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Discrete-Time Nonlinear Attitude Tracking Control of Spacecraft

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Abstract

Recent space programs require agile and large-angle attitude maneuvers for applications in various fields such as observational astronomy. To achieve agility and large-angle attitude maneuvers, it will be required to design an attitude control system that takes into account nonlinear motion because agile and large-angle rotational motion of a spacecraft in such missions represents a nonlinear system. Considerable research has been done about the nonlinear attitude tracking control of spacecraft, and these methods involve a continuous-time control framework. However, since a computer, which is a digital device, is employed as a spacecraft controller, the control method should have discrete-time control or sampled-data control framework. This chapter considers discrete-time nonlinear attitude tracking control problem of spacecraft. To this end, a Euler approximation system with respect to tracking error is first derived. Then, we design a discrete-time nonlinear attitude tracking controller so that the closed-loop system consisting of the Euler approximation system becomes input-to-state stable (ISS). Furthermore, the exact discrete-time system with a derived controller is indicated semiglobal practical asymptotic (SPA) stable. Finally, the effectiveness of proposed control method is verified by numerical simulations.

Keywords: spacecraft, attitude tracking control, discrete-time nonlinear control

1. Introduction

Recent space programs require agile and large-angle attitude maneuvers for applications in various fields such as observational astronomy [1–3]. To achieve agility and large-angle attitude maneuvers, it will be required to design an attitude control system that takes into account nonlinear motion because agile and large-angle rotational motion of a spacecraft in such missions represents a nonlinear system.

Considerable research has been done about the nonlinear attitude tracking control of spacecraft [4–12], and these methods involve a continuous-time control framework. However, since a computer, which is a digital device, is employed as a spacecraft controller, the control method should have discrete-time control or sampled-data control framework.

Although a sampled-data control method for nonlinear system did not advance because it is difficult to discretize a nonlinear system, a control method based on the Euler approximate model has been proposed in recent years [13, 14] and is applied to ship control [15]. Although our research group has proposed a sampled-data

control method using backstepping [16] and a discrete-time control method based on sliding mode control [17] for spacecraft control problem, these methods are disadvantageous because control input amplitude depends on the sampling period T as the control law is of the form $u = a(x) + (b(x)/T)$.

For these facts, about the spacecraft attitude control problem that requires agile and large-angle attitude maneuvers, this chapter proposed a discrete-time nonlinear attitude tracking control in which the control input amplitude is independent of the sampling period T . The effectiveness of proposed control method is verified by numerical simulations.

The following notations are used throughout the chapter. Let $\mathbb{R}$ and $\mathbb{N}$ denote the real and the integer numbers. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ are the sets of real vectors and matrices. For real vector $a \in \mathbb{R}^n$, a^T is the vector transpose, $\|a\|$ denotes the Euclidean norm, and $a^\times \in \mathbb{R}^{3 \times 3}$ is the skew symmetric matrix

$$a^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

derived from vector $a \in \mathbb{R}^3$. For real symmetric matrix A , $A > 0$ means the positive definite matrix. The identity matrix of size 3×3 is denoted by I_3 . $\lambda_A^{\max} \in \mathbb{R}$ and $\lambda_A^{\min} \in \mathbb{R}$ are the maximal and the minimal eigenvalues of a matrix A , respectively.

2. Relative equation of motion and discrete-time model for spacecraft

In this chapter, as the kinematics represents the attitude of the spacecraft with respect to the inertia frame $\{i\}$, the modified Rodrigues parameters (MRPs) [5] are used. The rotational motion equations of the spacecraft's body-fixed frame $\{b\}$ are given by the following equations:

$$\dot{\sigma}(t) = G(\sigma(t))\omega(t), \quad (1)$$

$$\begin{aligned} G(\sigma(t)) &= \frac{1}{2} \left\{ \frac{1 - \|\sigma(t)\|^2}{2} I_3 + \sigma(t)\sigma(t)^T + \sigma(t)^\times \right\}, \\ \dot{\omega}(t) &= J^{-1} \{ -\omega(t)^\times J \omega(t) + u(t) + w(t) \}, \end{aligned} \quad (2)$$

where Eq. (1) is the kinematics that represents the attitude of $\{b\}$ with respect to the $\{i\}$, Eq. (2) is the rotation dynamics, $\sigma(t) \in \mathbb{R}^3 [-]$ is the MRPs, $\omega(t) \in \mathbb{R}^3$ [rad/s] is the angular velocity, $u(t) \in \mathbb{R}^3$ [Nm] is the control torque (input), $w(t) \in \mathbb{R}^3$ [Nm] is the disturbance input, and $J \in \mathbb{R}^{3 \times 3}$ [kg m²] is the moment of inertia.

