laws of Bejan for their investigations. Their work was restricted to cases where two buoyancy forces aid each other; however, it was observed that heat diffusion is always more efficient than mass diffusion meaning that Lewis number is always greater than unity in most cases. It has been recommended in some previous works that more numerical or experimental results covering a wide range of Prandtl and Schmidt numbers are needed to be obtained by further investigations.

Some other research studies carried out were by Angirasa and Peterson [9], who considered free convection due to combined buoyancy forces for $N = 2$ in a thermally stratified medium, and, recently, other contributors have considered flow of power law fluids in saturated porous medium due to double diffusive free convection [10]. Other effects such as Soret and Dufour forces in a Darcy porous medium were considered by Krishna et al. [11]. The problem of mass transfer flow through an inclined plate has generated much interest from astrophysical, renewable energy system, and also hypersonic aerodynamics researchers for a number of decades [1]. It is important to note that combined heat and mass flow over an isothermal inclined wall has received little contributions from scholars [12, 13]. The key notable ones in the literature include the general model formulation of natural convection boundary layer flow over a flat plate with arbitrary inclination by Umemura and Law [14]. Their results showed that flow properties depend on both the degree of inclination and distance from the leading edge. Other investigations considered the problem of combined heat and mass transfer by MHD free convection from an inclined plate in the presence of internal heat generation of absorption [15], natural convection flow over a permeable inclined surface with variable temperature, momentum, and concentration [16], investigations on combined heat and mass transfer in hydro-magnetic dynamic boundary layer flow past an inclined plate with viscous dissipation in porous medium [17], a study on micro-polar fluid behavior in MHD-free convection with constant heat and mass flux [18] and investigations on mass transfer flow through an inclined plate with porous medium [19].

However, research conducted to critically analyze fluid behavior with the effect of species concentration and thermal diffusion on heat and mass transfer particularly for low Prandtl flows past an inclined wall is very rare. This gap has been captured in this study. The objective of this research is to investigate the effect of combined heat and species concentration involving a low Prandtl number fluid flow over an inclined wall using the method of scale analysis in formulation of the model along with the similarity transformation technique to convert partial differential equations to ordinary differential equation. The resulting dimensionless coupled and non-linear equations are solved using differential transform method. The numerics of the computation are discussed for different values of dimensionless parameters and are graphically presented.

2. Problem formulation and scale analysis

The problem of combined heat and mass transfer over a heated semi-infinite inclined solid wall is considered. The fluid is assumed to be steady, Newtonian, viscous, and incompressible. It is assumed that the wall is maintained at uniform surface temperature T_w and concentration C_w and it is immersed in fluid reservoir at rest which is kept at uniform ambient temperature T_∞ and concentration C_∞ such that $T_w > T_\infty$ and $C_w > C_\infty$. Boundary layer flow over an inclined wall driven by both thermal gradient and concentration gradient, respectively, are thereby set up due to the difference between wall values and quiescent fluid values. Hence, it is called combined heat and mass transfer phenomenon over an inclined wall (**Figure 1**).

This problem is governed by the non-linear and coupled conservation equations. Using the Boussinesq approximation and boundary layer simplifications, we have the following:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

Momentum equation

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \nu \frac{\partial^2 v}{\partial x^2} + \rho g \beta_T (T - T_\infty) \cos \alpha + \rho g \beta_c (C - C_\infty) \cos \alpha \quad (2)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_T \frac{\partial^2 T}{\partial x^2} \quad (3)$$

Species concentration equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial x^2} \quad (4)$$

Figure 1.
Physical model of double diffusive free convection over vertical wall.

