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Chapter

Corrosion Wear of Pipelines and Equipment in Complex Stress-Strain State

Vladimir A. Pukhliy

Abstract

An analytical approach is described to determine the resource of structural elements of hydropower, the nuclear industry, etc., under difficult stress conditions when exposed to a corrosive environment with certain parameters (degree of chemical activity, temperature, humidity, flow rate, etc.). The initial-boundary problem for structural elements (bimetallic pipelines, centrifugal pumps) is considered with a decrease in the thickness of the element due to the influence of a corrosive environment. In this case, the effect of a corrosive medium on an element is described by a differential equation with certain initial conditions. Equations describing the stress-strain state of an element are added to this equation. As the first object, the corrosion wear of bimetallic pipelines of nuclear energy is considered. The solution to the problem is to integrate the ordinary differential equation. The criterion for terminating the step-by-step process in time is the condition $\left| \sigma_{ij}^{\substack{max \\ min}} \right| \leq [\sigma_T]$, where σ_T is the yield strength of the material of the structural element. As the second object, the corrosion wear of the working blades of centrifugal pumps is considered. The stress-strain state of the blade is described by a system of partial differential equations of the 12th order, to the solution of which the method of integral relations by Dorodnitsyn is applied. At the next stage, the system of ordinary differential equations is integrated by the modified method of successive approximations developed by Professor V.A. Pukhliy. At each stage, the corrosion equation is attached to the solution of this problem. For bimetallic pipelines, a specific example of calculation according to the described algorithm has been implemented.

Keywords: general corrosion, pipelines of hydropower and nuclear energy, corrosion cracking, two-layer shells, initial-boundary value problem, centrifugal pumps, modified method of successive approximation

1. Introduction

In a number of industries, in particular in the chemical industry, axial and radial turbomachines (compressors, superchargers, gas turbine installations, pumps)

operate under aggressive environments, as a result of which rotor blades and disks are subject to corrosion wear.

Pipelines of hydropower are subject to corrosion wear, as a result of which their service life is significantly reduced. The problem of corrosion is especially acute in nuclear energy. Pipelines are made of low-alloy pearlite-grade carbon steel with stainless steel cladding on the inner surface. We also note that all parts and assemblies of the main circulation pumps in contact with the coolant, industrial cooling water, and locking water are made of special steels that are resistant to corrosion.

The coolant of the first circuit of a nuclear reactor is not just pure water, but water with boric acid dissolved in it (H_3BO_3), which contributes to significant corrosion of metal in pipelines. Steam generators of nuclear plants are made of pipes clad with anticorrosive austenitic surfacing.

It should be emphasized that the influence of the stress-strain state on the rate of corrosion and erosion wear also becomes important. For example, a strain of 1% increases the rate of corrosion of silicon iron in a 0.01% solution of sulfuric acid by 53% compared with undeformed metal [1].

Stress corrosion cracking of metals was previously studied. This phenomenon takes place at certain critical (threshold) values of tension determined by the acting stresses and potential energy. Stresses less than critical have an effect on general corrosion without causing cracking.

The impellers of radial and axial turbomachines subjected to corrosion are usually thin-walled plates and shells. The problem of the durability of the elements of the impellers of turbomachines is the problem of the durability of plates and shells of a variable thickness over time, under the influence of an aggressive environment that has certain parameters (degree of chemical activity, temperature, flow rate, etc.), and the stress-strain state.

One of the first works in this direction was an article by Kornishin [2], in which a joint solution of the corrosion equation is considered, which is a linear dependence of the corrosion rate on stress and equations describing the stress-strain state of a shell of variable thickness. The system of joint equations describing the behavior of the shell in a corrosive medium is then solved in finite differences according to a two-layer explicit scheme with a time step.

To date, there are a number of semi-empirical models that approximate corrosion wear taking into account the stress state. In T**able 1**, a number of models used in the calculations are given [3].
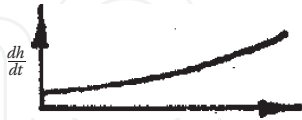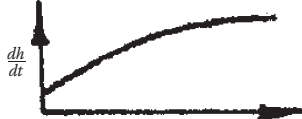
Table 1 indicates: h is the depth of the wear layer; t is the time; σ and ε are stress and strain; T is the temperature; k, α, β, and γ are constants; and φ (t) are some functions.

The issues of corrosion wear of centrifugal fan elements were investigated in a number of works by Pukhliy and Semenenko [1, 4, 5].

2. Corrosion cracking of bimetallic pipelines

Bimetallic structures are widely used in modern technology, in particular, in the manufacture of bimetallic elements in nuclear energy. These are, first of all, bimetallic pipelines, bends, etc. These elements are characterized by high strength, heat resistance, and corrosion resistance.

The structural elements of nuclear power units operate under complex loading conditions, in particular, under conditions of exposure to aggressive environments. In this regard, the determination of the time to destruction of structural elements (resource) is the most important in the study of corrosion wear of elements that are in a complex stress-strain state [6–8].

Corrosion equation wear	The law of the effect of stress on corrosion rate
$\frac{dh}{dt} = \varphi(t)(1 + k\sigma)$	
$\frac{dh}{dt} = \varphi(t)c^{\alpha\sigma/RT}$	
$\frac{dh}{dt} = \alpha + \beta(\sigma - \sigma_0)\varepsilon$	
$\frac{dh}{dt} = \alpha - \beta e^{-\gamma\sigma}$	

Table 1.
Corrosion wear models.

In the present paper, an analytical approach to determining the resource of structural elements of nuclear power units based on the theory of bimetallic shells, taking into account the stress-strain state and corrosion wear of the elements, is presented.

Consider a cylindrical bimetallic shell when exposed to a corrosive environment [4]. Such tasks are very important in relation to the design of pipelines of nuclear power plants (corrosion cracking).

The rate of change of thickness at a given point in the shell is taken in the form:

$$\frac{dh}{dt} = F(t, T, \sigma), 0 \leq t \leq t_k, h > 0. \tag{1}$$

with the initial condition:

$$h(x, y, 0) = h_0(x, y), \tag{2}$$

where x, y are the normal coordinates of the middle surface of the shell; T is the temperature; and σ is the function connecting the rate of change of the shell thickness with the stress state at a surface point.

Note that F is a known function whose form is determined from experiment and t_k is the final point in time.

We study the effect of the stress state on general corrosion under the assumption that the corrosion rate is a linear function of the stress intensity.

The equations of corrosion wear are written as follows:

$$\frac{dh}{dt} = -\alpha\varphi(t)\big(1 + k\sigma_{ij}\big), 0 \leq t \leq t_k, h > 0. \tag{3}$$

Here σ_{ij} is the stress intensity on the surface of the bimetallic shell; α and k are finite coefficients; and $\varphi(t)$ is a dimensionless function of time. As a rule, in most practical cases $\varphi(t)$ it is a constant or monotonically decreasing function.

It is necessary to add the equations of the theory of bimetallic shells to Eq. (1) or (3). As a result, we obtain an unrelated problem of the theory of shells, in view of

which it is possible to apply a finite-difference approximation in time to the solution of Eq. (3).

Thus, the algorithm for solving the initial-boundary-value problem is reduced to the joint solution of Eq. (3) under the initial conditions in Eq. (2) and the system of equations for bimetallic shells, in the general case of variable thickness under the corresponding boundary conditions. Moreover, at each time step from Eq. (3), we obtain numerical values of the thickness of the structural element, which are then used to construct spline functions [9]. Then the system of equations of bimetallic shells is solved, from the solution of which the values of σ_{ij} are determined.

The criterion for terminating the step-by-step process is the following condition:

$$\left| \sigma_{ij} \begin{matrix} \max \\ \min \end{matrix} \right| \leq [\sigma_T],$$

where σ_T is the yield strength of the material of the structural element.

The resource of structural elements of nuclear power units as a whole is determined by the summation of time steps.