We consider a control problem in which a spacecraft tracks a desired attitude (MRPs) $\sigma_d(t) \in \mathbb{R}^3$ and angular velocity $\omega_d(t) \in \mathbb{R}^3$ in fixed frame $\{d\}$. The MRPs of the relative attitude $\sigma_e(t) \in \mathbb{R}^3$ and the relative angular velocity $\omega_e(t) \in \mathbb{R}^3$ in the frame $\{b\}$ are given by

$$\sigma_e(t) = \frac{N_e(t)}{1 + \|\sigma(t)\|^2 \|\sigma_d(t)\|^2 + 2\sigma_d(t)^T \sigma(t)}, \quad (3)$$

$$N_e(t) = (1 - \|\sigma_d(t)\|^2) \sigma(t) - (1 - \|\sigma(t)\|^2) \sigma_d(t) + 2\sigma(t)^\times \sigma_d(t),$$

$$\omega_e(t) = \omega(t) - C(t)\omega_d(t), \quad (4)$$

where $C(t) \in \mathbb{R}^{3 \times 3}$ is the direction cosine matrix from $\{b\}$ to $\{d\}$ that expresses the following Eq. [7]:

$$C(t) = I_3 + \frac{8(\sigma_e(t)^\times)^2 - 4(1 - \|\sigma_e(t)\|^2)\sigma_e(t)^\times}{(1 + \|\sigma_e(t)\|^2)^2}. \quad (5)$$

Substituting Eqs. (3) and (4) into Eqs. (1) and (2) using the identity $\dot{C}(t) = -\omega_e(t)^\times C(t)$ yields the following relative motion equations:

$$\dot{\sigma}_e(t) = G(\sigma_e(t))\omega_e(t), \quad (6)$$

$$\begin{aligned} \dot{\omega}_e(t) = & J^{-1}[-\{\omega_e(t) + C(t)\omega_d(t)\}^\times J\{\omega_e(t) + C(t)\omega_d(t)\} \\ & - J\{C(t)\dot{\omega}_d(t) - \omega_e(t)^\times C(t)\omega_d(t)\} + u(t) + w(t)] \end{aligned} \quad (7)$$

Hereafter, we assume that the variables of spacecraft $\sigma(t)$ and $\omega(t)$ are directly measurable and J is known. In addition, regarding the desired states $\sigma_d(t)$, $\omega_d(t)$, $\dot{\omega}_d(t)$, and the disturbance $w(t)$, the following assumption is made.

Assumption 1: the desired states $\sigma_d(t)$, $\omega_d(t)$, and $\dot{\omega}_d(t)$ are uniformly continuous and bounded $\forall t \in [0, \infty)$. The disturbance $w(t)$ is uniformly bounded $\forall t \in [0, \infty)$.

From Eqs. (A4) and (A5) in Appendix, the exact discrete-time model of relative motion equations is obtained as

$$\sigma_{e,k+1} = \sigma_{e,k} + \int_{kT}^{(k+1)T} G(\sigma_e(s))\omega_e(s) ds, \quad (8)$$

$$\begin{aligned} \omega_{e,k+1} = & \omega_{e,k} + \int_{kT}^{(k+1)T} [-\{\omega_e(s) + C(s)\omega_d(s)\}^\times J\{\omega_e(s) + C(s)\omega_d(s)\} \\ & - J\{C(s)\dot{\omega}_d(s) - \omega_e(s)^\times C(s)\omega_d(s)\} + u_k + w_k] ds \end{aligned} \quad (9)$$

and the Euler approximate model of relative motion equations are obtained as

$$\sigma_{e,k+1} = \sigma_{e,k} + TG(\sigma_{e,k})\omega_{e,k}, \quad (10)$$

$$\begin{aligned} \omega_{e,k+1} = & \omega_{e,k} - TJ^{-1}[-\{\omega_{e,k} + C_k\omega_{d,k}\}^\times J\{\omega_{e,k} + C_k\omega_{d,k}\} \\ & - J\{C_k\dot{\omega}_{d,k} - \omega_{e,k}^\times C_k\omega_{d,k}\} + u_k + w_k]. \end{aligned} \quad (11)$$

3. Discrete-time nonlinear attitude tracking control

We derive a controller based on the backstepping approach that makes the closed-loop system consisting of the Euler approximate modes (10) and (11) become input-to-state stable (ISS), i.e., the state variable of closed-loop system

