We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,300

Open access books available

172,000

International authors and editors

190M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

The Study of Magneto-Convection Heat Transfer in a Partially Open Cavity Based on LBM

Raoudha Chaabane

Abstract

A free convection heat transfer in a sinusoidally heated enclosure filled with conducting fluid is presented in this chapter by Lattice Boltzmann Method (LBM). The horizontal walls in the enclosures are insulated and there is an opening part on the right wall. The right non-open parts of the vertical wall of the square cavity are maintained at constant cold temperature and the left wall of the cavity is sinusoidally heated. The cavity is get under a uniform in-plane magnetic field. The main aim of this study is to highlight the effectiveness of the LBM mesoscopic approach to predict the effects of pertinent parameters such as the Hartmann number varying from 0 to 150 where Rayleigh number is fixed at moderate value of 10^5 on flow patterns. This in-house numerical code used in this chapter is ascertained and a good agreement with literature is highlighted. The appropriate validation with previous numerical investigations demonstrated that this attitude is a suitable method and a powerful approach for engineering MHD problems. Findings and results show the alterations of Hartmann number that influence the isotherms and the streamlines widely.

Keywords: sinusoidal thermal boundary condition, MHD, partially open cavity, free convection, LBM

1. Introduction

Convective flow and heat transfer in an open cavity has been studied due to the extensive range of applications in engineering science and technology that consider various combinations of imposed temperature gradients and enclosure sketches [1–10]. Open cavity with a modified linear or sinusoidal thermal boundary condition is encountered in many practical engineering and industrial applications, such as solar energy collection, cooling of electronic devices, material processing, grain storage, flow and heat transfer in solar ponds, high-performance insulation for buildings, dynamics of lakes, reservoirs and cooling ponds, crystal growing, float glass production, metal casting, food processing, galvanizing, metal coating, and so on.

Besides, pertinent useful numerical research works had been conducted to simulate the MHD free convection under nonuniform thermic boundaries where recent attention has been intensively focused on the cases of mixed boundary conditions [11–14]. However, few results have been reported for free convection caused

simultaneously by both external magnetic field in partially open enclosures subjected to a sinusoidal temperature variation in the left vertical wall although problems of this type are frequently important, and their study is necessary for understanding the performance of complex magneto-convection flow and heat transfer.

MHD forces generated from the interaction of induced electric currents with an applied external magnetic field can alter the flow of an electrically conducting fluid in the presence of magnetic field. An externally imposed magnetic field is an important tool used to control melting flows that grow bulk crystal in semiconductor's applications. A main purpose of electromagnetic control is to stabilize the flow and suppress oscillatory instabilities, which degrades the resulting crystal. In literature, wide ranges of investigations were investigated by researchers in MHD free convection [15–30]. In such complex geometry, the balance is achieved by inertial, viscous, electromagnetic, and buoyancy forces; finding a numerical efficient tool to predict flow and heat pattern inside MHD cavities is a crucial aim for industrial related engineering applications.

The effect of Ra number on free convection MHD in an open cavity was investigated in [29]. Nonetheless several investigations in MHD free convection inside partially open enclosure with sinusoidally wall have been carried out yet.

The mesoscopic approach called Boltzmann method (LBM) joined the microscopic models and the macroscopic dynamics of a fluid. It can recover the Navier-Stokes equation by using the Chapman-Enskog expansion [39]. LB method is not very demanding in terms of memory requirement. In addition, in terms of computational speed, the algorithm is generally simpler and therefore faster than many traditional CFD schemes. It is easy for parallel computation and for implementation of irregular boundary conditions, which is a sought task in many-needed engineering geometry.

To our best knowledge, no previous study on effects of sinusoidally heated boundary on free convection in a partially open MHD enclosure cavity with the LBM had already been studied so far. The main aim of this chapter is to study the effects of linearly heated wall on flow field and temperature distribution in an open MHD cavity filled.

The main aim of the present study is to demonstrate the use of the Lattice Boltzmann Method (LBM) [31–40] for MHD with a simple and clear statement and also solve MHD free convection as a left sinusoidally heated side mixed with a partially open right wall filled with a conducting fluid. Hartmann number varies in a wide range from 0 to 150. First, the results of LBM are validated with previous numerical investigations. Effects of Rayleigh, Hartmann number, and various positions of the open side on flow field and temperature distribution are considered simultaneously.