Here, u is the velocity along x-axis, v is the velocity along y-axis or along the vertical wall, T is the temperature, and C is the concentration. These equations are subject to the boundary conditions given by

$$u = 0, v = 0, T = T_w, C = C_w \text{ at } x = 0 \quad (5)$$

$$u = 0, v = 0, T = T_\infty, C = C_w \text{ at } x = \infty \quad (6)$$

Following the procedures as outlined by Khair and Bejan [5], it can be clearly shown that for low Prandtl number flows, the velocity, concentration, and temperature boundary layer scales (for $N = 0$) are:

$$\delta_v \sim H Ra_*^{-\frac{1}{4}} Pr^{\frac{1}{4}} \quad (7)$$

$$\delta_T \sim H \left(Ra_*^{-\frac{1}{4}} Pr \right)^{-\frac{1}{4}} \quad (8)$$

$$\delta_C \sim H \frac{D}{\alpha} Ra_*^{-\frac{1}{4}} Pr^{-\frac{3}{4}} \quad (9)$$

While vertical velocity (v) scales as.

$$v \sim \frac{\alpha}{y} \left(Ra_*^{-\frac{1}{4}} Pr \right)^{\frac{1}{2}} \quad (10)$$

The similarity variable for velocity layer scales as

$$\eta = \frac{x}{\delta_v} = \frac{x}{y} \left(\frac{Ra_*}{Pr} \right)^{\frac{1}{4}} \quad (11)$$

And corresponding stream function ψ scales as

$$\psi \sim \alpha Ra_*^{\frac{1}{4}} Pr^{\frac{3}{4}} F(\eta) \quad (12)$$

where $F(\eta)$ is the velocity function.

Boussinesq approximation is used in Eq. (2), and the PDEs are reduced to a set of coupled ODEs using similarity variable η . It can easily be shown that for the inner layer of low Prandtl number flows, the dimensionless momentum, energy and concentration equations give:

$$\frac{-3}{4} f f'' + \frac{(f')^2}{2} = -f'''(\eta) + \theta(\eta) \cos \alpha + NC(\eta) \cos \alpha \quad (13)$$

$$\frac{3}{4} Pr f \theta' = \theta'' \quad (14)$$

$$\frac{3}{4} Pr Le f C' = C'' \quad (15)$$

Eqs. (13)–(15) are solved for temperature θ , velocity f' , and concentration C , respectively, subject to the boundary conditions in Eq. (16).

$$\begin{aligned} f'(0) = f(0) = 0, \theta(0) = \gamma(0) = 1 \text{ at } \eta = 0 \\ f'(\infty) = \theta(\infty) = \gamma(\infty) = 0 \text{ at } \eta = \infty \end{aligned} \quad (16)$$

$\bar{\xi}$ the similarity variable for concentration layer is defined as $\bar{\xi} = \frac{x}{\delta_c}$, for $\delta_c > \delta_v$ where δ_v is the thin viscous layer closest to the wall and δ_c is the concentration boundary layer. The attendant stream function is obtained as $\psi \sim DRa_*^{1/4} Pr^{-1/4} \bar{F}(\eta)$.

The ordinary differential equations governing the momentum, energy, and species concentration become:

$$\frac{-3}{4} \bar{F} \bar{F}'' + \frac{(\bar{F}')^2}{2} = -LeSc \bar{F}'''(\eta) + \bar{\theta}(\eta) \cos \alpha + N \bar{C}(\eta) \cos \alpha \quad (17)$$

$$\frac{3}{4} \bar{F} \bar{\theta}' = Sc \cdot \bar{\theta}'' \quad (18)$$

$$\frac{3}{4} \bar{F} \bar{C}' = Pr \cdot \bar{C}'' \quad (19)$$

In the outer layer, where there is inertia-buoyancy balance, ξ which is the similarity variable for thermal layer is defined as $\xi = \frac{x}{\delta_T}$, and the associated stream function obtained as $\psi \sim \alpha Ra_*^{1/4} Pr^{1/4} F(\eta)$.

The resulting dimensionless equations for low Prandtl number flow are:

$$\frac{-3}{4} F F'' + \frac{(F')^2}{2} = -Pr F'''(\eta) + \tilde{\theta}(\eta) \cos \alpha + N \tilde{C}(\eta) \cos \alpha \quad (20)$$

$$\frac{3}{4} F \tilde{\theta}' = \tilde{\theta}'' \quad (21)$$

$$\frac{3}{4} Le F \tilde{C}' = \tilde{C}'' \quad (22)$$

In further works, these equations will be solved asymptotically as $Pr \rightarrow 0$ to obtain approximate analytical results.

