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## Chapter

# Analytic Prognostic in the Linear Damage Case Applied to Buried Petrochemical Pipelines and the Complex Probability Paradigm

*Abdo Abou Jaoude*

## Abstract

In 1933, *Andrey Nikolaevich Kolmogorov* established the system of five axioms that define the concept of mathematical probability. This system can be developed to include the set of imaginary numbers by adding a supplementary three original axioms. Therefore, any experiment can be performed in the set  $\mathcal{C}$  of complex probabilities which is the summation of the set  $\mathcal{R}$  of real probabilities and the set  $\mathcal{M}$  of imaginary probabilities. The purpose here is to include additional imaginary dimensions to the experiment taking place in the “real” laboratory in  $\mathcal{R}$  and hence to evaluate all the probabilities. Consequently, the probability in the entire set  $\mathcal{C} = \mathcal{R} + \mathcal{M}$  is permanently equal to one no matter what the stochastic distribution of the input random variable in  $\mathcal{R}$  is; therefore the outcome of the probabilistic experiment in  $\mathcal{C}$  can be determined perfectly. This is due to the fact that the probability in  $\mathcal{C}$  is calculated after subtracting from the degree of our knowledge the chaotic factor of the random experiment. Consequently, the purpose in this chapter is to join my complex probability paradigm to the analytic prognostic of buried petrochemical pipelines in the case of linear damage accumulation. Accordingly, after the calculation of the novel prognostic model parameters, we will be able to evaluate the degree of knowledge, the magnitude of the chaotic factor, the complex probability, the probabilities of the system failure and survival, and the probability of the remaining useful lifetime; after that a pressure time  $t$  has been applied to the pipeline, which are all functions of the system degradation subject to random and stochastic influences.

**Keywords:** probability norm, complex probability set, degree of our knowledge, chaotic factor, remaining useful lifetime, degradation, analytic prognostic, linear damage

## 1. Introduction

*“An intellect which at any given moment knew all the forces that animate Nature and the mutual positions of the beings that comprise it, if this intellect were vast enough to submit its data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom: for such*

*intellect nothing could be uncertain; and the future just like the past would be present before its eyes”.*

*Marquis Pierre-Simon de Laplace.*

*“The Divine Spirit found a sublime outlet in that wonder of analysis, that portent of the ideal world, that amphibian between being and not-being, which we call the imaginary root of negative unity”.*

*Gottfried Wilhelm von Leibniz.*

The high availability of technological systems, like defense, aerospace, automobile industries, and petrochemistry, is a central major objective of previous and latest developments in the technology of system design. Pipelines are the primary component of the systems of hydrocarbon transport in petrochemical industries. They are vital for human activities because they serve to transport water, natural gases, and oil from sources to all consumer sites. A novel analytic prognostic model was established in my earlier research work and applied to the case of pipelines subject to the effects of corrosion, to soil loading, and to internal pressure. These will initiate micro-cracks in the body of the tubes that can spread suddenly and can lead to failure. The increase of pipeline availability and the reduction of their global mission cost and performance necessitate to elaborate a suitable process of prognostic. Accordingly, a novel strategy based on degradation analytic laws was applied to diverse dynamic systems and was developed in my research work [1–6]. Additionally, the remaining useful lifetime (*RUL*) was predicted and calculated from a predefined threshold of degradation. Based on a system of a physical petrochemical pipeline, my publications developed a strategy to design a model of failure prognostic that will be more elaborated and further enhanced in the present book chapter.

Moreover, prognostic is a process involving a prediction capacity. Using prognostic, we are able to evaluate the equipment remaining useful lifetime in terms of its future usage and its history of functioning. Predicting the remaining useful lifetime of industrial systems turns out to be presently a vital goal for industrialists knowing that the consequences of failure, which can occur suddenly, are usually very expensive. The traditional maintenance strategies [7, 8] founded on a static threshold of alarm are no more practical and efficient since they do not consider the instantaneous functioning state of a product. The establishment of a prognostic approach as an “intelligent” maintenance consists of the health follow-up, monitoring, and analysis, based on physical measurements utilizing sensors.

Also, earlier expert studies of prognostic belong in general to three categories of technical approaches: the first category is the “experience-based prognostic” [9] which is based on measurements taken from a machine health monitoring, for example, those based on stochastic model, expert judgment, Bayesian approach, reliability analysis, Markovian process, optimization of preventive maintenance, etc. Their methodology of prognostic shows to be simple but inflexible toward changes in the environment and in the system behavior. The second category is the “estimation-based or trending prognostic” based on the statistics of vast measured data. We can cite as illustrations the work relying on the behavior of degradation expressed by abaci and utilizing a system expert description (process-mission-environment) [10]; the work relying on artificial intelligence, machine learning [11], neural network [12], and fuzzy logic [13]; and additionally the work based on dissipativity-based fuzzy integral sliding mode control of continuous time T-S fuzzy systems, SMC design for robust stabilization of nonlinear Markovian jump singular systems, sliding mode control of fuzzy singularly perturbed systems with application to electric circuits, the stabilization of quantized sampled-data neural

network-based control systems, etc. Their methodologies are designated generally as not very precise, but they propose a powerful tool to the theory of prognostic. The third category is the “model-based prognostic” relying on the mathematical description of the process of degradation and its evolution level utilizing nondestructive inspection (NDI) monitoring. It is designated to be more precise and flexible than the two first categories. My earlier research illustrates a methodology of analytical prognostic relying on analytic laws of damage, such as the linear damage accumulation law of Palmgren-Miner and the fatigue crack propagation law of Paris-Erdogan. It fits in the third category of models. This approach is used whenever the law of damage of the studied system is analytically available. The advantage of this approach is consequently its precise and realistic features in evaluating the remaining useful lifetime of a system [14–17].

Additionally, pipes are petrochemical systems that transport natural gas and oil in huge quantities and over long distances. Their life prognostic is crucial in this industry because their availability has vital outcomes. Their major failures are due to soil settlements, seismic ground waves, deformations, buckling, internal and external corrosion, vibration and resonance, stress concentration in welding and fitting, and pressure fluctuation over long period. The failures due to fatigue by means of cracks propagation are noticed and measured by the tools of crack detection. Therefore, three case studies of pipelines were taken into consideration in my earlier publications [18, 19]: buried, unburied, and subsea (offshore pipelines). Each one of these situations necessitates different physical parameters like friction and soil pressure, atmospheric and water pressure, and corrosion. The buried pipes case will only be considered in the present chapter.

## 2. The purpose and the advantages of the present work

Computing probabilities is the main work of classical probability theory. Adding new dimensions to the stochastic experiments will lead to a deterministic expression of probability theory. This is the original idea at the foundations of this work. Actually, the theory of probability is a nondeterministic system in its essence; that means that the event outcomes are due to the chance and randomness. The addition of novel imaginary dimensions to the chaotic experiment occurring in the set  $\mathcal{R}$  will yield a deterministic experiment, and hence a stochastic event will have a certain result in the complex probability set  $\mathcal{C}$ . If the random event becomes completely predictable, then we will be fully knowledgeable to predict the outcome of stochastic experiments that arise in the real world in all stochastic processes. Consequently, the work that has been accomplished here was to extend the real probabilities set  $\mathcal{R}$  to the deterministic complex probabilities set  $\mathcal{C} = \mathcal{R} + \mathcal{M}$  by including the contributions of the set  $\mathcal{M}$  which is the imaginary set of probabilities. Therefore, since this extension was found to be successful, then a novel paradigm of stochastic sciences and prognostic was laid down in which all stochastic phenomena in  $\mathcal{R}$  was expressed deterministically. I called this original model “the Complex Probability Paradigm” that was initiated and illustrated in my 12 research publications. [20–31].

Furthermore, although the analytic linear prognostic laws are deterministic and very well-known in [14, 16], there are chaotic and stochastic influences and aspects (such as humidity, temperature, material nature, geometry dimensions, applied load location, water action, corrosion, soil pressure and friction, atmospheric pressure, etc.) that influence the buried pipeline system and make its function of degradation diverge from its computed trajectory modeled by these deterministic laws. An updated follow-up of the degradation performance and behavior with cycle number or time, which is subject to non-chaotic and chaotic influences, is

made possible by what I called the system failure probability due to its definition that estimates the jumps in the function of degradation  $D$ .

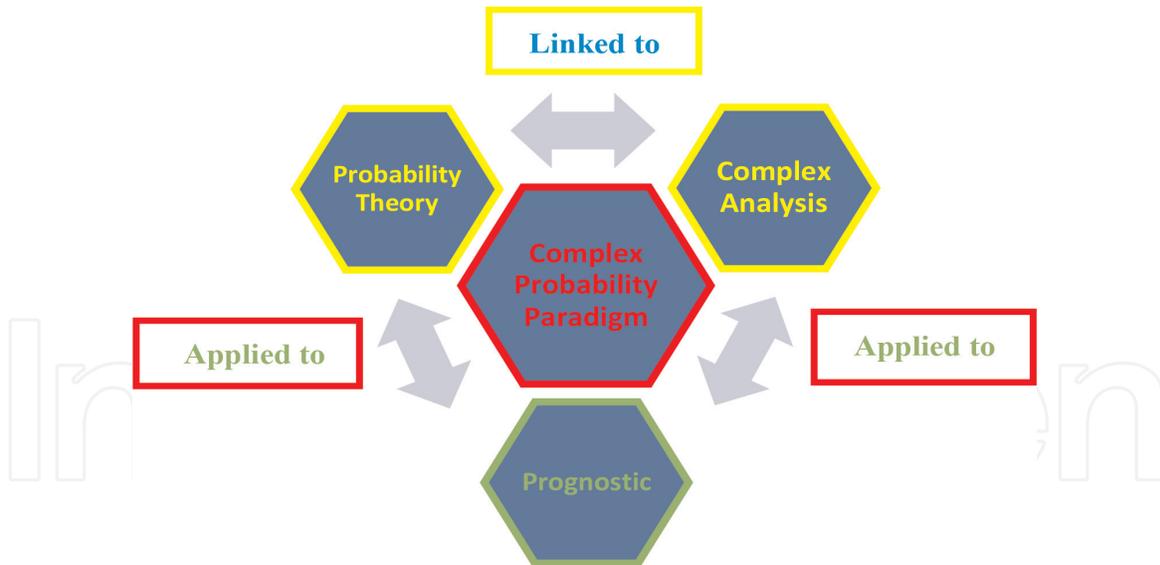
Additionally, my objective in this present work is to connect the complex probability paradigm to the buried pipeline system analytic prognostic in the case of linear damage accumulation which is subject to fatigue. In fact, the system failure probability derived from prognostic will be applied to and included in the complex probability paradigm. This will lead to the original and novel model of prognostic illustrated in this chapter. Thus, by determining the new prognostic model parameters, it becomes possible to evaluate the degree of our knowledge, the magnitude of the chaotic factor, the complex probability, the  $RUL$  probability, and the system failure and survival probabilities; after that a pressure cycle time  $t$  has been applied to the buried pipeline, which are all functions of the system degradation subject to chaotic and stochastic influences.

Accordingly, the advantages and the purpose of the current chapter are to:

1. Extend classical probability theory to the set of complex numbers and therefore to link the theory of probability to the field of complex variables and analysis. This job was started and elaborated in my previous 12 papers.
2. Do an updated follow-up of the degradation  $D$  performance and behavior with cycle number or time which is subject to chaos. This follow-up is accomplished by the real failure probability of the system due to its definition that evaluates the jumps in  $D$ , therefore linking a system degradation to probability theory in a novel and original way.
3. Apply the new axioms of probability and paradigm to system prognostic; thus, I will extend the prognostic concepts to the set of complex probabilities  $\mathcal{C}$ .
4. Show that all stochastic phenomena can be expressed deterministically in the set of complex probabilities  $\mathcal{C}$ .
5. Measure and compute both the degree of our knowledge and the chaotic factor of the system remaining useful lifetime and its degradation.
6. Draw and illustrate the graphs of the parameters and functions of the original paradigm corresponding to a buried pipeline prognostic.
7. Show that the classical concepts of random remaining useful lifetime and degradation possess a probability permanently equal to one in the complex set; hence, no randomness, no chaos, no uncertainty, no ignorance, no disorder, and no unpredictability exist in:

$$\mathcal{C} \text{ (complex set)} = \mathcal{R} \text{ (real set)} + \mathcal{M} \text{ (imaginary set)}.$$

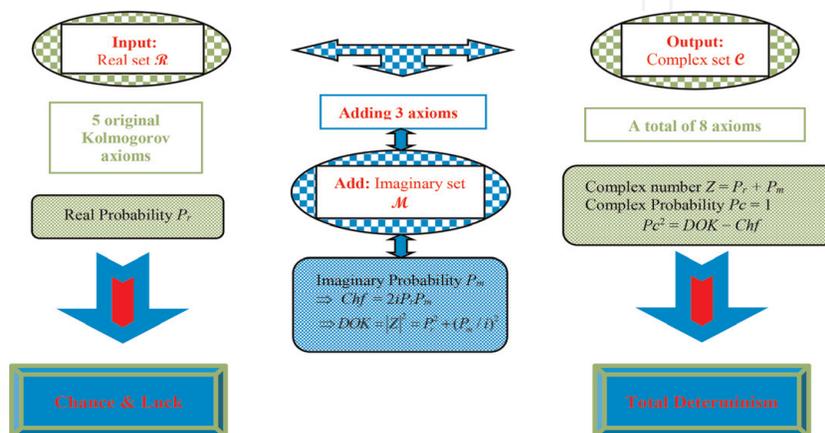
8. Show that by adding new and supplementary dimensions to any stochastic phenomenon, whether it is a pipeline system or any other random experiment, it becomes possible to do prognostic in a deterministic way in the set  $\mathcal{C}$  of complex probabilities.
9. Pave the way to implement this novel model to other areas in stochastic processes and to the field of prognostics in science and engineering. These will be the topics of my future research works.



**Figure 1.**  
 The diagram of the main purposes of the complex probability paradigm and research work.

Concerning some applications of the original elaborated paradigm and as a future work, it can be applied to a wide set of dynamic systems like vehicle suspension systems and offshore and buried petrochemical pipelines which are subject to fatigue and in the cases of nonlinear and linear damage accumulation. Furthermore, compared with existing literature, the main contribution of the present research work is to apply the novel paradigm of complex probability to the concepts of random remaining useful lifetime and degradation of a buried pipeline system hence to the case of analytic prognostic in the case of linear damage accumulation subject to fatigue. The following figure shows the main purposes of the complex probability paradigm (CPP) (Figure 1).

To conclude and to summarize, in the real probability universe  $\mathcal{R}$ , our degree of our certain knowledge is regrettably imperfect; therefore we extend our study to the complex set  $\mathcal{C}$  which embraces the contributions of both the real probabilities set  $\mathcal{R}$  and the imaginary probabilities set  $\mathcal{M}$ . Subsequently, this will lead to a perfect and complete degree of knowledge in the universe  $\mathcal{C} = \mathcal{R} + \mathcal{M}$  (since  $P_c = 1$ ). In fact, working in the complex universe  $\mathcal{C}$  leads to a certain prediction of any random event, because in  $\mathcal{C}$  we eliminate and subtract from the calculated degree of our knowledge the quantified chaotic factor. This will yield a probability in the universe  $\mathcal{C}$  equal to one ( $P_c^2 = DOK - Chf = DOK + MChf = 1 = P_c$ ). Many



**Figure 2.**  
 The EKA or the CPP diagram.

illustrations considering various continuous and discrete probability distributions in my 12 previous research papers verify this hypothesis and novel paradigm [20–31]. The extended Kolmogorov axioms (*EKA* for short) or the complex probability paradigm can be summarized and shown in the following figure (**Figure 2**).

### 3. Previous research work: analytic prognostic and linear damage accumulation for buried petrochemical pipelines

In this section a comprehensive summary of a part of my previously published PhD thesis [16] and of the formerly published IFAC conference paper [14] will be done, and the results that this current chapter needs will be just cited.

#### 3.1 A brief introduction to the adopted methodology

The objective of my earlier research study, which will be enhanced in the present chapter and will be linked to *CPP*, was to develop an analytic linear model of prognostic capable of predicting the remaining useful lifetime and the degradation  $D$  curves of a buried petrochemical pipeline system subject to fatigue starting from an initial known damage and under a given environment [14, 16]. This shows to be beneficial for many reasons which are fewer pipe bending; reduced plant congestion, wind, and other loads; and protection from ambient temperature changes. This work is restricted here to normal service loads that consist of only soil action and internal pressure.