$x_k = \begin{bmatrix} \sigma_{e,k}^T & \omega_{e,k}^T \end{bmatrix}^T$ satisfies the following equation:

$$\|x_{k+1}\| \leq \rho(\|x_0\|, k) + \gamma(\|w_k\|), \quad \forall x_k \in \mathbb{R}^3, \quad \forall w_k \in \mathbb{R}^3,$$

where $\rho(\cdot)$ is the class KL function and $\gamma(\cdot)$ is the class K function. To this end, assume that $\omega_{e,k}$ is the virtual input to subsystem (10), and derive the stabilizing function α_k that $\sigma_{e,k}$ is asymptotic convergence to zero. Then, derive the control

input u_k that closed-loop system becomes ISS. Here, regarding the variable $\sigma_{e,k}$, the following assumption is made.

Assumption 2: $\sigma_{e,k}$ lies in the region that satisfies the following equation:

$$0 \leq \|\sigma_{e,k}\| \leq 1, \quad \forall k.$$

Remark 1: from the relational expression

$$\sigma_{e,k} = \frac{\varepsilon_{e,k}}{1 + \eta_{e,k}},$$

where $\varepsilon_{e,k} \in \mathbb{R}^3$ and $\eta_{e,k} \in \mathbb{R}$ are the quaternion $\left(\left\| \begin{bmatrix} \varepsilon_{e,k}^T \\ \eta_{e,k} \end{bmatrix} \right\|^2 = 1, \|\varepsilon_{e,k}\| \leq 1, |\eta_{e,k}| \leq 1, \forall k \right)$. Assumption 2 is equivalent to $\eta_{e,k} \in [0, 1]$.

In addition, Lemmas when using the derivation of the control law are shown below.

Lemma 1: for all $\sigma \in \mathbb{R}^3$, the following equations hold [5]:

$$\sigma^T G(\sigma) = b\sigma^T, \quad G(\sigma)^T G(\sigma) = b^2 I_3, \quad \left(b = \frac{1 + \|\sigma\|^2}{4} > 0 \right).$$

Lemma 2: when the quadratic equation

$$ax^2 + bx + c = 0 (a, b, c \in \mathbb{R})$$

has two distinct real roots $x = \alpha, \beta (\alpha < \beta)$, if $a > 0$, then the solution of the quadratic inequality

$$ax^2 + bx + c < 0$$

is $\alpha < x < \beta$.

3.1 Derivation of virtual input α_k

Assume that $\omega_{e,k}$ is the virtual input to subsystem (10), and define the stabilizing function such that

$$\omega_{e,k} = \alpha_k = -f_1 \sigma_{e,k}, \quad (12)$$

where $f_1 \in \mathbb{R}$ is the feedback gain. The candidate Lyapunov function for (10) is defined as

$$V_1(k) = \|\sigma_{e,k}\|^2. \quad (13)$$

From Lemma 1, the difference of Eq. (13) along the trajectories of the closed-loop system is given by

$$\Delta V_1(k) = V_1(k+1) - V_1(k) = \left\{ (Tf_1 b_k)^2 - 2Tf_1 b_k \right\} \|\sigma_{e,k}\|^2. \quad (14)$$

From Lemma 2, $\Delta V_1(k)$ becomes negative, i.e., the range of f_1 that holds the following equation

$$(Tf_1b_k)^2 - 2Tf_1b_k < 0 \quad (15)$$

is obtained as

$$0 < f_1 < \frac{2}{Tb_k}. \quad (16)$$

In addition, since $2 \leq (1/b_k) \leq 4$ under Assumption 2, the range of f_1 that holds Eq. (15) is obtained as

$$0 < f_1 < \frac{4}{T}. \quad (17)$$

Therefore, if f_1 satisfies Eq. (17) and $\omega_{e,k} \rightarrow \alpha_k (k \rightarrow \infty)$, then $\sigma_{e,k} \rightarrow 0$.