We obtain the equilibrium equation of the bimetallic shell on the basis of the Lagrange variational principle:

$$\delta\Pi = \delta\Pi_1 + \delta\Pi_2 = 0. \tag{4}$$

Here $\delta\Pi_1$ is a variation of the potential energy of shell deformation; $\delta\Pi_2$ is a variation of the potential of external forces, equal to the variations of the work of external forces, taken with the opposite sign.

We write an expression for the variation of potential energy:

$$\delta\Pi_1 = \int_{a_1}^{a_2}\int_{b_1}^{b_2} \delta a_0 A_1 A_2 d\alpha_1 d\alpha_2 = \int_{a_1}^{a_2}\int_{b_1}^{b_2} \delta a_0 dS_1 dS_2, \tag{5}$$

where a_0 is the potential energy of deformation of a unit surface of the shell.
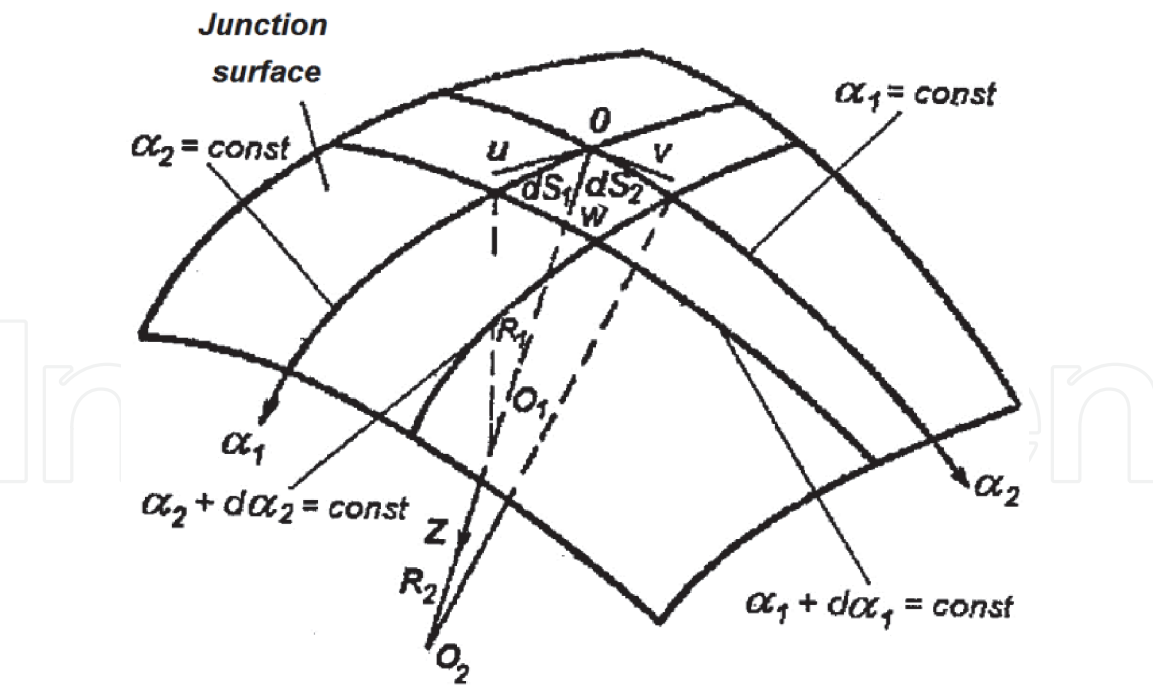
Integration extends to the entire surface of the junction (**Figure 1**): from $\alpha_1 = a_1$ to $\alpha_1 = a_2$ and from $\alpha_2 = b_1$ to $\alpha_2 = b_2$.

$$\delta a_0 = \int_0^{\delta_1} \left[\sigma_1^{(1)} \cdot \delta\varepsilon_1^{(1)} + \sigma_2^{(1)} \delta\varepsilon_2^{(1)} + \tau_{12}^{(1)} \delta\gamma_{12}^{(1)} \right] \left(1 - \frac{z}{R_1}\right)\left(1 - \frac{z}{R_2}\right) dz$$

$$+ \int_{-\delta_2}^{0} \left[\sigma_1^{(2)} \delta\varepsilon_1^{(2)} + \sigma_2^{(2)} \cdot \delta\varepsilon_2^{(2)} + \tau_{12}^{(2)} \delta\gamma_{12}^{(2)} \right] \left(1 - \frac{z}{R_1}\right)\left(1 - \frac{z}{R_2}\right) dz$$

In the analysis, the following assumptions are used (**Figure 1**).

The curvilinear coordinate system coincides with the lines of the main curvatures. This coordinate system is a Gaussian coordinate system, it is orthogonal.

The position of the point that does not belong to the junction surface determines the coordinates of the z-distance normal to the point from the junction surface (+ if it is directed along the internal normal to the junction surface).

Figure 1.
System of curvilinear coordinates on an undeformed junction surface.

The movement of u and v in are the direction of the tangents to α_1 and α_2 and w in the direction of the normal to the junction surface.

Deformations of the junction surface are determined by relative elongations ε_1 and ε_2 in both the α_1 and α_2 directions, and by a shift γ_{12}—a change in the angle between the tangents to the lines α_1 and α_2 (before deformation $\pi/2$, after $\frac{\pi}{2} - \gamma_{12}$).

Eq. (5) can be represented as follows:

$$\delta\Pi_1 = \int\limits_{a_1}^{a_2}\int\limits_{b_1}^{b_2} (N_1\delta\varepsilon_1 + N_2\delta\varepsilon_2 - M_1\delta\chi_1 - M_2\delta\chi_2 + T\delta\gamma_{12} - H\delta\chi_{12})A_1A_2 d\alpha_1 d\alpha_2 \quad (6)$$

Denote by:

p_1, p_2, p_3 the projection of external surface forces, referred to the unit of the junction surface, on the direction of the tangents to the lines of curvature α_1 and α_2 and normal to the junction surface;

$N_1^{\alpha_1}, T_1^{\alpha_1}, Q_1^{\alpha_1}, M_1^{\alpha_1}$ the normal, shear, shear forces, and bending moment for the section $\alpha_1 = const$;

$N_2^{\alpha_2}, T_2^{\alpha_2}, Q_2^{\alpha_2}, M_2^{\alpha_2}$ the same for section $\alpha_2 = const$.

Then the variation of the potential of external forces is equal to:

$$\delta\Pi_2 = -\int\limits_{a_1}^{a_2}\int\limits_{b_1}^{b_2} (p_1\delta u + p_2\delta v + p_3\delta w)A_1A_2 d\alpha_1 d\alpha_2 - \int\limits_{b_1}^{b_2} (N_1^{\alpha_1}\delta u + T_1^{\alpha_1}\delta v - M_1^{\alpha_1}\delta v_1$$

$$+ Q_1^{\alpha_1}\delta w)A_2 d\alpha_2 - \int\limits_{a_1}^{a_2} (N_2^{\alpha_2}\delta v + T_2^{\alpha_1}\delta u - M_2^{\alpha_2}\delta v_2 + Q_2^{\alpha_2}\delta w)A_1 d\alpha_1. \quad (7)$$

