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Introduction: Plasma Parameters and Simplest Models

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Abstract

Plasma is ionized gas (partially or fully). Overwhelming majority of matter in the universe is in plasma state (stars, Sun, etc.). Basic parameters of plasma state are given briefly as well as classification of plasma types: classic-quantum, ideal-nonideal, etc. Differences between plasma and neutral gas are presented. Plasma properties are determined by long distance electrostatic forces. If spatial dimensions of a system of charged particles exceed the so-called Debye radius, the system may be considered as plasma, that is, a medium with qualitatively new properties. The expressions for Debye radius for classical and quantum plasma are carried out. Basic principles of plasma description are presented. It is shown that plasma is a subject to specific electrostatic (or Langmuir) oscillations and instabilities. Simplest plasma models are given briefly: the model of “test” particle and model of two (electron and ion) fluids. As an example, Buneman instability is presented along with qualitative analysis of its complicate dispersion relation. Such analysis is typical in plasma theory. It allows to easily obtain the growth rate.

Keywords: plasma, quasi-neutrality, Langmuir frequency, Debye length, simplest plasma models, unstable plasma oscillations

1. Introduction

Everyone knows the three states of matter: solids, liquids, and gases. Plasma is often called the fourth state of matter; bear in mind that with increasing temperature, the following transitions take place: solids-liquids-gases-plasma. Under the last transition, atoms lose electrons. Plasma consists (along with neutral atoms) of charged particles: electrons and positively charged ions (single and/or multiple ionized). This definition of plasma is far from complete. The complete definition of plasma is, in fact, impossible. It must cover a very wide range of phenomena in a wide variety of conditions.

Plasma is very common in the universe. Most of the substance in it (more than 99%) is in plasma state. Media consisting of ionized atoms is found almost everywhere. The upper layers of the Earth's and stellar atmospheres, interstellar medium, etc. actually are in plasma state. Stellar plasma is another widespread example. In the plasma of stars, in particular the Sun, reactions of the synthesis of light elements, the so-called thermonuclear reactions, provide a huge release of energy and plasma heating. Currently, scientists from many countries around the world are studying the possibility of creating such a high-temperature plasma in terrestrial conditions, setting the task of implementing controlled thermonuclear fusion and providing humanity with an inexhaustible supply of energy.

There are two fundamentally different approaches to the implementation of controlled thermonuclear fusion. The first approach is obtaining the reactions in the so-called magnetic traps. Hot plasma should not come in contact with the walls of the chamber, as this will lead to its actual destruction. The confinement of the plasma by a magnetic field, in theory, should prevent the contact of the plasma with the walls of the chamber, since the magnetic field bends the trajectory of charged particles. However, despite tremendous efforts, the task of plasma confining has not been completely solved yet. Special configurations of magnetic field, magnetic traps help only partially. Plasma is an unstable medium in which small perturbations increase and destroy its given state. Instabilities are intrinsic for plasma. It turned out that any nonequilibrium initial distribution of particles is unstable. Below we show how instability follows from general electrodynamical consideration and give an example.

The second approach is the very quick heating of plasma up to thermonuclear temperatures. The reaction itself and energy removal also take place quickly. These processes recur very fast, and confinement of such plasma is not needed. This approach is called inertial fusion. It was proposed when very fast heating of plasma becomes possible by using intense laser beams and/or high-current relativistic electron beams.

The development of these studies is associated with the rebirth of the concept of plasma, which arises upon the investigations of gas-discharge processes. The processes in gas-discharge plasma have also been intensively studied. The studies were associated with the development of the needs of classical and quantum electronics, for which gas-discharge appliances play an important role. Finally, solid-state plasma should be noted: electron plasma of metals and electron-hole plasma of semiconductors.

The listed series can be continued almost unlimitedly, speaking about plasma in magnetohydrodynamic and thermionic converters of thermal energy into electrical energy, about plasma in solutions of electrolytes, etc. However, the above examples are sufficient to make sure the extremely wide prevalence of plasma in nature and the importance of studying its properties.

A vast literature has been grown to describe plasma state (see, e.g., [1–7] and many others). Our further presentation is based on the principles of plasma electrodynamics considering plasma as a continuous medium with a large number of free charged carriers.

2. Plasma parameters

Plasma is an ionized gas consisting of free electrons and various types of ions and neutrals. First of all, it is necessary to know the charge e_α and concentration n_α of plasma components (here the index takes different values corresponding to the types of particles in the plasma, $\alpha = e$ for electrons, $\alpha = i_1, i_2 \dots$ for ions of various types, and $\alpha = n$ for neutrals). All plasma particles are in chaotic motion, but full thermodynamic equilibrium is absent. Usually each component has its own temperature T_α , which is also necessary to know.

In solid-state and semiconductor plasma, the conception of temperature should be given more accurately. If the Fermi energy $E_{F\alpha}$ of α -type particles exceeds their thermal energy

$$E_{F\alpha} = \frac{(3\pi^2)^{2/3} \hbar^2 n_\alpha^{2/3}}{2m_\alpha} \gg T_\alpha \quad (1)$$

(here m_α is the mass of the particle, and $\hbar$ is the Planck constant), quantum effects should be accounted. In this case the Maxwell's distribution function does not describe the behavior of charged particles. It should be described by Fermi distribution function, and the $E_{F\alpha}$ (1) plays the role of temperature. In this case plasma is *degenerate*.

If neutrals are absent, plasma is *fully ionized*. In the opposite case, plasma is *partially ionized*, and it is necessary to know the level of plasma ionization. This is the ratio of neutrals' density to the density of charged particles (or to the full density of plasma).