2.1 Method of solution

The differential transform method is used to solve the non-linear similarity Eqs. (13)–(15) subject to boundary conditions in Eq. (16). The procedure to convert the PDEs to ODEs is outlined below.

$$\text{Let } Z_1 = \theta; Z_2 = Z'_1 = \theta'; Z_3 = F; Z_4 = F' = Z'_3; Z_5 = Z'_4 = F''; Z_6 = C; Z_7 = Z'_6 = C'. \quad (23)$$

Such that the governing equations of motion become:

$$\left. \begin{aligned} Z'_1 &= Z_2 \\ Z'_2 &= -\frac{3}{4} Pr \cdot Z_2 Z_3 \\ Z'_3 &= Z_4 \\ Z'_4 &= Z_5 \\ Z'_5 &= Z_1 \cos \alpha - \frac{1}{2} Z_4^2 + \frac{3}{4} Z_3 Z_5 + N Z_7 \cos \alpha \\ Z'_6 &= Z_7 \\ Z'_7 &= -\frac{3}{4} Le \cdot Pr \cdot Z_3 Z_7 \end{aligned} \right\} \quad (24)$$

Due to limitation of convergence of the classical DTM which is only valid near $\eta = 0$, the multi-step transformation is used. Carrying out multi-step differential transformations, we have:

$$\begin{aligned} F_i(\eta) &= \sum_{i=0}^k \left(\frac{\eta}{Hi}\right)^i \bar{\bar{F}}_i(k) \\ \theta_i(\eta) &= \sum_{i=0}^k \left(\frac{\eta}{Hi}\right)^i \bar{\bar{\theta}}_i(k) \\ C_i(\eta) &= \sum_{i=0}^k \left(\frac{\eta}{Hi}\right)^i \bar{\bar{C}}_i(k) \end{aligned} \tag{25}$$

Where $i = 0, 1, 2, 3 \dots n$ indicates the i^{th} sub-domain; $k = 0, 1, 2, \dots m$ represents the number of terms of the power series. Hi represents the sub-domain interval, and $\bar{\bar{F}}_i(k), \bar{\bar{\theta}}_i(k), \bar{\bar{C}}_i(k)$ are the transformed functions, respectively.

The transformation of the associated boundary conditions follows as:
 $Z_1(0) = 1, Z_2(0) = a, Z_3(0) = 0, Z_4(0) = 0, Z_5(0) = b, Z_6(0) = 1, Z_7(0) = c$
where a, b , and c are obtained by solving the system of algebraic simultaneous equations, and the results obtained are shown in **Table 1**.

$$\left. \begin{aligned} Z_1(k+1) &= \frac{Z_2(k)}{k+1} \\ Z_2(k+1) &= \frac{-\frac{3}{4}\text{Pr}}{k+1} \sum_{i=0}^k (Z_3(i).Z_2(k-i)) \\ Z_3(k+1) &= \frac{Z_4(k)}{k+1} \\ Z_4(k+1) &= \frac{Z_5(k)}{k+1} \\ Z_5(k+1) &= \frac{1}{k+1} \left[Z_1(k)\cos\alpha - \frac{1}{2} \sum_{i=0}^k Z_4(i)Z_4(k-i) + \frac{3}{4} \sum_{i=0}^k Z_3(i)Z_5(k-i) + N.Z_7(k)\cos\alpha \right] \\ Z_6(k+1) &= \frac{Z_7(k)}{k+1} \\ Z_7(k+1) &= \frac{-\frac{3}{4}.\text{Pr}.Le}{k+1} \sum_{i=0}^k [Z_3(i)Z_7(k-i)] \end{aligned} \right\} \tag{26}$$

Parameters Domain: $Pr < 1, Le \gg 1, Le \ll 1$, and $N \geq 0$
The local Nusselt number: $Nu = [Gr_*(y)]^{\frac{1}{4}} (\theta')_{\eta=0}$
The Local Sherwood number: $Sh = -[Gr_*(y)]^{\frac{1}{4}} (C')_{\eta=0}$
The shearing stress on the plate: $\tau_w = \frac{\infty}{y^2} Ra_*(y)^{3/4} Pr^{1/4} (f'')_{\eta=0}$
The coefficient of skin friction: $C_f = \frac{\alpha 9 Pr^{1/4} Ra_*(y)^{3/4}}{u^2 y^2} 2 (f'')_{\eta=0}$

Table 1.
Important dimensionless parameters of interest.

