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A Global Method for a Two-Dimensional Cutting Stock Problem in the Manufacturing Industry

Yao-Huei Huang, Hao-Chun Lu, Yun-Cheng Wang, Yu-Fang Chang and Chun-Kai Gao

Abstract

A two-dimensional cutting stock problem (2DCSP) needs to cut a set of given rectangular items from standard-sized rectangular materials with the objective of minimizing the number of materials used. This problem frequently arises in different manufacturing industries such as glass, wood, paper, plastic, etc. However, the current literatures lack a deterministic method for solving the 2DCSP. However, this study proposes a global method to solve the 2DCSP. It aims to reduce the number of binary variables for the proposed model to speed up the solving time and obtain the optimal solution. Our experiments demonstrate that the proposed method is superior to current reference methods for solving the 2DCSP.

Keywords: two-dimensional cutting stock problem (2DCSP), rectangular items, optimal solution, deterministic model

1. Introduction

Two-dimensional cutting stock problem (2DCSP) is a well-known problem in the fields of management science and operations research. The problem frequently arises in the manufacturing processes of different products such as wood, glass, paper, steel, etc. In the 2DCSP, a set of given rectangular items is cut from a set of rectangular materials with the aim of determining the minimum number of materials [1, 2]. These applications include sawing plates from wood stocks [3], reel and sheet cutting at a paper mill [4], cutting plates of thin-film-transistor liquid-crystal display (TFT-LCD) from glass substrate [5, 6], placing devices into a system-on-a-chip circuit [7], and container loading or calculation of containers [8, 9]. Minimizing the number of materials is normally the target in this type of the problem because it does not only reduce the overhead consumption but also enhances environmental protection. The problem in the literatures have been classified as one-dimensional, 1.5-dimensional, and 2DCSPs (Hinxman [10] and Lodi et al. [11]) and suggested two categories of approaches in solving the problems, namely, the heuristic and deterministic approaches (Belov [12], Burke et al. [13], Chen et al. [14], Hopper and Turton [15], Lin [16] and Martello et al. [17]).

Various heuristic approaches have been proposed and discussed in the literatures. The primary advantage of this approach is easier in solving the 2DCSP within an acceptable and economical timeframe [18, 19]. The feasible solution is obtained within a reasonable time, while the optimal solution cannot be guaranteed. Chazelle [20] first proposed a popular heuristic algorithm, called the bottom-left heuristic algorithm. Berkey and Wang [21] proposed a finite best strip heuristic algorithm to improve the original bottom-left method which packs the items directly into the bins with a best-fit policy. On the other hand, Lodi et al. [22] proposed an integrated heuristic approach that initiates the solution by paralleling the edges of the items and bins (i.e., materials) and utilizes a Tabu search [23, 24] to explore the neighborhood and refine the possible solution. In order to enhance the effectiveness of the algorithm used, Boschetti and Mingozzi [25, 26] consider empty bins in turn and fill the bins with items in a sequence defined by the prices attributed to the items and update them iteratively. Likewise, Monaci and Toth [27] initially used Lagrangian-based heuristic to generate a set of covering programming model to obtain a lower bound solution, in which the items cannot be rotated. They applied geometric analytical techniques and Dantzig-Wolfe decomposition to produce various lower bounds of the 2DCSP so that a better solution can be compared and obtained [28–31].

Despite the development of heuristic approaches can obtain possible solution in a reasonable time, however there is a scarcity of literature attempting to ensure the achievement of an optimal solution. Moreover, the distance between one random feasible solution and the actual global optimal solution can be enlarged with an increasing problem size. Only a few studies attempted to develop deterministic approaches for an optimal solution. For example, Chen et al. [32] formed a mathematical model for packing a set of given rectangular items into a rectangular space in which the dimension of the rectangular space is minimized. The packing problem is equal to the cutting problem, and the problem can also be called as an assortment problem. Moreover, Williams [33] formulated a mathematical model considering the increased generalization of 2DCSP, to solve the problem with various sizes of bins. However, Williams' model contains an excessive number of binary variables as indicated by Pisinger and Sigurd [31] who showed that Williams' model has difficulty in solving a standard 2DCSP by their computational experiments. The subsequent studies by Li and Chang [34], Li et al. [35, 36], Hu et al. [37], and Tsai et al. [38] (these approaches are called Li's approach in this study) enhanced Chen's model with reformulation techniques based on reducing binary variables and piecewise linearization technique. The deterministic approaches can guarantee the achievement of global optimization with an acceptable tolerance; however, these approaches are only suitable for the assortment problem (i.e., cutting rectangular items from one material only), while many manufacturing situations require considering minimal number of materials.