3.2 Derivation of control input u_k

The error variable between the state $\omega_{e,k}$ and α_k is defined as

$$z_k := \omega_{e,k} - \alpha_k. \quad (18)$$

The control input u_k that makes the closed-loop system becomes ISS is derived. From Eq. (18), subsystem (10) becomes

$$\sigma_{e,k+1} = \sigma_{e,k} + TG(\sigma_{e,k})(z_k + \alpha_k). \quad (19)$$

From Eqs. (18) and (19) and the following equation

$$\alpha_k - \alpha_{k+1} = Tf_1\{G(\sigma_{e,k})z_k - f_1b_k\sigma_{e,k}\},$$

the discrete-time equation with respect to z_k is

$$\begin{aligned} z_{k+1} = & z_k + Tf_1\{G(\sigma_{e,k})z_k - f_1b_k\sigma_{e,k}\} \\ & + TJ^{-1}[-\{z_k + \alpha_{k,k} + C_k\omega_{d,k}\} \times J\{z_k + \alpha_{k,k} + C_k\omega_{d,k}\} \\ & - J\{C_k\dot{\omega}_{d,k} - (z_k + \alpha_{k,k}) \times C_k\omega_{d,k}\} + u_k + w_k]. \end{aligned} \quad (20)$$

Now, by setting u_k to

$$\begin{aligned} u_k = & \{z_k + \alpha_{k,k} + C_k\omega_{d,k}\} \times J\{z_k + \alpha_{k,k} + C_k\omega_{d,k}\} \\ & + J\{C_k\dot{\omega}_{d,k} - (z_k + \alpha_{k,k}) \times C_k\omega_{d,k}\} \\ & - f_1J\{G(\sigma_{e,k})z_k - f_1b_k\sigma_{e,k}\} - f_2Jz_k, \end{aligned}$$

Eq. (20) becomes

$$z_{k+1} = (1 - Tf_2)z_k + TJ^{-1}w_k, \quad (21)$$

where $f_2 \in \mathbb{R}$ is the feedback gain. The candidate Lyapunov function for Eqs. (19) and (21) is defined as

$$V_2(k) = V_1(k) + \|z_k\|^2 = \|X_k\|^2, X_k = [\sigma_{e,k}^T z_k^T]^T. \quad (22)$$

As Eq. (14) is given by

$$\Delta V_1(k) = (Tb_k)^2 \|z_k\|^2 + \left\{ (Tf_1 b_k)^2 - 2Tf_1 b_k \right\} \|\sigma_{e,k}\|^2 + 2Tb_k(1 - Tf_1 b_k) z_k^T \sigma_{e,k}^T$$

from Eq. (18), by using completing square, the difference of Eq. (22) along the trajectories of the closed-loop system is given by

$$\begin{aligned} \Delta V_2(k) &= (T^2 f_2^2 - 2Tf_2 + T^2 b_k^2) \|z_k\|^2 + \left\{ (Tf_1 b_k)^2 - 2Tf_1 b_k \right\} \|\sigma_{e,k}\|^2 \\ &\quad + 2Tb_k(1 - Tf_1 b_k) z_k^T \sigma_{e,k}^T + T^2 w_k^T J^{-2} w_k + 2T(1 - Tf_2) w_k^T J^{-1} z_k \\ &\leq (2T^2 f_2^2 - 4Tf_2 + T^2 b_k^2 + 1) \|z_k\|^2 + \left\{ (Tf_1 b_k)^2 - 2Tf_1 b_k \right\} \|\sigma_{e,k}\|^2 \\ &\quad + 2Tb_k(1 - Tf_1 b_k) z_k^T \sigma_{e,k}^T + 2\left(\frac{T}{\lambda_J}\right)^2 \|w_k\|^2 \\ &= X_k^T Q_k X_k + 2\left(\frac{T}{\lambda_J}\right)^2 \|w_k\|^2, \end{aligned} \quad (23)$$

where

$$\begin{aligned} \lambda_J &= \|J\|, Q_k = \begin{bmatrix} Q_{11,k} & Q_{12,k} \\ Q_{12,k}^T & Q_{22,k} \end{bmatrix}, Q_{11,k} = \left\{ (Tf_1 b_k)^2 - 2Tf_1 b_k \right\} I_3, \\ Q_{12,k} &= Tb_k(1 - Tf_1 b_k) I_3, Q_{22,k} = (2T^2 f_2^2 - 4Tf_2 + T^2 b_k^2 + 1) I_3. \end{aligned}$$

In Eq. (23), if $Q_k < 0$, then

$$\Delta V_2(k) \leq -\left|\lambda_{Q_k}^{\min}\right| \|X_k\|^2 + 2\left(\frac{T}{\lambda_J}\right)^2 \|w_k\|^2,$$

where $\lambda_{Q_k}^{\min} < 0 \in \mathbb{R}$ is the minimum eigenvalue of Q_k and the condition of ISS holds [18]. Hereafter, conditions of f_1 and f_2 which the matrix Q_k holds $Q_k < 0$ are derived under Assumption 2.