Substituting Eqs. (6) and (7) in Eq. (4) we obtain:

$$\delta\Pi = -\int\limits_{a_1}^{a_2}\int\limits_{b_1}^{b_2}\left\{\left[\frac{\partial N_1 A_2}{\partial\alpha_1} - N_2\frac{\partial A_2}{\partial\alpha_1} + \frac{\partial T_2 A_1}{\partial\alpha_2} + T_1\frac{\partial A_1}{\partial\alpha_2} - \frac{A_1 A_2}{R_1}Q_1 + A_1 A_2 p_1\right]\delta u + \right.$$

$$+\left[\frac{\partial N_2 A_1}{\partial\alpha_2} - N_1\frac{\partial A_1}{\partial\alpha_2} + T_2\frac{\partial A_2}{\partial\alpha_1} - \frac{A_1 A_2}{R_2}Q_2 + A_1 A_2 p_2\right]\delta v +$$

$$+\left[\frac{\partial Q_1 A_2}{\partial\alpha_1} + \frac{\partial Q_2 A_1}{\partial\alpha_2} + A_1 A_2\left(\frac{N_1}{R_1} + \frac{N_2}{R_2}\right) + A_1 A_2 p_3\right]\delta w\right\}d\alpha_1 d\alpha_2 +$$

$$+\int\limits_{b_1}^{b_2}\left\{[N_1 - N_1^{\alpha_1}]\,\delta u + \left[T_1 - \frac{H_1}{R_2} - T_1^{\alpha_2}\right]\delta v - [M_1 - M_1^{\alpha_1}]\,\delta\left(\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1} + \frac{u}{R_1}\right) + \right.$$

$$+\left.\left[Q_1 + \frac{1}{A_2}\frac{\partial H_1}{\partial\alpha_2} - Q_1^{\alpha_1}\right]\delta w\right\}A_2 d\alpha_2 + \int\limits_{a_1}^{a_2}\left\{\left[T_2 - \frac{H_2}{R_1} - T_2^{\alpha_2}\right]\delta u + [N_2 - N_2^{\alpha_2}]\,\delta v - \right.$$

$$-[M_2 - M_2^{\alpha_2}]\,\delta\left(\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2} + \frac{v}{R_2}\right) + \left.\left[Q_2 + \frac{1}{A_1}\frac{\partial H_2}{\partial\alpha_1} - Q_2^{\alpha_2}\right]\delta w\right\} - \{[H_1 + H_2]\,\delta w\}_{b_1,a_1} -$$

$$-\{[H_1 + H_2]\,\delta w\}_{b_2,a_2} + \{[H_1 + H_2]\,\delta w\}_{b_2,a_2} + \{[H_1 + H_2]\,\delta w\}_{b_1,a_2}.$$

$$(8)$$

Here Q_1 and Q_2 are the transverse forces arising in the shell:

$$Q_1 = \frac{1}{A_1 A_2}\left[\frac{\partial M_1 A_2}{\partial\alpha_1} - M_2\frac{\partial A_2}{\partial\alpha_1} + H_1\frac{\partial A_2}{\partial\alpha_2} + \frac{\partial H_2 A_1}{\partial\alpha_2}\right]; \qquad (9)$$

$$Q_2 = \frac{1}{A_1 A_2}\left[\frac{\partial M_2 A_1}{\partial\alpha_2} - M_1\frac{\partial A_1}{\partial\alpha_2} + H_2\frac{\partial A_2}{\partial\alpha_1} + \frac{\partial H_1 A_2}{\partial\alpha_1}\right]. \qquad (10)$$

The last four terms in Eq. (8) is the work of concentrated forces along the edges of the shell $\alpha_1 = const$, $\alpha_2 = const$.

From Eq. (8), we obtain the equilibrium equation and boundary conditions.
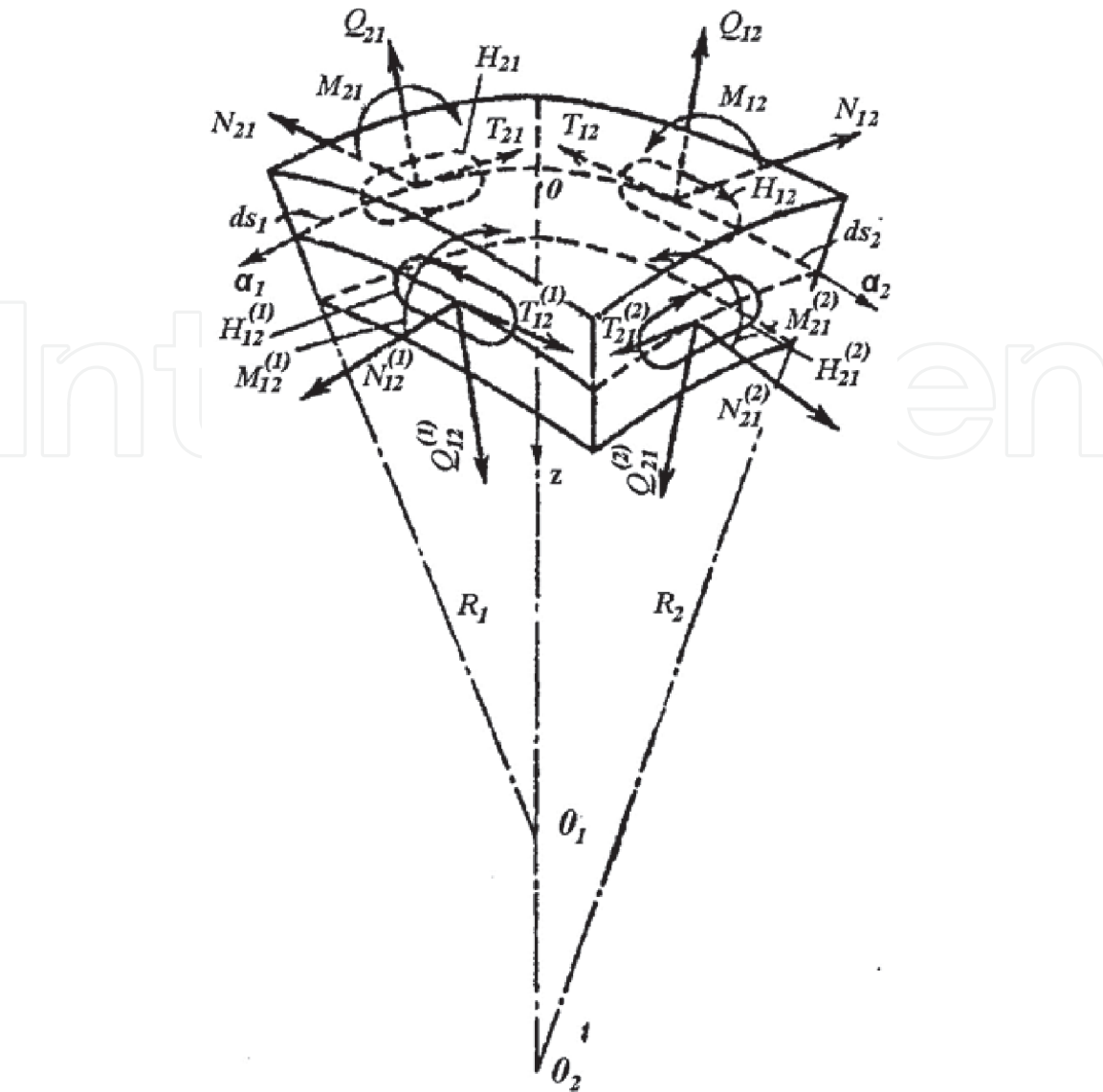
2.1 Equilibrium equations

In the mechanics of a solid deformable body, equilibrium equations can be obtained by making up for the main vector and the main moment of all the forces acting on the element for the infinitely small element extracted from the shell under the influence of external and internal forces (**Figure 2**). Here, the equilibrium equations and boundary conditions are obtained from the variational Lagrange principle in Eq. (4).

Note that in the case of dynamics, it is necessary to apply the variational Hamilton-Ostrogradsky principle.