3. Results and discussion

Table 1 shows the expressions for the dimensionless parameters that are of interest in this work. The solutions to Eqs. (13)–(15) subjected to Eq. (16) solved using the multi-step DTM method are presented graphically in the figures below. The results shown are for Prandtl numbers of 0.01, 0.1, 0.5, and 0.72, respectively.

Figure 2 shows the profiles of local skin friction against Prandtl number for various angles of inclination at a constant Lewis number. It could be observed from the plots that the shearing stress decreases as the Prandtl number increases for all the plate angles considered. More importantly, it is illustrated by the graphs that the local skin friction also decreases with increase in the buoyancy ratio and also with respect to the angle of inclination of the wall.

Figure 3 shows the plots of local Nusselt number against Prandtl number for various angles of inclination at a constant Lewis number. It is clearly seen from the results that the rate of heat transfer (Nusselt number) increases as the Pr increases for all the wall inclination angles. Also to note is the fact that Nusselt number increases as the buoyancy ratio is increased.

Figure 4 shows the results of how the local Sherwood number changes as the Lewis number is increased for various angles of inclination at a constant Prandtl number. It is noted from the graphs that the local Sherwood number increases as the Lewis number increases for all angles of inclination. Also, it is observed from the figures that the rate of increase of Sherwood number is dependent on N as well as the angle of inclination of the wall.

In **Figure 5**, the similarity profiles of the effect of the buoyancy ratio and the angles of inclination on the dimensionless velocities at fixed Prandtl and Lewis numbers is presented. It could be interpreted from the results for $N = 0$ that a maximum velocity is obtained for a vertical wall while at an angle of 60° , the minimum velocity. Also, it is clearly seen from the plots that aside from the vertical wall possessing the maximum velocity, its vertical velocity value for $N = 1$ is higher when compared to the case when $N = 0$. When the buoyancy ratio N is further increased to 1.5, the velocity for the vertical wall also increases. The trend in the figure also shows that increasing the inclination angle increases the velocity boundary layer thickness for all buoyancy ratio.

In **Figure 6**, the similarity profiles of dimensionless temperature for the wall angles of inclinations considered in this study at constant Lewis number are presented. The temperature profiles show that despite the varying buoyancy ratio, the thermal boundary layer thickness increases as the inclination angle of the wall increases.

Figure 7 presents the similarity profiles of dimensionless species concentration distributions for the angles of inclination of the wall at a constant Lewis number. It is clearly seen that as the buoyancy ratio is increased, the concentration boundary layer thickness has a decreasing trend under this same flow configuration but as the angle of inclination increases, δ_c increases.

Figure 8 shows the plots of the coupled similarity profiles of dimensionless temperature, species concentration, and velocity for wall inclination angle of 60° under the constant Lewis number of 10. It can be seen from the results that increasing N has negligible effect on the trio of velocity, concentration and temperature boundary layer thicknesses for a fixed wall angle. However, the effect of N is very noticeable in the vertical velocity values which is clearly higher when $N = 1.5$ compared to when $N = 0$.

Figure 2. Similarity profiles of effects of Prandtl number on skin friction for (a) $N = 0$; (b) $N = 1$; (c) $N = 1.5$ at constant Lewis number of 10.

Figure 3.
Similarity profiles of effects of Prandtl number on Nusselt number for (a) $N = 0$; (b) $N = 1$; (c) $N = 1.5$ at constant Lewis number of 10.

Figure 4. Similarity profiles of effects of Lewis number on Sherwood number for (a) $N = 0$; (b) $N = 1$; (c) $N = 1.5$ at constant Pr of 0.1.