From Schur complement, condition $Q_k < 0$ is equivalent to the following equations:

$$(Tf_1 b_k)^2 - 2Tf_1 b_k < 0, \quad (24)$$

$$2T^2 f_2^2 - 4Tf_2 + c_k < 0 \quad \left(c_k = \frac{Tb_k f_1^2 - 2f_1 - Tb_k}{Tb_k f_1^2 - 2f_1} \right). \quad (25)$$

Condition (24) is the same as Eq. (15), and assume that Eq. (24) holds. From Lemma 2, the range of f_2 that holds for Eq. (25) is obtained as

$$\frac{2 - \sqrt{2(2 - c_k)}}{2T} < f_2 < \frac{2 + \sqrt{2(2 - c_k)}}{2T}, \quad (26)$$

and the following Eq.

$$2 - c_k > 0 \Rightarrow \frac{Tb_k f_1^2 - 2f_1 + Tb_k}{Tb_k f_1^2 - 2f_1} > 0 \quad (27)$$

must hold true in order to obtain a real number. As the denominator of Eq. (27) is the same as Eq. (24), the following equation must hold

$$Tb_k f_1^2 - 2f_1 + Tb_k < 0 \quad (28)$$

in order to hold Eq. (27). From Lemma 2, the range of f_1 that holds for Eq. (28) is obtained as

$$\frac{1 - \sqrt{1 - (Tb_k)^2}}{Tb_k} < f_1 < \frac{1 + \sqrt{1 - (Tb_k)^2}}{Tb_k}, \quad (29)$$

and the following Eq.

$$1 - (Tb_k)^2 > 0 \Rightarrow 0 < T < \frac{1}{b_k} \quad (30)$$

must hold in order to have the real number. As $2 \leq (1/b_k) \leq 4$ under Assumption 2, T must satisfy the condition

$$0 < T < 2. \quad (31)$$

In addition, since

$$\begin{aligned} \max_{b_k} \frac{1 - \sqrt{1 - (Tb_k)^2}}{Tb_k} &= \frac{2 - \sqrt{4 - T^2}}{T}, \\ \min_{b_k} \frac{1 + \sqrt{1 - (Tb_k)^2}}{Tb_k} &= \frac{2 + \sqrt{4 - T^2}}{T} \end{aligned}$$

under Assumption 2, the condition (29) is given by

$$\frac{2 - \sqrt{4 - T^2}}{T} < f_1 < \frac{2 + \sqrt{4 - T^2}}{T} \quad (0 < T < 2). \quad (32)$$

Therefore, if f_1 satisfies Eq. (32) under Assumption 2, Eqs. (27) and (28) hold. Furthermore, since

$$\begin{aligned} \max_{b_k} \frac{2 - \sqrt{2(2 - c_k)}}{2T} &= \frac{1}{T} - \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2f_1(Tf_1 - 4)}}, \\ \min_{b_k} \frac{2 + \sqrt{2(2 - c_k)}}{2T} &= \frac{1}{T} + \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2f_1(Tf_1 - 4)}}, \end{aligned}$$

under Assumption 2, the condition (26) is given by.

$$\frac{1}{T} - \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2f_1(Tf_1 - 4)}} < f_2 < \frac{1}{T} + \sqrt{\frac{Tf_1^2 - 4f_1 + T}{2T^2f_1(Tf_1 - 4)}} \quad (0 < T < 2). \quad (33)$$

Therefore, if f_1 and f_2 satisfy Eqs. (32) and (33) under Assumption 2, then $Q_k < 0$.

Summarizing the above, the following theorem can be obtained.

Theorem 1: if sampling period T and feedback gains f_1 and f_2 satisfy Eqs. (31), (32), and (33) under Assumption 2, then the closed-loop systems (10) and (11) with the following control law

$$\begin{aligned}
 u_k &= \{z_k + \alpha_{,k} + C_k \omega_{d,k}\}^\times J \{z_k + \alpha_{,k} + C_k \omega_{d,k}\} \\
 &\quad + J \{C_k \dot{\omega}_{d,k} - (z_k + \alpha_{,k})^\times C_k \omega_{d,k}\} \\
 &\quad - f_1 J \{G(\sigma_{e,k}) z_k - f_1 b_k \sigma_{e,k}\} - f_2 J z_k \\
 &= \omega_k^\times J \omega_k + J \{C_k \dot{\omega}_{d,k} - (z_k + \alpha_{,k})^\times C_k \omega_{d,k}\} \\
 &\quad - f_1 f_2 J \sigma_{e,k} - J \{f_1 G(\sigma_{e,k}) + f_2 I_3\} \omega_{e,k}
 \end{aligned} \tag{34}$$

becomes ISS.