Aiming to close the knowledge gap, this study modifies the two programs of the assortment problem proposed by Chen et al. [32] and Li and Chang [34] to be two corresponding deterministic models for the 2DCSP. As an innovative approach, a global approach of the 2DCSP with a logarithmic number of binary variables and extra constraints is proposed and demonstrated.

The remainder of this study is organized as follows: Section 2 discusses the 2DCSP formulations. Section 3 proposes the 2DCSP models with logarithmic number of binary variables and extra constraints. Numerical examples are given in Section 4 to demonstrate the theoretical advances and advantages of the proposed global approach. Section 5 gives the concluding remarks.

2. Problem formulations

Given n small rectangular items, the 2DCSP is to cut all items within large rectangular materials with the objective of minimizing the number of materials used. Denote x and y as the width and the length of the enveloping rectangle. By referring to the method of Chen et al. [32], a mathematical program can be formed with the objective of minimizing the volume (i.e., Min xy) as discussed in Section 2.1. In the 2DCSP, the minimal number of materials can reduce the manufacturing costs. Thus Section 2.2 explains how to reformulate two new 2DCSP programs based on the original model in Section 2.1. Firstly, the terminologies, including decision variables and parameters, are introduced in **Tables 1** and **2**.

2.1 Cutting problem in one material

The cutting problem considering one material is also called the assortment problem, which considers cutting a set of given rectangular items within a rectangular material of minimum area. Avoiding the overlapping of items is the core requirements. Chen et al. [32] and Li and Chang [34] use four binary variables $(a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j})$ and two binary variables $(u_{i,j}, v_{i,j})$, respectively, to handle the non-overlapping conditions, as shown in **Table 3**.

The following assortment program is proposed by Chen et al. [32]:

Original (a)

Min xy

$$s.t. \; x_j + p_j s_j + q_j(1 - s_j) \leq x_i + \overline{x}(1 - a_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \qquad (1)$$

Parameter	Meaning
n	The number of given rectangular items needed to be cut
m	The number of rectangular materials with the same size
(p_i, q_i)	The width and length of given rectangular item i for $i = 1, ..., n$
$(\overline{x}, \overline{y})$	$\overline{x}$ and $\overline{y}$ are the upper bounds of x and y, respectively. These items also denote the width and length of a given rectangular material

Table 1.
Parameters in the 2DCSP.

Variable	Meaning
(x_i, y_i)	The bottom-left coordinate of rectangular item i
(x, y)	The top-right coordinate of the rectangular material
$(a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j})$	A set of binary variables expressing the non-overlapping conditions for a pair of rectangular items i and rectangular items j for $i < j$, which are defined by Chen et al. [32]
$(u_{i,j}, v_{i,j})$	A pair of binary variables expressing the non-overlapping conditions for a pair of rectangular items i and rectangular items j for $i < j$, which are defined by Li and Chan [34]
s_i	An orientation indicator for a given rectangular item i. $s_i = 1$ if p_i is parallel to the x-axis; otherwise, $s_i = 0$ if p_i is parallel to the y-axis (s_i is a binary variable)
Y	The accumulated length of all materials used.

Table 2.
Decision variables in the 2DCSP.

Method	Chen et al. [32]				Li and Chang [34]		Condition
Case	$a_{i,j}$	$b_{i,j}$	$c_{i,j}$	$d_{i,j}$	$u_{i,j}$	$u_{i,j}$	
1	1	0	0	0	0	0	j i
2	0	1	0	0	1	0	i j
3	0	0	1	0	0	1	i / j
4	0	0	0	1	1	1	j / i

Table 3.
Four cases of non-overlapping conditions.

$$x_i + p_i s_i + q_i(1 - s_i) \leq x_j + \overline{x}(1 - b_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (2)$$

$$y_j + q_j s_j + p_j(1 - s_j) \leq y_i + \overline{y}(1 - c_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (3)$$

$$y_i + q_i s_i + p_i(1 - s_i) \leq y_j + \overline{y}(1 - d_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (4)$$

$$a_{i,j} + b_{i,j} + c_{i,j} + d_{i,j} = 1 \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (5)$$

$$x_i + p_i s_i + q_i(1 - s_i) \leq x \leq \overline{x} \text{ for } i = 1, ..., n, \quad (6)$$

$$y_i + q_i s_i + p_i(1 - s_i) \leq y \leq \overline{y} \text{ for } i = 1, ..., n, \quad (7)$$

where $a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j},$ and s_i are binary variables; x_i and y_i are nonnegative continuous variables; Constraints (1)–(5) ensure that the rectangular items are non-overlapping, and Constraints (6) and (7) are to cut all of the rectangular items within an enveloping rectangular material $(\overline{x}, \overline{y})$.