An important characteristic peculiarity of plasma state is a very wide range of values of these (and other) parameters. For example, plasma in some stars (white dwarfs) has a density of $10^{25}\text{--}10^{26}\text{ cm}^{-3}$, but in the interstellar space, plasma has a density of $1\text{--}10\text{ cm}^{-3}$. The ratio is 10^{26} . The ratio of other parameter values is a bit less. This leads to important consequences. In the example above, different approaches may be required to describe the plasma inside the stars and in the interstellar space. The most interesting cases will be mentioned below.

An important condition for plasma existence is its quasi-neutrality. The condition of quasi-neutrality has the form

$$\sum_{\alpha} e_{\alpha} n_{\alpha} \approx 0 \tag{2}$$

where the summation is made over all types of charged particles $\alpha = e, i_1, i_2, i_3 \dots$. When it is violated, strong electric fields arise, which restore plasma quasi-neutrality. Violations of quasi-neutrality are possible only in spatial and temporal scales, small in comparison with the characteristic plasma scales. The temporal characteristic scale of plasma is determined by its proper oscillations, but the spatial scale is determined by the length of plasma shielding (Debye length; see below).

2.1 Plasma oscillations

2.1.1 Langmuir frequency

Plasma, as a medium with a large number of free charged particles, is a subject to oscillations. Consider in detail the oscillations of uniform electron plasma. Ions are heavy (immobile) and serve for charge neutralization. Let a small displacement of an electron layer relative to the ions take place (see **Figure 1**). We denote the displacement vector by $\mathbf{X}$. The density of the uncompensated electron charge at the displacement $\mathbf{X}$ may be found from the continuity equation:

$$\rho = \nabla n_e e \mathbf{X} = n_e e \nabla \mathbf{X} \tag{3}$$

Figure 1.
Oscillations of the electron layer.

This charge creates an electric field $\mathbf{E}$, the value of which may be determined from Poisson's equation:

$$\nabla \cdot \mathbf{E} = 4\pi\rho = 4\pi n_e |e| \nabla \cdot \mathbf{X} \quad (4)$$

Hence, we can write (given that for $\mathbf{X} = 0$, we have $\mathbf{E} = 0$)

$$\mathbf{E} = 4\pi n_e e \mathbf{X} \quad (5)$$

Thus, the field $\mathbf{E}$ is parallel to the displacement of electrons and acts on each electron with a force

$$\mathbf{F} = -e\mathbf{E} = -4\pi n_e e^2 \mathbf{X} \quad (6)$$

tending to return the electron to its original equilibrium position. As a result, we have the equation of motion of an electron in the form

$$m \frac{d^2 \mathbf{X}}{dt^2} = -e\mathbf{E} = -4\pi n_e e^2 \mathbf{X} \quad (7)$$

This equation describes the oscillations of plasma electrons near the equilibrium position ($\mathbf{X} = 0$) with a frequency

$$\omega = \omega_{Le} = \sqrt{\frac{4\pi n_e e^2}{m}}, \quad (8)$$

which is known as the electron Langmuir frequency. If one uses MKS units, the expression for electron Langmuir frequency is

$$\omega_{Le} = \sqrt{\frac{n_e e^2}{\varepsilon_0 m}} \quad (9)$$

where ε_0 is the dielectric permittivity of vacuum. Violations of plasma quasi-neutrality are possible only on a temporal scale, small in comparison with time $\tau \sim 1/\omega_{Le}$.

2.2 Gas parameter. Debye length

The behavior of an ionized gas is determined by long-distance electrostatic forces. These forces significantly influence on the plasma behavior and, actually, determine its parameters. First of all it is necessary to find out under what conditions a system of electrostatically interacting particles can be considered as a gas. The main peculiarity of a gas is the following: its particles interact during very small time intervals only (during collisions); the rest time every particle moves independently on others. At distances exceeding the size of the gas molecules, there is no interaction (its potential is equal to zero). Or, in other words, the potential energy of a particle is much lesser than their kinetic energy. In this case, the ratio of the distance, at which the interaction between the particles is significant to the average distance between particles, is small [8]:

$$\Lambda_G = \frac{a}{\langle r \rangle} \approx a n^{1/3} \ll 1 \quad (10)$$

(here a is the molecule size, $\langle r \rangle$ is the average distance between particles, and n is the density). The condition (10) also holds for the interaction of electrons with

neutrals and ions with neutrals. However, if we consider long-distance interaction between charge particles, the parameter Λ_G (gas-like parameter) requires rethinking. Its physical meaning becomes slightly different. The gas approximation is valid if the energy of the interaction between particles $U(\langle r \rangle)$ is smaller than the average thermal energy T of the particles itself, i.e.

$$U(\langle r \rangle) \approx \frac{e^2}{\langle r \rangle} \ll T \quad (11)$$

In other words, the following parameter, determining the plasma state

$$\Lambda_G \rightarrow \Lambda_P = \frac{U(\langle r \rangle)}{T} \approx \frac{e^2 / \langle r \rangle}{T} \approx \frac{e^2 n^{1/3}}{T} \ll 1 \quad (12)$$

must be small. The first condition (for a neutral gas) means that in a sphere with a radius equal to the radius of interaction, there are few particles. The meaning of a similar condition for plasma is the opposite. To prove this we, first of all, determine the interaction radius in plasma. For the determination we consider in detail the potential of a test particle in plasma. Let a particle with a charge q be placed at the point $r = 0$. We intend to find its potential φ from Poisson's equation. Assuming, for simplicity, that the charge of the single type of singly charged ions is not changed by the test particle (i.e., $e_i = -e$), we can write Poisson's equation in the form