So, from the first integral of expression in Eq. (8), the first three equations of equilibrium follow:

$$\left.\begin{array}{l}\dfrac{1}{A_1 A_2}\left[\dfrac{\partial N_1 A_2}{\partial\alpha_1} - N_2\dfrac{\partial A_2}{\partial\alpha_1} + \dfrac{\partial T_2 A_1}{\partial\alpha_2} + T_1\dfrac{\partial A_1}{\partial\alpha_2}\right] - \dfrac{Q_1}{R_1} + p_1 = 0; \\[3mm] \dfrac{1}{A_1 A_2}\left[\dfrac{\partial T_1 A_2}{\partial\alpha_1} - N_1\dfrac{\partial A_1}{\partial\alpha_2} + \dfrac{\partial N_2 A_1}{\partial\alpha_2} + T_2\dfrac{\partial A_2}{\partial\alpha_1}\right] - \dfrac{Q_2}{R_2} + p_2 = 0; \\[3mm] \dfrac{1}{A_1 A_2}\left[\dfrac{\partial Q_1 A_2}{\partial\alpha_1} + \dfrac{\partial Q_2 A_1}{\partial\alpha_2}\right] - \dfrac{N_1}{R_1} + \dfrac{N_2}{R_2} + p_3 = 0.\end{array}\right\} \qquad (11)$$

Figure 2.
Internal forces acting on the edge of the element.

From the expressions in Eqs. (9)–(10), two more equations follow:

$$\frac{1}{A_1 A_2}\left[\frac{\partial M_1 A_2}{\partial \alpha_1} - M_2 \frac{\partial A_2}{\partial \alpha_1} + \frac{\partial H_2 A_1}{\partial \alpha_2} + H_1 \frac{\partial A_1}{\partial \alpha_2}\right] - Q_1 = 0;$$

$$\frac{1}{A_1 A_2}\left[\frac{\partial M_2 A_1}{\partial \alpha_2} - M_1 \frac{\partial A_1}{\partial \alpha_2} + \frac{\partial H_1 A_2}{\partial \alpha_1} + H_2 \frac{\partial A_2}{\partial \alpha_1}\right] - Q_2 = 0.$$

(12)

The sixth equation is an identity expressing the equality of the moments of all forces acting on the element to zero relative to the axis normal to the surface of the element junction:

$$T_1 - T_2 - \frac{H_1}{R_1} + \frac{H_2}{R_2} = 0.$$

This equation was used to obtain Eq. (11).

2.2 Border conditions

The second and third integrals of expression in Eq. (8) give boundary conditions for the edges $\alpha_1 = const$ and $\alpha_2 = const$, that is, for lines of principal curvatures.

We emphasize that if one of the lines of the main curvatures is closed, then the displacements along this line will be periodic functions.

2.3 Axisymmetric deformation of a cylindrical shell

We introduce the following notation (**Figure 3**): R is the radius of the surface of the junction of a cylindrical bimetallic shell; l is the shell length; δ_1 and δ_2 are the thickness of the inner and outer layers; x is the distance from the left edge of the cylinder to the current section; σ_1 and σ_2 are normal stresses; τ_{13} and is shear stress.

$$M_1 = \int_0^{\delta_1} \sigma_1^{(1)} z dz + \int_{-\delta_2}^{0} \sigma_1^{(2)} z dz; N_1 = \int_0^{\delta_1} \sigma_1^{(1)} dz + \int_{-\delta_2}^{0} \sigma_1^{(2)} dz;$$

$$M_2 = \int_0^{\delta_1} \sigma_2^{(1)} z dz + \int_{-\delta_2}^{0} \sigma_2^{(2)} z dz; N_2 = \int_0^{\delta_1} \sigma_2^{(1)} z dz + \int_{-\delta_2}^{0} \sigma_2^{(2)} z dz;$$

$$Q_1 = \int_0^{\delta_1} \tau_{13}^{(1)} \left(1 - \frac{z}{R}\right) dz + \int_{-\delta_2}^{0} \tau_{13}^{(2)} \left(1 - \frac{z}{R}\right) dz.$$
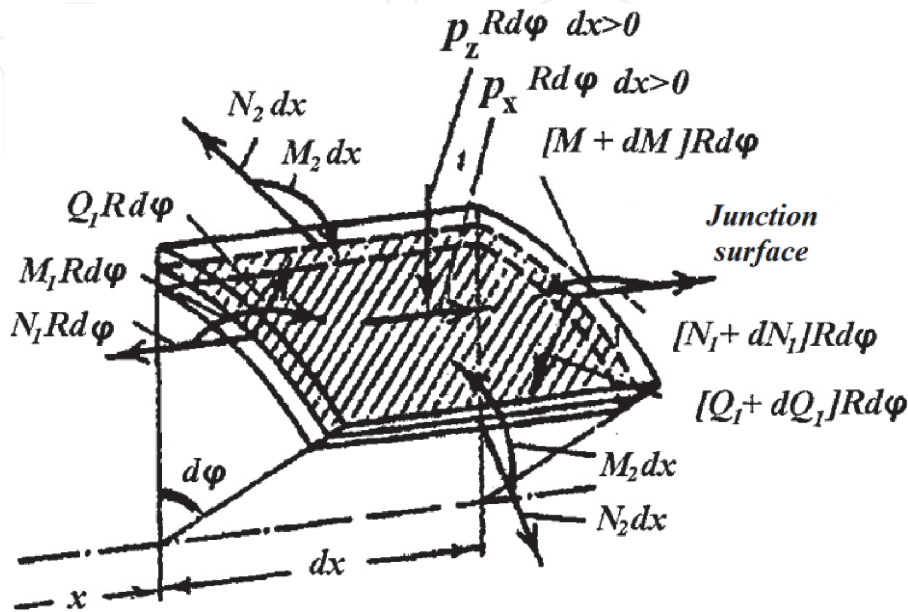
Relative deformations of a surface located at a distance z from the junction surface (**Figure 4**):

$$\varepsilon_{1z} = \varepsilon_1 - z\chi_1; \quad \chi_1 = w''; \quad \varepsilon_{2z} = \varepsilon_2 - z\chi_2; \quad \chi_2 = \frac{w}{R^2}.$$
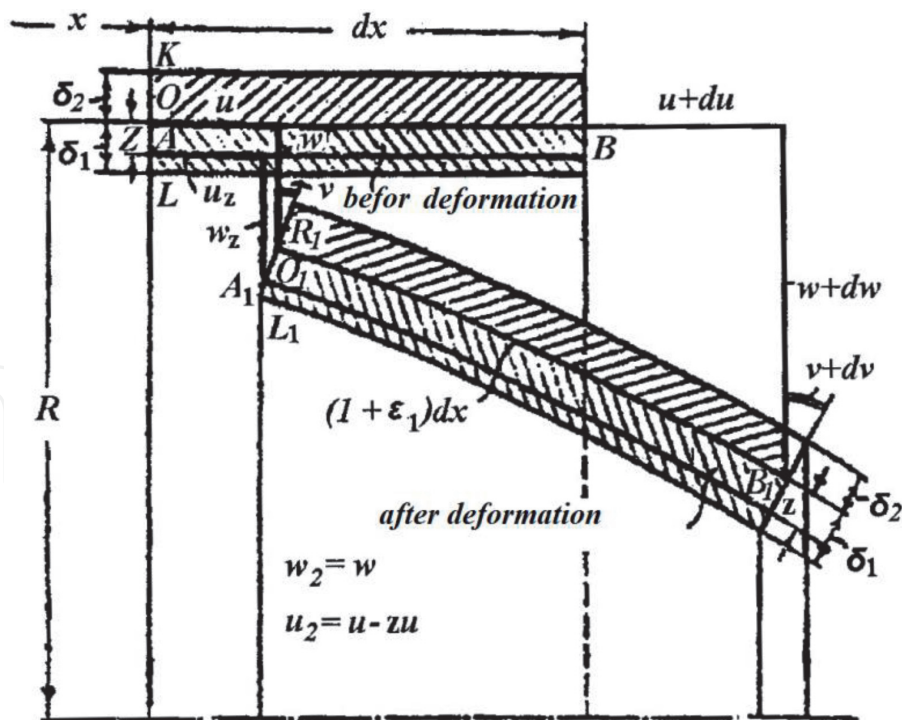
Normal stresses according to Hooke's law:

$$\sigma_1^{(1)} = \frac{E_1}{1 - \mu_1^2} \left[\varepsilon_1 + \mu_1\varepsilon_2 - z(\chi_1 + \mu_1\chi_2) - (1 + \mu_1)\beta_1\overline{T}\right]; \sigma_2^{(1)}$$
$$= \frac{E_1}{1 - \mu_1^2} \left[\varepsilon_2 + \mu_1\varepsilon_1 - z(\chi_2 + \mu_1\chi_1) - (1 + \mu_1)\beta_1\overline{T}\right]; (0 \leq z \leq \delta_1)$$

Figure 3.
Element of cylindrical shell.