(a)

(b)

(c)

Figure 5.
Similarity profiles of dimensionless velocity for (a) $N = 0$, (b) $N = 1$, (c) $N = 1.5$ at $Pr = 0.1$ and $Le = 10$.

Figure 6. Similarity profiles of dimensionless temperature for (a) $N = 0$, (b) $N = 1.5$ at $Pr = 0.1$ and $Le = 10$.

4. Conclusion

The problem of double diffusive convection and its associated boundary layer flow is of tremendous interest in academic research and various manufacturing and process industries because of its implications in energy and mass transfer efficiency in engineering and scientific applications. In this study, scale analysis and double diffusive free convection of low Prandtl fluid flow over an inclined wall is investigated in the presence of species concentration and thermal diffusion. The governing boundary layer equations obtained by scale analysis are numerically solved using differential transform method (DTM). The conclusions reached as a result of the parametric study conducted are presented below:

- i. The velocity boundary layer thickness is maximum when the plate is in the vertical position, while it is minimum when the plate is at the inclined angle of 60° to the vertical for any value of Lewis number but within a certain buoyancy ratio.
- ii. The thermal boundary thickness is maximum when the wall is inclined at 60° to the vertical and minimum when in a vertical position while keeping N constant.
- iii. Thermal boundary thickness decreases with increase in Prandtl number for all angles of inclination while keeping both the Lewis number and buoyancy ratio constant.

(a)

(b)

Figure 7.
Similarity profiles of dimensionless concentration for (a) $N = 0$, (b) $N = 1.5$ at $Pr = 0.1$ and $Le = 10$.

(a)

(b)

Figure 8. Similarity profiles of dimensionless velocity, concentration, and temperature at inclination angle of 60° . (a) $N = 1.5$, (b) $N = 0$ ($Pr = 0.1$ and $Le = 10$).

- iv. The concentration decreases with the increase in Lewis number for all range of values considered for N and all angles of inclination which is in agreement with Akter et al. [1]. Also, as N increases, both concentration and temperature increase while velocity decreases with increase in plate angles.
- v. The velocity, concentration, and thermal boundary layer thicknesses increase with an increase in the angle of inclination of the wall.

Nomenclature

C	chemical species concentration
D	chemical species diffusivity
N	buoyancy ratio
Nu	Nusselt number
g	gravity constant
k	thermal conductivity
T	temperature
Pr	Prandtl number
Ra_T	thermal Rayleigh number
Le	Lewis number
u	velocity component in x-direction
v	velocity component in y-direction
x	horizontal axis
y	vertical axis

Greek Symbols

β_T	coefficient of thermal expansion
β_C	coefficient of specie expansion
	thermal diffusivity of fluid
$c'(0)$	wall derivative of dimensionless concentration
δ_v	velocity boundary layer thickness
δ_T	thermal boundary layer thickness
δ_C	concentration boundary layer thickness
ΔC	concentration difference ($C - C_\infty$)
ΔT	temperature difference ($T - T_\infty$)
η	similarity variable
θ	dimensionless temperature
$\theta'(0)$	wall derivative of dimensionless temperature
ϑ	kinematic viscosity
ρ	density of fluid
$\theta'(0)$	constant wall dimensionless heat flux
μ	dynamic viscosity
ψ	stream function

Subscript

∞	condition at infinity
w	condition at the wall

IntechOpen

IntechOpen

Author details

Mubashir O. Quadri, Matthew N. Ottah, Olayinka Omowunmi Adewumi*
and Ayowole A. Oyediran
Department of Mechanical Engineering, University of Lagos, Lagos, Nigeria