Petrochemical pipelines are systems that are used to transport natural gas and oil between sites. We believe that pipeline tubes are a major element in petrochemical industries. As a matter of fact, the prognostic of their life is essential in this industry since their availability has decisive and critical consequences on the cost of exploitation. Fatigue, which is due to internal pressure-depression variation along time, is the major failure cause of these systems. These pipelines are typically devised for ultimate limit states (resistance). Additionally, due to soil aggression influences, buried pipelines are subject to corrosion. Pipelines are designed as cylindrical tubes of thickness  $e$  and radius  $R$ .

A target failure probability of about  $10^{-5}$  for pipelines is suggested by the DNV 2000 rules. Their major failure causes are soil settlements, seismic ground waves, deformations, buckling, stress concentration in welding and fitting, internal and external corrosion, pressure fluctuation over a long period, and vibration and resonance. Moreover, crack detection tools detect the crack propagation caused by failures due to fatigue.

An important part of the main pipes is exposed to external cracking, which is a dangerous setback for the industry of pipes, for example, in the USA, Canada, and Russia. External crack identification is accomplished using diverse nondestructive evaluation (NDE) methods. If cracks were detected during inspection, we should evaluate their influence on the remaining useful lifetime of the pipeline in order to select the action of maintenance that should be applied: do nothing/repair/replace. We judge the integrity of pipes by assuming that some defects after in-line inspection (ILI) can be still undetected; detected, but not measured; detected and measured.

Moreover, the objective in my publications was to assess the evolution of the lifetime of a system at each instant. Consequently, and for this purpose, the trajectories of degradation had been utilized in terms of the time of operation or cycles' number. Hence, we deduce the *RUL* variations from these trajectories of degradation. Thus, I have considered many industrial illustrations in the simulation of my

model in these earlier publications and work to prove the effectiveness of my model [1–6, 13–19]. Three case studies of pipelines were taken into consideration: buried, unburied, and subsea (offshore pipes). Each one of these situations necessitates different physical parameters like friction and soil pressure, corrosion, and atmospheric and water pressure. One of these cases is elaborated here which is the system of buried petrochemical pipes where three modes of pressure profiles (mode 1 = high, mode 2 = middle, and mode 3 = low-pressure conditions) were examined and simulated. My model showed that it presented a useful tool for a prognostic analysis and that it is very convenient in such industrial systems. Furthermore, it proved that it is less expensive than other models that require a huge number of measurements and data.

### 3.2 Fatigue crack growth

The stress intensity factor was introduced to calculate the correlation between the crack growth rate,  $da/dN$ , and the stress intensity factor range,  $\Delta K$ . The Paris-Erdogan's law [7] allows to evaluate the rate of propagation of the crack length  $a$  after its detection. This damage growth law is expressed by the following equation:

$$\frac{da}{dN} = C(\Delta K)^m \quad (1)$$

where

$\frac{da}{dN}$ , the crack growth rate = the increase of the crack length  $a$  per cycle  $N$ .

$\Delta K(a) = Y(a)\Delta\sigma\sqrt{\pi a}$ , the intensity factor of the stress.

$Y(a)$ , the component's crack geometry function.

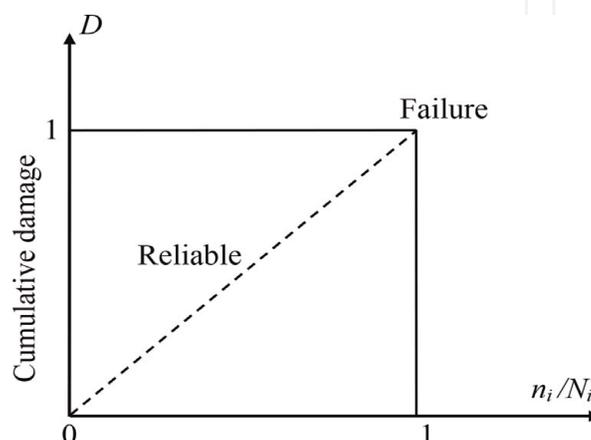
$\Delta\sigma$ , the range of the applied stress in a cycle.

$m$  and  $C$ , the constants of materials obtained experimentally; ( $2 \leq m \leq 4$ ) and ( $0 < C \ll 1$ ).

### 3.3 The modeling of linear cumulative damage

To do the prognostic of a degrading element, my approach was to evaluate and to predict the end of life of the element by modeling and tracking the function of degradation. My model of damage, whose progress is up to the macro-crack initiation point, is illustrated in **Figure 3** by the damage linear rule of Palmgren-Miner.

As a matter of fact, this law [7] is used to calculate the cumulative damage  $d_i$  of different stress levels  $\sigma_i$  ( $i = 1, i = 2, \dots, i = k$ ) applied for  $n_i$  cycles. Knowing that  $N_i$  is



**Figure 3.**  
 Palmgren-Miner's linear rule of damage.

the total cycle's number of stress  $\sigma_i$  to be applied and that lead to failure. The linear cumulative damage corresponding to the applied stresses ( $i = 1$  to  $k$ ) is provided by

$$D_k = \sum_{i=1}^k d_i = \sum_{i=1}^k \frac{n_i}{N_i} \quad (2)$$

The initial detectable crack  $a_0$  at the cycle  $N_0$ , the crack length  $a_N$  at any cycle  $N$ , and the crack length  $a_C$  at the failure cycle  $N_C$  are estimated by a sensor, and their values are included in the model of damage prognostic in the equation of damage. It is expressed in my model by the resulting relation

$$D_N = \frac{a_N}{a_C - a_0} \quad (3)$$

Or in terms of the pressure cycle time  $t$ , the relation is given by

$$D_t = \frac{a_t}{a_C - a_0} \quad (4)$$

To simplify the study, it is suitable to adopt a measurement of damage denoted by  $D \in [0, 1]$  which is computed by the Palmgren-Miner's law of linear cumulative damage. The damage level in a system at a specific cycle which is due to fatigue is illustrated by a scalar function of damage denoted by  $D(t)$  or  $D(N)$ . "No damage" corresponds to the value  $D = 0$ , and "total damage" or the appearance of the first macro-crack corresponds to  $D = 1$ .

### 3.4 An expression for degradation

Therefore, my general prognostic analytic linear model function, which is a recursive relation for the sequence of  $D$ , is given by [16]

$$\begin{aligned} D_N &= D(N) = P_{rog}(a_N) \\ &= \frac{a_{N-1}}{a_C - a_0} + \frac{C}{a_C - a_0} \times (\pi a_{N-1})^{3/2} \times \left[ 0.6 \times \frac{1 + 2(a_{N-1}/e)}{(1 - a_{N-1}/e)^{3/2}} \right]^3 \times (P_j R/e)^3 \end{aligned} \quad (5)$$

where  $C$ , the environment parameter;  $e$ , the pipe thickness;  $R$ , the pipe radius;  $a_0$ , the initial crack length at the cycle  $N_0$ ;  $a_{N-1}$ , the crack length at the load cycle  $N-1$ ;  $a_C$ : the crack length at the failure cycle  $N_C$ . It was assumed in the model that  $a_C = e/8$  for justified reasons [16];  $P_j$ : the pipe internal pressure.

Or in terms of the pressure cycle time  $t$ , the recursive relation for the sequence of  $D$  is given by

$$\begin{aligned} D_t &= D(t) = P_{rog}(a_t) \\ &= \frac{a_{t-1}}{a_C - a_0} + \frac{C}{a_C - a_0} \times (\pi a_{t-1})^{3/2} \times \left[ 0.6 \times \frac{1 + 2(a_{t-1}/e)}{(1 - a_{t-1}/e)^{3/2}} \right]^3 \times (P_j R/e)^3 \end{aligned} \quad (6)$$

Consequently, the previous recursive relation leads to a sequence of  $D_t$  values with  $N_0 \leq N \leq N_C$  or  $t_0 \leq t \leq t_C$  whose limit is  $D_C = 1$ :

$$D_0 = \frac{a_0}{a_C - a_0}; D_1 = \frac{a_1}{a_C - a_0}; D_2 = \frac{a_2}{a_C - a_0}; \dots; D_{t-1} = \frac{a_{t-1}}{a_C - a_0}; D_t = \frac{a_t}{a_C - a_0} \quad (7)$$

We will consider three different levels of internal pressure to take into account the diverse states of pressure conditions which are low, middle, and high. Moreover, as the stress load is a function of the cycles  $N$  or of time  $t$ , then we can draw the trajectories of degradation of  $D(N)$  or  $D(t)$  in addition to the trajectories of  $RUL(N)$  or  $RUL(t)$  in terms of the total number of loading cycles  $N$  or in terms of the pressure time  $t$ . Therefore, my developed model of linear damage will be applied in order to compute the pipeline system prognostic.

### 3.5 The three levels of internal pressure simulations

We will consider in our current work a pipeline transporting natural gas of radius  $R = 240$  mm and of thickness  $e = 8$  mm. The parameters in this case are  $C = 1.3 \times 10^{-14}$  (under soil, buried pipelines) and  $m = 3$  (metal). The initial crack length is considered to be  $a_0 = 0.02$  mm. The crack length  $a_C$  at the failure cycle time  $t_C$  was assumed in the model to be equal to  $e/8$  for justified reasons [16]. Hence, from Eqs. (5) and (6), we get

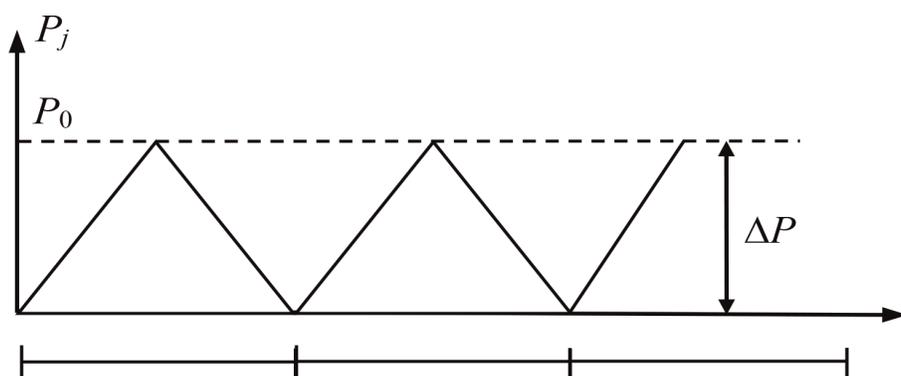
$$D_0 = \frac{a_0}{a_C - a_0} = \frac{a_0}{(e/8) - a_0} = \frac{0.02}{(8/8) - 0.02} = \frac{0.02}{0.98} = 0.020408163$$

The soil specific weight is  $\gamma = 9.843$  kN/m<sup>3</sup>. The weight per linear meter of pipe and gas content is  $W_p = 203.27$  kg/m. The specific gravity of the pipe material and of the natural gas are, respectively,  $\gamma_{pipe} = 7850$  kg/m<sup>3</sup> and  $\gamma_{gas} = 600$  kg/m<sup>3</sup>. The depth of the pipe is taken as  $H = 7R$ , and the friction coefficient interval is  $0.5 \leq \mu \leq 0.7$  [16].

The internal pressure  $P_j$  is modeled following a triangular form and distribution in order to be similar to the real case of pipeline operating condition (pressure-depression) (**Figure 4**).

We will consider three maximal levels of  $P_j$  which are  $P_0 = 3, 5, \text{ and } 8$  MPa and with a period of repetition  $T$ . This repetition period varies depending on the conditions of exploitation; it is considered to be equal to 20 h. We note that these three levels are supposed to be the extreme conditions of the pipeline exploitations and are mean estimations of the real and actual random period and pressure rates. A trajectory of degradation  $D(N)$  is inferred at each of these three levels in terms of the cycle number  $N$  or pressure cycle time  $t$ . When  $D_t$  or  $D_N$  attains the unit value, therefore the corresponding  $t = t_C$  or  $N = N_C$  is the lifetime of the pipeline in the fatigue case.

For the purposes of simulations, in **Table 1**, the values of pressure  $P_j$  are considered to be equal to the maximal values  $P_0$ . The analytic linear prognostic model



**Figure 4.**  
 Triangular variation of internal pressure.

Pressure mode	$P_j$ (MPa)	Model
High (mode 1)	8	Triangular
Middle (mode 2)	5	Triangular
Low (mode 3)	3	Triangular

**Table 1.**  
Characteristics of each internal pressure mode.

(Eqs. (5) and (6)) simulation is achieved for each internal pressure level (low, middle, and high).

A huge amount of pressure simulations of the order of hundreds of millions are required to estimate the real system lifetime; hence, we have used an approximated model of lifetime simulation of the order of 10,000,000 iterations. Accordingly, we have considered for this purpose a high-capacity computer system: a workstation computer with parallel microprocessors, a 64-Bit operating system, a 64 GB RAM, as well as a 64-Bit MATLAB version 2019 software.

### 3.6 RUL computation

The evaluation of the remaining useful lifetime of the system is the major objective in a prognostic study. Since the  $RUL$  is the complement of the damage curve  $D(t)$ , it can be deduced from it. Accordingly, at each time  $t$ , the required  $RUL$  is the length from cycle time  $t$  to the critical cycle time  $t_C$  that corresponds to the threshold  $D = 1$ . The entire  $RUL$  is inferred using the following relation:

$$RUL = t_C - t_0 \quad (8)$$

where  $t_C$  is the necessary cycle time for the appearance of the first macro-cracks that means to reach failure, and  $t_0$  is the initial cycle time considered in general to be equal to 0.

Consequently, my prognostic model computes the  $RULs$  for the three internal pressure modes that can be now simply inferred from these three curves at any instant  $t$  or at any active cycle  $N$  in this manner:

For mode 3,  $RUL_3(t) = t_{C3} - t$ .

For mode 2,  $RUL_2(t) = t_{C2} - t$ .

For mode 1,  $RUL_1(t) = t_{C1} - t$ .

### 3.7 The effects of environment in the suggested prognostic model

Two parameters which are  $C$  and  $m$  embody the effects of the environment. These two parameters are associated to the material environment.  $C$  and  $m$  depend on the initial crack length, on the geometry and size of the specimen, and on the testing conditions (such as the loading ratio  $\sigma$ ). These two parameters affect the performance of the material during the process of fatigue through the crack propagation. The influencing parameters on this fatigue process, like humidity, temperature, material nature, geometry dimensions, applied load location, corrosion, water action, soil pressure and friction, atmospheric pressure, etc., can be stochastic and can be also embodied by  $C$  and  $m$ . Furthermore, it is crucial to note here that these two parameters can be as well random variables and hence can be represented by probability distributions materializing the environment stochastic and chaotic influences on the system. It is also important to mention that these two parameters are computed by the mean of experiments in real conditions. We give here some

examples from several and other prognostic studies [7, 8]:  $C = 5.2 \times 10^{-13}$  (free air, unburied pipelines),  $C = 1.3 \times 10^{-14}$  (under soil, buried pipelines),  $C = 2 \times 10^{-11}$  (for offshore pipelines), and  $m = 3$  (metal).

#### 4. The complex probability paradigm applied to prognostic

In this section, the novel complex probability paradigm will be presented after applying it to prognostic.

##### 4.1 The basic parameters of the new model

It is very well-known that in systems engineering, the remaining useful lifetime and the degradation prediction is profoundly linked to many aspects (like humidity, temperature, material nature, geometry dimensions, applied load location, water action, corrosion, soil pressure and friction, atmospheric pressure, etc.) that usually have a stochastic and chaotic behavior which reduces the degree of our certain system knowledge [32–35]. Consequently, the lifetime of the system becomes a random variable and is computed by the arbitrary time  $t_C$  which is evaluated when sudden failure occurs due to these stochastic causes and chaotic factors. We can deduce from the CPP that we can foretell the exact probabilities of *RUL* and *D* with certitude in the whole set  $\mathcal{C} = \mathcal{R} + \mathcal{M}$  if we add to the probability measure of a random variable in the real set  $\mathcal{R}$  the corresponding imaginary counterpart  $\mathcal{M}$  since  $P_c = 1$  perpetually and constantly. In fact, prognostic is based on the forecast of a system remaining useful lifetime at any cycle  $N$  or instant  $t$  and during the system operation. Therefore, we can make use of this novel idea and procedure to do the prognostic analysis of the system *RUL* and degradation prediction and evolution.

