We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,300

Open access books available

172,000

International authors and editors

190M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Noise Characteristic Analysis of Multi-Port Network in Phased Array Radar

Yu Hongbiao

Abstract

Noise figure and noise power are detailedly analyzed and deduced in theory for multi-port network in active phased array radar. The mathematical expressions of output noise power and noise figure of network are given out under various conditions. Accordingly, this provides a basis of theories for multi-port network and radar receiver system design, the test method of array noise figure. Finally, two application examples are given to verify the accuracy of the formulae. Making use of these formulas, the designer can use to calculate the dynamic range of the radar receive system, and the designer can also constitute a measure scheme of the array noise figure for active phased array radar.

Keywords: noise power, noise figure, active network, passive network, active phased array radar

1. Introduction

In modern active phased array radar, the active antenna array is generally composed of dozens to tens of thousands of active transmit/receive (T/R) modules. However, the feeding of T/R modules (receiving echo signal and transmitting excitation power) is usually realized by a multi-port feeding network. The calculation of noise power and the measurement method of system noise figure of active antenna array including multi-port feed network are essential work for radar system designers and receiver designers. Understanding the analysis and calculation of noise power and noise figure of multi-port network is the basis for design specification such as system dynamic range, so how to correctly analyze and calculate noise power and noise figure of active antenna array is an important factor in radar system design. Next, the analysis and calculation of noise power for multi-port network and the calculation and measurement method of system noise figure in active phased array radar will be described in detail.

2. Noise power and noise figure of two-port network

For a two-port linear network as shown in **Figure 1**, suppose that $G(L)$ in the figure is the gain (loss) of the two-port network and NF is the noise figure of the two-port linear network. BW is the signal bandwidth, then the equivalent noise temperature of the two-port linear network is [1].

Figure 1.
Noise characteristics of a two-port network.

$$T_e = (NF - 1)T_0 \quad (1)$$

where T_e is the equivalent noise temperature and T_0 is the room temperature, equal to 290 K.

The white noise power PN_0 is [2]:

$$PN_0 = kT_0BW \quad (2)$$

where k is the Boltzmann constant, equal to 1.381×10^{-23} J/K.

Assuming that the noise temperature of the input port is T_i , the input noise power PN_i is:

$$PN_i = kT_iBW \quad (3)$$

The noise power PN_a of a two-port linear network caused by equivalent noise temperature is:

$$PN_a = kT_eBW \quad (4)$$

The noise figure NF is expressed by noise power as:

$$NF = 1 + \frac{PN_a}{PN_0} \quad (5)$$

The output noise power PN_o of the two-port linear network is:

$$PN_o = GPN_i + GPN_a \quad (6)$$

Note that T_i in the formula is not necessarily equal to T_0 .

3. Analysis of noise characteristics of passive two-port linear network

We first analyze the noise characteristics of the passive two-port linear network as shown in **Figure 2**. In the figure, L is the insertion loss of the passive two-port network, BW is the operating bandwidth of the passive two-port linear network, T_i is the noise temperature at the input of the passive two-port network, PN_i is the

Figure 2.
Passive two-port linear network.

noise power at the input of the passive two-port network, and PN_o is the noise power at the output of the passive two-port network.

Let:

$$T_i = T_0 + T_{ei} \quad (7)$$

Then:

$$PN_i = kT_i BW = k(T_0 + T_{ei}) BW \quad (8)$$

The noise power generated by PN_i at the network output is:

$$PN_{oi} = \frac{kT_i BW}{L} \quad (9)$$

The equivalent noise temperature of the passive two-port lossy network converted to the input of the two-port network is:

$$T_{eL} = (L - 1)T_0 \quad (10)$$

The noise power generated by the passive two-port lossy network at the output is:

$$PN_{oL} = \left(1 - \frac{1}{L}\right) kT_0 BW \quad (11)$$

Therefore, the total noise power generated by the passive two-port lossy network at the output is $PN_o = PN_{oi} + PN_{oL}$

$$\begin{aligned} PN_o &= \frac{kT_i BW}{L} + \left(1 - \frac{1}{L}\right) kT_0 BW = \frac{kT_0 BW}{L} + \frac{kT_{ei} BW}{L} + \left(1 - \frac{1}{L}\right) kT_0 BW \\ &= kT_0 BW + \frac{kT_{ei} BW}{L} \end{aligned} \quad (12)$$

4. Analysis of noise characteristics of multi-port linear passive networks

Next, we will analyze the noise characteristics of the multi-port linear passive network as shown in **Figure 3**. It is assumed that the multi-port linear passive network has n input ports and one output port, the active power loss of the network is L , and the signal bandwidth is BW .