Figure 4.
Deformation of surface junction shell.

$$\sigma_1^{(2)} = \frac{E_2}{1-\mu_2^2}\left[\varepsilon_1 + \mu_2\varepsilon_2 - z(\chi_1 + \mu_2\chi_2) - (1+\mu_2)\beta_2\overline{T}\right]; \sigma_2^{(2)}$$

$$= \frac{E_2}{1-\mu_2^2}\left[\varepsilon_2 + \mu_2\varepsilon_1 - z(\chi_2 + \mu_2\chi_1) - (1+\mu_2)\beta_2\overline{T}\right].(-\delta_2 \leq z \leq 0)$$

Power factors:

$$M_1 = c_1\varepsilon_1 + c_2\varepsilon_2 - D_1\chi_1 - D_2\chi_2 - g; N_1 = B_1\varepsilon_1 + B_2\varepsilon_2 - c_2\chi_2 - f;$$

$$M_2 = c_2\varepsilon_1 + c_1\varepsilon_2 - D_2\chi_1 - D_1\chi_2 - g; N_2 = B_2\varepsilon_1 + B_1\varepsilon_2 - c_2\chi_1 - c_1\chi_2 - f.$$

Here:

$$B_1 = \frac{E_1\delta_1}{1-\mu_1^2} + \frac{E_2\delta_2}{1-\mu_2^2}; c_1 = \frac{1}{2}\frac{E_1\sigma_1^2}{1-\mu_1^2} - \frac{1}{2}\frac{E_2\delta_2^2}{1-\mu_2^2}; D_1 = \frac{1}{3}\frac{E_1\delta_1^3}{1-\mu_1^2} + \frac{1}{3}\frac{E_2\delta_2^3}{1-\mu_2^2};$$

$$f = \frac{E_1\delta_1 m}{1-\mu_1} + \frac{E_2\delta_2 m}{1-\mu_2}; g = \frac{1}{2}\frac{E_1\delta_1^2 n_1}{1-\mu_1} + \frac{1}{2}\frac{E_2\delta_2^2 n_2}{1-\mu_2};$$

$$m_1 = \frac{1}{\delta_1}\int_0^{\delta_1}\beta_1 t dz; m_2 = \frac{1}{\delta_2}\int_{-\delta_2}^0\beta_2 t dz; n_1 = \frac{2}{\delta_1^2}\int_0^{\delta_1}\beta_1 t z dz; n_2 = \frac{2}{\delta_2^2}\int_{-\delta_2}^0\beta_2 t z dz.$$

Equations of an infinitesimal element:

$$\left.\begin{array}{l} [N_1(1+w')]' + p_x = 0; \\[2mm] Q_1' + \dfrac{M_2}{R^2} + [N_1 w']' + \dfrac{N_2}{R} + p_z = 0; \\[2mm] M_1' - Q_1 = 0. \end{array}\right\}$$

The problem of axisymmetric deformation of an elastic bimetallic cylindrical shell for any relations between thicknesses, different mechanical characteristics of the material of the layers, and arbitrary heating along the thickness and axial direction is described by the equation:

$$w^{IV} + 2aw'' + b^2w = \theta(x). \tag{13}$$

$$a = \frac{1}{R}\frac{c_2B_1 - c_1B_2 - \frac{1}{2}B_1RN_1}{D_1B_1 - c_1^2}; b^2 = \frac{1}{R^2}\frac{B_1^2 - B_2^2}{D_1B_1 - c_1^2};$$

$$\theta(x) = \frac{B_1}{D_1B_1 - c_1^2}\left\{\frac{c_1}{B_1}(N_1 + f)'' + \frac{1}{R}\frac{B_2}{B_1}\left[N_1 + \left(1 - \frac{B_1}{B_2}\right)f\right] - g'' - p_z\right\}.$$

Consider the case:

$$c_1 = c_2 = 0; \mu_1 = \mu_2 = \mu; E_1\delta_1^2 = E_2\delta_2^2.$$

Then the force factors are written as follows:

$$M_1 = -D[w'' + (1 + \mu)n]; N_1 = B\left[\varepsilon_1 - \mu\frac{w}{R} - (1 + \mu)m\right];$$

$$M_2 = -D[\mu w'' + (1 + \mu)n]; N_2 = \mu N_1 - \frac{B}{R}(1 - \mu^2)(w + mR).$$

Here

$$B = \frac{\delta\sqrt{E_1E_2}}{1 - \mu^2}; D = \frac{1}{3}\frac{E_1E_2\delta^3}{(1 - \mu^2)\left(\sqrt{E_1} + \sqrt{E_2}\right)^2};$$

$$m = \frac{m_1 + m_2\sqrt{E_2/E_1}}{1 + \sqrt{E_2/E_1}}; n = \frac{3}{2}\frac{n_1 + n_2}{\delta};$$

$$k^4 = \sqrt[4]{\frac{(1 - \mu^2)B}{4DR^2}} = \frac{\sqrt[4]{3(1 - \mu^2)}}{\sqrt{2R\delta}}\frac{\sqrt{\sqrt{E_1} + \sqrt{E_2}}}{\sqrt[3]{E_1E_2}}.$$

In this case, instead of Eq. (13), we obtain the following equation:

$$w^{IV} - \frac{RN_1}{D}w'' + 4k^4w = \frac{4k^4B}{(1 - \mu^2)B}(\mu N_1 + p_zR) - 4k^4Rm - (1 + \mu)n''. \tag{14}$$

The boundary conditions for bimetallic shells coincide with similar conditions for homogeneous shells.

So, for a hard-pressed edge we get:

$$w = \frac{\partial w}{\partial n} = 0$$

For the free edge we have:

$$M_1 = Q_1 = 0$$

2.4 Infinitely long cylindrical shell (piping)

We write the solution of the homogeneous Eq. (13) in the following form:

$$w = k_1 ch\alpha x \cos \beta x + k_2 sh\alpha x \sin \beta x + k_3 sh\alpha x \cos \beta x + k_4 ch\alpha x \sin \beta x, \qquad (15)$$

$k_1, \dots k_4$ are arbitrary constants; $\alpha = \sqrt{\frac{b-a}{2}}$; $\beta = \sqrt{\frac{b+a}{2}}$.

For an infinitely long shell in solution in Eq. (15), $sh\alpha x$ and $ch(\alpha x)$ we express it in terms of exponential functions. As $x \to \infty$, terms containing exp. $\alpha x \to \infty$ and integration constants $\to 0$.

Then:

$$w = k_1 A_1(\alpha x, \beta x) + T_2 A_2(\alpha x, \beta x) + w_0.$$

Here

$$A_1 = A_1(\alpha x, \beta x) = e^{-\alpha x} \sin \beta x; A_2 = A_2(\alpha x, \beta x) = e^{-\alpha x} \cos \beta x.$$

Example 1. Consider a bimetallic cylindrical shell under the influence of internal pressure q and a corrosive medium (**Figure 5**).