Let us consider a system degradation trajectory  $D(t)$  where we study a specific instant (or cycle)  $t_k$ . The system age is measured by the number of years and by the variable  $t_k$  (Figure 5). From the illustrated figures (Figures 5a and 5b), we can infer that at the system age  $t_k$  of the prognostic study must give the prediction of the failure instant  $t_C$ . Therefore, the *RUL* predicted here at the instant  $t_k$  has the following value:

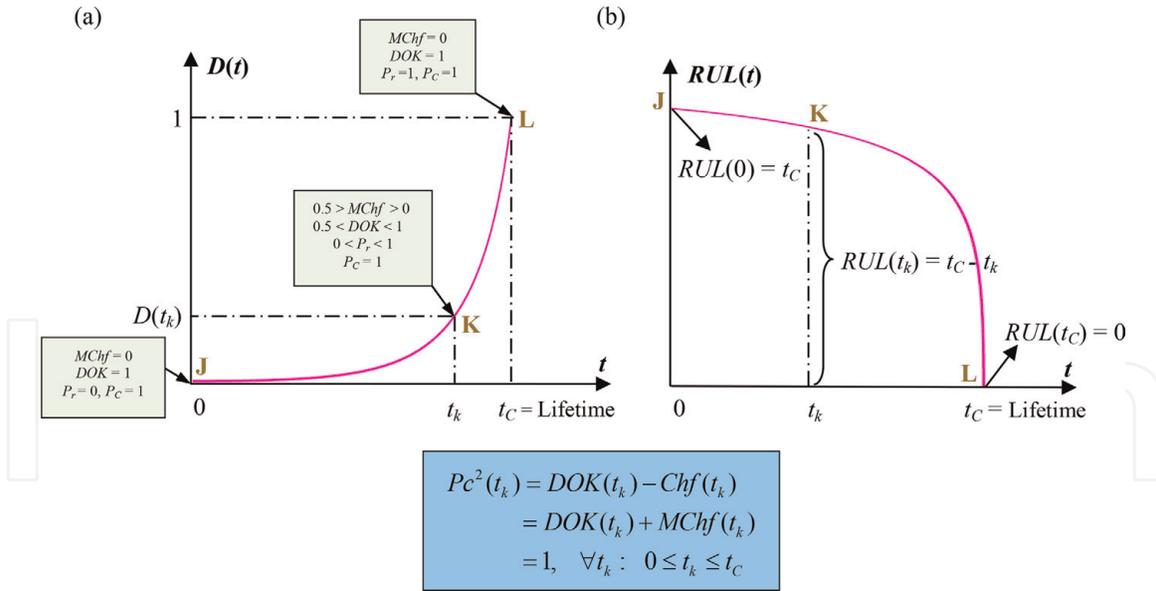
$$RUL(t_k) = t_C - t_k \quad (9)$$

As a matter of fact, at  $t_k = 0$  (at the beginning) (point J), the system is intact, then the failure probability of the system is  $P_r = 0$ , the chaotic factor in our prognostic is null ( $MChf = 0$ ) because no chaos exists yet, and our knowledge of the unharmed and undamaged system is complete and certain ( $DOK = 1$ ); consequently,

$$RUL(0) = t_C - t_k = t_C - 0 = t_C.$$

If  $t_k = t_C$  (point L), the system is completely damaged, then  $RUL(t_C) = t_C - t_C = 0$ , and therefore the failure probability of the system is one ( $P_r = 1$ ). Failure occurs at this point. Thus, our knowledge of the totally worn-out system is perfect ( $DOK = 1$ ) and the harmful task of chaos has finished; hence it is no more applicable ( $MChf = 0$ ).

If  $0 < t_k < t_C$  (point K, where  $J < K < L$ ), the probability of occurrence of this instant and the probabilities of prediction of *RUL* and *D* are both less than 1 and are imperfect in  $\mathcal{R}$  ( $0 < P_r < 1$ ). This is the result of non-zero chaotic factors influencing the system ( $MChf > 0$ ). The system degree of our knowledge which is subject to chaos is thus uncertain and is consequently less than one in  $\mathcal{R}$  ( $0.5 < DOK < 1$ ).



**Figure 5.** CPP. (a) The prognostic of degradation and (b) The prognostic of RUL.

Furthermore, by applying here the CPP paradigm, we can therefore determine at any instant  $t_k$  ( $0 \leq t_k \leq t_C$ ) and, at any point between J and L inclusively, the RUL and  $D$  of the system with certitude in the set  $\mathcal{C} = \mathcal{R} + \mathcal{M}$  because in  $\mathcal{C}$  we have  $P_C = 1$  permanently.

Additionally, we can express two complementary phenomena or events  $E$  and  $\bar{E}$  by their respective probabilities as follows:

$$P_{rob}(E) = p \text{ and } P_{rob}(\bar{E}) = q = 1 - p.$$

Therefore, let the probability  $P_{rob}(E)$  as a function of the time  $t_k$  be defined by

$$P_{rob}(E) = P_{rob}(t \leq t_k) = F(t_k) \quad (10)$$

where the classical and usual cumulative distribution function (CDF) of the random variable  $t$  is denoted by the term  $F(t)$ .

Since  $P_{rob}(E) + P_{rob}(\bar{E}) = 1$ , therefore, we deduce at an instant  $t = t_k$ :

$$P_{rob}(\bar{E}) = 1 - P_{rob}(E) = 1 - P_{rob}(t \leq t_k) = P_{rob}(t > t_k) = 1 - F(t_k) \quad (11)$$

In addition, two particular instants can be defined:

$t = t_0 = 0$  which corresponds to the system raw state and which is assumed to be the initial time of functioning where  $D = D_0$ .

$t = t_C$  which corresponds to the system wear-out state and which is the failure instant where  $D = D_C = 1$ .

Consequently, we can state the boundary conditions as follows:

For  $t = t_0 = 0$ , we have  $D = D_0 \approx 0$  (the initial damage that may be nearly 0) and  $F(t) = F(t_0) = P_{rob}(t \leq 0) = 0$ .

For  $t = t_C$ , we have  $D = D_C = 1$  and  $F(t) = F(t_C) = P_{rob}(t \leq t_C) = 1$ .

We note also that since  $F(t_k)$  is defined as a cumulative probability function, then  $F(t_k)$  is a non-decreasing function that varies between 0 and 1. In addition, since  $RUL(t_k) = t_C - t_k$  and  $t_k$  is always increasing ( $0 \leq t_k \leq t_C$ ), then  $RUL(t_k)$  is a non-increasing remaining useful lifetime function (**Figure 5b**).

## 4.2 The new prognostic model

The novel model of prognostic basic assumption will be presented now [36–53]. We assume first the cumulative probability distribution function  $F(t)$  of the random variable time  $t$  as being equal to the function of degradation itself, which means

$$F(t_k) = P_{rob}(t_0 \leq t \leq t_k) = \sum_{t=t_0}^{t=t_k} P_{rob}(t) = D(t_k) \quad (12)$$

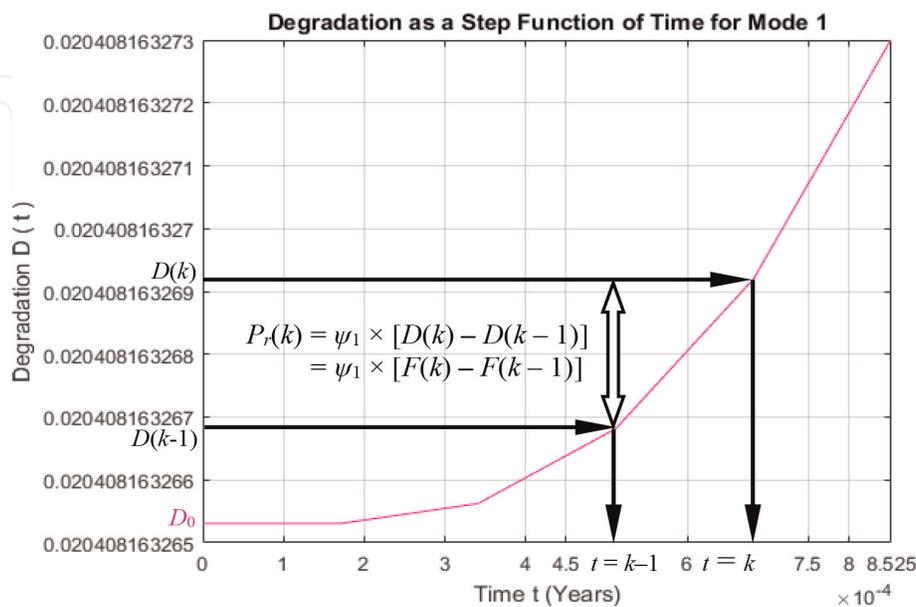
We mention here that we are working with discrete random functions that depend on the discrete random time  $t$  of pressure cycles.

This basic assumption is reasonable because:

1. Both  $D$  and  $F$  are cumulative functions starting from zero and ending with one.
2. Both are non-decreasing functions.
3. Both functions are without measure units:  $D$  is an indicator quantifying system damage and degradation, as well as  $F$  which is an indicator quantifying randomness and chance.

Afterward, we suppose that, at the instant  $t = t_k$ , the term  $P_r(t)/\psi_j$  is the real probability of system failure and is computed as follows:

$$\begin{aligned} P_r(t_k) &= \psi_j \times [P_{rob}(t \leq t_k) - P_{rob}(t \leq t_{k-1})] = \psi_j \times [F(t_k) - F(t_{k-1})] \\ &= \psi_j \times [D(t_k) - D(t_{k-1})] \\ &= \psi_j \times \left[ \sum_{t=t_0}^{t=t_k} P_{rob}(t) - \sum_{t=t_0}^{t=t_{k-1}} P_{rob}(t) \right] \\ &= \psi_j \times \sum_{t=t_{k-1}}^{t=t_k} P_{rob}(t) = \psi_j \times P_{rob}(t_{k-1} \leq t \leq t_k) \end{aligned} \quad (13)$$



**Figure 6.**  
 $P_r$ , degradation, and the CDF step function.

=  $\psi_j$  times the jump in  $F(t)$  or  $D(t)$  from  $t = t_{k-1}$  to  $t = t_k$  (**Figures 6 and 7**).

where  $t = [0, 1, 2, \dots, t_{k-1}, t_k, t_{k+1}, \dots, t_C]$  is the time of pressure cycles and  $t_0 = 0$  is the initial time of pressure cycles at the simulation beginning. It corresponds to a degradation  $D = D(t_0) = D_0$  which is generally considered to be nearly equal to 0.

Hence, since  $F(t_k) = D(t_k)$  then  $F(t_0) = D(t_0) = 0.020408 \approx 0$ , but  $F(t_0)$  is taken all over this research work as being equal to 0;

$t_1 = 1 =$  the first pressure cycle time ...  $t_k =$  the  $k^{\text{th}}$  pressure cycle time ...  $t_C =$  the pressure cycles time that leads to system failure = the critical pressure time. It corresponds to  $D = D_C = 1$ . It follows directly that  $F(t_C) = D(t_C) = D_C = 1$ .

$\psi_j$  is the simulation magnifying factor that depends on the pressure profile. It is  $\psi_1 = 5082$  for the high-pressure mode ( $j = 1$ , mode 1),  $\psi_2 = 6737$  for the middle-pressure mode ( $j = 2$ , mode 2), and  $\psi_3 = 9151$  for the low-pressure mode ( $j = 3$ , mode 3).

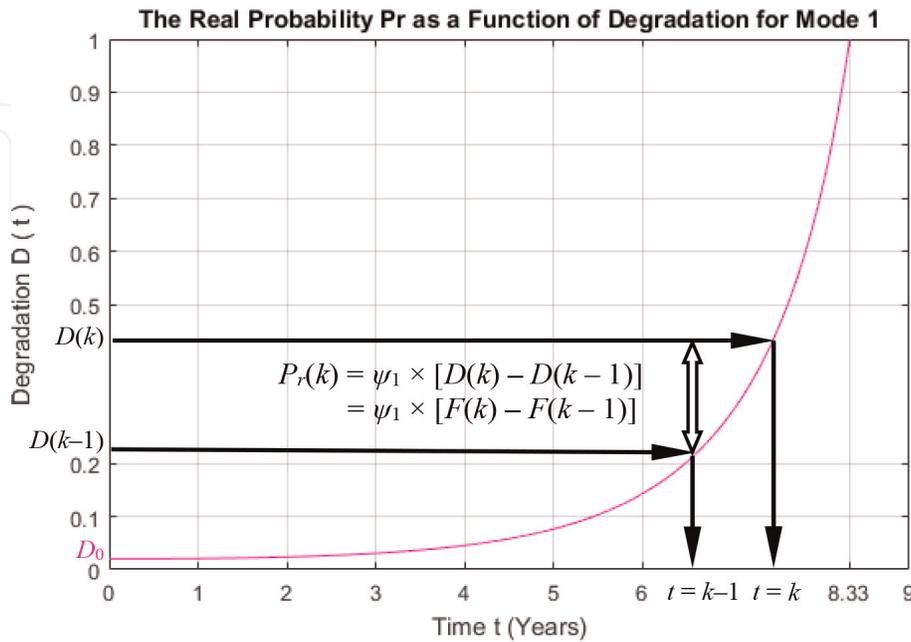
Thus, initially we have

$$P_r(t_k = t_0 = 0) = \psi_j \times F(t_0) = \psi_j \times 0 = 0$$

Moreover,

$$P_r(t_k) = \psi_j \times f_j(t_k) \Rightarrow P_r(t_k)/\psi_j = f_j(t_k), \quad (14)$$

where  $1/\psi_j$  is a normalizing constant that is used to reduce  $P_r(t_k)$  function to a probability density function (PDF) with a total probability equal to one.  $1/\psi_j$  is a function of the pressure mode and conditions, and it depends on the parameters in the degradation (Eqs. (5) and (6)). The decreasing values of  $1/\psi_j$  are logical since pipeline failure probabilities are decreasing with the decreasing pressure modes; hence,  $1/\psi_1 > 1/\psi_2 > 1/\psi_3$ . Consequently, we deduce that  $f_j(t_k)$  is the usual probability density function (PDF) for each pressure mode  $j$ . Knowing that, from classical probability theory, we have always:



**Figure 7.**  
 $P_r$  as a function of degradation  $D(t)$ .

$$\sum_{t_k=t_0}^{t_k=t_C} f_j(t_k) = \sum_{t_k=t_0}^{t_k=t_C} P_r(t_k)/\psi_j = 1 \text{ for any pressure profile } j = 1, 2, 3.$$

This result is reasonable since  $P_r(t_k)/\psi_j$  is here a probability density function (**Figure 6**).

Therefore, we can deduce that

$$\begin{aligned} \sum_{t_k=t_0}^{t_k=t_C} P_r(t_k) &= \psi_j \times \sum_{t=t_0}^{t=t_C} P_{rob}(t) = \psi_j \times P_{rob}(t_0 \leq t \leq t_C) \\ &= \psi_j \times [F(t = t_C) - F(t = t_0)] = \psi_j \times [D(t = t_C) - D(t = t_0)] \\ &= \psi_j \times F(t_C) \approx \psi_j \times D(t_C), \end{aligned}$$

since  $D(t_C) = 1$  and  $D(t_0) = 0.020408 \approx 0$  and  $F(t_0)$  is taken as  $= 0$  (15)

$$= \psi_j \times \sum_{t_k=t_0}^{t_k=t_C} f_j(t_k) = \psi_j \times 1 = \psi_j$$

$$\Rightarrow \sum_{t_k=t_0}^{t_k=t_C} P_r(t_k)/\psi_j = 1, \text{ for any pressure profile } j = 1, 2, 3$$

We can understand that  $F(t) = D(t)$  is a discrete CDF where the amount of the jump is  $P_r(t)/\psi_j$ ; then,  $P_r(t)/\psi_j$  is a damage evolution and degradation function (**Figures 6 and 7**). And we can infer from the preceding computations that  $P_r(t)/\psi_j$  is a probability density function. Accordingly, we can realize now that  $P_r(t)/\psi_j$  quantifies and measures the system degradation or failure probability. Consequently, what we have achieved at this point is that we have linked degradation measure to probability theory.