*Address all correspondence to: oadewunmi@unilag.edu.ng

IntechOpen

© 2020 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

References

- [1] Akter F, Islam MM, Islam A, Khan MS, Hossain MS. Chemical reaction and thermal diffusion effects on mass transfer flow through an inclined plate. *Open Journal of Fluid Dynamics*. 2016;**6**:62. DOI: 10.4236/ojfd.2016.61006
- [2] Schlichting H. *Boundary-Layer Theory*. 7th ed. New York: McGraw-Hill; 1968
- [3] Rajua KV, Reddyb PBA, Suneetha S. Thermophoresis effect on a radiating inclined permeable moving plate in the presence of chemical reaction and heat absorption. *International Journal of Dynamics of Fluids*. 2017;**13**:89-112
- [4] Gebhart B, Pera L. The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion. *International Journal of Heat and Mass Transfer*. 1971;**14**:2025-2050. DOI: 10.1016/0017-9310(71)90026-3
- [5] Khair KR, Bejan A. Mass transfer to natural convection boundary layer flow driven by heat transfer. *Journal of Heat Transfer*. 1985;**107**:979-981. DOI: 10.1115/1.3247535
- [6] Mongrue A, Cloitre M, Allain C. Scaling of boundary-layer flows driven by double-diffusive convection. *International Journal of Heat and Mass Transfer*. 1996;**39**:3899-3910. DOI: 10.1016/0017-9310(96)00054-3
- [7] Bejan A, Khair KR. Heat and mass transfer by natural convection in a porous medium. *International Journal of Heat and Mass Transfer*. 1985;**28**:909-918. DOI: 10.1016/0017-9310(85)90272-8
- [8] Allain C, Cloitre M, Mongrue A. Scaling in flows driven by heat and mass convection in a porous medium. *EPL (Europhysics Letters)*. 1992;**20**:313. DOI: 10.1209/0295-5075/20/4/005
- [9] Angirasa D, Srinivasan J. Natural convection flows due to the combined buoyancy of heat and mass diffusion in a thermally stratified medium. *Journal of Heat Transfer*. 1989;**111**:657-663. DOI: 10.1115/1.3250733
- [10] Angirasa D, Peterson GP, Pop I. Combined heat and mass transfer by natural convection with opposing buoyancy effects in a fluid saturated porous medium. *International Journal of Heat and Mass Transfer*. 1997;**40**:2755-2773. DOI: 10.1016/S0017-9310(96)00354-7
- [11] Murthy SK, Kumar BR, Chandra P, Sangwan V, Nigam M. A study of double diffusive free convection from a corrugated vertical surface in a Darcy porous medium under Soret and Dufour effects. *Journal of Heat Transfer*. 2011;**133**:092601. DOI: 10.1115/1.4003813
- [12] Ferdows M, Khan MS, Bég OA, Azad MAK, Alam MM. Numerical study of transient magnetohydrodynamic radiative free convection nanofluid flow from a stretching permeable surface. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*. 2014;**228**:181-196. DOI: 10.1177/0954408913493406
- [13] Khan MS, Karim I, Ali LE, Islam A. Unsteady MHD free convection boundary-layer flow of a nanofluid along a stretching sheet with thermal radiation and viscous dissipation effects. *International Nano Letters*. 2012;**2**:24. DOI: 10.1186/2228-5326-2-24
- [14] Umemura A, Law CK. Natural-convection boundary-layer flow over a heated plate with arbitrary inclination. *Journal of Fluid Mechanics*. 1990;**219**:571-584. DOI: 10.1017/S0022112090003081
- [15] Chamka A, Khaled ARA. Simultaneously heat and mass transfer

in free convection. Industrial Engineering Chemical. 2001;**49**: 961-968. DOI: 10.1021/ie50570a025

[16] Reddy MG, Reddy NB. Mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium. Journal of Applied Fluid Mechanics. 2011;**4**:7-11

[17] Singh PK. Heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. International Journal of Scientific and Engineering Research. 2012;**3**:1-11

[18] Ali LE, Islam A, Islam N. Investigate micropolar fluid behavior on MHD free convection and mass transfer flow with constant heat and mass fluxes by finite difference method. American Journal of Applied Mathematics. 2015;**3**:157-168. DOI: 10.11648/j.ajam.20150303.23

[19] Islam M, Akter F, Islam A. Mass transfer flow through an inclined plate with porous medium. American Journal of Applied Mathematics. 2015;**3**:215-220. DOI: 10.11648/j.ajam.20150305.12