In this case, the stress intensity is constant for all points of the shell and is equal to [5]:

$$\sigma_i = \frac{qR_a^2}{R_c^2 - R_a^2} - \frac{Q}{R_c^2 - R_b^2}; Q = q \frac{\left(\frac{1}{E_1} - \frac{1}{E_2}\right)\frac{R_b^2 - R_a^2}{R_c - R_a^2}}{E_1^{-1}(R_c^2 - R_b^2)^{-1} + E_2^{-1}(R_b^2 - R_a^2)^{-1}}; \qquad (16)$$

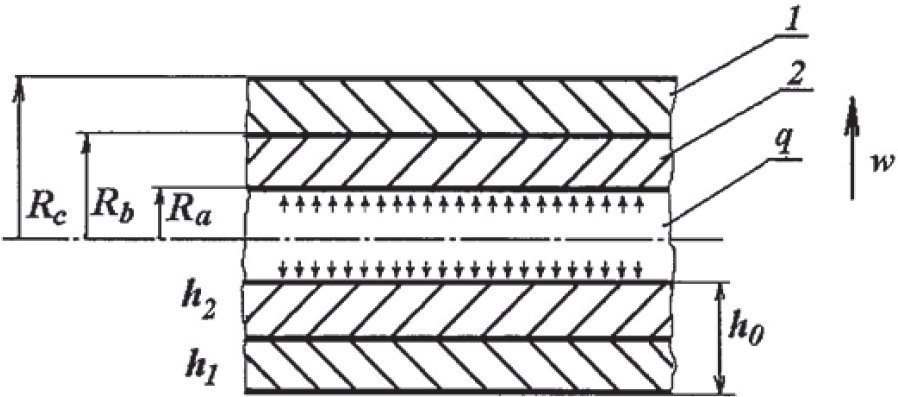Given Eq. (16), Eq. (3) takes the form:

$$\frac{dh}{dt} = -\alpha\varphi(t)\left(1 + \frac{kA}{h}\right); \qquad (17)$$

$$A = q(R_c - R_a)\left\{\frac{R_a^2}{R_c^2 - R_a^2} - \frac{\left(\frac{1}{E_1} - \frac{1}{E_2}\right)\left(\frac{R_b^2 - R_a^2}{R_c^2 - R_a^2}\right)}{\frac{1}{E_1(R_c^2 - R_b^2)} + \frac{1}{E_2(R_b^2 - R_a^2)}} \cdot \frac{1}{R_c^2 - R_b^2}\right\}$$

with the initial condition:

$$h(0) = h_0, h_0 = R_c - R_a. \qquad (18)$$

The initial thickness h_0 is taken constant for all points of the two-layer shell.

Figure 5.
Geometry and loads acting on an element of two-layer shell: 1—outer layer; 2—inner layer; q_a—internal pressure.

Eq. (17) with the initial condition Eq. (18) is integrated in quadratures:

$$h = h_0 - kP \ln \frac{h_0 kA}{h + kA} - \alpha \int_0^\tau \varphi(t)dt, h > 0. \tag{19}$$

In case $\varphi \equiv 1$, expression in Eq. (19) takes the form:

$$h = h_0 - \alpha t - kP \ln \frac{h_0 + kA}{h + kA}, h > 0. \tag{20}$$

We note that Eq. (20) differs from linear equation $h = h_0 - \alpha t$ by the presence of an additional term that takes into account the effect of the stress state of the two-layer shell on the corrosion rate.

Let us determine the durability of a steel two-layer cylindrical shell with R_c = 80.0 cm; $h_0 = R_c - R_a = 0.8$ cm; $h_1 = R_c - R_b = 0.5$ cm; $h_2 = R_b - R_a = 0, 3$cm; q = 10 kg/cm^2; and $[\sigma_T]$ = 2200 kg/cm^2; $E_1 = E_2 = 2 \cdot 10^6$ kg/cm^2.

Corrosion rate, $\left|\frac{dh}{dt}\right|_{\sigma_i} = \alpha = 0.03$ cm/year [2]; $\left|\frac{dh}{dt}\right|_{\sigma_i=[\sigma_T]} = \alpha_1 = 0.05$cm/year [2], where $[\sigma_T]$ is the allowable value of stress intensity at the end of the service life.

Given Eq. (3) we get:

$$k = \frac{\alpha_1/\alpha - 1}{[\sigma_T]}.$$

The final value of the thickness of the shell h_T is found from the conditions of achievement $\sigma_{ij} = [\sigma_T]$:
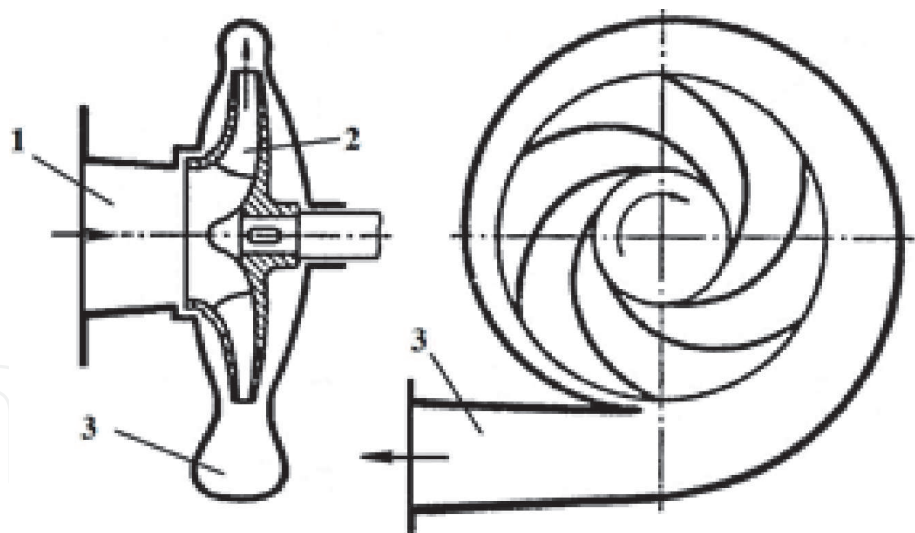
$$h_T = \frac{A}{[\sigma_T]} = 0.315 \text{ cm.}$$

Substituting the values of the coefficients in Eq. (3), we find the durability T = 11.6 years. In conventional calculations, the durability is calculated by the formula: $T = \frac{h_0 - h_T}{\alpha_1}$, where T = 9.7 years.

In conclusion, it should be noted that in the general case it is necessary to solve the unrelated problem of the theory of shells [10, 11], when at each step of integration over time it is necessary to solve the problem of the stress-strain state of a bimetallic shell by a variable thickness, for which it is necessary to use methods of integrating partial equations derivatives [12, 13].
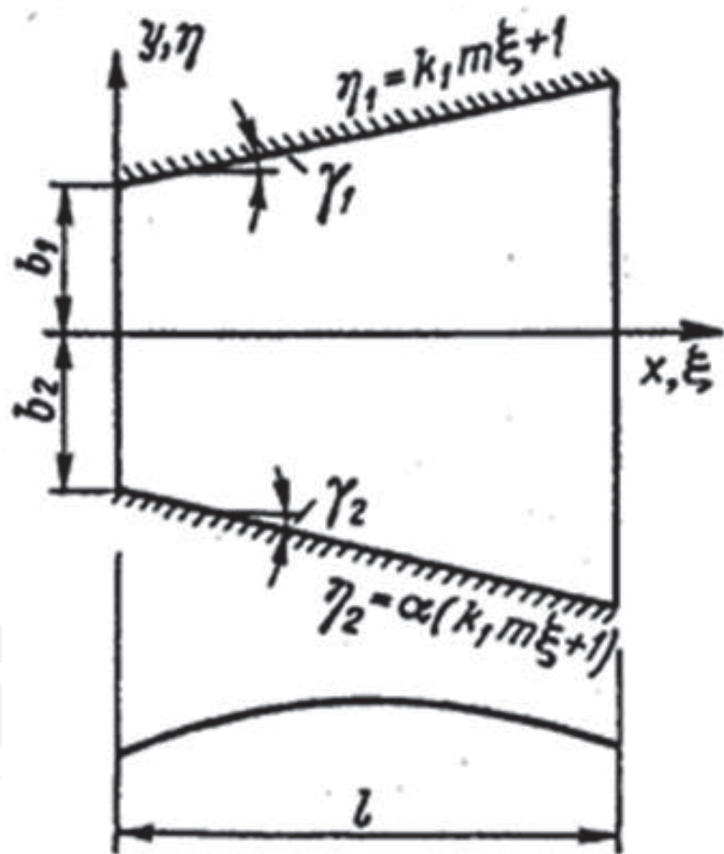
3. The durability of the rotor blades of centrifugal pumps when exposed to corrosion wear

The impellers of centrifugal pumps subjected to corrosion are usually thin-walled plates and shells. The problem of the durability of the elements of the impellers of centrifugal pumps is the problem of the durability of the plates and shells of a variable thickness over time, under the influence of an aggressive environment with certain parameters (degree of chemical activity, temperature, flow rate, etc.), and the stress-strain state.