$$\Delta \varphi = -4\pi q \delta(r) - 4\pi e \left[n_e \exp \frac{-e\varphi}{T_e} - n_i \exp \frac{e\varphi}{T_i} \right] \quad (13)$$

where Δ is the Laplace operator and $\delta(r)$ is the Dirac function. Assuming that $|e\varphi| \ll T_e, T_i$, we find

$$\varphi(r) = \frac{q}{r} \exp \left(-\frac{r}{r_D} \right) \quad (14)$$

where

$$r_D = \left\{ \sum_{\alpha} (v_{T\alpha}^2 / \omega_{L\alpha}^2)^{-1} \right\}^{-\frac{1}{2}} \quad (15)$$

is the so-called Debye radius. It shows the distance, in which the Coulomb forces are acting in plasma. Outside of the Debye radius, the interaction between charged particles is exponentially small and may be neglected. Comparative characteristics of the two curves are given in **Figure 2**. Curve (a) presents Debye potential, and curve (b) presents the vacuum potential $\sim 1/r$.

The electrostatic forces are, in fact, shielded. Now we can compare the average distance between charged particles with the Debye radius and make sure that the number of particles in the Debye sphere is large. For a simple case of plasma with singly charged ions and $T_e \approx T_i \approx T$, we have

$$r_D n^{1/3} = n^{1/3} \sqrt{\frac{T_e T_i}{4\pi(e^2 n_e T_i + e_i^2 n_i T_e)}} \approx \frac{\sqrt{T}}{en^{1/6}} = \sqrt{\frac{1}{\Lambda_P}} \gg 1 \quad (16)$$

This condition is essentially the opposite of analogous condition for gas (10). In a gas, the particles generally do not interact. The interaction takes place only at very

Figure 2.

Comparative characteristic of the two curves. Curve (a) presents Debye potential, and curve (b) presents the vacuum potential.

short intervals during collisions. In plasma, on the contrary, particles experience an interaction almost always. But, at the same time, the interaction is weak. It does not outrage their movement.

Debye radius, in particular, for electrons is

$$r_{De} = \frac{v_{Te}}{\omega_{Le}} = \sqrt{\frac{T_e}{4\pi n_e e^2}} \quad (17)$$

For the quasi-neutrality of plasma, it is necessary that its characteristic dimensions L be much larger than the Debye radius $L \gg r_D$. Moreover, under this condition a system of charged particles can be considered as plasma, i.e., a material medium with qualitatively new properties. Otherwise, it is a simple collection of individual charged particles, to which vacuum electrodynamics is applicable.

2.3 Degenerate plasma

It remains to determine the gas-like parameter for degenerate plasma as well as to answer the question of whether is there in quantum plasma Debye screening. For this we first recall that the expression for average energy of the particles, which is valid both for classical and quantum cases, may be written in the following form:

$$\langle E \rangle = \begin{cases} T & \text{if } T > E_F \\ E_F & \text{if } E_F > T \end{cases} \quad (18)$$

i.e., in the quantum case, the average energy of the state is equal to Fermi energy E_F (see (1)). The gas-like parameter for degenerate plasma may be obtained if we replace $T \rightarrow E_F$ in the expression (12). It becomes

$$\Lambda_P^{(D)} = \frac{e^2 n^{1/3}}{E_F} \ll 1 \quad (19)$$

Now we show that in quantum (degenerate) plasma of metals, shielding of electrostatic field also takes place and derives an expression for characteristic length for the Debye radius in degenerate plasma.

The energy of free electrons is $p^2/2m$. In the presence of a field with the potential $\Phi(r)$, it is $p^2/2m + \Phi(r)$. As a result, Fermi particles become distributed uniformly in the spherical layer between $p_{\min} = \sqrt{2me\Phi}$ and $p_{\max} = \sqrt{p^2 + 2me\Phi}$. Given this circumstance, one can find the expression for the electron density:

$$n_e = n_{0e} \left(1 + \frac{e\Phi}{E_{Fe}} \right)^{3/2}, \tag{20}$$

where n_{0e} is the density in the absence of a field (which coincides with the density of the neutralizing ion background). Now it is not difficult to write Poisson's equation for the potential of the test particle placed at the point $r = 0$, and the charge of which is q . The equation is

$$\Delta\Phi = -4\pi q\delta(\mathbf{r}) - 4\pi en_{0e} \left[\left(1 + \frac{e\Phi}{E_{Fe}} \right)^{3/2} - 1 \right] \tag{21}$$

The solution of this equation in the limit of weak fields, $|e\Phi| \ll E_{Fe}$, gives the shielded Coulomb potential:

$$\Phi(r) = \frac{q}{r} \exp \left(-\frac{r}{r_{De}} \right) \tag{22}$$

with a Debye radius r_{De} , the expression for which is

$$r_{De} = \sqrt{\frac{E_{Fe}}{6\pi e^2 n_{0e}}} \tag{23}$$

The gas parameter for plasma (12) is similar to condition (10) for neutral gas in the following sense. Both of these conditions are fulfilled better for fewer densities of plasma and neutral gas. The better the gas condition is satisfied, the more ideal is plasma. For degenerate plasma (in which particles need quantum description), in the contrary, the gas condition depends on density inversely, i.e., with the increase in density, the ideality becomes better (see (19)). As $E_F \sim n^{2/3}$, it turns out that with the increase in density, the average energy of Coulomb interaction increases slower. As a result $\Lambda_G^{(D)} \sim n^{-1/3}$. So, the denser the degenerate metal component, the better gas condition is fulfilled for it.

The diagram below presents the areas of the charge carriers' degeneracy and areas of the gas approximation applicability.