The proposed configuration with sinusoidal temperatures on the left side wall of a partially open cavity in the presence of a magnetic field has not been focused. A major objective of the present study is to examine the magneto-convection in this configuration filled with a conducting fluid confined between two horizontal walls, which are thermally insulated. The effect of the open side on fluid flow and heat transfer is studied numerically.

2. Governing equations and mathematical formulation

2.1 Problem statement

The considered geometries of the present problem are shown in **Figure 1**. They display a two-dimensional closed, open or partially open east side cavity with the height of H . A constant, linear, or sinusoidal temperature is imposed along the left vertical wall. Then opening side boundaries are correlated with temperature

Figure 1.
 The standard D_2Q_9 LBM lattice.

conducting fluid at the cold temperature (T_c). The north and south horizontal boundaries are adiabatic. We consider a horizontal uniform magnetic field applied to a two-dimensional Newtonian, laminar, and incompressible conducting fluid. The radiation effects, the viscous dissipation, and Joule heating are neglected in the present study. The thermophysical properties of the conducting fluid are constant, and the density variation in the liquid gallium is approximated by the standard Boussinesq (**Figure 2**).

2.2 Governing equations

Governing equations for MHD free convection are written in terms of the macroscopic variable depending on position x,y as:
 Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equations

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F_x \tag{2}$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + F_y \tag{3}$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}$$

where $\nu = \mu/\rho$ is the kinematic viscosity and F_x and F_y are the body forces at horizontal and vertical directions, respectively, and they are defined as follows [30]:

Figure 2.
Used configurations C1–4 with different boundary conditions.

$$F_x = R(v \sin \gamma \cos \gamma - u \sin^2 \gamma) \quad (5)$$

$$F_y = R(u \sin \gamma \cos \gamma - v \cos^2 \gamma) + \rho g \beta (T - T_m) \quad (6)$$

where the Ha number is defined as:

$$Ha = H B_x \sqrt{\sigma / \mu} \quad (7)$$

The LBM method [31–40] with standard, two-dimensional, nine velocities (D2Q9) for flow and temperature is used in this chapter (**Figure 1a**); for completeness, only a brief discussion is given in the following paragraphs. The Bhatnagar-Gross-Krook (BGK) approximation Lattice Boltzmann equation with external forces F_{tot} can be written as:

$$f_\alpha(\mathbf{r} + \mathbf{e}_\alpha \Delta t, t + \Delta t) = f_\alpha(\mathbf{r}, t) - \frac{\Delta t}{\tau_f} [f_\alpha(\mathbf{r}, t) - f_\alpha^{eq}(\mathbf{r}, t)] + \Delta t F_{tot} \quad (8)$$

where $f_\alpha(\mathbf{r}, t)$ is the particle distribution defined for the finite set of the discrete particle velocity vectors $\mathbf{e}_\alpha$. $\mathbf{r}$ and t are the coordinates of Eulerian node and time.

where τ_f is the relaxation time and $f_\alpha^{eq}(\mathbf{r}, t)$ is the local equilibrium distribution function.

The equilibrium distribution can be formulated as:

$$f_\alpha^{eq} = w_\alpha \rho \left(1 + \frac{\mathbf{e}_\alpha \cdot \mathbf{u}}{c_s^2} + \frac{(\mathbf{e}_\alpha \cdot \mathbf{u})^2}{2c_s^4} - \frac{\mathbf{u} \cdot \mathbf{u}}{2c_s^2} \right) \quad (9)$$

where u and ρ are the macroscopic velocity and density, respectively, and w_α are the values of the weighting constant factors for e_α that must be assigned as $w_0 = 4/9$ (rest-particle) for $|e_0| = 0$, $w_{1-4} = 1/9$ for $|e_{1-4}| = 1$, and $w_{5-9} = 1/36$ for $|e_{5-9}| = \sqrt{2}$.

where $c = \Delta x / \Delta t$, Δx and Δt are the lattice space and the lattice time step size, respectively, which are set to unity.