Let the noise temperature of the j th input port of the multi-port linear passive network be:

$$T_{ij} = T_0 + T_{eij} \quad (13)$$

Then the noise power generated by the j th input port at the output of the multi-port linear passive network is:

$$PN_{oj} = \frac{k(T_0 + T_{eij}) BW}{nL} \quad (14)$$

Figure 3.
Multi-port linear passive network.

Then the total noise power generated by the n input ports at the network output is:

$$PN_{on} = \sum_{j=1}^n PN_{oj} = \frac{kBW}{nL} \sum_{j=1}^n (T_0 + T_{ej}) = \frac{kT_0BW}{L} + \frac{kBW}{nL} \sum_{j=1}^n T_{ej} \quad (15)$$

The equivalent noise temperature of the multi-port lossy passive network converted to each input is of the same formula (10), and then the noise power generated by the lossy network at the output is:

$$PN_{oL} = \frac{nkT_{eL}BW}{nL} = \left(1 - \frac{1}{L}\right) kT_0BW \quad (16)$$

The total noise power generated at the output of the multi-port linear passive network is:

$$PN_o = PN_{on} + PN_{oL} = kT_0BW + \frac{kBW}{nL} \sum_{j=1}^n T_{ej} \quad (17)$$

If T_{ej} of each input port of the multi-port linear passive network is the same as T_{ei} , then:

$$PN_o = kT_0BW + \frac{kT_{ei}BW}{L} \quad (18)$$

By comparing Eq. (12) with Eq. (18), we can find that when $T_{ei} = 0$, i.e., each input port of the passive lossy network is connected to a matching load with a noise temperature of T_0 , the noise power generated by the passive lossy network at the output port is equal, i.e., $PN_o = kT_0BW$.

5. Analysis of network noise characteristics after cascade of two-port active network and two-port passive network

If the two-port active network and the two-port passive network are cascaded, as shown in **Figure 4**, what is the noise characteristic of the cascaded two-port network? For the convenience of analysis, we make the noise figure of the two-port active network to be NF_1 , the gain G , and the insertion loss L . For the convenience of analysis, it is assumed that the operating signal bandwidths of both are the same and both are BW .

Figure 4.
 Cascade of two-port active network and two-port passive network.

The equivalent noise temperature of the two-port active network converted to its input is [3]:

$$T_{ea} = (NF_1 - 1)T_0 \quad (19)$$

The equivalent noise temperature of a two-port passive network converted to its input is of the same formula (10). The noise power at the output of the two-port active network is thus:

$$PN_a = kT_0BWG + kT_{ea}BWG \quad (20)$$

The noise power PN_a at the output of the two-port active network produces the following noise power at the output of the passive network:

$$PN_{oa} = \frac{kT_0BWG}{L} + \frac{kT_{ea}BWG}{L} \quad (21)$$

The noise power generated by the two-port passive network itself at the output is shown in Eq. (11), so the total noise power at the output of the two-port synthetic network is:

$$\begin{aligned} PN_o &= PN_{oa} + PN_{oL} = \frac{kT_0BWG}{L} + \frac{kT_{ea}BWG}{L} + \left(1 - \frac{1}{L}\right) kT_0BW \\ &= kT_0BW \left(1 + \frac{G-1}{L}\right) + \frac{kT_{ea}BWG}{L} \end{aligned} \quad (22)$$

The total noise figure of the synthetic network after the two-port active network and the two-port passive network are cascaded is:

$$NF = \frac{PN_o}{kT_0BWG/L} = \frac{L+G-1}{G} + \frac{T_{ea}}{T_0} = NF_1 + \frac{L-1}{G} \quad (23)$$

6. Analysis of network noise characteristics after cascaded two-port active network and multi-port passive network

Cascade n two-port active linear networks and a passive linear network with n input ports, as shown in **Figure 5**. Next, let's analyze the noise characteristics of n two-port active networks and multi-port passive networks after cascading.