Remark 1. Original (a) uses $2n^2 - n$ binary variables $(a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j})$ and $2.5n$ $(n - 1) + 2n$ constraints to formulate an assortment problem with n rectangular items.

By referring to Li and Chang [34], an alternative mathematical model can be expressed as follows:

Original (b)
Min xy
s.t. (6) and (7),

$$x_j + p_j s_j + q_j(1 - s_j) \leq x_i + \overline{x}(u_{i,j} + v_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (8)$$

$$x_i + p_i s_i + q_i(1 - s_i) \leq x_j + \overline{x}(1 - u_{i,j} + v_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (9)$$

$$y_j + q_j s_j + p_j(1 - s_j) \leq y_i + \overline{y}(1 + u_{i,j} - v_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (10)$$

$$y_i + q_i s_i + p_i(1 - s_i) \leq y_j + \overline{y}(2 - u_{i,j} - v_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (11)$$

where $u_{i,j}, v_{i,j},$ and s_i are binary variables; x_i and y_i are nonnegative continuous variables; and Constraints (8)–(11) ensure that the rectangular items are non-overlapping.

Remark 2. Original (b) uses n^2 binary variables $(u_{i,j}, v_{i,j})$ and $2n^2$ constraints to formulate an assortment problem with n rectangular items.

However, these two models are inappropriate for directly solving the general 2DCSP because the objective of the 2DCSP must minimize the number of materials used for cutting all items. By referring to the two models above, two corresponding 2DCSP models are proposed in Section 2.2.

2.2 General deterministic models of 2DCSP

As mentioned above, we need to find out the minimal number of materials used for cutting all items. Original (a) is then reformulated as a general deterministic model of 2DCSP, where cutting n rectangular items from m materials, as shown in P1 (a):

P1 (a)
Min Y
s.t. (1), (2) and (5) in Original (a),

$$y_j + q_j s_j + p_j (1 - s_j) \leq y_i + m\bar{y}(1 - c_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \tag{12}$$

$$y_i + q_i s_i + p_i (1 - s_i) \leq y_j + m\bar{y}(1 - d_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \tag{13}$$

$$x_i + p_i s_i + q_i (1 - s_i) \leq \bar{x} \text{ for } i = 1, ..., n, \tag{14}$$

$$y_i + q_i s_i + p_i (1 - s_i) \leq \sum_{k=1}^{m} k\bar{y} Q_{i,k} \text{ for } i = 1, ..., n, \tag{15}$$

$$y_i \geq \sum_{k=1}^{m} (k-1)\bar{y} Q_{i,k} \text{ for } i = 1, ..., n, \tag{16}$$

$$\sum_{k=1}^{m} Q_{i,k} = 1 \text{ for } i = 1, ..., n, \tag{17}$$

$$y_i + q_i s_i + p_i (1 - s_i) \leq Y \text{ for } i = 1, ..., n, \tag{18}$$

where $s_i, a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j},$ and $Q_{i,k}$ are binary variables; x_i, y_i and Y are nonnegative continuous variables; Constraints (1), (2), (5), (12) and (13) ensure that the rectangular items are non-overlapping; Constraints (15)–(17) mean that each rectangular item is fitly cut from one of the m materials; Constraint (18) obtains the accumulated length of materials used; and the objective function minimizes the accumulated length of materials used.

There are nm new binary variables (i.e., $Q_{i,k}$ for $i = 1, 2, ..., n$ and $k = 1, 2, ..., m$) in Constraints (15)–(17) of P1 (a) model. It aims to cut the ith rectangular item from the kth material if $Q_{i,k} = 1$, and Constraint (17) forces any rectangular item to be cut from one of such materials. Supposing that rectangular item i' is cut from the k'th material, then $Q_{i',k'} = 1$ and $Q_{i',k} = 0$ for $k \neq k'$ and $k = 1, 2, ..., m$. Constraints (15)–(17) will force the y-axis position of rectangular item i' cut from the k'th material as shown below:

$$y_{i'} + q_{i'} s_{i'} + p_{i'}(1 - s_{i'}) \leq k'\bar{y}, \tag{19}$$

$$(k' - 1)\bar{y} \leq y_{i'}. \tag{20}$$

Remark 3. P1 (a) requires $2n^2 - n(1 - m)$ binary variables and $(5n^2 + 3n)/2$ constraints to form a 2DCSP program.