We can notice the following:

$$0 \leq P_r(t_k)/\psi_j \leq 1, 0 \leq F(t_k) \leq 1, \text{ and } (D_0 \approx 0) \leq D(t_k) \leq (D_C = 1),$$

$$\text{for every } t_k : 0 \leq t_k \leq t_C.$$

and

$$\text{If } t_k \rightarrow 0 \Rightarrow D \rightarrow D_0 = 0.020408 \approx 0 \Rightarrow F \rightarrow 0 \Rightarrow P_r(t_k) \rightarrow 0$$

$$\text{if } t_k \rightarrow t_C \Rightarrow D \rightarrow D_C = 1 \Rightarrow F \rightarrow 1 \Rightarrow P_r(t_k) \rightarrow 1.$$

This, since the degradation is very flat near 0 and starts increasing with  $t$ , becoming very acute at  $t = t_C$ , hence, near  $t_C$ ,  $P_r$  is the greatest and is equal to 1 (**Figures 7 and 8**).

Furthermore, we have:

$$RUL(t_k) = t_C - t_k \text{ and it corresponds to a degradation of } D(t_k).$$

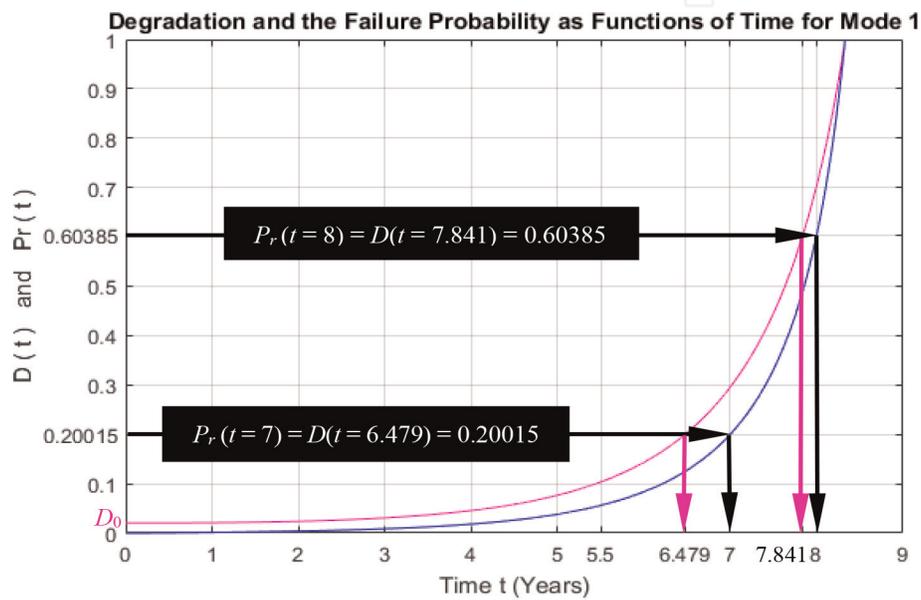
$$RUL(t_{k-1}) = t_C - t_{k-1} \text{ and it corresponds to a degradation of } D(t_{k-1}).$$

This implies that (**Figure 9**)

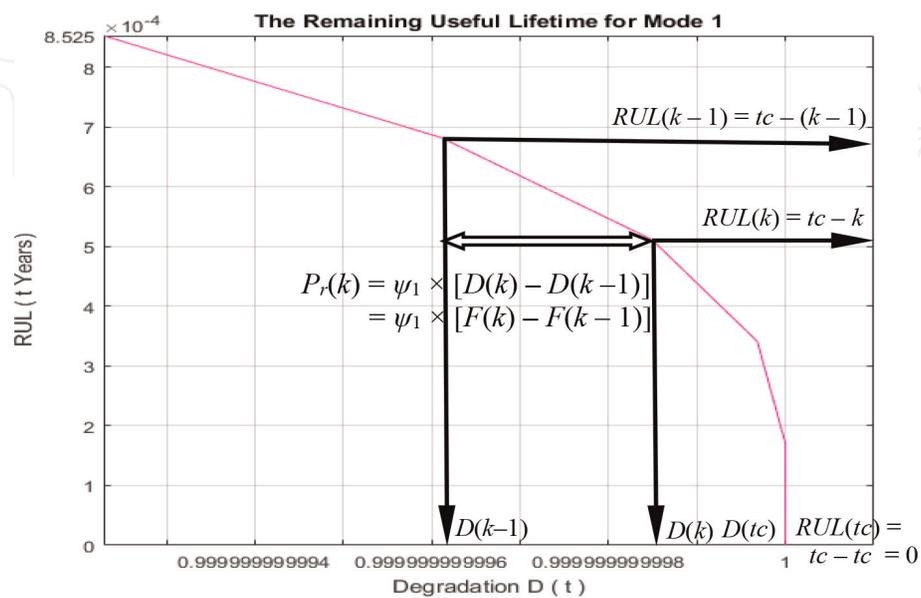
$$\begin{aligned} P_r(t_k) &= \psi_j \times [D(t_k) - D(t_{k-1})] \\ &= \psi_j \times \{D[t_C - RUL(t_k)] - D[t_C - RUL(t_{k-1})]\} \end{aligned} \quad (16)$$

### 4.3 Analysis and extreme chaotic and random conditions

Although the analytic linear laws of prognostic are very well-known and deterministic in [14, 16], there are general influences and aspects that can be chaotic and stochastic (like humidity, temperature, material nature, geometry dimensions, applied load location, water action, corrosion, soil pressure and friction, atmospheric pressure, etc.). Moreover, various variables in the expressions (5) and (6) of degradation which are considered as deterministic can also have a random aspect, such as the magnitude of applied pressure (due to the different conditions of pressure profile) and the length of the initial crack (potentially existing from the process of manufacturing). All those stochastic factors, embodied in the model by their mean values, influence the buried pipeline system and make its function of



**Figure 8.**  
Degradation and  $P_r$ .



**Figure 9.**  
 $P_r$ ,  $D$ , and  $RUL$ .

degradation diverge from its computed trajectory modeled by these deterministic laws. An updated follow-up of the degradation performance and behavior with cycle number or time, which is subject to non-chaotic and chaotic influences, is made possible by  $P_r(t_k)/\psi_j$  due to its definition that evaluates the jumps in  $D$ . In fact, chaos modifies and affects all the environment and system parameters included in the degradation equations (Eqs. (5) and (6)). Consequently, chaos total effect on the pipelines contributes to shape the degradation curve  $D$  and is materialized by and counted in the pipeline system failure probability  $P_r(t_k)/\psi_j$ . Actually,  $P_r(t_k)/\psi_j$  quantifies the resultant of all the nonrandom (deterministic) and random (nondeterministic) parameters and aspects which are contained in the equation of  $D$ , which affect the system and which lead to the consequent final curve of degradation. Consequently, an accentuated influence of chaos on the pipeline can lead to a smaller (or bigger) jump in the trajectory of degradation and therefore to a smaller (or bigger) failure probability  $P_r(t_k)/\psi_j$ . If, for example, due to extreme deterministic causes and random factors,  $D$  jumps directly from  $D_0 \approx 0$  to 1 then  $RUL$  goes straight from  $t_C$  to 0 and consequently  $P_r(t_k)/\psi_j$  jumps instantly from 0 to 1:

$$P_r(t_k)/\psi_j = D(t_k) - D(t_{k-1}) = D(t_C) - D(0) \approx 1 - 0 = \sum_{t=0}^{t=t_C} P_{rob}(t) = 1$$

where  $t$  jumps directly from 0 to  $t_C$ .

In the extreme ideal case, if the pipeline system never deteriorates (no stresses or pressure) and with zero random causes and chaotic factors, then the resultant of all the nondeterministic and deterministic influences is null (like in the pipeline isolated and idle state). Accordingly, the system remains indefinitely at  $D_0 \approx 0$  and  $RUL$  stays equal to  $t_C$ . So consequently, the jump in  $D$  is constantly zero. Hence, the failure probability remains ideally 0:

$$P_r(t_k)/\psi_j = [D(t_k) - D(t_{k-1})] = [D_0 - D_0] = 0$$

where  $D(t_0) = D(t_1) = \dots = D(t_{k-1}) = D(t_k) = D(t_{k+1}) = \dots = D_0 = 0.020408 \approx 0$ , for  $k = 0, 1, 2, 3, \dots \infty$ .

**Figure 6** illustrates the real probability of failure  $P_r(t)$  in terms of the random degradation step CDF of the pipeline as a function of the cycle time  $t$  of pressure for mode 1.

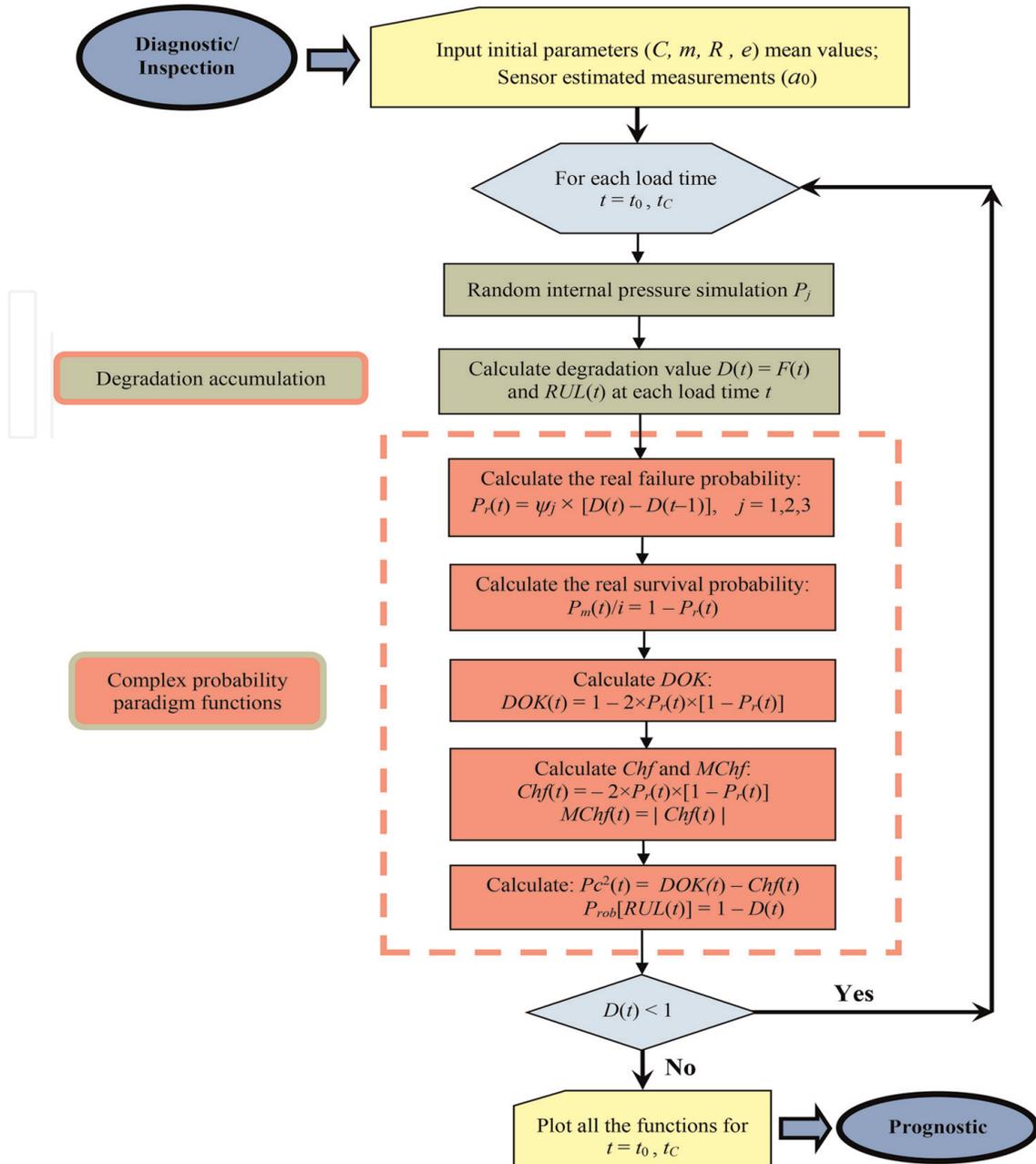
**Figure 7** illustrates the real probability of failure  $P_r(t)$  in terms of the random degradation of the pipeline as a function of the cycle time  $t$  of pressure for mode 1.

**Figure 8** illustrates the real probability of failure  $P_r(t)$  and the random degradation  $D(t)$  of the pipeline in terms of the number of cycle time  $t$  of pressure for mode 1.

**Figure 9** illustrates the real probability of failure  $P_r(t)$  in terms of the random degradation  $D(t)$  of the pipeline and the random  $RUL(t)$  of the pipeline as a function of the cycle time  $t$  (in years) of pressure for mode 1.

#### 4.4 The flowchart of the complex probability analytic linear prognostic model

The following flowchart summarizes all the procedures of the proposed complex probability prognostic model:



#### 4.5 The evaluation of the new paradigm parameters

We can infer from what has been elaborated previously the following:

The real probability is  $P_r(t_k) = \psi_j \times [D(t_k) - D(t_{k-1})]$ , for pressure modes  $j = 1, 2, 3$

$$(17)$$

The imaginary probability is  $P_m(t_k) = i \times [1 - P_r(t_k)] = i \times \{1 - \psi_j \times [D(t_k) - D(t_{k-1})]\}$

$$(18)$$

The complementary probability is  $P_m(t_k)/i = 1 - P_r(t_k) = 1 - \psi_j \times [D(t_k) - D(t_{k-1})]$

$$(19)$$

The complex probability vector is  $Z(t_k) = P_r(t_k) + P_m(t_k) = P_r(t_k) + i \times [1 - P_r(t_k)]$

$$(20)$$

The degree of our knowledge

$$\begin{aligned} DOK(t_k) &= |Z(t_k)|^2 = 1 + 2iP_r(t_k)P_m(t_k) = 1 - 2P_r(t_k)P_m(t_k)/i = 1 - 2P_r(t_k)[1 - P_r(t_k)] \\ &= 1 - 2P_r(t_k) + 2P_r^2(t_k) \end{aligned} \quad (21)$$

The chaotic factor

$$\begin{aligned} Chf(t_k) &= 2iP_r(t_k)P_m(t_k) = -2P_r(t_k)P_m(t_k)/i = -2P_r(t_k)[1 - P_r(t_k)] \\ &= -2P_r(t_k) + 2P_r^2(t_k) \end{aligned} \quad (22)$$

$Chf$  is null when  $P_r(N_k) = P_r(0) = 0$  (point J) and when  $P_r(t_k) = P_r(t_C) = 1$  (point L) (**Figures 5a and 5b**).

The magnitude of the chaotic factor  $MChf$ :

$$\begin{aligned} MChf(t_k) &= |Chf(t_k)| = -2iP_r(t_k)P_m(t_k) = 2P_r(t_k)P_m(t_k)/i = 2P_r(t_k)[1 - P_r(t_k)] \\ &= 2P_r(t_k) - 2P_r^2(t_k) \end{aligned} \quad (23)$$

$MChf$  is null when  $P_r(t_k) = P_r(0) = 0$  (point J) and when  $P_r(t_k) = P_r(t_C) = 1$  (point L) (**Figures 5a and 5b**).

At any instant  $t_k$   $0 \leq \forall t_k \leq t_C$ , the probability expressed in the complex set  $\mathcal{C}$  is the following:

$$\begin{aligned} Pc(t_k)^2 &= [P_r(t_k) + P_m(t_k)/i]^2 = |Z(t_k)|^2 - 2iP_r(t_k)P_m(t_k) \\ &= DOK(t_k) - Chf(t_k) \\ &= DOK(t_k) + MChf(t_k) \\ &= 1 \end{aligned} \quad (24)$$

then,  $Pc(t_k) = P_r(t_k) + P_m(t_k)/i = P_r(t_k) + [1 - P_r(t_k)] = 1$  always.