For scalar function (temperature), another distribution is defined:

$$g_\alpha(r + e_\alpha \Delta t, t + \Delta t) = g_\alpha(r, t) - \frac{\Delta t}{\tau_g} [g_\epsilon(r, t) - g_\alpha^{eq}(r, t)] \quad (10)$$

The equilibrium distribution function can be written as:

$$g_\alpha^{eq} = w_\alpha T \left(1 + \frac{e_\alpha \cdot u}{c_s^2} + \frac{(e_\alpha \cdot u)^2}{2c_s^4} - \frac{u \cdot u}{2c_s^2} \right) \quad (11)$$

For momentum equation, we have:

$$\tau_f = 0.5 + (3\nu/c^2 \Delta t) \quad (12)$$

where ν is the kinematic viscosity and for the scalar:

$$\tau_g = 0.5 + (3\alpha/c^2 \Delta t) \quad (13)$$

where α is the diffusion coefficient (thermal diffusion coefficient) and Δt is the lattice time step. The buoyancy force term is added as an extra source term to Eq. (1) as:

$$F_\alpha = 3w_\alpha g_y \beta \Delta T \quad (14)$$

where g_y , β , and ΔT are gravitational acceleration, thermal expansion coefficient, and temperature difference, respectively.

In the LBM, the total force is

$$F = F_x + F_y \quad (15)$$

$$F_x = 3w_k \rho [R(v \sin \gamma \cos \gamma) - u \sin^2 \gamma] \quad (16)$$

$$F_y = 3w_k \rho [g_y \beta (T - T_m) + R(u \sin \gamma \cos \gamma) - v \cos^2 \gamma] \quad (17)$$

where $R = \mu H a^2$ and γ is the direction of the magnetic field.

After completing streaming and collision processes where the Boussinesq approximation is considered for free convection, the macroscopic fluid quantities, namely, the macro density, velocity (obtained through moment summations in the velocity space), and temperature are computed.

$$\rho(r, t) = \sum_k f_k(r, t) \quad (18)$$

$$u(r, t) = \sum_k e_k f_k(r, t) / \rho(r, t) \quad (19)$$

$$T = \sum_k g_k(r, t) \quad (20)$$

3. Results and discussion

In all cases, both the computations for flow and temperature fields are based on the D2Q9 LBM approach.

We have validated our in-house Fortran computer code for the free convection in a square open cavity with insulated horizontal walls filled with air with a uniform right (west) vertical temperature by reference [29]. The obtained numerical results are compared with the numerical ones reported in [29]. It can be seen from **Figure 3** that there is a good agreement for the distribution of streamlines and isotherms among the present solution and literature and this for $Pr = 0.71$, $Ha = 0$, and $Ra = 10^5$. The used configuration is C1 (**Figure 2**).

In **Figure 4**, the considered matter is MHD free convection in an open cavity with linearly heated west wall (C2), which is filled with liquid gallium ($Pr = 0.025$) and $Ha = 150$. As shown in **Figure 4**, an increase in Rayleigh number makes the thermal boundary layer to become narrower for the constant parameters $Pr = 0.025$ and $Ha = 150$.

Figure 5 displays steady-state contour maps for the isotherms and the streamline contours at $Pr = 0.025$ for a high Rayleigh number $Ra = 10^6$ and high Hartmann

Figure 3.
Comparison of the steady state isotherms (a) and streamlines (b) at $Pr = 0.71$ for $Ha = 0$ and $Ra = 10^5$ between [29] and the present work.

Figure 4.
Steady-state isotherms (b) at $Pr = 0.025$ and $Ha = 150$ for different Rayleigh numbers.

(a)

(b)

Figure 5.
Steady-state isotherms (a) and isotherms (b) at $Pr = 0.025$, $Ra = 106$, and $Ha = 150$.

number $Ha = 150$. Configuration C2 is considered for this case. The effect of the open right side wall is depicted in **Figure 5a** via streamline display.

In this chapter, we aim to test the ability of the LBM to deal with a future complex configuration (C4), which is a MHD partially open cavity with sinusoidal input excitation on the west wall. The horizontal walls are adiabatic. The cavity is

(a)

(b)

Figure 6. Steady-state isotherms (a) and streamlines (b) of sinusoidally heated side walls in a partially open MHD cavity for $Ha = 150$, $Pr = 0.025$ and $Ra = 10^5$.

filled with liquid gallium, and the simulation is done for $Ha = 50$ and a moderate Rayleigh number of $Ra = 10^5$. Many researchers were considering a sinusoidal heating wall in their simulation [11–14]. For brevity, we resume their important findings.

Figure 7.
Steady-state isotherms (a) and streamlines (b) of sinusoidally heated right side wall partially open MHD cavity for $Ha = 150$, $Pr = 0.025$ and $Ra = 10^6$.

In literature, we find that the heat transfer rate was increased as the amplitude ratio of the sinusoidal excitation increases with Rayleigh and Hartmann numbers.

For a uniform heating wall, the heat transfer rate remains low, which makes that the nonuniform heating of both walls is advised for enhancing and improving heat transfer. It is proven that the heat transfer rate is increased first and then decreased on increasing the phase deviation from 0 to π [11–14].