Referring to the Original (b), another corresponding 2DCSP program can be formulated as follows:

P1 (b)
Min Y
s.t. (8), (9), (14)–(18),

$$y_j + q_j s_j + p_j(1 - s_j) \leq y_i + m\bar{y}(1 + u_{i,j} - v_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \tag{21}$$

$$y_i + q_i s_i + p_i(1 - s_i) \le y_j + m\bar{y}(2 - u_{i,j} - v_{i,j}) \text{ for } i, j = 1, ..., n \text{ and } i < j, \quad (22)$$

where $s_i, u_{i,j}, v_{i,j}$, and $Q_{i,k}$ are binary variables and x_i, y_i, and Y are nonnegative continuous variables.

Remark 4. P1 (b) requires $n^2 + nm$ binary variables and $2n(n + 1)$ constraints to formulate a 2DCSP program.

Although both P1 (a) and P1 (b) can obtain a minimal number of materials used, there is mainly an issue needed to be addressed. That is, an excessive number of binary variables $Q_{i,k}$ is used to assign rectangular item i into one of the materials; such that the computational load becomes a serious burden as the size of the problem grows.

As indicated by Li et al. [39], reducing the number of binary variables can accelerate the solving speed. Hence, we can roughly estimate the number of materials by the following remark.

Remark 5. The number of materials can be reduced from m to f where $f \le m$ by the following initial calculating:

$$f \cong \lceil \sum_{i=1}^{n} x_i y_i / \bar{x} \times \bar{y} \rceil, \quad (23)$$

where if f value is not big enough, i.e., in solving P1(a) and P1(b) are infeasible, then we can accumulate f, i.e., $f = f + 1$, until feasible solutions exist.

By referring to Remark 5, the number of binary variables in Constraints (15)–(18) can be reduced from nm to nf where $f \le m$. Moreover, this study proposes a reformulation technique using logarithmic number of binary variables for the P1 (a) and P1 (b) models. The detail of technique is then discussed in Section 3.

3. Logarithmic reformulation technique of 2DCSP

After considering Remark 5, for a 2DCSP with n rectangular items and f materials, the P1 (a) and P1 (b) models will require nf binary variables ($Q_{i,k}$) to cut each rectangular item from one of the materials. The computational efficiency of the P1 (a) and P1 (b) models become a serious burden when an increasing size of the 2DCSP. For any rectangular item i, Constraint (17) ($\sum_{k=1}^{f} Q_{i,k} = 1$) is an SOS1 constraint [40], which is an ordered set of variables where only one variable may be one. An SOS1 constraint model with size f will generally require f binary variables. However, Vielma and Nemhauser [41] use SOS1 constraint with a logarithmic number of binary variables and constraints. This section utilizes the concept of Vielma and Nemhauser [41] and introduces the binary variables $Q_{i,r}$ ($i = 1, ..., n$ and $r = 1, ..., \lceil \log_2 f \rceil$) to replace the original binary variables ($Q_{i,k}$) of the P1 (a) and P1 (b) models. Thus, the number of required binary variables can be reduced from nf to $n\lceil \log_2 f \rceil$. The following remarks and propositions discuss the logarithmic reformulation technique of the 2DCSP.

Remark 6. Let $K = \{1, 2, ..., f = 2^\theta\}$, $\theta = \lceil \log_2 f \rceil$, and $k \in K$ be the injective function for $B : \{1, 2, ..., 2^\theta\} \to \{0, 1\}^\theta$, which can be expressed as follows:

$$B(k) = [w_1, w_2, ..., w_\theta]^T \text{ and } w_r = 1 - \left(\lceil \frac{k}{2^{r-1}} \rceil \% 2\right) \text{ for } r = 1, ..., \theta. \quad (24)$$

Proposition 1. Let $K = \{1, ..., f\}$, $\theta = \lceil \log_2 f \rceil$ and $k \in K$; the original SOS1 Constraint in (25) which requires f binary variables (Q_k) can be replaced by the Constraint set (26) and (27):

$$\sum_{k=1}^{f} Q_k = 1 \text{ and } Q_k \in \{0, 1\}. \tag{25}$$

The Constraint set (25) and (26) only requires θ binary variables (Q_r), 2θ additional constraints, and f additional continuous variables (λ_k):

$$\sum_{k=1}^{f} \lambda_k = 1, \tag{26}$$

$$\sum_{k \in S^+(r)} \lambda_k = Q_r \text{ for } r = 1, ..., \theta, \tag{27}$$

where

i. $Q_r \in \{0, 1\}$.

ii. $B(k)$ is an injective function based on Remark 6 (i.e., $B(k) = [w_1, w_2, ..., w_\theta]^T$ with $w_r \in \{0, 1\}$ for $r = 1, ..., \theta$).

iii. $S^+(r) = \{k \in K | \forall B(k) \text{ where } w_r = 1\}$ and $S^-(r) = \{k \in K | \forall B(k) \text{ where } w_r = 0\}$.