As before, we assume that the active power loss of the multi-port passive network is L , the noise figure of the i th two-port active network is NF_i , and the gain is G_i . To facilitate analysis, if the operating signal bandwidth of both networks is BW , the equivalent noise temperature of the i th two-port active network is:

$$T_{eai} = (NF_i - 1)T_0 \quad (24)$$

Figure 5.
Cascade of n two-port active networks and multi-port passive networks.

The noise power of the i th two-port active network at its output is:

$$PN_{ai} = kT_0BWG_i + kT_{eai}BWG_i \quad (25)$$

The noise power generated by the i th two-port active network at the output of the passive network is:

$$PN_{oai} = \frac{kT_0BWG_i + kT_{eai}BWG_i}{nL} \quad (26)$$

The total noise power at the output of the synthesis network is obtained from Eqs. (16) and (26):

$$\begin{aligned} PN_o &= PN_{oL} + \sum_{i=1}^n PN_{oai} = \left(1 - \frac{1}{L}\right) kT_0BW + \sum_{i=1}^n \frac{kT_0BWG_i + kT_{eai}BWG_i}{nL} \\ &= \left(1 - \frac{1}{L}\right) kT_0BW + \left[\frac{kBW}{nL} \sum_{i=1}^n (T_0 + T_{eai})G_i \right] \end{aligned} \quad (27)$$

We can calculate the total noise figure NF of the synthetic network as follows:

$$\begin{aligned} NF &= \frac{PN_o}{kT_0BWG_\Sigma} = \frac{PN_o}{\left(kT_0BW \sum_{i=1}^n \frac{G_i}{nL}\right)} \\ &= \left\{ \left(1 - \frac{1}{L}\right) kT_0BW + \left[\frac{kBW}{nL} \sum_{i=1}^n (T_0 + T_{eai})G_i \right] \right\} / \left(kT_0BW \sum_{i=1}^n \frac{G_i}{nL} \right) \\ &= n(L-1) / \sum_{i=1}^n G_i + \left[\sum_{i=1}^n \left(1 + \frac{T_{eai}}{T_0}\right) G_i \right] / \sum_{i=1}^n G_i \end{aligned} \quad (28)$$

where G_Σ is the gain of the synthetic network.

If the gain and noise figure of the two-port active network are the same, i.e., $G_i = G$ and $NF_i = NF_1$, then Eq. (28) is simplified as:

$$NF = 1 + \frac{T_{eai}}{T_0} + \frac{L-1}{G} = NF_1 + \frac{L-1}{G} \quad (29)$$

6.1 Analysis of noise characteristics of active and passive synthetic networks with $n - 1$ input ports connected to a matching load with a noise temperature of T_0

The previous analysis is to analyze the noise characteristics of n two-port active networks under normal operation. If the $n - 1$ input ports of the synthesis network are connected to a matching load with a noise temperature of T_0 and all two-port active networks operate normally, what will happen to the noise characteristics of the synthesis network? At this time, the multi-port synthesis network degenerates into a two-port network, but the total noise power at its output remains unchanged, the same as Eq. (27). At this time, the total noise figure NF of the synthesis network is:

$$\begin{aligned} \text{NF} &= \frac{\text{PN}_o}{kT_0\text{BW}G_\Sigma} = \text{PN}_o / \left(\frac{kT_0\text{BW}G_i}{nL} \right) \\ &= \left\{ \left(1 - \frac{1}{L} \right) kT_0\text{BW} + \left[\frac{k\text{BW}}{nL} \sum_{i=1}^n (T_0 + T_{\text{eai}}) G_i \right] \right\} / \left(\frac{kT_0\text{BW}G_i}{nL} \right) \\ &= n(L - 1)/G_i + \left[\sum_{i=1}^n \left(1 + \frac{T_{\text{eai}}}{T_0} \right) G_i \right] / G_i \end{aligned} \quad (30)$$

If the gain and noise figure of the two-port active network are the same ($G_i = G$, $\text{NF}_i = \text{NF}_1$), the above equation becomes:

$$\text{NF} = n \left(1 + \frac{T_{\text{eai}}}{T_0} + \frac{L - 1}{G} \right) = n \left(\text{NF}_1 + \frac{L - 1}{G} \right) \quad (31)$$

6.2 It is stated in Section 6.1 that if $n - 1$ two-port active networks are not operating, the noise characteristics of the synthetic network are analyzed