Then, we show that the pair $(u_k, V_2(k))$ is semiglobal practical asymptotic (SPA) stabilizing pair for the Euler approximate systems (10) and (11). Hereafter, suppose that sampling period T and feedback gains f_1 and f_2 satisfy Eqs. (31), (32), and (33) under Assumption 2. By using the following coordinate transformation

$$X_k = \begin{bmatrix} 1 & 0 \\ f_1 & 1 \end{bmatrix} \begin{bmatrix} \sigma_{e,k} \\ \omega_{e,k} \end{bmatrix} = Z \bar{X}_k,$$

Lyapunov function $V_2(k)$ and its difference $\Delta V_2(k)$ can be rewritten as

$$\begin{aligned}
 V_2(k) &= \bar{X}_k^T Z^T Z \bar{X}_k = \bar{X}_k^T R \bar{X}_k, \\
 \Delta V_2(k) &= \bar{X}_k^T Z^T Q_k Z \bar{X}_k + 2 \left(\frac{T}{\lambda_J} \right)^2 \|w_k\|^2 = \bar{X}_k^T \bar{Q}_k \bar{X}_k + 2 \left(\frac{T}{\lambda_J} \right)^2 \|w_k\|^2.
 \end{aligned}$$

Since $R > 0$ and $\bar{Q}_k < 0$, $V_2(k)$ and $\Delta V_2(k)$ satisfy following equations:

$$\lambda_R^{\min} \|\bar{X}_k\|^2 \leq V_2(k) \leq \lambda_R^{\max} \|\bar{X}_k\|^2, \tag{35}$$

$$\Delta V_2(k) \leq -\left| \lambda_{\bar{Q}_k}^{\min} \right| \|\bar{X}_k\|^2 + 2 \left(\frac{T}{\lambda_J} \right)^2 \|w_k\|^2. \tag{36}$$

In addition, $\bar{X}_k$ is bounded, and $V_2(k)$ is radially unbounded from Eqs. (35) and (36). Hence, the control input (34) satisfies the following equation under Assumption 1:

$$\|u_k\| \leq M, \tag{37}$$

where M is a positive constant. Furthermore, $V_2(k)$ also satisfies the following equation for all $x, z \in \mathbb{R}^6$ with $\max\{\|x\|, \|z\|\} \leq \Delta$:

$$\begin{aligned}
 |V_2(x) - V_2(z)| &= |x^T R x - z^T R z| = |(x + z)^T R (x - z)| \\
 &= \lambda_R^{\max} \|x + z\| \|x - z\| \leq 2\Delta \lambda_R^{\max} \|x - z\|,
 \end{aligned} \tag{38}$$

where Δ is a positive constant. Therefore, from Eqs. (35) to (38), Lyapunov function $V_2(k)$ and control input u_k satisfied Eqs. (A8)–(A11) in Definition 2 under Assumptions 1 and 2, and the pair $(u_k, V_2(k))$ becomes SPA stabilizing pair for the

Euler approximate systems (10) and (11). Then, the following theorem can be obtained by Theorem A.1 in Appendix.

Theorem 2: control input (34) is SPA stabilizing for exact discrete-time systems (8) and (9).

4. Numerical simulation

The properties of the proposed method are discussed in the numerical study. For this purpose, parameter setting of simulation is as follows:

$$J = \begin{bmatrix} 7050.0 & -0.536 & 43.9 \\ -0.536 & 2390 & 1640.0 \\ 43.9 & 1640.0 & 6130.0 \end{bmatrix} \text{kgm}^2, \sigma(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \omega(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{rad/s}$$

$$T = \begin{cases} 1.0 : \text{Case 1} \\ 0.5 : \text{Case 2}, f_1 = 0.6, f_2 = 0.8. \\ 0.1 : \text{Case 3} \end{cases}$$

The moment of inertia J is from [1]. The initial values $\sigma(0)$ correspond to Euler angles of 1-2-3 system of $\theta(0) = [\theta_1(0)\theta_2(0)\theta_3(0)]^T = [0 \ 0 \ 0]^T$ [deg]. The feedback gains f_1 and f_2 satisfy Eqs. (25) and (28) for all cases of T . The desired states $\sigma_d(t)$, $\omega_d(t)$, and $\dot{\omega}_d(t)$ in this simulation are the switching maneuver as shown in **Figure 1**.