Proof: Following Li et al. [39], Constraints (26) and (27) are used to construct the SOS1 property.

Following Proposition 1, we then have Proposition 2 that uses $\lceil \log_2 f \rceil$ binary variables to determine whether rectangular item i could be exactly cut from one of the given materials.

Proposition 2. Let f be the number of materials, $\bar{y}$ the length of material, and y_i the y-axis position of rectangular item i. The original Constraint set (15)–(17) of the P1 (a) and P1 (b) models will be re-expressed by the following linear system, which holds the rectangular item i to be cut from one of the given materials:

$$y_i + q_i s_i + p_i (1 - s_i) \le \sum_{k=1}^{f} k \bar{y} \lambda_{i,k} \text{ for } i = 1, ..., n, \tag{28}$$

$$y_i \ge \sum_{k=1}^{f} (k - 1) \bar{y} \lambda_{i,k} \text{ for } i = 1, ..., n, \tag{29}$$

$$\sum_{k=1}^{f} \lambda_{i,k} = 1 \text{ for } i = 1, ..., n, \tag{30}$$

$$\sum_{k \in S^+(r)} \lambda_{i,k} = Q_{i,r} \text{ for } i = 1, ..., n \text{ and } r = 1, ..., \theta = \lceil \log_2 f \rceil, \tag{31}$$

where $S^+(r)$ and $S^-(r)$ are the same as the notations in Proposition 1 and $s_i, Q_{i,r} \in \{0, 1\}$.

Proof: According to Proposition 1, the continuous variables $\lambda_{i,k}$ with the Constraint set (30) and (31) have the characteristics of binary variables. Therefore, the Constraint set (28)–(30) is equivalent to the Constraint set (15)–(17).

Two types of 2DCSP models are formulated by utilizing Proposition 2 as the following P2 (a) and P2 (b), respectively:

Items	P1(a)	P1(b)	P2(a)	P2(b)
Concept of non-overlapping	Chen et al. [32]	Li and Chang [34]	Chen et al. [32]	Li and Chang [34]
Constraints for assigning rectangular items into materials	$\sum_{k=1}^{f} Q_{i,k} = 1$	$\sum_{k=1}^{f} Q_{i,k} = 1$	Proposition 2	Proposition 2
No. of binary variables	$2n^2 - n(1-f)$	$n^2 + nf$	$n(2n + \theta - 1)$	$n(n + \theta)$
No. of continuous variables	$2n + 1$	$2n + 1$	$2n + nf + 1$	$2n + nf + 1$
No. of constraints	$(5n^2 + 5n)/2$	$2n^2 + 3n$	$n(5n + 10) + 4\theta$	$n(2n + 2\theta + 3)$
where $\theta = \lceil \log_2 f \rceil$				

Table 4.
Comparison of the four ways of expressing the 2DCSP.

P2 (a)
Min Y
s.t. (1), (2), (5), (12)–(14), (18) and (28)–(31),
where $Y, x_i, y_i, \lambda_{i,k} \geq 0$ and $a_{i,j}, b_{i,j}, c_{i,j}, d_{i,j}, s_i, Q_{i,r} \in \{0,1\}$ for $i, j = 1, ..., n, i < j$, $k = 1, ..., f$, and $r = 1, 2, ... \lceil \log_2 f \rceil$.

Remark 7. P2 (a) requires $n(2n + \lceil \log_2 f \rceil - 1)$ binary variables and $5n^2 + 10n + \lceil \log_2 f \rceil$ constraints to express a 2DCSP model.

P2 (b)
Min Y
s.t. (8), (9), (14), (18), (28)–(31),
where $Y, x_i, y_i, \lambda_{i,k} \geq 0$ and $s_i, u_{i,j}, v_{i,j}, Q_{i,r} \in \{0,1\}$ for $i, j = 1, ..., n, i < j, k = 1, ..., f$ and $r = 1, ..., \lceil \log_2 f \rceil$.

Remark 8. P2 (b) requires $n(n + \lceil \log_2 f \rceil)$ binary variables and $n(2n + 2\lceil \log_2 f \rceil + 3)$ constraints to express another 2DCSP model.

Table 4 shows a comparison of the four ways of expressing the 2DCSP, and it clearly lists the number of binary variables, auxiliary continuous variables, and constraints.