Following the previous analysis, when the $n - 1$ two-port active network input ports in the synthetic network are connected to a matching load with a noise temperature of T_0 and the $n - 1$ active networks do not operating, what will happen to the noise characteristics of the synthetic network at this time? In order to facilitate the analysis, it is assumed that the two-port active network matches the passive network in both operating and nonoperating states. At this time, the synthetic network degenerates into a two-port network with nL loss. As mentioned in Section 5, the noise power of the two-port active network at the output port of the synthetic network is:

$$\text{PN}_{\text{oa}} = \frac{kT_0\text{BW}G}{nL} + \frac{kT_{\text{ea}}\text{BW}G}{nL} \quad (32)$$

As mentioned in Section 4, the noise power of the passive network at the output port of the synthetic network is:

$$\text{PN}_{\text{oL}} = \frac{nk(L - 1)T_0\text{BW}}{nL} + \frac{(n - 1)kT_0\text{BW}}{nL} = kT_0\text{BW} \left(1 - \frac{1}{nL} \right) \quad (33)$$

Then the total noise power at that output port of the synthetic network is:

$$\text{PN}_o = \text{PN}_{\text{oa}} + \text{PN}_{\text{oL}} = \frac{k(T_0 + T_{\text{ea}})\text{BW}G}{nL} + \left(1 - \frac{1}{nL} \right) kT_0\text{BW} \quad (34)$$

The noise figure NF of the synthetic network is:

$$NF = \frac{PN_o}{kT_0BWG_\Sigma} = PN_o / \left(\frac{kT_0BWG}{nL} \right) = \left(1 + \frac{T_{eai}}{T_0} + \frac{nL-1}{G} \right) = NF_1 + \frac{nL-1}{G} \quad (35)$$

when $n = 1$, the synthetic network degrades to section 5 state, that is, the cascade of two-port active network and two-port passive network.

7. Design, application, and verification

7.1 Calculation of array receiving dynamics of phased array radar

An active phased array radar is composed of 64 identical T/R modules and a 64:1 multi-port passive in-phase power synthesis network. Its structure is similar to that of **Figure 5**. In the engineering design, the design specification of the gain and noise figure of all T/R modules are the same, so we use the same noise figure NF_1 and gain G in the analysis and calculation. The error caused by the inconsistent indexes of different T/R modules is always acceptable and reasonable in the engineering design and calculation. Then the total noise power of the synthetic output received by the radar array can be calculated from Eq. (27):

$$PN_o = kT_0BW \left(\frac{NF_1G}{L} + 1 - \frac{1}{L} \right) \quad (36)$$

Using Eq. (29), NF_1 is expressed by the total noise figure NF of the synthesis network and substituted into the simplified equation above to obtain:

$$PN_o = kT_0BW \frac{NFG}{L} \quad (37)$$

Assuming that the baseband signal bandwidth of the receiver is 4 MHz, the noise figure of the T/R module is 2 dB, the gain is 25 dB, and the active power loss of the 64:1 power synthesis network is 5 dB, the total output noise power of the synthesis network can be calculated by using Eq. (36) as follows:

$$PN_o = -114 + 6 + 22.02 = -85.98 \text{ dBm} \quad (38)$$

In order to facilitate calculation in engineering application, we use T/R module noise figure NF_1 to replace the total noise figure NF of the synthesis network and use Eq. (37) to calculate the total output noise power of the synthesis network, then:

$$PN_o = -114 + 6 + 2 + 25 - 5 = -86 \text{ dBm} \quad (39)$$

We compare the calculation results of the above two different methods and find that the difference between them is only 0.02 dB. Therefore, as long as the gain of the active network is much larger than the active power loss of the passive network in engineering application, the error caused by using the noise figure of the active network instead of the noise figure of the synthesis network to calculate the total output noise power of the synthesis network can be ignored, which is enough to meet the requirements of engineering design.

Next, we will calculate the dynamic range of the output signal of the synthesis network.

We assume that the input signal power received by each T/R module of the array is -105 dBm and the phases of the input signals are the same, then the output signal power of the synthesis network is:

$$S_o = -105 + 25 - 5 + 18 = -67 \text{ dBm} \quad (40)$$

The input dynamic range of signal power relative noise power (regardless of noise introduced by antenna) is:

$$DR_i = -105 - (-14 + 6) = 3 \text{ dB} \quad (41)$$

After the signal is synthesized by the network, the output dynamics of the signal power relative noise power is as follows:

$$DR_o = -67 - (-86) = 19 \text{ dB} \quad (42)$$

Note that when calculating the input and output noise power above, the bandwidth of both must be the same; both are 4 MHz.