The degeneracy condition for electron plasma has the form $E_F > T$ (E_F ; see (1)). In the diagram $\ln n$ vs $\ln T$, this condition gives line 1 dividing the region of the degenerate plasma from the nondegenerate (classical) state. The condition for the applicability of the gas approximation in a nondegenerate state is $\Lambda_P = \frac{e^2 n^{1/3}}{T} \ll 1$. In the same diagram, the condition $\Lambda_P = 1$ gives line 2. In the degenerate state, the condition for applicability of gas approximation is $\Lambda_P^{(D)} = e^2 n^{1/3} / E_F \ll 1$, in which E_F does not depend on T . In these conditions $\Lambda_P^{(D)} = 1$ gives line 3 passing through the point, where lines 1 ($E_F = T$) and 2 ($\Lambda_P = 1$) intersect. Therefore, region I is the region of a nondegenerate plasma with weak interaction (the gas approximation is applicable). Region II is the region in which the plasma is nondegenerate with strong interaction, i.e., classic fluid. In region III, the plasma is degenerate with strong interaction, i.e., quantum fluid. Both in region II and region III, the gas approximation is not applicable. Finally, region IV of the parameter variations characterizes degenerate plasma with a weak interaction (gas approximation is applicable).

In conclusion, we give an estimate of the applicability conditions for the gas approximation (10) and (12) for various plasmas. First of all, we note that the size of atoms and molecules is of order $a \sim 10^{-7} - 10^{-8}$ cm and the condition for gas approximation (10) is satisfied up to $n < 10^{21} - 10^{22}$ cm $^{-3}$, i.e., in gases at normal temperature up to a pressure of hundred atmospheres. It is obvious that in gas plasma both in the ionosphere and in the laboratory, this condition is fulfilled perfectly, with a large margin.

A somewhat different situation holds for the condition of gas approximation in plasma (12). In the ionosphere plasma, where $n_e \sim 10^6 - 10^7$ cm $^{-3}$ and $T_e \approx 10^4$ K, we have $\Lambda_P \approx 10^{-4} \ll 1$, i.e., the condition is well satisfied. In ordinary gas-discharge fluorescent lamps, as well as in discharges used in laboratory experiments, where $n_e \sim 10^{10} - 10^{14}$ cm $^{-3}$ and $T_e \sim 10^4 - 10^5$ K, the value of $\Lambda_P \ll 1$. However, at the discharge in dense gases used in the light sources for laser pumping, as a rule, $n_e \ll 10^{18} - 10^{19}$ cm $^{-3}$, and $T_e \ll 1 - 10$ eV. Herewith $\Lambda_P \approx 0.1 - 0.5$, which indicates a significant violation of the applicability condition for the gas approximation and a significant manifestation of the properties of non-ideal plasma or, as one says, liquid effects.

In a thermonuclear plasma, in facilities with magnetic confinement, $n_e \approx 10^{14} - 10^{15}$ cm $^{-3}$, and $T_e \approx T_i \approx 10^8$ K. As a result, we have $\Lambda_P \approx 10^{-5} \ll 1$, i.e., the ideality of plasma is guaranteed. However, in the inertial thermonuclear reactors, where experimentators strive to obtain plasma with $n_e \sim n_i \sim 10^{24} - 10^{25}$ cm $^{-3}$ at the temperature of $T \approx 10^8$ K, it turns out that $\Lambda_P \geq 0.01$ and may be even more. This, apparently, will require a consideration of weakly non-ideal plasma, especially in conditions of pollution (the presence of multiply charged ions).

Finally, a brief summary on ideality of plasma in solids is presented. Even in good conductors, such as copper, where $n_e \approx 5 \cdot 10^{22}$ cm $^{-3}$ and $E_F \sim 1$ eV, we have $\Lambda_P \equiv \Lambda_P^{(D)} \sim 1$, i.e., plasma of metals is always non-ideal, and it is more correct to consider it as electron liquid. Nevertheless, it turns out that the application of the gas approximation to metals leads to good results from the point of view of the comparison with experiments. As for the electron-hole plasma of semiconductors, they are not degenerate at normal temperature, due to the small density of the carriers. For this reason, the condition of gas approximation (19) is well satisfied. Exceptions can occur only at very low temperatures.

3. Plasma description

3.1 Self-consistent approach

The main feature of plasma and plasma-like media, such as gas plasma, plasma of metals, semimetals, and semiconductors, is the presence of a large number of free

charge carriers. Here we present general principles of their description as a continuous media. The term “plasma-like media” was first introduced in [6] (see also [7]), the authors of which understood such seemingly different states of matter as an ionized gas, or actually plasma; metals, semiconductors, and even molecular colloidal crystals and electrolytes may be described based on similar principles – the principles of electrodynamics of plasma-like media. In this section we briefly formulate these principles.

A self-consistent interaction of the electromagnetic field and charge carriers takes place in plasma-like media. Field equations are the equations of Maxwell in which the current and the charge must be represented by a sum over all carriers (charged particles) in the plasma:

$$\begin{aligned}\nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{j} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \sum_{\alpha} n_{\alpha} \mathbf{v}_{\alpha} e_{\alpha} \\ \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \\ \nabla \mathbf{E} &= 4\pi \rho = 4\pi \sum n_{\alpha} e_{\alpha}; \quad \nabla \mathbf{B} = 0\end{aligned}\tag{24}$$

where $\mathbf{E}$ is the electric field strength, $\mathbf{B}$ is the magnetic induction, and n_{α} , e_{α} , and $\mathbf{v}_{\alpha}$ are the density, charge, and the velocity of α -th carrier $\alpha = e, i_1, i_2, i_3 \dots$

The equations for fields are written in this form (i.e., in terms of $\mathbf{E}$ and $\mathbf{B}$), because these quantities have direct physical meaning: they determine the Lorentz force $\mathbf{F}_n$ that acts on n -th carrier of α type (it may be an electron or ion of arbitrary type):

$$\mathbf{F}_n = e_{\alpha} \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}_n \times \mathbf{B}] \right\}\tag{25}$$

According to charge conservation law, the continuity equation must be satisfied for electrons and all types of ions, i.e., for $\alpha = e, i_1, i_2, \dots$. The continuity equation is

$$\frac{\partial n_{\alpha}}{\partial t} + \nabla(n_{\alpha} \mathbf{v}_{\alpha}) = 0\tag{26}$$

Here and in consideration below, we do not take into account the processes of ionization and recombination.