4. Numerical examples

There are two examples modified from Tsai et al. [38]. The detail sizes of rectangular items and materials are listed in **Table 5**. We implement a Java program, which embedded an optimization package GUROBI (2011) as an MIP solver

Problem	Size of material	Qty. of items	f	Size of items (p_i, q_i)
1	(40, 69)	8	2	(25, 20), (16, 20), (15, 20), (14, 20), (20, 18), (15, 17), (30, 16), (30, 14)
2	(25, 150)	12	2	(32, 24), (26, 20), (25, 20), (24, 20), (40, 18), (35, 17), (20, 16), (18, 16), (38, 15), (50, 15), (18, 4), (25, 5)

Table 5.
Sizes of rectangular items and materials.

| Items		P1 (a)	P1 (b)	P2 (a)	P2 (b)
Problem 2 Y = 83	No. of 0–1 variables	136	80	128	72
	No. of cont. variables	17	17	33	33
	No. of constraints	180	152	404	168
	Iterations	621,821	686,982	263,296	293,432
	Nodes	176,776	211,564	70,154	111,173
	Solving time	18.8	18.0	7.5	9.1
Problem 2 Objective = 293	No. of 0–1 variables	300	168	288	156
	No. of cont. variables	25	25	49	49
	No. of constraints	390	324	844	348
	Iterations	31,166,357	1,017,922	1,114,911	805,136
	Nodes	9,766,654	244,444	266,672	229,701
	Solving time	856.5	24.5	34.9	19.4

Table 6.
Experiment results of two problems.
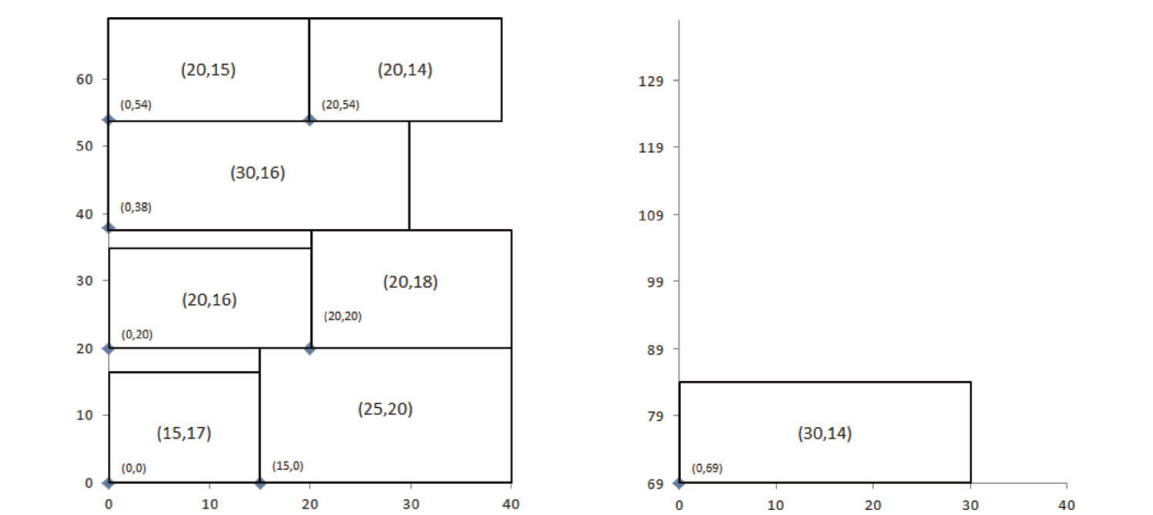
Figure 1.
Visualization result of Example 1.

for solving the two examples with the four proposed models (P1 (a), P1 (b), P2 (a), P2 (b)). The experimental tests were run on a PC equipped with an Intel® Core™ 2 Duo CPU, 4GB RAM, and 32 bit Windows 7 operating system.