In addition, with a phase deviation from 0 to $3\pi/4$, heat transfer rate is enhanced for all Rayleigh numbers, and the average Nusselt number reaches its highest value at $3\pi/4$.

A further finding proves that when both walls are in the same temperature distribution in the absence of phase deviation, the heat transfer rate is low for all values of Hartmann and Rayleigh numbers. The right wall is widely influenced by the variation of the amplitude ratio and the phase deviation of the sinusoidal temperature distribution. However, those physical variations have very little effect on the left wall. Besides, an increase in Hartmann number decreases the heat transfer [11–14].

In the present chapter, we seek to deal with a new configuration C4. For this goal, we deal first with the configuration (C3) of **Figure 2**. The sidewalls of the cavity have spatially varying sinusoidal temperature distributions. The horizontal walls are adiabatic. Simulation is established for MHD cavity for $Ha = 150$, $Pr = 0.025$, and $Ra = 10^6$. The heat transfer rate is highlighted within the evolution of isotherms and streamlines inside the cavity in **Figure 6**.

Figure 7 shows the dynamic and thermic behavior of configuration C4. Numerical results in terms of flow and thermic structure show that the flow within the cavity takes place owing to the thermic buoyancy effects caused by the sinusoidally heated right wall. In C3, the flow is characterized by a symmetric multicellular behavior in which the recirculating eddies or cells of relatively high velocity are formed within the enclosure and this in the presence of a high Hartmann $Ha = 150$ and a high Rayleigh number $Ra = 10^5$.

In C4, the presence of the partially open sidewall changes the flow and heat transfer. We note the presence of a one dominant cell in the core region of the cavity and a little cell that occurs at upper left side.

As forecasted, because of the partially open side effects, the temperature field was sketched by a noticeable drop in its behavior near the open side of the cavity and these both in flow and heat behaviors. Besides, we highlight that the temperature contour maps lose the sinusoidal behavior as they move to the partially open side.

We infer that a partially open sidewall has the tendency to control efficiently the movement of the fluid in such given configuration (C4).

We subject in the convergence criterion that the relative change in two successive iterates of the solution (temperate and velocities) at each computational point be below a prescribed small value of 10^{-6} .

For all simulations, the criterion convergence is considered to be reached, for velocity and temperature when the following convergence is satisfied:

$$|\xi^{\sigma+1} - \xi^{\sigma}| \leq 10^{-6} \quad (21)$$

where ξ is velocity or temperature and σ is the iteration number.

4. Conclusions

To the author's knowledge, studies have thus far addressed a mesoscopic approach in an MHD open cavity with sinusoidally heated wall (**Figure 2**). The objective of the

present chapter is therefore to predict dynamic and thermic heat transfer in a crucial engineering application. The cavity is investigated at the high Ra number of 10^5 , high Ha number ($Ha = 50$), and Pr number of 0.025. The present chapter extends the study to deal with free convection in MHD open cavity with sinusoidal heated west wall which is filled with liquid gallium with $Ha = 50$ for $Ra = 10^5$. Numerical study has been made of free convection in a square enclosure with spatially varying sinusoidal temperature distributions on the vertical left sidewall, whereas the horizontal walls are thermally insulated. The right wall is a partially open one. Lattice Boltzmann method is considered for flow and heat transfer simulation of this problem inside the cavity. After validation, the present in-house Fortran code is extended to deal with the present complex geometry in order to highlight the workability and the ability of LBM to deal with such mixed boundary condition sketch. This investigation demonstrated ability of LBM for simulation of different boundary conditions at various elements affecting the stream in a partially open cavity with sinusoidal heating vertical wall. An analysis of the opening mass flow is highlighted in the dynamic and thermal behavior of the streamlines and the isotherms.