Eqs. (24)–(26) describe the self-consistent interaction between electromagnetic fields and statistically large numbers of charged particles (plasma). According to the set, electromagnetic fields determine the motion of charged particles. In its turn, the same electromagnetic fields are induced by moving plasma particles.

If the plasma (or plasma-like media) are in external fields (electric $\mathbf{E}_0$ and/or magnetic $\mathbf{B}_0$), the equations must be written in somewhat other form. External fields should be singled out. If the external fields are excited by external current $\mathbf{j}_0$ and charge ρ_0 densities, the set (24) should be written as

$$\begin{aligned}\nabla \times \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} (\mathbf{j}_0 + \mathbf{j}) = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \left\{ \mathbf{j}_0 + \sum_{\alpha} n_{\alpha} \mathbf{v}_{\alpha} e_{\alpha} \right\} \\ \nabla \mathbf{E} &= 4\pi(\rho + \rho_0) = 4\pi \sum n_{\alpha} e_{\alpha} + 4\pi \rho_0 \\ \nabla \mathbf{B} &= 0; \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}\end{aligned}\tag{27}$$

The external current $\mathbf{j}_0$ and charge density ρ_0 do not depend on the processes in plasma. Their values, along with $\mathbf{E}_0$ and $\mathbf{B}_0$, satisfy Maxwell's equations. These fields also influence on the motion of plasma particles. The expression for Lorentz force looks like (25)

$$\mathbf{F}_n = e_\alpha \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}_n \times \mathbf{B}] \right\} \quad (28)$$

However, here fields $\mathbf{E}$ and $\mathbf{B}$ are induced by external charge and current also.

An important conclusion follows from the sets (24) and (27). Only one additional vector quantity appears in the field equations – the current in the plasma $\mathbf{j}$ (the charge density may be expressed in terms of $\mathbf{j}$ by solving the continuity equation):

$$\mathbf{j} = \mathbf{j}(\mathbf{E}) = \sum_\alpha n_\alpha e_\alpha \mathbf{v}_\alpha \quad (29)$$

In this expression $\mathbf{v} = \mathbf{v}(\mathbf{E})$. The expression (29) shows that the current, induced in plasma, depends on velocities $\mathbf{v}_\alpha$ which are found independently (e.g., from equations of motion).¹ In the following, we will consider linear phenomena only. This implies linear dependence $\mathbf{j}(\mathbf{E})$, which is true, if the fields are comparatively small. It is to the point to note that in spite of our (and most other) consideration, nonlinear effects reveal themselves, first of all, in plasma and plasma-like media. Under linear consideration in isotropic media (media with no preferred directions), we actually have proportional dependence of the plasma current on the electric field or

$$\mathbf{j} = \sigma \mathbf{E} \quad (30)$$

This dependence represents Ohm's law for plasma, and σ is plasma conductivity for the considered case of isotropic plasma. However, if the plasma is in the external field, it loses the isotropy, and the relationship between $\mathbf{j}$ and $\mathbf{E}$ becomes much more complicated.

3.2 Waves in plasma and plasma-like media

The most important solutions of formulated set of equations are the solutions in the form of traveling waves, i.e., the solutions that depend on space coordinate $\mathbf{r}$ and time t as

$$\sim \exp(-i\omega t + i\mathbf{k}\mathbf{r}) \quad (31)$$

Solutions of this type are the simplest solutions. In this case the initial equations may be essentially simplified. In (24) derivations may be replaced by multiplication:

$$\frac{\partial}{\partial t} \rightarrow -i\omega \quad \nabla \equiv \frac{\partial}{\partial \mathbf{r}} \rightarrow i\mathbf{k} \quad (32)$$

¹ The expression for current $\mathbf{j}$ in plasma depends on the model, which is chosen for plasma description (see examples below). If one explores the most complete kinetic consideration, the expression for the plasma current changes and becomes

$$\mathbf{j} = \sum_\alpha e_\alpha \int \mathbf{v} d\mathbf{v} f_\alpha(\mathbf{v})$$

where $f_\alpha(\mathbf{v})$ is the distribution function for α -type plasma particles.

And the initial set reduces to a set of linear algebraic equations:

$$\begin{aligned} [\mathbf{kE}] &= \frac{\omega}{c} \mathbf{B}; & i[\mathbf{kB}] &= \frac{4\pi}{c} \mathbf{j} - i \frac{\omega}{c} \mathbf{E} \\ i\mathbf{kE} &= 4\pi\rho; & \mathbf{kB} &= 0 \end{aligned} \quad (33)$$

Any other, more complicate, solution of the initial equations in linear theory can be presented as a superposition of the simplest solutions with various amplitudes. As follows from the general principle of electrodynamics, this superposition is also a solution of the initial set. This emphasizes the importance of consideration of the solutions in the form of traveling waves.