The two problems with the number of materials firstly estimated to be 2 (i.e., $f = 2$) are solved by using the four models including P1(a), P1(b), P2(a), and P2 (b). **Table 6** shows the experiment results of two problems. Both of **Figures 1** and **2** depict the visualization solutions. In solving four models, we obtain the same objective values of (83) and (293) in Problem 1 and Problem 2, respectively. The results clearly indicate that solving P2(a) and P2(b) is much more computationally efficient than that of P1(a) and P1(b). By observing the four models, we know that both P2(a) and P2(b) use proposed approach to reduce the numbers of binary variables. The results demonstrate that the adoption of a smaller number of binary variables can enhance the solving efficiency in solving 2DCSP.
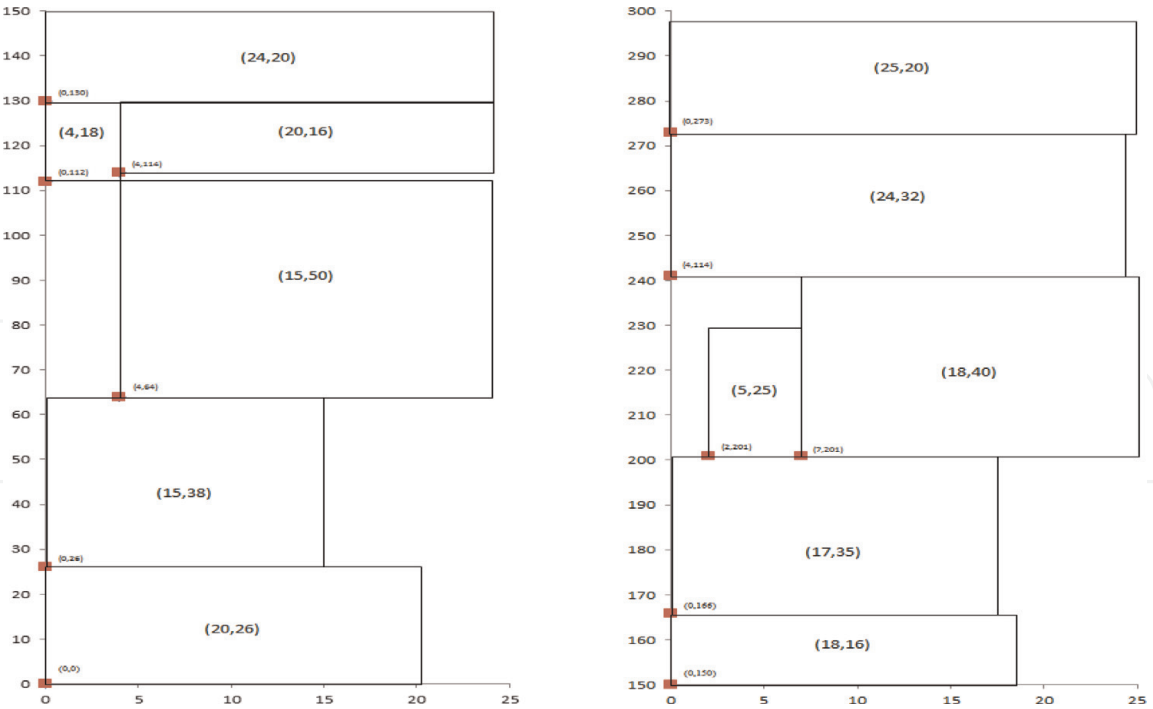
Figure 2.
Visualization result of Example 2.

5. Conclusions

This study develops a logarithmic reformulation technique for reducing the required binary variables of the mixed integer program for two-dimensional cutting stock problem in the manufacturing industry. A reformulated logarithmic technique in the deterministic method reduces the number of binary variables to speed up the solving time. The deterministic methods are guaranteed to find a global optimal solution, but the computational complexity grows rapidly by increasing the number of variables and constraints. Future studies are suggested to enhance the computational efficiency for globally solving large-scale 2DCSP, such as column generation, cloud computing and meta-heuristic algorithms.

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References

[1] Dyckhoff H. A typology of cutting and packing problems. European Journal of Operational Research. 1990;**44**:145-159

[2] Wäscher G, Haußner H, Schumann H. An improved typology of cutting and packing problems. European Journal of Operational Research. 2007;**183**:1109-1130

[3] Reinders MP. Cutting stock optimization and integral production planning for centralized wood processing. Mathematical and Computer Modelling. 1992;**16**:37-55

[4] Correia MH, Oliveira JF, Ferreira JS. Reel and sheet cutting at a paper mill. Computers and Operations Research. 2004;**21**:1223-1243

[5] Tsai JF, Hsieh PL, Huang YH. An optimization algorithm for cutting stock problems in the TFT-LCD industry. Computers and Industrial Engineering. 2009;**57**:913-919

[6] Lu HC, Huang YH, Tseng KA. An integrated algorithm for cutting stock problems in the thin-film transistor liquid crystal display industry. Computers and Industrial Engineering. 2013;**64**:1084-1092

[7] Lim SK. Physical design for 3D system on package. IEEE Design and Test of Computers. 2005;**22**:532-539

[8] Pisinger D. Heuristics for the container loading problem. European Journal of Operational Research. 2002;**141**:382-392

[9] Wang Z, Li KW, Levy JK. A heuristic for the container loading problem: A tertiary-tree-based dynamic space decomposition approach. European Journal of Operational Research. 2008;**191**:86-99

[10] Hinxman AI. The trim-loss and assortment problem: A survey. European Journal of Operational Research. 1980;**5**:8-18

[11] Lodi SM, Vigo D. Recent advances on two-dimensional bin packing problems. Discrete Applied Mathematics. 2002;**123**:379-396

[12] Belov G. Problems, Models and Algorithms in One- and Two-Dimensional Cutting [Ph.D. Thesis]. Technischen University Dresden; 2003