Figure 1.
 Switching maneuver.

Figure 2.
Time histories of MRPs $\sigma(t)$ and $\sigma_e(t)$ (solid line, case 1; dashed-dotted line, case 2; dashed line, and case 3; dotted line, $\sigma_d(t)$).

Figure 3.
Time histories of attitude angles $\theta(t)$ and $\theta_e(t)$ (solid line, case 1; dashed-dotted line, case 2; dashed line, and case 3; dotted line, $\theta_d(t)$).

Figure 4.
Time histories of angular velocities $\omega(t)$ and $\omega_e(t)$ (solid line, case 1; dashed-dotted line, case 2; dashed line, and case 3; dotted line, $\omega_d(t)$).

Figure 5.
 Time histories of control input $u(t)$ (solid line, case 1; dashed-dotted line, case 2; and dashed line, case 3).

The results of the numerical simulation are shown in **Figures 2–5**. The relative attitude $\sigma_e(t)$ and relative angular velocity $\omega_e(t)$ converge to the neighborhood of $(\sigma_e(t), \omega_e(t)) = (0, 0)$, and the control input amplitude $u(t)$ does not depend on the sampling period T although there is a slight difference in the maximal value of $u(t)$.

5. Conclusion

This chapter considers the spacecraft attitude tracking control problem that requires agile and large-angle attitude maneuvers and proposed a discrete-time nonlinear attitude tracking control that the amplitude of the control input does not depend on the sampling period T . The effectiveness of proposed control method is verified by numerical simulations. Extension to the guarantee of stability as sampled-data control system will be subject to future work.

Appendix: sampled-data control of nonlinear system

This section shows preliminary results for nonlinear sampled-data control [13, 14, 19].

Let us consider the following nonlinear system:

$$\dot{x}(t) = f(x(t), u(t)), x(0) = x_0, f(0, 0) = 0, \quad (A1)$$

where $x(t) \in \mathbb{R}^n$ is the state variable and $u(t) \in \mathbb{R}^m$ is the control input. The function $f(x(t), u(t))$ in Eq. (A1) is assumed to be such that, for each initial condition and each constant control input, there exists a unique solution defined on some intervals of $x[0, \tau)$.

The nonlinear system (A1) is assumed to be between a sampler (A/D converter) and zero-order hold (D/A converter), and the control signal is assumed to be piecewise constant, that is,

$$u(t) = u(kT) =: u(k), \forall t \in [kT, (k+1)T], k \in \{0\} \cup \mathbb{N}, \quad (A2)$$

where $T > 0$ is a sampling period. In addition, assume that the state variable

$$x(k) := x(kT) \quad (\text{A3})$$

is measurable at each sampling instance. The exact discrete-time model and Euler approximate model of the nonlinear sampled-data systems (A1)–(A3) are expressed as follows, respectively:

$$x_{k+1} = x_k + \int_{kT}^{(k+1)T} f(x(s), u_k) ds =: F_T^e(x_k, u_k), \quad (\text{A4})$$

$$x_{k+1} = x_k + Tf(x_k, u_k) =: F_T^{Euler}(x_k, u_k), \quad (\text{A5})$$

where we abbreviate $x(k)$ and $u(k)$ to x_k and u_k . For the stability of the exact discrete-time model (A4) (F_T^e) and Euler approximate model (A5) (F_T^{Euler}), the following definitions are used [13, 14, 19].

Definition 1: consider the following discrete-time nonlinear system:

$$x_{k+1} = F_T(x_k, u_T(x_k)), \quad (\text{A6})$$

where $x_k \in \mathbb{R}^n$ is the state variable and $u_T(x_k) \in \mathbb{R}^m$ is a control input. The family of controllers $u_T(x_k)$ SPA stabilizes the system (A6) if there exists a class KL function $\beta(\cdot)$ such that for any strictly positive real numbers (D, ν) , there exists $T^* > 0$, and such that for all $T \in (0, T^*)$ and all initial state x_0 with $\|x_0\| \leq D$, the solution of the system satisfies

$$\|x_k\| \leq \beta(\|x_0\|, kT) + \nu, \forall k \in \{0\} \cup \mathbb{N}. \quad (\text{A7})$$