Figure 6 shows the layout of a centrifugal pump. Consider the durability of the working blades of centrifugal pumps, which are a trapezoidal shell of variable stiffness (**Figure 7**). The blade is subject to the combined action of centrifugal load and corrosion wear.

Figure 6.
Scheme of centrifugal pump: 1—confuser; 2—impeller; 3—diffuser.

Figure 7.
Geometry and coordinate system of centrifugal pump blade.

The rate of change of thickness at a given point of the blade is taken in the form of a functional relationship:

$$\frac{dh}{dt} = F(t, T, \sigma) \tag{21}$$

with the initial condition:

$$h(x, y, 0) = h_0(x, y), \tag{22}$$

where x, y are the coordinates of the middle surface of the scapula; T is the temperature; and σ is the function connecting the rate of change of thickness with the stress state at a surface point. Function F should be determined from experiment.

Assuming that the rate of change in corrosion wear is a linear function of stress intensity, we write Eq. (21) in the form:

$$\frac{dh}{dt} = \varphi(t)\left(1 + k\sigma_{ij}\right). \tag{23}$$

Eq. (23) must be supplemented with shell theory equations of variable thickness.

Omitting the intermediate calculations, we present a system of partial differential equations of the type Margherra [14] with respect to the normal deflection w and the stress function F of the eighth order, describing the stress state of the blade of variable thickness, taking into account the temperature effect:

$$D\nabla^4 w + 2\frac{\partial D}{\partial x}\frac{\partial}{\partial x}\nabla^2 w + 2\frac{\partial D}{\partial y}\frac{\partial}{\partial y}\nabla^2 w + \nabla^2 D\nabla^2 w - (1-\nu)L(D, W) - h\nabla_k^2 F$$

$$= q - \nabla^2 M_T; \frac{1}{B}\nabla^4 F + 2\frac{\partial}{\partial x}\left(\frac{1}{B}\right)\frac{\partial}{\partial x}\nabla^2 F + 2\frac{\partial}{\partial y}\left(\frac{1}{B}\right)\frac{\partial}{\partial y}\nabla^2 F + \nabla^2\left(\frac{1}{B}\right)\nabla^2 F$$

$$- (1+\nu)L\left(F, \frac{1}{B}\right) + \frac{(1-\nu^2)}{h}\nabla_k^2 w$$

$$= -(1-\nu)\frac{1}{h}\nabla^2\left(\frac{N_T}{B}\right). \tag{24}$$

Here T = T(x, y, z) is the temperature field of a general form; α is the coefficient of linear expansion of the material of the blade. ∇^2 and ∇^4 harmonic and biharmonic operators; and ∇_k^2 metaharmonic operator:

$\nabla_k^2 = k_{11}\frac{\partial^2}{\partial x^2} + k_{22}\frac{\partial^2}{\partial y^2}$; L(D, w) and $L\left(F, \frac{1}{B}\right)$ are second-order linear differential operators.

Power factors due to temperature exposure are recorded as:

$$N_T = \frac{\alpha E}{1-\nu}\int_{-h/2}^{h/2} T(x,y,z)\,dz; M_T = \frac{\alpha E}{1-\nu}\int_{-h/2}^{h/2} T(x,y,z)\,zdz. \tag{25}$$

We introduce the dimensionless coordinate system:

$$\xi = x/l; \ \eta = y/b_1; \ m = l/b_1 \tag{26}$$

and dimensionless unknown functions

$$\overline{w} = w/h_0; \ \overline{F} = \frac{F}{E^* h_0^2}. \tag{27}$$

Here h_0 is the characteristic thickness of the scapula and E^* is the modulus of elasticity of the material of the blade at a temperature of T = 20°C.

The boundary conditions at the edges of the blade adjacent to the disks $\eta_1 = k_1 m\xi + 1$ and $\eta_2 = \alpha(k_1 m\xi + 1)$ are considered as follows:

$$\overline{w} = \overline{w}_{,n} = \overline{F} = \overline{F}_{,n} = 0. \tag{28}$$

The boundary conditions at the inlet and outlet edges of the blade correspond to the free edge. In this case we have:

$$N_\xi = N_{\xi\eta} = M_\xi = 0; Q_\xi - \frac{\partial M_{\xi\eta}}{\partial \eta} = 0. \tag{29}$$

The important issue is to specify the function of changing the blade thickness h = h (x, y) in the process of erosion-corrosion wear.

In our studies, the function of changing the blade thickness was set in the form of cubic splines [9].

In the general case, the blade thickness can be represented as two-dimensional spline interpolations:

$$h(\xi_i, \eta_i) = h_0 \sum_{\alpha=0}^{3} \sum_{\beta=0}^{3} A_{\alpha\beta}^{(ij)} (\xi - \xi_i)^\alpha \left(\eta - \eta_j\right)^\beta \text{ (i = 0, 1, ..., n; j = 0, 1, ..., m).} \tag{30}$$

This function on each element of the surface of the scapula

$$\left[\xi_i, \quad \xi_{i+1}; \quad \eta_j, \eta_{j+1}\right]$$

is a bicubic polynomial, continuous, and has continuous partial derivatives up to and including $\frac{\partial^4 h(\xi,\eta)}{\partial\xi^2\partial\eta^2}$, that is $h(\xi, \eta) \in c^{2,2}$.

We represent the system of Eq. (24) in a dimensionless form:

$$\left.\begin{array}{l} L_{11}(\overline{w}) + L_{12}(\overline{F}) = L_1; \\ L_{21}(\overline{w}) + L_{22}(\overline{F}) = L_2, \end{array}\right\} \tag{31}$$

where $L_{ik}(i, k = 1, 2)$ are dimensionless differential operators of the theory of shells, referred to the lines of curvature of the surface; L_m (m = 1, 2) are components of a given surface and temperature load.

The analytical solution of the system of Eq. (31) with boundary conditions in Eq. (28) is based on the application of the method of integral relations by Dorodnitsyn [13].

In accordance with the method, we write the initial system of Eq. (31) in divergent form:

$$\frac{\partial \overline{X}}{\partial \xi} + \frac{\partial \overline{Y}}{\partial \eta} + \overline{L} = 0, \tag{32}$$

where $\overline{X} = \{X_i\} = \{\overline{w}, z_1, z_2, z_3, \overline{F}, z_4, z_5, z_6\}$;

$$\overline{Y} = B_0\overline{X} + B_1\frac{\partial \overline{X}}{\partial \eta} + B_2\frac{\partial^2 \overline{X}}{\partial \eta^2} + B_3\frac{\partial^3 \overline{X}}{\partial \eta^3}; \overline{L} = B\overline{X} + \overline{b}.$$

Through $z_1, ..., z_6$ marked:

$$z_1 = \overline{w}_{,1}; z_2 = \overline{w}_{,11}; z_3 = \overline{w}_{,111}; z_4 = \overline{F}_{,1}; z_5 = \overline{F}_{,11}; z_6 = \overline{F}_{,111}.$$

In Eq. (32) $B_r = \{b_{mn,s}\}$ and $B = \{b_{mn}\}; (s = 0, 1, 2, 3; m, n = 1, 2, ..., 8)$ are transformation matrices.