[13] Burke EK, Kendall G, Whitewell G. A new placement heuristic for the orthogonal stock-cutting problem. Operations Research. 2004;**52**:655-671

[14] Chen CS, Lee SM, Shen QS. An analytical model for the container loading problem. European Journal of Operational Research. 1995;**80**:68-76

[15] Hopper E, Turton B. A genetic algorithm for a 2D industrial packing problem. Computers and Industrial Engineering. 1999;**37**:375-378

[16] Lin CC. A genetic algorithm for solving the two-dimensional assortment problem. Computers and Industrial Engineering. 2006;**50**:175-184

[17] Martello S, Pisinger D, Vigo D. The three-dimensional bin packing problem. Operations Research. 2000;**48**:256-267

[18] Jakobs S. On genetic algorithms for the packing of polygons. European Journal of Operational Research. 1996;**88**:165-181

[19] Leung TW, Chan CK, Troutt MD. Application of a mixed simulated annealing-genetic algorithm for a 2D packing problem. European Journal of Operational Research. 2003;**145**:530-542

[20] Chazelle B. The bottom-left bin-packing heuristic: An efficient implementation, computers. IEEE Transactions on Computers. 1983;**100**: 697-707

[21] Berkey JO, Wang PY. Two-dimensional finite bin-packing algorithms. Journal of the Operational Research Society. 1987;**38**:423-429

[22] Lodi SM, Vigo D. Heuristic and metaheuristic approaches for a class of two-dimensional bin packing problems. INFORMS Journal of Computing. 1999; **11**:345-357

[23] Glover F, McMillan C. The general employee scheduling problem: An integration of MS and AI. Computers and Operations Research. 1986;**13**: 563-573

[24] Glover F, Laguna M. Tabu Search. Boston: Kluwer Academic Publishers; 1997

[25] Boschetti MA, Mingozzi A. The two-dimensional finite bin packing problem. Part I: New lower bounds for the oriented case, 4OR-Q. Journal of the Operational Research Society. 2003a;**1**: 274-294

[26] Boschetti MA, Mingozzi A. The two-dimensional finite bin packing problem. Part II: New lower and upper bounds, 4OR: 4OR-Q. Journal of the Operational Research Society. 2003b;**1**: 135-147

[27] Monaci M, Toth P. A set-covering-based heuristic approach for bin-packing problems. INFORMS Journal of Computing. 2006;**18**:71-85

[28] Dell'Amico M, Martello S, Vigo D. A lower bound for the non-oriented two-dimensional bin packing problem. Discrete Applied Mathematics. 2002; **118**:13-24

[29] Fekete SP, Schepers J. New classes of lower bounds for the bin packing problem. Mathematical Programming. 2001;**91**:11-31

[30] Martello S, Vigo D. Exact solution of the two-dimensional finite bin packing problem. Management Science. 1998; **44**:388-399

[31] Pisinger D, Sigurd M. The two-dimensional bin packing problem with variable bin sizes and cost. Discrete Optimization. 2005;**2**:154-167

[32] Chen CS, Sarin S, Balasubramanian R. A mixed-integer programming model for a class of assortment problems. European Journal of Operational Research. 1993;**65**: 362-367

[33] Williams HP. Model Building in Mathematical Programming. fourth ed. Chichester: Wiley; 1999

[34] Li HL, Chang CT. An approximately global optimization method for assortment problems. European Journal of Operational Research. 1998;**105**: 604-612

[35] Li HL, Chang CT, Tsai JF. Approximately global optimization for assortment problems using piecewise linearization techniques. European Journal of Operational Research. 2002; **140**:584-589

[36] Li HL, Tsai JF, Hu NZ. A distributed global optimization method for packing problems. The Journal of the Operational Research Society. 2003;**54**: 419-425

[37] Hu NZ, Tsai JF, Li HL. A global optimization method for packing problems. Journal of the Chinese Institute of Industrial Engineers. 2002; **19**:75-82

[38] Tsai JF, Wang PC, Lin MH. An efficient deterministic optimization approach for rectangular packing problems. Optimization. 2013;**62**: 989-1002

[39] Li HL, Huang YH, Fang SC. A logarithmic method for reducing binary variables and inequality constraints in solving task assignment problems. INFORMS Journal of Computing. 2013; **25**(4):643-653

[40] Beale EML, Forrest JJH. Global optimization using special ordered sets. Mathematical Programming: Series A and B. 1976;**10**:52-69

[41] Vielma JP, Nemhauser G. Modeling disjunctive constraints with a logarithmic number of binary variables and constraints. Mathematical Programming. 2011;**128**:49-72