The motions and continuity equations also may be reduced to algebraic form. Here we present the reduced form of continuity equation only as the motion equations for plasma particles depend on the model, chosen for plasma description (see below):

$$\omega\rho = \mathbf{kj} \quad (34)$$

So, the initial equations (consisting of Maxwell's, continuity, and motion equations) reduce to linear algebraic set. The condition for the existence of nonzero solutions of the set is called *dispersion relation*. It, actually, presents a certain relation between frequency ω and components of wave vector $\mathbf{k}$. This relation helps determine ω for the given $\mathbf{k}$ and, vice versa, to determine $\mathbf{k}$ (one of its components) if ω and other components of $\mathbf{k}$ are given. These statements of the problem are called initial and boundary problems accordingly. These statements are widely used in plasma physics. Herewith in plasma many cases are encountered, in which solution of the initial problem gives complex frequency $\omega = \omega' + i\omega''$ for real $\mathbf{k}$. In this case the real part of the frequency $\text{Re } \omega = \omega'$ shows the frequency of the wave, but the imaginary part shows (depending on its sign) either the increasing of the wave's amplitude if $\omega'' > 0$ or decreasing if $\omega'' < 0$ in accordance to

$$\exp(-i\omega t) = \exp(\omega''t) \exp\{-i\omega't + i\mathbf{k}\mathbf{r}\} \quad (35)$$

The solution of the boundary problem should be interpreted in the same manner. If the solution of the problem gives a complex component of the wave vector $\mathbf{k}$, its imaginary part shows either amplification of the given wave in a given direction or its quenching.

3.3 Electrostatic waves in plasma

Plasma is a medium, where propagation of specific, electrostatic (or plasma) waves is possible. These waves have no oscillating magnetic field. These waves are also called space charge or Langmuir waves. In the waves electric field is parallel to its propagation direction $\mathbf{k} \parallel \mathbf{E}$. Oscillations of plasma particles also are parallel to the propagation direction, i.e., the waves are purely longitudinal. These waves play the most important role in plasma and strongly influence on its stability, much more than the usual electromagnetic waves (propagation of which in plasma is also possible).

The explicit expression for plasma conductivity σ helps to obtain the dispersion relation for longitudinal waves. When we consider solutions of Maxwell's equations in the form of traveling waves, the conductivity σ depends on frequency ω and wave vector $\mathbf{k}$, i.e., $\sigma \equiv \sigma(\omega, \mathbf{k})$. The dispersion relation for electrostatic waves in plasma can be expressed in terms of $\sigma(\omega, \mathbf{k})$. It has the following form:

$$1 + i \frac{4\pi}{\omega} \sigma(\omega, \mathbf{k}) \equiv \varepsilon(\omega, \mathbf{k}) = 0 \quad (36)$$

where $\varepsilon(\omega, \mathbf{k})$ is the well-known dielectric permittivity of the given media.

Propagation of usual electromagnetic waves in plasma is also possible. The dispersion relation for this case is

$$k^2 = \frac{\omega^2}{c^2} \varepsilon(\omega, \mathbf{k}) \quad (37)$$

In a particular case of vacuum, this expression gives propagation of usual vacuum electromagnetic waves. For vacuum $\varepsilon(\omega, \mathbf{k}) \equiv 1$, and the dispersion relation (37) takes a familiar look, $\omega = kc$.

For further development of the properties of plasma-like media, it is necessary to specify the plasma models.

4. The simplest plasma models

Each model of plasma specifies how its particles interact with the electromagnetic field, as well as specifies the behavior of plasma particles inside and between plasma species. Here we consider the simplest models only, leaving aside the most rigorous kinetic consideration. On examples of simple models, we show how the models work as well as some of their advantages and disadvantages.

4.1 One-particle model

We begin with the model of one, “average” (or test) particle. In this model particles interact via electromagnetic field, and the interaction, in fact, is weak (it weakly perturbs the motion of the particles). Collisions inside and between species are also taken into account. This model describes the oscillatory properties of gas-discharge and ionosphere plasma well enough. In particular, the model was successfully used for the description of radio-frequency wave propagation through the ionosphere [9].

The initial set of equations in the model of “average” particle includes Newton equations for the “average” electron and for “average” ion along with equations for electromagnetic field and continuity equation:

$$\begin{aligned} \frac{d}{dt} \mathbf{v}_e &= \frac{e}{m} \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}_e \times \mathbf{B}] \right\} - \nu_{en} \mathbf{v}_e - \nu_{ei} (\mathbf{v}_e - \mathbf{v}_i) \\ \frac{d}{dt} \mathbf{v}_i &= \frac{e_i}{M} \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}_i \times \mathbf{B}] \right\} - \nu_{in} \mathbf{v}_i - \nu_{ie} (\mathbf{v}_i - \mathbf{v}_e) \end{aligned} \quad (38)$$

Here $\mathbf{v}_e$ and $\mathbf{v}_i$ are the velocities of electrons and ions; m and M are their masses; ν_{en} , ν_{ei} and ν_{in} , ν_{ie} are the frequencies of their collisions, which determine the friction forces inhibiting their motion; ν_{en} is the frequency of collisions of electrons with neutral atoms (molecules) and ν_{ei} with ions, respectively; and for ions, this is ν_{in} and ν_{ie} . According to Newton’s third law, $m\nu_{ei} = M\nu_{ie}$. A similar system of equations is also used to describe the dynamics of solid-state plasma, but in this case the meaning of the collision frequencies differs from the abovementioned. The frequencies, actually, are the inverse lifetimes of electrons and holes, respectively.