Through the calculation of the above practical examples, we can draw a conclusion that when calculating the dynamic range of the network output signals synthesized by the active network and the multi-port passive in-phase network, we must remember that the total noise power output by the network is not added, only the in-phase signals can be added, and the dynamic range of the signal to noise will increase after passing through the synthesized network.

7.2 Method for testing noise figure of synthesis network after cascading two-port active network and multi-port passive network

In the active phased array radar, we design a T/R module, which consists of four identical receiving channels. Finally, the four receiving channels are output through a 4:1 power synthesis network. How to measure the noise figure of the T/R module in practical engineering application? Our common noise figure instruments, such as HP8970B and Agilent N8975A, have only one noise source. At first, engineers measured the noise figure of each receiving channel to be about 8 dB under the condition of normal operation of the four channels. This measurement data is quite different from the actual design specification, and there are obvious problems. Later, when measuring the noise figure of one receiving channel, we turned off the other three receiving channels and measured the noise figure of each channel in turn. At this time, the noise figure of each channel was measured to be about 2 dB, and the result basically met the design requirements.

It is not difficult for us to understand the above phenomena by using the previous analysis and derivation results. Obviously, it can be seen from Eq. (29) that the noise figure of the synthetic network is basically close to that of a single active channel (when G is much larger than nL). Looking at Eq. (31), we find that when all four channels are operating, if we measure the noise figure of one channel, it will increase by n times than the theoretical value, where n is 4, i.e. 6 dB, so the noise figure we measure is about 8 dB. When one receiving channel is measured and the other three receiving channels are turned off, the noise figure measured at this time is the result given by Eq. (35), and the measurement result is close to the noise figure of a single active channel (when G is much larger than nL). Therefore, when

	f_1	f_2	f_3	f_4
Noise figure in single-channel operation/dB	2.28	2.20	2.26	2.24
Noise figure in four-channel operation/dB	8.20	8.12	8.16	8.15

Table 1.
Noise figure of multi-port active synthesis network under different test conditions.

G is much larger than nL (which can be realized in engineering), it can be considered that Eq. (35) is close to Eq. (29). See **Table 1** for noise figure of multi-port active synthesis network under different test conditions.

In engineering applications, we use the existing noise figure test instruments and adopt the above method to measure the noise figure of the multi-port active synthesis network. We must remember that there is a condition that the gain of a single active channel is much larger than the loss of the passive synthesis network (including the distribution loss at this time); otherwise the measurement result will be greatly different from the theoretical value. We can also average the measured values of each channel to characterize the noise figure of the whole synthetic network. For example, the active channel gain G is only 15 dB, while the passive network is 32:1. When the noise figure of the synthesis network is measured by the above test method, the result will cause a large error. The specific reason can be seen in the previous correlation analysis and calculation formula (35). Of course, we can also correct the measurement by setting the loss of the DUT in the noise figure testing instrument, so that we can also obtain the correct measurement value. For specific operation settings, please refer to the relevant operating instructions of the noise figure test instrument.

In this chapter, the mathematical expressions of the total output noise power and noise figure of the multi-port network in many common cases are given. Using these formulas, designers can calculate the dynamic range of the active phased array radar receiving system and can also use the calculation formula of noise figure to formulate the testing scheme of the active phased array radar noise figure [4, 5].

IntechOpen

Author details

Yu Hongbiao
Nanjing Research Institute of Electronics Technology (NRIET), China

*Address all correspondence to: sssyhb@126.com

IntechOpen

© 2020 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

References

- [1] Connor FR. Noise. Beijing: Science Press; 1982. p. 72
- [2] Sklar B. In: PingPing X et al., editors. Digital Communications Fundamentals and Applications. 2nd ed. Beijing: Electronics Industry Press; 2002. pp. 207-213
- [3] Guangyi Z. Phased Array Radar System. Beijing: National Defense Industry Press; 2000. p. 218
- [4] Lin GM, Yuan LS. Noise measurement of active phased array receiver system. Modern Radar, China. 2004;**26**(3):54-57
- [5] Chuandong H. Measurement of equivalent noise temperature of phased array radar receiving system. In: National Conference on Microwave Measurement. 2000