Nomenclature

C	lattice speed
c_i	discrete particle speeds
c_p	specific heat at constant pressure
F	external forces
f^{eq}	equilibrium density distribution functions
g^{eq}	equilibrium internal energy distribution functions
g	gravity
G	buoyancy per unit mass
H	enclosure height
Ma	Mach number
Nu	Nusselt number
Pr	Prandtl number
Ra	Rayleigh number
T	temperature
x,y	Cartesian coordinates
X	horizontal length of the cavity
Y	vertical length of the cavity
Ha	Hartmann number

Greek letters

ω_i	weighted factor indirection i
β	thermal expansion coefficient
τ	relaxation time
ν	kinematic viscosity
Δt	time increment
α	thermal diffusivity

Subscripts

av	average
H	hot
C	cold
b	bottom

IntechOpen

Author details

Raoudha Chaabane^{1,2}

1 Laboratory of Thermal and Energetic Systems Studies (LESTE), National School of Engineering of Monastir, University of Monastir, Tunisia

2 Preparatory Institute of Engineering Studies of Monastir (IPEIM), University of Monastir, Tunisia

*Address all correspondence to: raoudhach@gmail.com

IntechOpen

© 2019 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/3.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

References

- [1] Lauriat G, Desrayaud G. Effect of surface radiation on conjugate natural convection in partially open enclosures. *International Journal of Thermal Sciences*. 2006;**45**(4):335-346
- [2] Bilgen E, Oztop H. Natural convection heat transfer in partially open inclined square cavities. *International Journal of Heat and Mass Transfer*. 2005;**48**(8):1470-1479
- [3] Chan YL, Tien CL. A numerical study of two-dimensional laminar natural convection in shallow open cavities. *International Journal of Heat Mass Transfer*. 1985;**28**(3):603-612
- [4] Xia JL, Zhou ZW. Natural convection in an externally heated partially open cavity with a heated protrusion. *FED-vol. 143/HTD, Vol. 232. Measurement and Modeling of Environmental Flows—ASME*. 1992;**232**:201-208
- [5] Angirasa D, Eggels JGM, Nieuwstadt FTM. Numerical simulation of transient natural convection from an isothermal cavity open on a side. *Numerical Heat Transfer, Part A: Applications*. 1995;**28**(6):755-768
- [6] Mohamad AA. Natural convection in open cavities and slots. *Numerical Heat Transfer, Part A: Applications*. 1995;**27**:705-716
- [7] Kennedy P, Zheng R. *Flow Analysis of Injection Molds*. Munich: Hanser; 2013
- [8] Mohamad AA, El-Ganaoui M, Bennacer R. Lattice Boltzmann simulation of natural convection in an open ended cavity. *International Journal of Thermal Sciences*. 2009;**48**:1870-1875
- [9] Mohamad AA, Bennacer R, El-Ganaoui M. Double dispersion natural convection in an open end cavity simulation via Lattice Boltzmann method. *International Journal of Thermal Sciences*. 2010;**49**:1944-1953
- [10] Polat O, Bilgen E. Laminar natural convection in inclined open shallow cavities. *International Journal of Thermal Sciences*. 2002;**41**:360-368
- [11] Bilgen E, Yedder RB. Natural convection in enclosure with heating and cooling by sinusoidal temperature profiles on one side. *International Journal of Heat and Mass Transfer*. 2007;**50**:139-150
- [12] Sarris IE, Lekakis I, Vlachos NS. Natural convection in a 2D enclosure with sinusoidal upper wall temperature. *Numerical Heat Transfer A*. 2002;**42**:513-530
- [13] Varol Y, Oztop HF, Pop I. Numerical analysis of natural convection for a porous rectangular enclosure with sinusoidally varying temperature profile on the bottom wall. *International Communications in Heat and Mass Transfer*. 2008;**35**:56-64
- [14] Saeid NH, Yaacob Y. Natural convection in a square cavity with spatial side wall temperature variation. *Numerical Heat Transfer A*. 2006;**49**:683-697
- [15] Fidaros D, Grecos A, Vlachos N. Development of numerical tool for 3D MHD natural convection. *Annex xx*. pp. 73-74
- [16] Sathiyamoorthy M, Chamkha A. Effect of magnetic field on natural convection flow in a liquid gallium filled square cavity for linearly heated side wall(s). *International Journal of Thermal Sciences*. 2010;**49**:1856-1865
- [17] Alchaar S, Vasseur P, Bilgen E. Natural convection heat transfer in a rectangular enclosure with a transverse