First we show how this model works on the simplest example of plasma oscillations, as well as how easy the Langmuir frequency follows from this model. Consider pure electron plasma. Ions are heavy and immobile. They serve only for neutralization of electrons' charge. Upon derivation of equations, which describe plasma oscillations, one should recall that we consider linear plasma phenomena. This means that the equations should be linearized, i.e., we consider small perturbations of physical quantities from their basic (equilibrium) state. For example, the density of electrons is considered as $n_e = n_{e0} + n'_e$ where $n'_e \ll n_{e0}$, and in the resulting expressions we retain the first-order terms only and neglect the terms of second-order smallness (multiplication of the first-order terms). We also take into account that the Langmuir oscillations are potential and use Poisson's equation instead of a full set of Maxwell's equations. For one more simplification, we consider one-dimensional case: let electrons oscillate along the z axis. All this leads to the following: the initial equations (motion, Poisson's, and continuity) are reduced to simple form presented below:

$$\frac{\partial}{\partial t} v'_e = \frac{eE}{m} - \nu_{ei} v'_e; \quad \frac{\partial}{\partial t} n'_e = -n_{e0} \frac{\partial}{\partial z} v'_e; \quad \frac{\partial}{\partial z} E = 4\pi e n'_e \quad (39)$$

Here v'_e is the velocity of electrons, e and m are their charge and mass, ν_{ei} is the frequency of electron-ion collisions, and t is the time.

For the solutions of (39) that depend on z and t in the form $\exp(-i\omega t + ikz)$, we have the equations

$$-i(\omega + i\nu_e) v'_e = \frac{eE}{m}; \quad \omega n'_e - ik n_{e0} v'_e = 0; \quad ikE = 4\pi e n'_e \quad (40)$$

Thus, we arrive to the set of simple algebraic equations, from which the following expression for plasma current results

$$j = e n'_e v'_e = i \frac{e^2 n_{e0}}{m} E \equiv \sigma E, \quad \sigma = i \frac{e^2 n_{e0}}{m} \quad (41)$$

From (41) one can easily obtain the corresponding expression for dielectric permittivity as well as the dispersion relation, which is

$$\epsilon = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu_{ei})} = 0 \quad (42)$$

If $\nu_{ei} \ll \omega$ (realizes in most cases), the relation (42) leads to $\omega = \pm \omega_p$, i.e., we have free plasma oscillations. If one takes into account plasma collisions, he obtains small negative imaginary correction to the frequency: $\omega_p \rightarrow \omega_p - i\nu_e/2$. This shows the decay of the oscillations. The decay takes place as a result of collisions.

Obtained results on plasma oscillations and their decay coincide to experimental data. In fact, the model of "average" particle describes plasma well in the considered range of frequencies. Namely this model was used by Langmuir to describe the oscillatory properties of gas-discharge plasma. Also the model was successfully used especially for describing the propagation of radio-frequency waves through the ionosphere [9]. Thereby, the model of "average" particle is justified for high-frequency range. However, in the opposite limit of low frequencies, this model does not lead to reasonable results. That is why, new, more complicate models have been explored.

4.2 Two-fluid hydrodynamic: relative electron-ion motion

The idea to consider plasma as a system consisting of electron and ion fluids arose long time ago. In this model plasma species are described by hydrodynamic equation and interact through the electromagnetic field and through the collisions. The interaction leads to various effects. In particular instability can follow from the interaction. The general theory of plasma instabilities shows that instability is a result of thermodynamically nonequilibrium initial distribution of plasma components.

The initial equations describing electron and ion fluids and their interaction are somewhat more complex than the previous case of “average” particle. They contain additional terms that follow from classical hydrodynamics:

$$\begin{aligned}\frac{d}{dt}\mathbf{v}_e &\equiv \left[\frac{\partial}{\partial t} + (\mathbf{v}_e \nabla) \right] \mathbf{v}_e = -\frac{\nabla(n_e T_e)}{m n_e} + \frac{e}{m} \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}_e \times \mathbf{B}] \right\} - \nu_{en} \mathbf{v}_e - \nu_{ei} (\mathbf{v}_e - \mathbf{v}_i) \\ \frac{d}{dt}\mathbf{v}_i &\equiv \left[\frac{\partial}{\partial t} + (\mathbf{v}_i \nabla) \right] \mathbf{v}_i = -\frac{\nabla(n_i T_i)}{M n_i} + \frac{e_i}{M} \left\{ \mathbf{E} + \frac{1}{c} [\mathbf{v}_i \times \mathbf{B}] \right\} - \nu_{in} \mathbf{v}_i - \nu_{ie} (\mathbf{v}_i - \mathbf{v}_e)\end{aligned}\quad (43)$$

Here m and M are electron and ion mass, T_e , T_i , and n_e , n_i are their temperatures and densities. Other denotations coincide to the previous case of “average” particle above.

The equations describing the electron and ion fluid motion (43) should be supplemented by Maxwell’s and continuity equations. Equations for T_e and T_i (energy balance equations or equations for heat) are also needed. Specific forms of these equations depend on the physical meaning of the problem, which is considered. For simplicity, it may be assumed as $T_e, T_i = \text{const}$. These assumptions greatly simplify further analysis.