Following the method of integral relations of Dorodnitsyn [13], we look for a solution to the system of Eq. (32) in the form of an expansion:

$$X_i(\xi, \eta) = \begin{cases} \sum\limits_{j=1}^{n} X_{ij}(\xi) P_j(\xi, \eta) & (i = 3, 6, 7, 8) \\[2mm] \sum\limits_{j=1}^{n} X_{ij}(\xi) P_{j,2}(\xi, \eta) & (i = 1, 2) \\[2mm] \sum\limits_{j=1}^{n} X_{ij}(\xi) P_{j,22}(\xi, \eta) & (i = 4, 5) \end{cases} \tag{33}$$

As approximating and weighting functions, we choose the Jacobi system of orthogonal polynomials [15, 16] and their derivatives:

$$P_j(\xi, \eta) = P_1(\xi, \eta) \sum_{j=1}^{n} \left[\eta - \frac{(1+\alpha)r}{2} \right]^{j-1};$$

$$P_1(\xi, \eta) = \eta^4 - 2(1+\alpha)r\eta^3 + (1 + 4\alpha + \alpha^2)r^2\eta^2 - 2\alpha(1+\alpha)r^3\eta + \alpha^2 r^4.$$

Here $r = 1 + k_1 m\xi$ is the equation of the inclined side of the scapula.

Note that polynomials $P_j(\xi, \eta)$ are orthogonal on the interval $[\eta_1, \eta_2]$ and forming a system of linearly independent functions, they satisfy the boundary conditions at the oblique edges of the blade in Eq. (28).

We also emphasize that their derivatives $P_{j,2}(\xi, \eta)$ and $P_{j,22}(\xi, \eta)$ also have the property of orthogonality.

Restricting ourselves to the two-term approximation and also choosing power polynomials $P_j(\xi, \eta)$ and their derivatives as weight functions, after applying the procedure of the method of integral relations to the original system of Eq. (32), we obtain a system of ordinary differential equations of order 8n with variable coefficients:

$$\frac{dX_m}{d\xi} = \sum_{\nu=1}^{s} B_{\nu,m} X_\nu + f_m, \quad m = 1, 2, \ldots, s \tag{34}$$

It should be noted that in the general case there is no exact solution of such equations in mathematics, with the exception of individual special cases, for example, the Bessel equation.

Here, the modified method of successive approximations developed by Professor Pukhliy and published by him in the Academic Press [12, 17] is applied to the solution of the boundary-value problem.

Later, the method was extended to the solution of initial-boundary value problems [4], and to accelerate the convergence of the solution, the method of telescopic shift of the power series of K. Lanczos [18] was used. For this, we used the possibility of representing any power series in terms of shifted Chebyshev polynomials [15, 16]. For the first time, such an approach was presented in the works of V.A. Pukhliy [19, 20].

In accordance with the method, variable coefficients $B_{\nu,m}$ and free terms f_m can be represented through shifted Chebyshev polynomials:

$$f_m = \sum_{r=0}^{q} f_{m,r}(d_r \cdot r!)^{-1} \sum_{k=0}^{r} a_k T_k^*(\xi), \quad B_{\nu,m} = \sum_{r=0}^{q} b_{\nu,m,r} d_r^{-1} \sum_{k=0}^{r} a_k T_k^*(\xi) \tag{35}$$

Here q is the degree of the interpolation polynomial and a_k are coefficients of expansion of ξ^r in a series of Chebyshev polynomials $T_k^*(\xi)$. In expressions of Eq. (35), $d_r = 1$ for $r = 0$ and $d_r = 2^{2r-1}$ for the remaining r.

The general solution of the system of Eq. (34) has the form [10, 19, 20]:

$$X_m = \sum_{\mu=1}^{s} C_\mu \left[d_0^{-1} a_0 T_0^*(\xi)\, \delta + \sum_{n=1}^{\infty} X_{m,\mu,n} \right] + \sum_{j=0}^{q} t_{m,j,0} \left[d_{j+1}(j+1)! \right]^{-1} \sum_{k=0}^{j+1} a_k T_k^*(\xi)$$

$$+ \sum_{n=2}^{\infty} X_{m,n},$$

(36)

where $t_{m,j,0} = f_{m,r}$ for $j = r$; μ is the number of the fundamental function; and C_μ are integration constants. In solution in Eq. (36), $\delta = 1$ if $m = \mu$ and $\delta = 0$ for the remaining μ.

The first approximation $X_{m,\mu,1}$ is obtained by substituting the zeroth approximation: $d_0^{-1} a_0 T_0^*(\xi)\, \delta$ into the right-hand side of system $\frac{dX_m}{d\xi} = \sum_{\nu=1}^{s} B_{\nu,m} X_\nu$.

Subsequent approximations are carried out according to the formulas:

$$X_{m,\mu,n} = \sum_{j=1}^{\beta} t_{m,\mu,n,j} \left[d_{n+j-1}(n+j-1)! \right]^{-1} \sum_{k=0}^{n+j-1} a_k T_k^*(\xi);$$

$$X_{m,n} = \sum_{j=1}^{\beta} t_{m,n,j} \left[d_{n+j-1}(n+j-1)! \right]^{-1} \sum_{k=0}^{n+j-1} a_k T_k^*(\xi), \quad \text{where } \beta = n(q+3))-2$$

(37)

The systems of fundamental functions in Eq. (37) are uniformly converging series, and the coefficients $t_{m,\mu,n,j}$ and $t_{m,n,j}$ are determined through the coefficients of the previous approximation using recurrence formulas:

$$t_{m,\mu,n,j} = \sum_{\nu=1}^{s} \sum_{r=0}^{q} b_{\nu,m,r} t_{\nu,\mu,n-1,j-r}(n+j-1)^{-1} \prod_{\gamma=0}^{r}(n+j-1-\gamma),$$

$$t_{m,n,j} = \sum_{\nu=1}^{s} \sum_{r=0}^{q} b_{\nu,m,r} t_{\nu,n-1,j-r}(n+j)^{-1} \prod_{\gamma=0}^{r}(n+j-\gamma).$$

The constants C_μ included in the general solution in Eq. (36) are found from the boundary conditions along the inlet and outlet edges of the blade in Eq. (29).

Thus, the problem reduces to the joint solution of Eq. (23) and the system of Eq. (24) under initial conditions in Eq. (22) and boundary conditions in Eqs. (28) and (29). Moreover, at each time step, from Eq. (23) we obtain the numerical values of the thickness, which are used to construct spline functions in Eq. (30). Then the system of Eq. (24) is solved, from the solution of which the values σ_{ij} are obtained.

The criterion for terminating the step-by-step process is the condition:

$$\sigma_{ij} \leq [\sigma_T],$$

where σ_T is the yield strength of the material.

The durability of the impeller element of centrifugal pumps is obtained by summing the steps in time.

4. Conclusions

The theory of corrosion wear of structural elements of hydropower and nuclear energy in the form of plates and shells is developed taking into account the stress state and corrosion wear.

Numerous factors affecting the speed of the corrosion wear process (degree of aggressiveness of the media, temperature, humidity, etc.) are taken into account in a generalized way by drawing up a differential equation for the rate of change of the thickness of the impeller element.

The criterion for the ultimate state of structural elements is the achievement by the structural element of the yield strength of the material σ_T.

An algorithm has been developed for solving the problem of corrosion wear of bimetallic pipelines of nuclear energy, taking into account the stress-strain state of the elements.

An algorithm has been developed for the analytical solution of the problem of corrosion wear of rotor blades of centrifugal pumps based on a combination of the method of integral relations and the modified method of successive approximations in displaced Chebyshev polynomials.

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