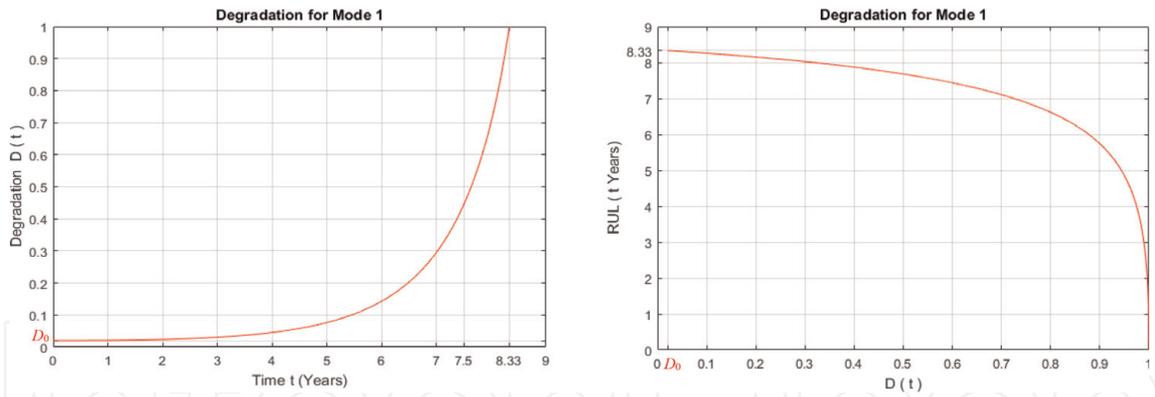
Therefore, the prognostic of  $RUL(t_k)$  and  $D(t_k)$  of the pipeline in the set  $\mathcal{C}$  is forever certain. The buried pipeline system is considered thereafter under three modes of pressure in order to simulate the cumulative distribution function  $D(t_k) = F(t_k)$  and hence in order to visualize, to quantify, as well as to draw all the prognostic parameters and  $CPP$ .

## 5. The simulation of the new paradigm

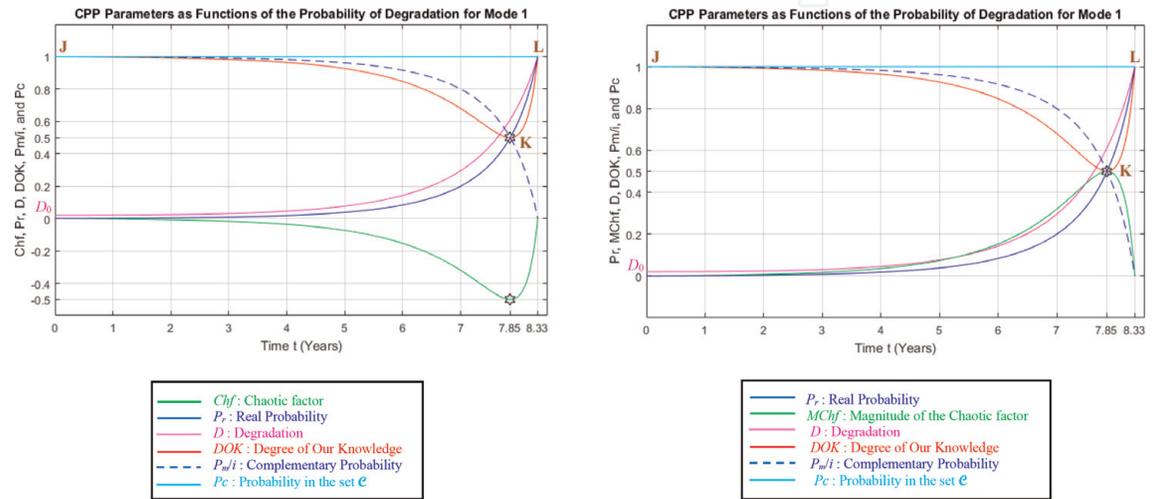
We will simulate in this section the original model of prognostic for the three internal pressure modes. We note that we have used the 64-Bit MATLAB version 2019 software to evaluate and find all the numerical values of the paradigm functions analysis.

### 5.1 The parameter simulation in the pipeline prognostic for mode 1

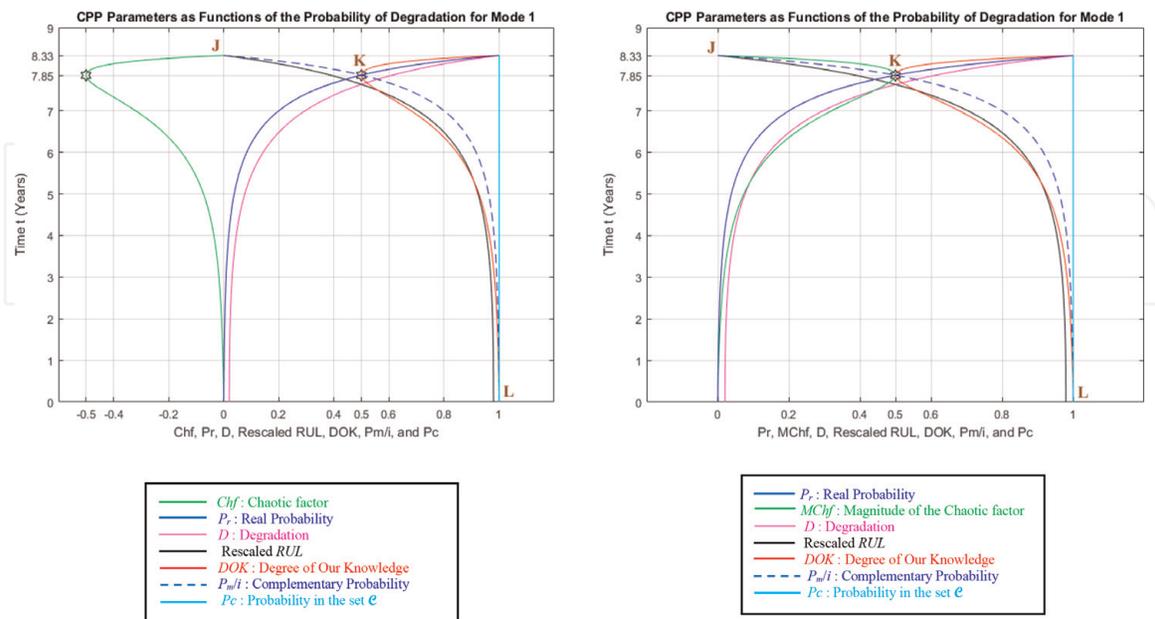
See **Figures 10–12**.



**Figure 10.** Pipeline degradation (a) and RUL (b) under linear damage law for high-pressure mode of excitation (mode 1).



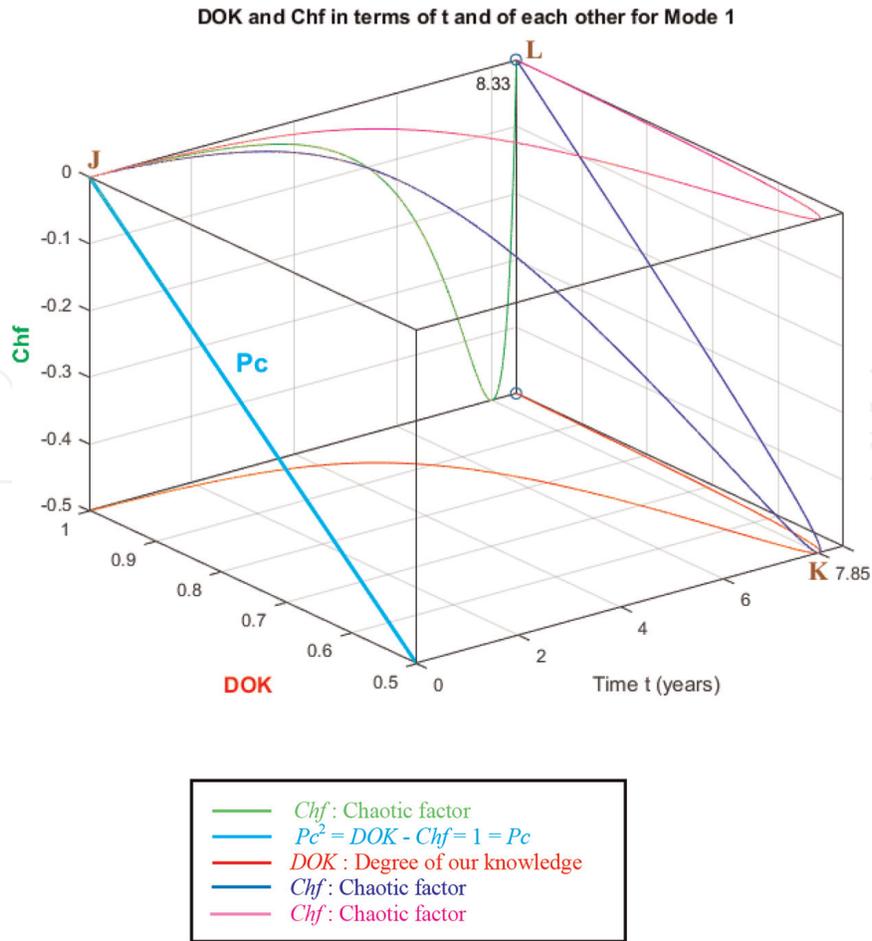
**Figure 11.** Degradation and CPP parameters with Chf (a) and with MChf (b) for mode 1.



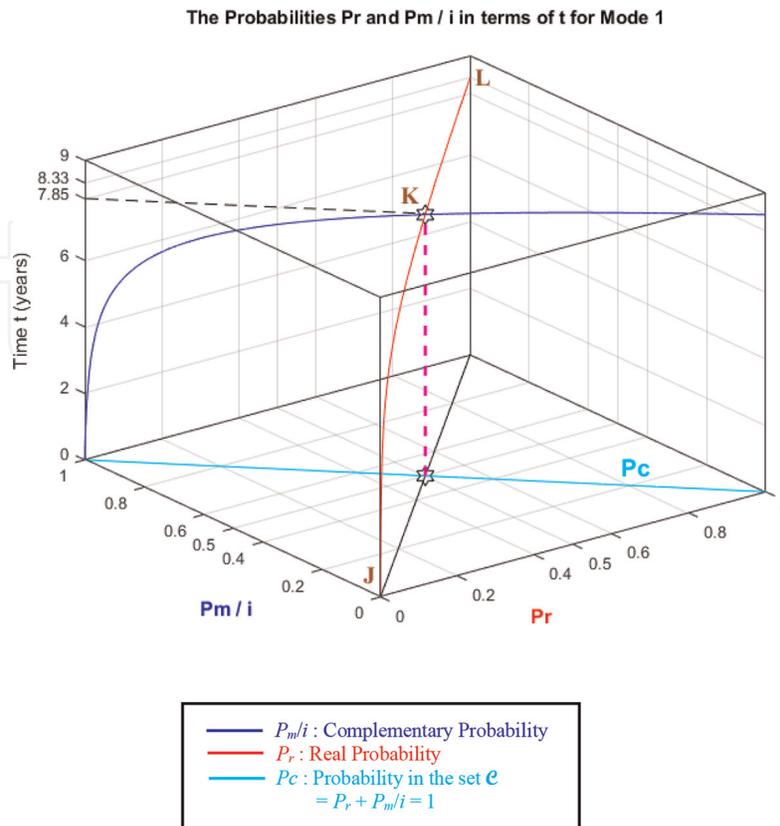
**Figure 12.** Degradation, rescaled RUL, and CPP parameters with Chf (a) and with MChf (b) for mode 1.

### 5.1.1 The complex probability cubes for mode 1

See Figures 13–15.



**Figure 13.**  
 DOK and Chf in terms of t and of each other for mode 1.



**Figure 14.**  
 $P_r$  and  $P_{m/i}$  in terms of t and of each other for mode 1.

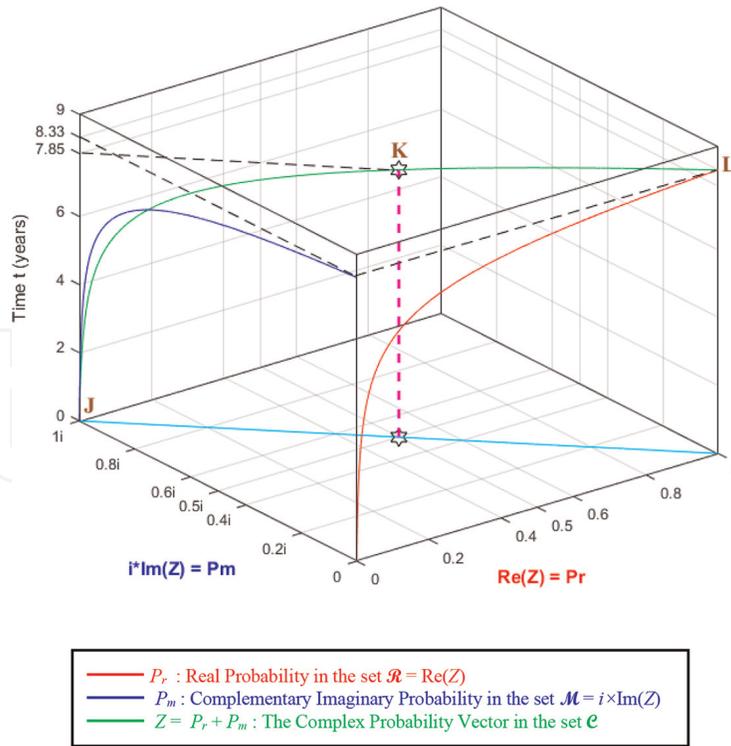


Figure 15. The complex probability vector  $Z$  in terms of  $t$  for mode 1.

## 5.2 The parameter simulation in the pipeline prognostic for mode 2

See Figures 16–18.

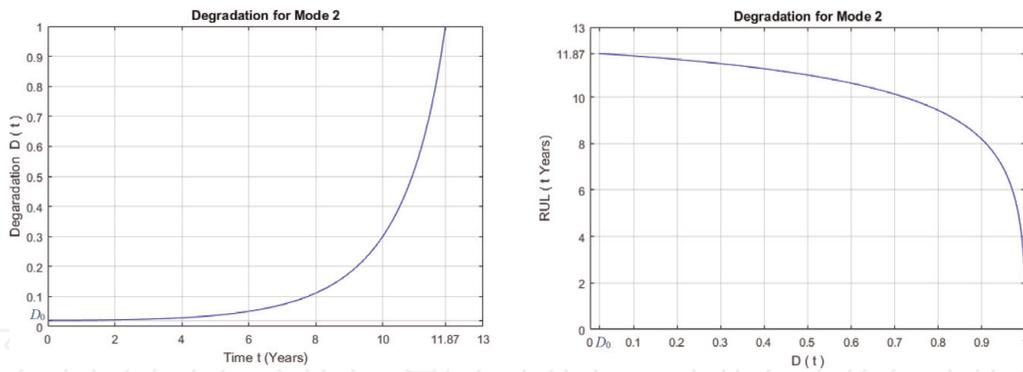


Figure 16. Pipeline degradation (a) and RUL (b) under linear damage law for middle-pressure mode of excitation (mode 2).