Here we briefly consider a simple example of two-fluid model. In order to show the role of ions and how this role can lead to instability, we consider a case in which electron fluid moves relative to ions in rest. Let $\mathbf{u}$ be the constant velocity of moving electrons. Neutrals are absent. The initial equations of electron and ion fluids (43) and Maxwell’s and continuity equations after linearization are reduced to the following set of equations (we assume $T_e, T_i = 0$ and consider the potential oscillation in one-dimensional system and choose the z axis along $\mathbf{u}$):

$$\begin{aligned}\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right) \mathbf{v}_e' &= \frac{eE}{m} - \nu_{ei} \mathbf{v}_e'; & \frac{\partial}{\partial t} \mathbf{v}_i' &= \frac{e_i E}{M} \\ \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial z} \right) n_e' &= -n_{0e} \frac{\partial}{\partial z} \mathbf{v}_e'; & \frac{\partial}{\partial t} n_i' &= -n_{0i} \frac{\partial}{\partial z} \mathbf{v}_i' \\ \frac{\partial}{\partial z} E &= 4\pi(e n_e' + e_i n_i')\end{aligned}\quad (44)$$

Here $\mathbf{v}_e'$ and $\mathbf{v}_i'$ are the perturbations of electron and ion fluid velocities accordingly, n_e' and n_i' are the perturbations of their densities, n_{0e} and n_{0i} are their unperturbed densities, ν_{ei} is the frequency of electron-ion collisions, and E is the electric field. We look for solutions of the set (44) in the form of waves propagating along the z axis $\sim \exp(-i\omega t + ikz)$. In this case the equations (44) are reduced to algebraic set. If one performs further steps (determination of induced plasma current, finding plasma conductivity and dielectric permittivity) by analogy to the previous case, he arrives to the expression for dielectric permittivity of considered system, which, in this case, consists of three terms:

$$\varepsilon = 1 + \Delta\varepsilon_e + \Delta\varepsilon_i \quad (45)$$

Here the second and third terms are electron and ion contributions in the dielectric permittivity:

$$\Delta\varepsilon_e = -\frac{\omega_{Le}^2}{(\omega - ku)(\omega - ku + i\nu_{ei})}; \quad \Delta\varepsilon_i = -\frac{\omega_{Li}^2}{\omega^2} \quad (46)$$

but the first term is, in fact, the vacuum unit. In (46) $\omega_{Le} = \sqrt{4\pi e^2 n_{0e}/m}$ and $\omega_{Li} = \sqrt{4\pi e^2 n_{0i}/M}$ are the electron and ion Langmuir frequencies. The dispersion relation of the considered system

$$1 - \frac{\omega_{Le}^2}{(\omega - ku)(\omega - ku + i\nu_{ei})} - \frac{\omega_{Li}^2}{\omega^2} = 0 \quad (47)$$

is an algebraic equation of the fourth order. Despite the solutions of the fourth-order equations are known (see any reference book on mathematics, e.g., [10]), they are not suitable for our purposes. Very cumbersome expressions with many radicals cannot give physical information, and we choose another way of finding the solutions of (47) and their analysis. The way presented below is typical for plasma theory.

For the analysis we first consider the case of the ion's absence, i.e., if there are streaming electrons only. The dispersion relation is of the second order, with the solution

$$\omega_{\pm} = ku \pm \omega_{Le} \quad (48)$$

This solution represents the waves in the electron stream: fast (+) and slow (−) beam waves. An important fact is that the energy of the slow beam wave is negative [11]. This means that for excitation of the slow wave in the electron beam, one should withdraw energy, but not put energy into the beam. This circumstance plays an important role in theory of streaming instabilities. It lies on the basis of explanation of physical meaning of the most well-known plasma instabilities, beam-plasma instability, as well as the meaning of dissipative beam instabilities. In the last case, dissipation leads not to quenching of the oscillations (as one expects) but to their amplification. The matter is in the following: dissipation serves as a channel for energy withdrawal for excitation of the beam negative-energy wave.

Continuing the analysis of the dispersion relation (47), one can easily see its important peculiarity, which appears as a result of the inequality $\omega_{Li} \ll \omega_{Le}$; ions play a role under small ω only:

$$\omega_{Li} \gg \omega \rightarrow 0 \quad (49)$$

In the opposite case, the contribution of the ions in (47) results in small corrections to the roots that describe the proper oscillations of streaming electron (48). However, if the condition (49) holds, two additional roots of (46) (from a total of four) are approximately equal:

$$\omega = \pm \frac{\omega_{Li}}{\sqrt{1 - \frac{\omega_{Le}^2}{k^2 u^2}}} \quad (50)$$

This expression shows that the instability (caused by the presence of ions) exists if $ku < \omega_{Le}$. In this case the imaginary part of the root (growth rate of unstable oscillations) attains its maximum if $ku \approx \omega_{Le}$. The instability in this case is called resonance

instability. If the two conditions ($\omega \rightarrow 0$ and $ku \approx \omega_{Le}$) are realized, one can at once calculate the roots of the dispersion relation (47), if he rewrites it in the form

$$\omega \left(\frac{\partial(\Delta\epsilon_e)}{\partial\omega} \right)_{\omega \rightarrow 0} = \frac{\omega_{Li}^2}{\omega^2} \quad (51)$$

From this expression the growth rate of resonant instability follows. It is

$$\delta_{Bn} = \frac{\sqrt{3}}{2^{4/3}} \omega_{Le} \left(Z^2 \frac{m}{M} \right)^{1/3} \quad (52)$$

In the given case, the growth rate is of the same order as the frequency of the unstable oscillations in plasma. This instability was first discovered by Buneman [12]. It (as well as other low-frequency instabilities) plays an important role in many scenarios in space physics and geophysics. The physical essence of this instability lies in the fact that the proper space charge oscillations of moving electrons (beam slow wave) in the frame, associated with the ions in rest due to the Doppler effect, experience red shift, and this greatly reduced frequency becomes close to the proper frequency of ions. Actually the instability is due to the resonance of the negative-energy wave with the ion oscillations.

The models of plasma that have been considered above are very simplified. In spite of this, we have seen that the models describe some phenomena in plasma well enough. For more detailed consideration, one should use the most complete kinetic consideration.

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