- magnetic field. *Journal of Heat Transfer*. 1995;117:668-673
- [18] Garandet J, Alboussiere T, Moreau R. Buoyancy drive convection in a rectangular enclosure with a transverse magnetic field. *International Journal of Heat and Mass Transfer*. 1992;35:741-748
- [19] Rudraiah N, Barron R, Venkatachalappa M, Subbaraya C. Effect of a magnetic field on free convection in a rectangular enclosure. *International Journal of Engineering Science*. 1995;33:1075-1084
- [20] Cowley M. Natural convection in rectangular enclosures of arbitrary orientation with magnetic field vertical. *Magnetohydrodynamics*. 1996;32:390-398
- [21] Teamah M. Hydro-magnetic double-diffusive natural convection in a rectangular enclosure with imposing an inner heat source or sink. *Alexandria Engineering Journal*. 2006;45(4):401-415
- [22] Ozoe H, Okada K. The effect of the direction of the external magnetic field on the three-dimensional natural convection in a cubic enclosure. *International Journal of Heat and Mass Transfer*. 1989;32:1939-1953
- [23] Ece M, Buyuk E. Natural convection flow under a magnetic field in an inclined rectangular enclosure heated and cooled on adjacent walls. *Fluid Dynamics Research*. 2006;38(5):546-590
- [24] Al-Najem N, Khanafer K, El-Refaee M. Numerical study of laminar natural convection in tilted enclosure with transverse magnetic field. *International Journal of Numerical Methods for Heat and Fluid Flow*. 1998;8:651-672
- [25] Jalil J, Al-Taey K. MHD turbulent natural convection in a liquid metal filed square enclosure. *Emirates Journal for Engineering Research*. 2007;12(2):31-40
- [26] Gelfgat A, Bar-Yoseph P. The effect of an external magnetic field on oscillatory instability of convective flows in a rectangular cavity. *Physics of Fluids*. 2001;13(8):2269-2278
- [27] Aleksandrova S, Molokov S. Three-dimensional buoyant convection in a rectangular cavity with differentially heated walls in a strong magnetic field. *Fluid Dynamics Research*. 2004;35:37-66
- [28] Kahveci K, Oztuna S. MHD natural convection flow and heat transfer in a laterally heated partitioned enclosure. *European Journal of Mechanics-B/ Fluids*. 2009;28:744-752
- [29] Kefayati GHR, Gorji M, Ganji DD, Sajjadi H. Investigation of Prandtl number effect on natural convection MHD in an open cavity by Lattice Boltzmann method. *Engineering Computations*. 2013;30:97-116
- [30] Martinez D, Chen S, Matthaeus W. Lattice Boltzmann magneto hydrodynamics. *Physics of Plasmas*. 1994;(6):1850-1867
- [31] Chaabane R, Askri F, Nasrallah SB. Parametric study of simultaneous transient conduction and radiation in a two-dimensional participating medium. *Communications in Nonlinear Science and Numerical Simulation*. 2011;16(10):4006-4020
- [32] Saha LK, Hossain MA, Gorla RSR. Effect of Hall current on the MHD laminar natural convection flow from a vertical permeable flat plate with uniform surface temperature. *International Journal of Thermal Science*. 2007;46:790-801
- [33] Chaabane R, Askri F, Nasrallah SB. Analysis of two-dimensional transient conduction-radiation problems in an

anisotropically scattering participating enclosure using the Lattice Boltzmann method and the control volume finite element method. *Journal of Computer Physics Communications*. 2011;**182**(7): 1402-1413

[34] Lamsaadi M, Naimi M, Hasnaoui M, Mamou M. Natural convection in a vertical rectangular cavity filled with a non-Newtonian power law fluid and subjected to a horizontal temperature gradient. *Numerical Heat Transfer Part A: Application*. 2006;**49**:969-990

[35] Chaabane R, Askri F, Ben Nasrallah S. Application of the Lattice Boltzmann method to transient conduction and radiation heat transfer in cylindrical media. *Journal of Quantitative Spectroscopy and Radiative Transfer*. 2011;**112**(12):2013-2027

[36] Chaabane R, Askri F, Jemni A, Nasrallah SB. Numerical study of transient convection with volumetric radiation using an hybrid lattice Boltzmann BGK-control volume finite element method. *Journal of Heat Transfer*. 2017;**139**(9):092701-072017

[37] Chaabane R, Askri F, Jemni A, Nasrallah SB. Analysis of Rayleigh-Bénard convection with thermal volumetric radiation using Lattice Boltzmann formulation. *Journal of Thermal Science and Technology*. 2017;**12**(2)

[38] Mohamad AA. *Applied Lattice Boltzmann Method for Transport Phenomena, Momentum, Heat and Mass Transfer*. Calgary: Sure; 2007

[39] Succi S. *The Lattice Boltzmann Equation for Fluid Dynamics and Beyond*. Oxford, London: Clarendon Press; 2001

[40] Series RW, Hurle DTJ. The use of magnetic fields in semiconductor crystal growth. *Journal of Crystal Growth*. 1991;**133**:305-328