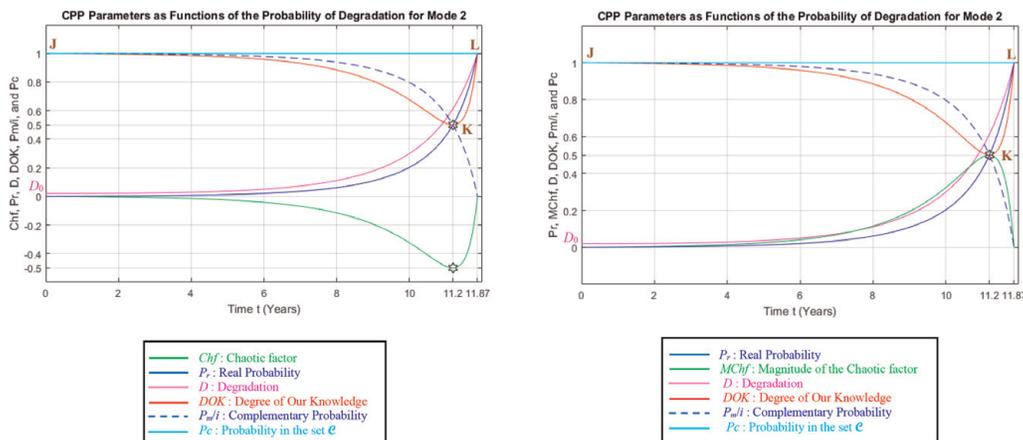
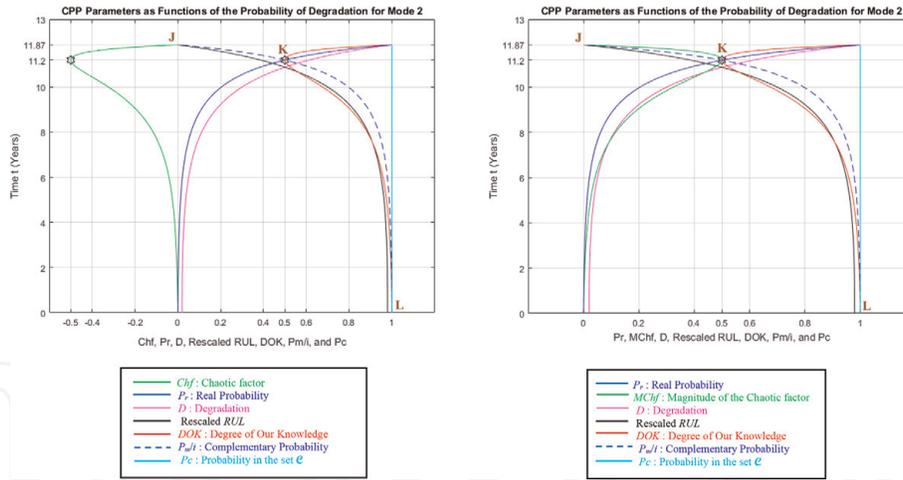


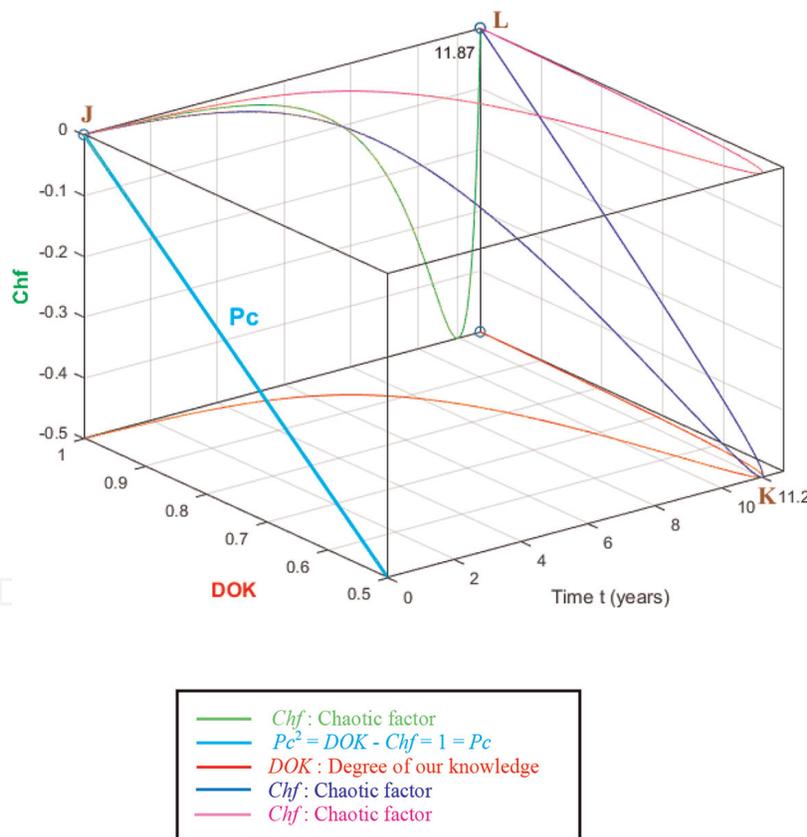
Figure 17. Degradation and CPP parameters with Chf (a) and with MChf (b) for mode 2.



**Figure 18.** Degradation, rescaled RUL, and CPP parameters with Chf (a) and with MChf (b) for mode 2.

### 5.2.1 The complex probability cubes for mode 2

See Figures 19–21.



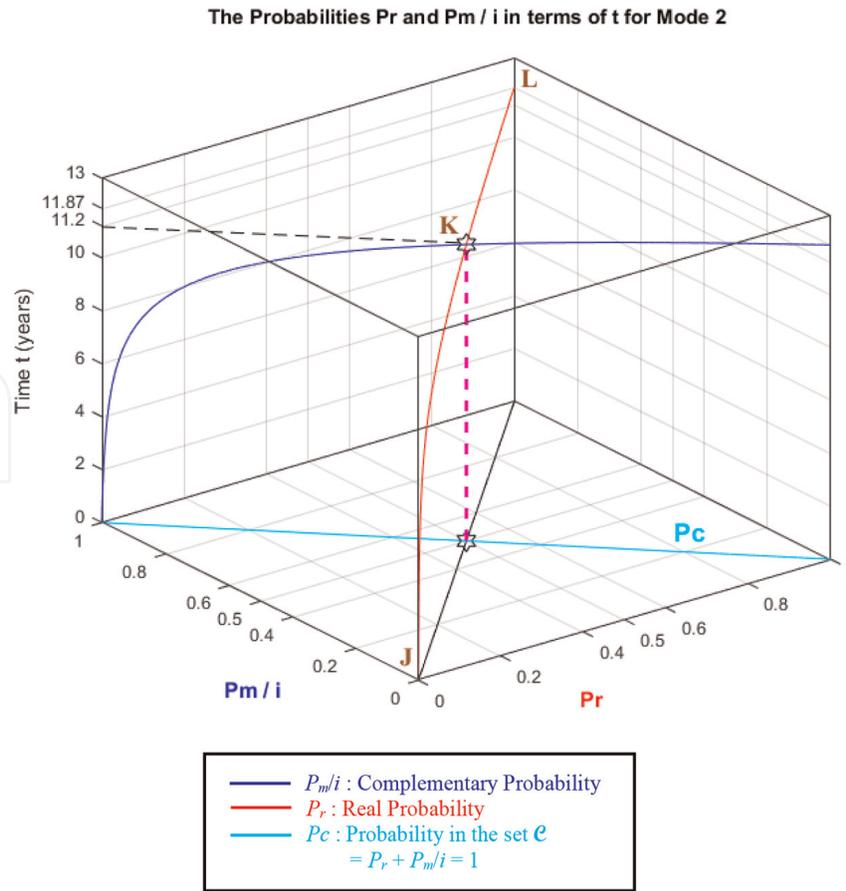
**Figure 19.** DOK and Chf in terms of t and of each other for mode 2.

### 5.3 The parameter simulation in the pipeline prognostic for mode 3

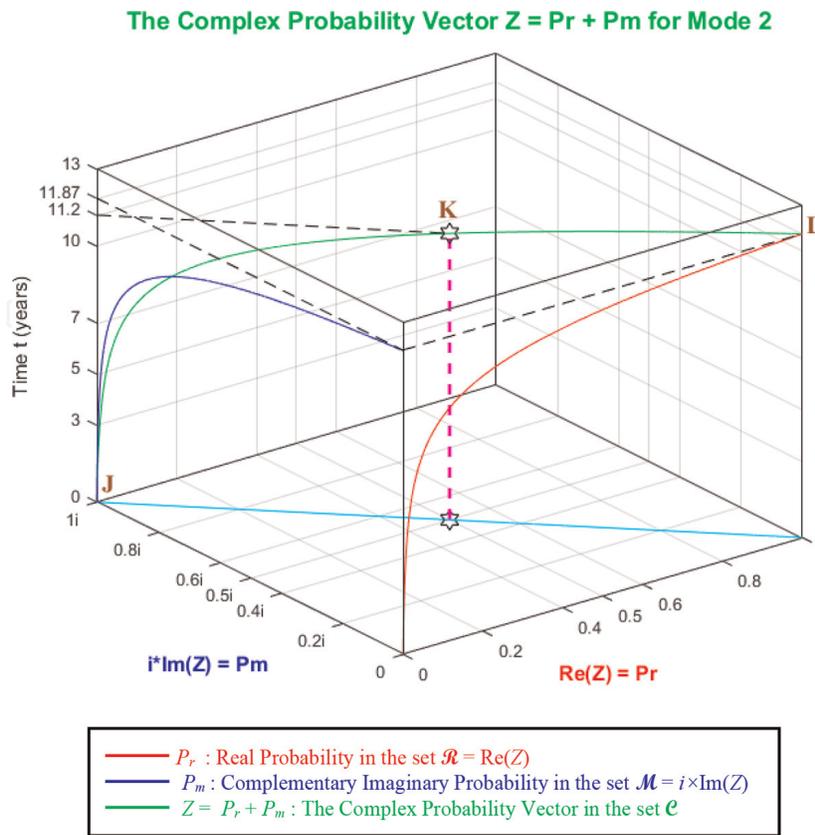
See Figures 22–24.

#### 5.3.1 The complex probability cubes for mode 3

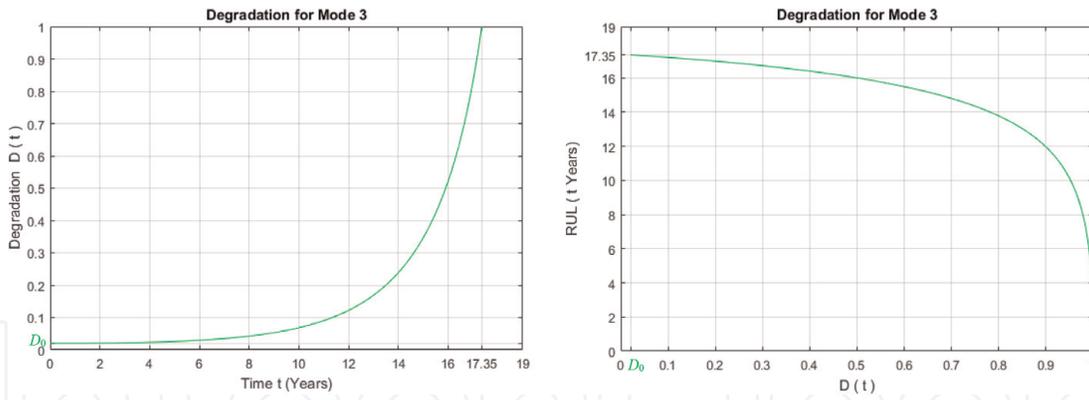
See Figures 25–27.



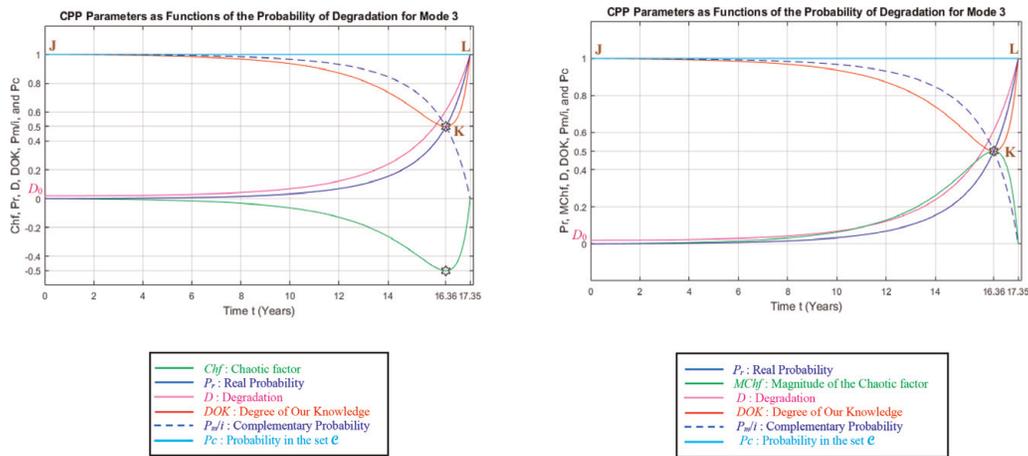
**Figure 20.**  
 $P_r$  and  $P_m/i$  in terms of  $t$  and of each other for mode 2.



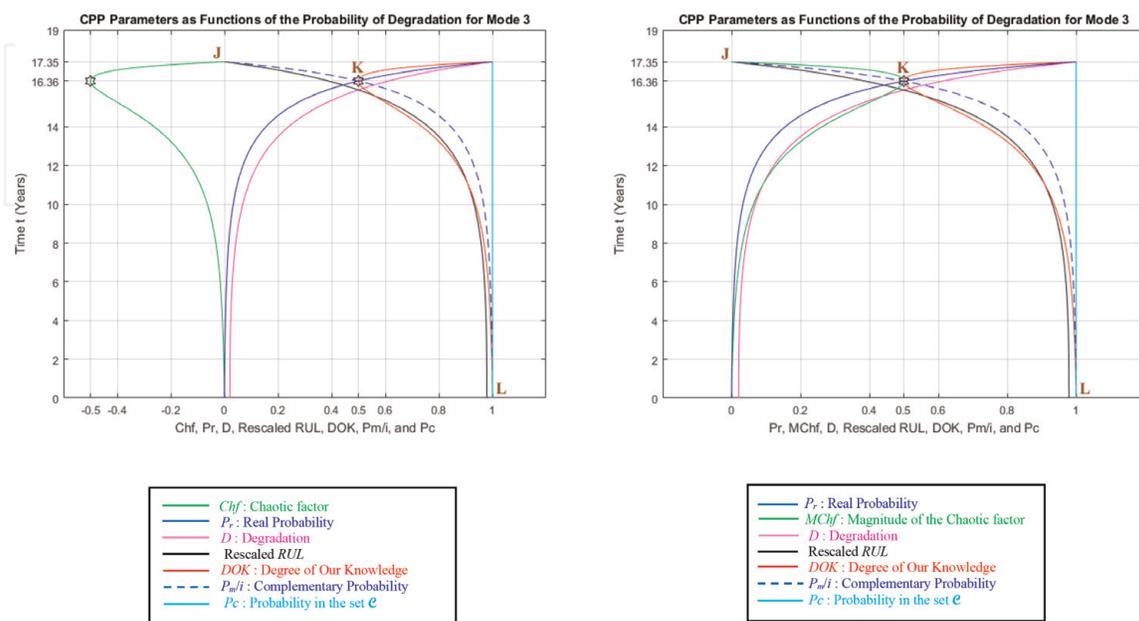
**Figure 21.**  
 The complex probability vector  $Z$  in terms of  $t$  for mode 2.



**Figure 22.** Pipeline degradation (a) and RUL (b) under linear damage law for low-pressure mode of excitation (mode 3).



**Figure 23.** Degradation and CPP parameters with  $Chf$  (a) and with  $MChf$  (b) for mode 3.



**Figure 24.** Degradation, rescaled RUL, and CPP parameters with  $Chf$  (a) and with  $MChf$  (b) for mode 3.

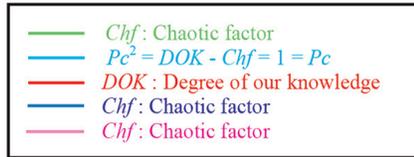
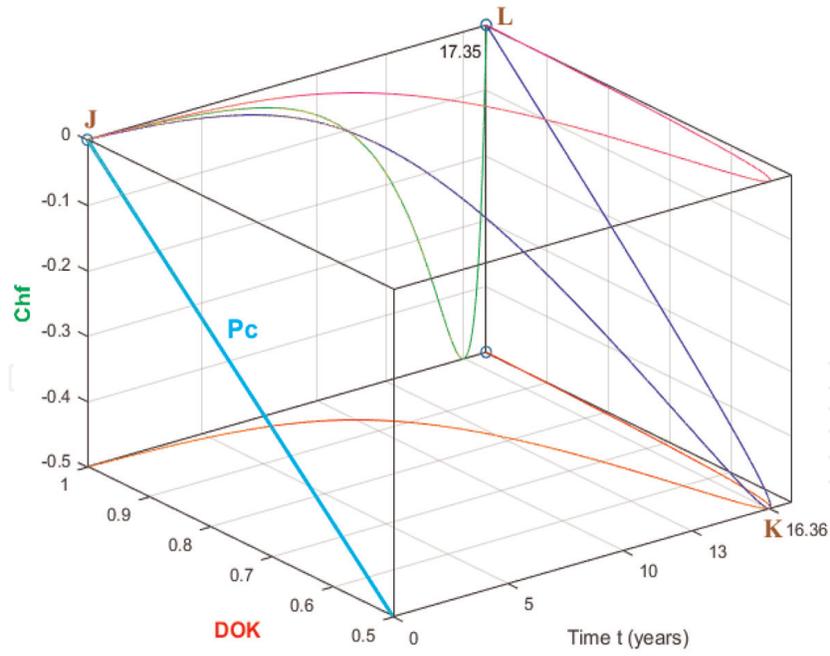


Figure 25.  $DOK$  and  $Chf$  in terms of  $t$  and of each other for mode 3.

The Probabilities  $Pr$  and  $Pm/i$  in terms of  $t$  for Mode 3

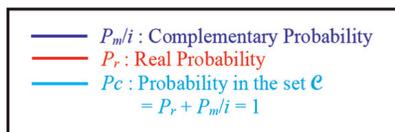
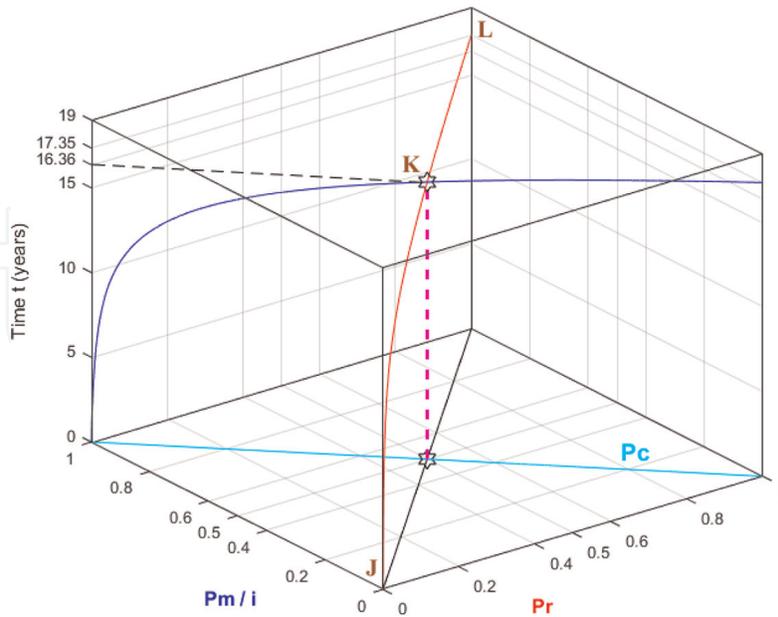


Figure 26.  $Pr$  and  $Pm/i$  in terms of  $t$  and of each other for mode 3.

The Complex Probability Vector  $Z = P_r + P_m$  for Mode 3

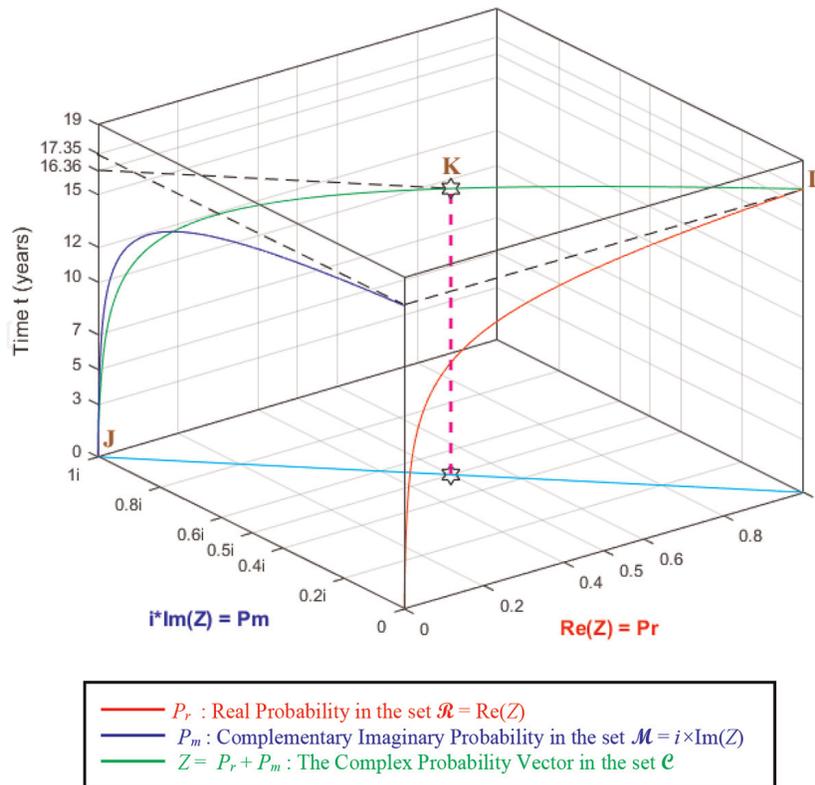


Figure 27.  
 The complex probability vector  $Z$  in terms of  $t$  for mode 3.

## 6. Final analysis: explanation and the general prognostic equations

We will present in this section the original general prognostic equations, we will interpret all the achieved simulations and the obtained data, and we will do a final analysis. Also, we will illustrate the results and a detailed discussion of the all the previous simulations and figures and of the following corresponding tables.

Firstly, we have linked prognostic characterized by the degradation  $D(t)$  with probability theory characterized by the CDF  $F(t)$  by supposing that  $D(t) = F(t)$  and the justification for this assumption were given. Consequently, the deterministic  $D(t)$  computed from deterministic analytic linear prognostic becomes a nondeterministic cumulative probability distribution function. Therefore, the deterministic and discrete variable of pressure cycles time  $t$  becomes a random and discrete variable. Thus, the resultant of all the factors influencing the system which was deterministic becomes a stochastic resultant because  $D(t)$  quantifies now the random degradation of the pipeline in terms of the random cycle time  $t$ . Accordingly, all the parameters' exact values of the  $D(t)$  expression (Eq. 6) become now the mean values of the stochastic factors influencing the pipeline and are embodied by PDFs as functions of the stochastic variable of pressure cycle time  $t$  (refer to Section 3.5). As a matter of fact, this is the real-world case where randomness is omnipresent in one form or another. What we consider and judge as a deterministic phenomenon is nothing in reality but a simplification and an approximation of an actual chaotic and stochastic phenomenon and experiment due to the impact of a huge number of nondeterministic and deterministic forces and factors (a good example is a lottery machine).

Subsequently, we do an updated follow-up of the performance of the random degradation in terms of time or cycle number, which is subject to non-chaotic and

chaotic influences, by using the quantity  $P_r(t_k)/\psi_j$  due to its definition that evaluates the jumps in the stochastic degradation CDF  $D(t)$ . Hence,

$$P_r(t_k) = \psi_j \times [D(t_k) - D(t_{k-1})], \text{ for any pressure mode } j = 1, 2, 3.$$

Referring to classical probability theory, this makes  $P_r(t_k)/\psi_j$  the system probability of failure at  $t = t_k$ , with  $0 \leq P_r(t_k)/\psi_j \leq 1$  and  $\sum_{t=t_0}^{t=t_C} P_r(t)/\psi_j = [\text{sum of all the jumps in } D \text{ from } t_0 \text{ to } t_C] = D_C = 1$ , just like any probability density function (PDF).

In addition, in the simulations, a constant and very small increments in  $t$  have been taken which lead to very small increments in  $D$  and hence in  $P_r(t_k)/\psi_j$ . So, we have multiplied those very small jumps in  $D$  by a simulation magnifying factor that we called  $\psi_j$ . Note that  $1/\psi_j$  is a normalizing constant that is used to reduce  $P_r(t_k)$  function to a probability density function with a total probability equal to one.  $1/\psi_j$  is a function of the pressure mode and conditions, and it depends on the parameters in the degradation (Eq. (6)). We have from the simulations  $\psi_1 = 5082$  for the high-pressure mode ( $j = 1$ , mode 1),  $\psi_2 = 6737$  for the middle-pressure mode ( $j = 2$ , mode 2), and  $\psi_3 = 9151$  for the low-pressure mode ( $j = 3$ , mode 3). So we get the following: if  $t$  tends to  $t_0 = 0$ , then  $P_r(t_k)$  tends to 0, and if  $t$  tends to  $t_C$  then  $P_r(t_k)$  tends to 1, so  $0 \leq P_r(t_k) \leq 1$  and  $\sum_{t=t_0}^{t=t_C} P_r(t) = \psi_j \times D_C = \psi_j \times 1 = \psi_j$  as if  $P_r(t_k)$  was a CDF although mathematically speaking it is not at all. This, since  $P_r(t_k)$  is not cumulative, it is just  $\psi_j$  times the probability of failure at  $t = t_k$ . Hence, in the simulations,  $P_r(t_k)$  becomes now the probability that the system failure occurs at  $t = t_k$  and is used accordingly to compute all the CPP parameters.

Therefore,  $D(t_k) = F(t_k) = P_{rob}(0 \leq t \leq t_k) = P_{rob}(t = 0 \text{ or } t = 1 \text{ or } t = 2 \text{ or } \dots \text{ or } t = t_k) = \text{sum of all failure probabilities between } 0 \text{ and } t_k = \text{probability that failure will occur somewhere between } 0 \text{ and } t_k$ . So, if  $t_k = 0$  then  $P_{rob}(t \leq 0) = D(0) = D_0$

= probability that failure will occur at  $t = 0$  and before. If  $t_k = t_C$  then  $P_{rob}(0 \leq t \leq t_C) = D(t_C) = 1 = \text{sum of all failure probabilities between } 0 \text{ and } t_C = \text{probability that failure will occur somewhere between } 0 \text{ and } t_C$ . If  $t_k > t_C$  then  $P_{rob}(t > t_C) = D(t_C) = 1 = \text{probability that failure will occur beyond } t_C$ . We can see that failure probability increases with the increase of the pressure cycles time  $t_k$  until at the end it becomes 1 when  $t_k \geq t_C$ .

Hence, if  $t_0 = 0$  and  $D(t_0) = 0$  then

$$D(t_k) = P_{rob}(0 \leq t \leq t_k) = \sum_{t=0}^{t=t_k} P_{rob}(t) = \sum_{t=0}^{t=t_k} P_r(t)/\psi_j$$

This implies that  $D(t_C) = P_{rob}(0 \leq t \leq t_C) = \sum_{t=0}^{t=t_C} P_{rob}(t) = \sum_{t=0}^{t=t_C} P_r(t)/\psi_j = 1$  and

$$D(0) = P_{rob}(t \leq 0) = \sum_{t=0}^{t=0} P_{rob}(t) = \sum_{t=0}^{t=0} P_r(t)/\psi_j = P_r(0)/\psi_j = 0.$$

If  $t_0 \neq 0$  and  $D(t_0) \neq 0$ , then the prognostic equation in the new model is

$$D(t_k) = P_{rob}(t_0 \leq t \leq t_k) = \sum_{t=t_0}^{t=t_k} P_{rob}(t) = \sum_{t=t_0}^{t=t_k} P_r(t)/\psi_j \quad (25)$$

for any mode  $j$  of pressure profile and with  $P_r(t_0)/\psi_j = D_0$ .

Moreover, since  $P_r(t_k) = \psi_j[D(t_k) - D(t_{k-1})]$ , this leads to the following recursive relation:

$$D(t_k) = D(t_{k-1}) + P_r(t_k)/\psi_j; \text{ for every } t_k, t_0 \leq t_k \leq t_C. \quad (26)$$

In the case of general prognostic, if we possess the *PDF* of system failure then it can be included in Eqs. (25) and (26) and hence evaluate at any instant  $t_k$  the system degradation and vice versa. Consequently, all the other *CPP* model parameters (*DOK*, *Chf*, *MChf*,  $P_r$ ,  $P_m$ ,  $P_m/i$ ,  $Z$ ,  $P_c$ ) will follow. This would be our new prognostic model general equation:

$$D(t_k) = P_{rob}(t_0 \leq t \leq t_k) = \sum_{t=t_0}^{t=t_k} P_{rob}(t) = \sum_{t=t_0}^{t=t_k} PDF_{failure}(t) \quad (27)$$

And the recursive relation

$$D(t_k) = D(t_{k-1}) + PDF_{failure}(t_k) \quad (28)$$

with  $PDF_{failure}(t_0) = D_0$ .

It is crucial to indicate here that the  $PDF_{failure}$  function of the system failure has all the mathematical characteristics and all the possible features of a probability density function whether it is a continuous or a discrete stochastic function and it can follow any imaginable probability distribution in condition only that it characterizes the failure function and the random degradation of the studied system whether it is a petrochemical pipe in the buried, unburied, or offshore case or a vehicle suspension system or any nondeterministic system under the effect of randomness and chaos. In fact, the function  $PDF_{failure}$  inherits all the attributes and features of the failure system function and of the nondeterministic degradation.

Furthermore, by applying *CPP* to the pipe prognostic, and in the three simulations of pressure modes, we were successful in the original prognostic model to quantify in  $\mathcal{R}$  (our real laboratory) both our chaos embodied by *Chf* and *MChf* and our certain knowledge embodied by *DOK*. These three parameters of *CPP* are evaluated and caused by the resultant of all the nonrandom (deterministic) and random (nondeterministic) aspects influencing the system of pipeline. Knowing that, in the novel paradigm, the factors' resultant effect on *RUL* and *D* is materialized by the jumps in their curves and is accordingly expressed and concretized in  $\mathcal{R}$  by  $P_r$  and in  $\mathcal{M}$  by  $P_m$ . As it was defined in *CPP*,  $\mathcal{M}$  is an imaginary probability extension of the real probability set  $\mathcal{R}$ , and the complex probability set  $\mathcal{C}$  is the sum of both probability sets; thus,  $\mathcal{C} = \mathcal{R} + \mathcal{M}$ . Because  $P_m = i(1 - P_r)$ , therefore it is the complementary probability of  $P_r$  in  $\mathcal{M}$ . Hence, if  $P_r$  is identified as the failure probability of the system in  $\mathcal{R}$  at the pressure cycle time  $t = t_k$ , then  $P_m$  is identified as the corresponding probability in the set  $\mathcal{M}$  that the system failure will not occur at the same pressure time  $t = t_k$ . So,  $P_m$  is the associated probability in the set  $\mathcal{M}$  of the system survival at  $t = t_k$ . It follows that  $P_m/i = 1 - P_r$  is the associated probability but in the set  $\mathcal{R}$  of the system survival at the same pressure cycles time. Accordingly, we know that the sum in  $\mathcal{R}$  of both complementary probabilities is surely 1 from classical probability theory. This sum is nothing but  $P_C$  which is equal to  $P_r + P_m/i = P_r + (1 - P_r) = 1$  always. The sum in  $\mathcal{C}$  of both complementary probabilities is the complex random number and vector  $Z$  which is equal to  $P_r + P_m = P_r + i(1 - P_r)$ . And as the complex probability cubes show and illustrate, we realize that  $Z$  is the sum in  $\mathcal{C}$  of the real probability of failure and of the imaginary probability of survival in the complex probability plane that has the equation

$P_r(t) = iP_m(t) + 1$  for  $\forall t : 0 \leq t \leq t_C$ ,  $\forall P_r : 0 \leq P_r \leq 1$ , and  $\forall P_m : 0 \leq P_m \leq i$ . What is interesting is that the square of the norm of  $Z$  which is  $|Z|^2$  is nothing but  $DOK$ , as it was proved in  $CPP$  and in the new model. Moreover, since  $MChf = -2iP_rP_m = 2P_rP_m/i$ , therefore it is twice the product in  $\mathcal{R}$  of both the probability of failure and the probability of survival, and it quantifies the magnitude of chaos since it is always 0 or positive. All the simulations show and prove all these facts.

We can conclude from all the above that since  $D(t)$  is a  $CDF$ , since the factor resultant is random, and since the jumps in  $D$  are the simulations failure probabilities  $P_r(t_k)$ , then we are dealing with a random experiment, thus the natural appearance of  $Chf$ ,  $MChf$ ,  $DOK$ ,  $Z$ , and hence  $Pc$ . So, we get in the simulations:

$$Chf(t_k) = -2P_r(t_k)P_m(t_k)/i = -2\left\{\psi_j[D(t_k) - D(t_{k-1})]\right\}\left\{1 - \psi_j[D(t_k) - D(t_{k-1})]\right\}. \quad (29)$$

$$MChf(t_k) = |Chf(t_k)| = 2\left\{\psi_j[D(t_k) - D(t_{k-1})]\right\}\left\{1 - \psi_j[D(t_k) - D(t_{k-1})]\right\}. \quad (30)$$

$$\begin{aligned} DOK(t_k) &= 1 - 2P_r(t_k)P_m(t_k)/i \\ &= 1 - 2\left\{\psi_j[D(t_k) - D(t_{k-1})]\right\}\left\{1 - \psi_j[D(t_k) - D(t_{k-1})]\right\}. \end{aligned} \quad (31)$$

$$Z(t_k) = P_r(t_k) + P_m(t_k) = \psi_j[D(t_k) - D(t_{k-1})] + i\left\{1 - \psi_j[D(t_k) - D(t_{k-1})]\right\}. \quad (32)$$

$$Pc^2(t_k) = DOK(t_k) - Chf(t_k) = DOK(t_k) + MChf(t_k) = 1; \text{ for every } t_k, 0 \leq t_k \leq t_C. \quad (33)$$

Furthermore, in the new model, we have

$$RUL(t_k) = t_C - t_k.$$

Note that since  $t$  and  $D$  are random, then  $RUL$  is also a random function of  $t$ . Thus, we have in the set  $\mathcal{R}$ :

$$\begin{aligned} P_{rob}[RUL(t_k)] &= P_{rob}(\text{the system will survive for } t_k < t \leq t_C) \\ &= 1 - P_{rob}(\text{the system will fail for } t \leq t_k) \\ &= 1 - D(t_k) \\ &= \text{Rescaled } [RUL(t_k)] \text{ in all the three pressure modes simulations} \end{aligned} \quad (34)$$

Then, we get always  $P_{rob}[RUL(t_k)] + D(t_k) = 1$  everywhere.

This implies that  $P_{rob}[RUL(t_k = 0)] = 1 - D(t_k = 0) = 1 - D_0 \approx 1$ .

and  $P_{rob}[RUL(t_k = t_C)] = 1 - D(t_k = t_C) = 1 - D_C = 1 - 1 = 0$ .

Hence, we reach a new and general prognostic equation for  $RUL$ . If  $t_0 \neq 0$  and  $D(t_0) \neq 0$  then

$$\begin{aligned} P_{rob}[RUL(t_k)] &= P_{rob}(\text{Survival} : t_k < t \leq t_C) = 1 - P_{rob}(\text{Failure} : t_0 \leq t \leq t_k) \\ &= 1 - \sum_{t=t_0}^{t=t_k} P_r(t)/\psi_j; \quad \text{with } P_r(t_0)/\psi_j = D_0 \end{aligned} \quad (35)$$

$$= 1 - D(t_k) = \sum_{t=t_{k+1}}^{t=t_C} P_r(t)/\psi_j \quad (36)$$

$$= 1 - \sum_{t=t_0}^{t=t_k} PDF_{failure}(t); \quad \text{with } PDF_{failure}(t_0) = D_0$$

$$= \sum_{t=t_{k+1}}^{t=t_C} PDF_{failure}(t) \quad (37)$$

for any mode  $j$  of pressure profile.

Moreover, from Eqs. (25), (26), (27), and (28) and for any mode  $j$  of pressure profile, we have the following recursive relations:

$$P_{rob}[RUL(t_k)] = 1 - D(t_k) = 1 - \left\{ D(t_{k-1}) + P_r(t_k)/\psi_j \right\} \quad (38)$$

$$= 1 - \left\{ D(t_{k-1}) + PDF_{failure}(t_k) \right\} \quad (39)$$

$$= 1 - \left\{ 1 - P_{rob}[RUL(t_{k-1})] + P_r(t_k)/\psi_j \right\} \quad (40)$$

$$= P_{rob}[RUL(t_{k-1})] - P_r(t_k)/\psi_j \quad (41)$$

$$= P_{rob}[RUL(t_{k-1})] - PDF_{failure}(t_k) \quad (42)$$

where  $P_{rob}[RUL(t_{k-1})] = 1 - D(t_{k-1})$ .

In the ideal case, if all the factors are 100% deterministic, then we have in  $\mathcal{R}$  the probability of failure for  $t_k < t_C$  is 0 and is 1 for  $t_k \geq t_C$ ; accordingly the probability of system survival for  $t_k < t_C$  is 1 and is 0 for  $t_k \geq t_C$ , since certain failure will occur only at  $t_k = t_C$ . So, degradation is determined surely everywhere in  $\mathcal{R}$ , and its CDF is replaced by a deterministic function and curve. Therefore, chaos is null, and hence  $Chf = MChf = 0$ , and  $DOK = 1$  always for all  $0 \leq t_k \leq t_C$ . Thus,  $P_{rob}[RUL(t_k < t_C)] = 1$  and  $P_{rob}[RUL(t_k \geq t_C)] = 0$ .

Furthermore, at each instant  $t$  in the original prognostic paradigm, the stochastic  $RUL(t)$  and  $D(t)$  are predicted with certitude in the complex probability set  $\mathcal{C}$  with  $Pc^2 = DOK - Chf = DOK + MChf$  maintained as equal to 1 through a continuous compensation between  $Chf$  and  $DOK$ . This compensation is from the instant  $t = 0$  where  $D(t) = D_0 = 0.020408 \approx 0$  until the instant of failure  $t_C$  where  $D(t_C) = 1$ . Moreover, we can realize that  $DOK$  does not include any uncertain knowledge (with a probability less than 100%); it is the measure of our certain knowledge (probability = 100%) about the expected event. We can understand that we have eliminated and subtracted in the equation above all the random factors and chaos ( $Chf$ ) from our random experiment when computing  $Pc^2$ ; hence no chaos exists in  $\mathcal{C}$ , and it only exists (if it does) in  $\mathcal{R}$ ; consequently, this has led to a 100% deterministic outcome and experiment in  $\mathcal{C}$  since the probability  $Pc$  is constantly equal to 1. This is one of the advantages of extending  $\mathcal{R}$  to  $\mathcal{M}$  and therefore of working in

For Any Internal Pressure Mode	$D$	$P_{rob}[RUL(t)]$	$DOK$	$Chf$	$MChf$	$P_m / i$	$Z$	$Pc$
$t = 0 \Rightarrow P_r = 0$	$= D_0$	$= 1 - D_0$	$= 1$	$= 0$	$= 0$	$= 1$	$= i$	$= 1$
$0 < P_r < 0.5$ $t \uparrow \Rightarrow P_r \uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$Re(Z) \uparrow$ $Im(Z) \downarrow$	$= 1$
$t \uparrow$ $P_r = 0.5$	$\uparrow$	$\downarrow$	$= \text{Min}$ $= +0.5$	$= \text{Min}$ $= -0.5$	$= \text{Max}$ $= +0.5$	$=$ $+0.5$	$=$ $0.5 + 0.5i$	$= 1$
$0.5 < P_r < 1$ $t \uparrow \Rightarrow P_r \uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$Re(Z) \uparrow$ $Im(Z) \downarrow$	$= 1$
$t = t_C \Rightarrow P_r = 1$	$= 1$	$= 0$	$= 1$	$= 0$	$= 0$	$= 0$	$= 1$	$= 1$

**Table 2.**  
 The new prognostic model parameters for any pipeline internal pressure mode.

Relative Pressure Modes	$t \uparrow$ and $0 < P_r < 0.5$		
	Mode2 / Mode1	Mode3 / Mode2	Mode3 / Mode1
$D$	↓	↓	↓
$P_{rob}[RUL(t)]$	↑	↑	↑
$DOK$	↑	↑	↑
$Chf$	↑	↑	↑
$MChf$	↓	↓	↓
$P_r$	↓	↓	↓
$P_m / i$	↑	↑	↑
$Z$	Re(Z) ↓ Im(Z) ↑	Re(Z) ↓ Im(Z) ↑	Re(Z) ↓ Im(Z) ↑
$P_c$	= 1	= 1	= 1

**Table 3.**  
The new prognostic model and the relative pipeline pressure mode comparisons for  $0 < P_r < 0.5$ .

Relative Pressure Modes	$t \uparrow$ and $0.5 < P_r < 1$		
	Mode2 / Mode1	Mode3 / Mode2	Mode3 / Mode1
$D$	↓	↓	↓
$P_{rob}[RUL(t)]$	↑	↑	↑
$DOK$	↓	↓	↓
$Chf$	↓	↓	↓
$MChf$	↑	↑	↑
$P_r$	↓	↓	↓
$P_m / i$	↑	↑	↑
$Z$	Re(Z) ↓ Im(Z) ↑	Re(Z) ↓ Im(Z) ↑	Re(Z) ↓ Im(Z) ↑
$P_c$	= 1	= 1	= 1

**Table 4.**  
The new prognostic model and the relative pipeline pressure mode comparisons for  $0.5 < P_r < 1$ .

$\mathcal{C} = \mathcal{R} + \mathcal{M}$ . Thus, in the original prognostic paradigm, our knowledge of all the indicators and parameters ( $RUL$ ,  $P_{rob}$ ,  $D$ , etc.) is totally predictable, always perfect, and constantly complete because  $P_c = 1$  permanently, independently of any random factors or any pressure profile (Table 2).

Finally, we say that we have applied for pressure modes 2 and 3 the same analysis, logic, and methodology that we have used for pressure mode 1 regarding the remaining useful lifetime, the degradation, as well as all the CPP parameters (Tables 3 and 4). Therefore, we can accordingly infer that whatever the pressure conditions and environment are, then the results and conclusions are analogous. This demonstrates the strength and soundness of the novel axioms adopted and of the new prognostic paradigm developed.

## 7. Conclusion and perspectives

The high availability of technological systems, like defense, aerospace, automobile industries, and petrochemistry, is a central major objective of previous and latest developments in the technology of system design where it is very well-known

that expensive failure may in general happen unexpectedly. A novel model of analytic prognostic was established in my earlier work and publications as a counterpart of existent classical strategies of maintenance in order to take into account the evolving environment and product state and in order to make them more efficient. We have applied this model to systems of petrochemical pipes that are exposed to fatigue failure under cyclic repetitive triangular pressure. It is known that the effects of fatigue will initiate micro-cracks that can spread rapidly and hence will lead to failure. This model is founded on existing laws of damage in fracture mechanics which are the law of Palmgren-Miner of linear damage accumulation and the law of Paris-Erdogan of crack propagation. This prognostic model estimates the system *RUL* from a predefined threshold of degradation  $D_C$ . The model of degradation established in this earlier work is founded on the damage measurement  $D$  accumulation after each cycle time of pressure. The system is judged to be in wear-out state when this measured and predefined threshold  $D_C$  is reached. Moreover, to make the model more realistic and accurate, we have taken into consideration the stochastic influences afterward as well here. We have applied this model to the industry of pipelines; therefore, a prognostic study of the pipeline system enables us to enhance its strategies of maintenance.

In the present research work, the novel extended Kolmogorov paradigm of eight axioms (*EKA*) was applied and bonded to the analytic and linear prognostic of buried petrochemical pipeline systems subject to fatigue. Hence, a tight link between the remaining useful lifetime or degradation and the original paradigm was made. Therefore, the model of “complex probability” was more elaborated beyond the scope of my previous 12 research works on this subject.

Although the analytic linear laws of prognostic are very well-known and deterministic in [14, 16], there are general influences and aspects that can be chaotic and stochastic (like humidity, temperature, material nature, geometry dimensions, applied load location, water action, corrosion, soil pressure and friction, atmospheric pressure, etc.). Moreover, various variables in the expressions (5) and (6) of degradation which are considered as deterministic can also have a random aspect, such as the magnitude of applied pressure (due to the different conditions of pressure profile) and the length of the initial crack (potentially existing from the process of manufacturing). All those stochastic factors, embodied in the model by their mean values, influence the buried pipeline system and make its function of degradation diverge from its computed trajectory modeled by these deterministic laws. An updated follow-up of the degradation performance and behavior with cycle number or time, which is subject to non-chaotic and chaotic influences, is made possible by  $P_r(t_k)/\psi_j$  due to its definition that evaluates the jumps in  $D$ . In fact, chaos modifies and affects all the environment and system parameters included in the degradation equations (Eqs. (5) and (6)). Consequently, chaos total effect on the pipelines contributes to shape the degradation curve  $D$  and is materialized by and counted in the pipeline system failure probability  $P_r(t_k)/\psi_j$ . Actually,  $P_r(t_k)/\psi_j$  quantifies the resultant of all the nonrandom (deterministic) and random (nondeterministic) parameters and aspects which are contained in the equation of  $D$ , which affect the system and which lead to the consequent final curve of degradation. Consequently, an accentuated influence of chaos on the pipeline can lead to a smaller (or bigger) jump in the trajectory of degradation and therefore to a smaller (or bigger) failure probability  $P_r(t_k)/\psi_j$ .

Additionally, as it was verified and shown in the novel model, when the degradation index is 0 or 1 and correspondingly the *RUL* is  $t_C$  or 0, then the chaotic factor (*Chf* and *MChf*) is zero, and the degree of our knowledge (*DOK*) is 1 since the system state is totally known. During the process of degradation ( $0 < D < 1$ ), we

have  $-0.5 \leq Chf < 0$ ,  $0 < MChf \leq 0.5$ , and  $0.5 \leq DOK < 1$ . Notice that during this whole process, we have always  $Pc^2 = DOK - Chf = DOK + MChf = 1 = Pc$ , which means that the phenomenon which looked to be stochastic and random in the set  $\mathcal{R}$  is now certain and deterministic in the set  $\mathcal{C} = \mathcal{R} + \mathcal{M}$ , and this after the addition of the contributions of  $\mathcal{M}$  to the phenomenon occurring in  $\mathcal{R}$  and thus after subtracting and eliminating the chaotic factor from the degree of our knowledge. Moreover, the probabilities of the system survival and of failure corresponding to each instant  $t$  have been evaluated, in addition to the probability of *RUL* after a pressure cycles time  $t$ , which are all functions of the stochastic degradation jump. Consequently, at each instance of  $t$ , all the novel *CPP* parameters  $D$ , *RUL*,  $P_r$ ,  $P_m$ ,  $P_m/i$ , *DOK*, *Chf*, *MChf*,  $Pc$ , and  $Z$  are certainly and perfectly predicted in the complex probability set  $\mathcal{C}$  with  $Pc$  maintained as equal to 1 constantly and permanently. Furthermore, using all these illustrated simulations and drawn graphs all over the whole research work, we can quantify and visualize both the certain knowledge (expressed by *DOK* and  $Pc$ ) and the system chaos and random effects (expressed by *Chf* and *MChf*) of the pipeline system. This is definitely very fascinating, fruitful, and wonderful and proves once again the advantages of extending the five probability axioms of Kolmogorov and thus the novelty and benefits of this original field in prognostic and applied mathematics that can be called verily “The Complex Probability Paradigm.”

As a prospective and future work and challenges, and concerning some applications to practical engineering, it is planned to more elaborate the original created prognostic paradigm and to implement it to a varied set of nondeterministic and dynamic systems like vehicle suspension systems and offshore and buried petrochemical pipes which are under the influence of fatigue and in the cases of nonlinear and linear damage accumulation. Furthermore, we will apply also *CPP* to other random experiments in classical probability theory and in stochastic processes and to the field of prognostic in engineering using the first order reliability method (FORM) as well as to the random walk problems which have enormous applications in physics, in economics, in chemistry, in applied and pure mathematics.

## Conflict of interest

No potential conflict of interest was reported by the author.

## Nomenclature

$\mathcal{R}$	The set of real probabilities of events
$\mathcal{M}$	The set of imaginary probabilities of events
$\mathcal{C}$	The set of complex probabilities of events
$i$	The imaginary number where $i^2 = -1$ and $i = \sqrt{-1}$
<i>EKA</i>	extended Kolmogorov axioms
<i>CPP</i>	Complex probability paradigm
$P_{rob}$	any event probability
$P_r$	system failure probability, probability in the real set $\mathcal{R}$
$P_m$	system survival probability in $\mathcal{M}$ , probability in the imaginary set $\mathcal{M}$ corresponding to the real probability in $\mathcal{R}$
$P_m/i$	system survival probability in $\mathcal{R}$
$Pc$	probability in the complex set $\mathcal{C}$ , probability of an event in $\mathcal{R}$ with its associated event in $\mathcal{M}$

$Z$	the sum of $P_r$ and $P_m$ , complex probability number and vector
$DOK$	$=  Z ^2$ , the square of the norm of $Z$ , degree of our knowledge of the random event and experiment
$Chf$	chaotic factor
$MChf$	magnitude of the chaotic factor
$t$	pressure cycle time
$t_C$	pressure cycle time till system failure
$P_j$	pipelines internal triangular pressure
$f_j(t)$	probability density function for each pressure mode $j$
$F(t)$	cumulative probability distribution function
$\psi_j$	simulation magnifying factors for each pressure mode $j$
$1/\psi_j$	the normalizing constant of $P_r(t)$ for each pressure mode $j$
$D$	degradation indicator of a system
$RUL$	remaining useful lifetime of a system
$P_{rob}[RUL(t)]$	probability of $RUL$ after a pressure cycle time $t$ .

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