Definition 2: let $\hat{T} > 0$ be given, and for each $T \in (0, \hat{T})$, let functions $V_T : \mathbb{R}^n \rightarrow \mathbb{R}$ and $u_T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be defined. The pair of families (u_T, V_T) is a SPA stabilizing pair for the system (A7) if there exist a class K_∞ functions α_1, α_2 , and α_3 such that for any pair of strictly positive real numbers (Δ, δ) , there exists a triple of strictly positive real numbers (T^*, L, M) ($T^* \leq \hat{T}$) such that for all $x, z \in \mathbb{R}^n$ with $\max\{\|x\|, \|z\|\} \leq \Delta$, and $T \in (0, T^*)$:

$$\alpha_1(\|x\|) \leq V_T(x) \leq \alpha_2(\|x\|), \quad (\text{A8})$$

$$V_T(F_T(x, u_T(x))) - V_T(x) \leq -\alpha_3(\|x\|) + T\delta, \quad (\text{A9})$$

$$|V_T(x) - V_T(z)| \leq L\|x - z\|, \quad (\text{A10})$$

$$\|u_T(x)\| \leq M. \quad (\text{A11})$$

In addition, if there exists $T^{**} > 0$ such that Eqs. (A8)–(A11) with $\delta = 0$ hold for all $x \in \mathbb{R}^n$ and $T \in (0, T^{**})$, then the pair (u_T, V_T) is globally asymptotic (GA) stabilizing pair for the system (A6).

Using the above definitions, the following theorem is obtained by literatures [13, 14, 19].

Theorem A.1: if the pair (u_T, V_T) is SPA stabilizing for F_T^{Euler} , then u_T is SPA stabilizing for F_T^e .

Hence, if we can find a family of pairs of (u_T, V_T) that is a GA or SPA stabilizing pair for F_T^{Euler} , then the controller u_T will stabilize the exact model F_T^e .

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References

- [1] Kamiya T, Maeda K, Ogura N. Preshaping profile for flexible spacecraft rest-to-rest maneuver. In: AIAA Guidance, Navigation, and Control Conference and Exhibit; AIAA 2006-6181; 2006
- [2] Somov Y, Butyrin S, Somov S. Guidance and robust gyromoment precise attitude control of agile observation spacecraft. In: 17th IFAC World Congress; 2008. pp. 3422-3427
- [3] Nagashio T, Kida T, Mitani S, Ohtani T, Yamaguchi I, Kasai T, Hamada H. A preliminary controller design study on precise switching maneuver of flexible spacecraft astro-G. In: Proceedings of 25th Guidance Control Symposium (in Japanese); 2008. pp. 25-32
- [4] Dalsmo M, Egeland O. State feedback H_∞ -suboptimal control of a rigid spacecraft. IEEE Transactions on Automatic Control. 1997;42(8): 1186-1189
- [5] Tsiotras P. Further passivity results for the attitude control problem. IEEE Transactions on Automatic Control. 1998;43(11):1597-1160
- [6] DeVon DA, Fuentes RJ, Fausz JL. Passivity-based attitude control for an integrated power and attitude control system using variable speed control moment gyroscopes. In: Proceedings of the 2004 American Control Conference; 2004. pp. 1019-1024
- [7] Meng Z, Ren W, You Z. Decentralized cooperative attitude tracking using modified rodriguez parameters. In: Proceedings of the 48th IEEE Conference on Decision and Control Held Jointly with 28th Chinese Control Conference; 2009. pp. 853-858
- [8] Schlanbusch R, Loria A, Kristiansen R, Nicklasson P. J. PD+ attitude control of rigid bodies with improved performance. In: Proceedings of the 49th IEEE Conference on Decision and Control; 2010. pp. 7069-7074
- [9] Liu S, Sun J, Geng Z. Passivity-based finite-time attitude control problem. In: 9th Asian Control Conference; 2013. pp. 1-6
- [10] Cong BL, Chen Z, Liu XD. Robust attitude control with improved transient performance. In: 19th IFAC World Congress; 2014. pp. 463-468
- [11] Ikeda Y, Kida T, Nagashio T. Nonlinear tracking control of rigid spacecraft under disturbance using PID type H_∞ adaptive state feedback. Transactions of the Japan Society for Aeronautical and Space Sciences. 2015; 58(5):289-297
- [12] Ikeda Y, Kida T, Nagashio T. Stabilizing nonlinear adaptive control of spacecraft before and after capture. Transactions of the Japan Society for Aeronautical and Space Sciences. 2016; 59(1):1-9
- [13] Nesi'c D, Teel AR, Kokotovi'c PV. Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximation. Systems & Control Letters. 1999;38(4-5): 259-270
- [14] Nesi'c D, Teel AR. Stabilization of sampled-data nonlinear system via backstepping on their Euler approximate model. Automatica. 2006; 42(10):1801-1808
- [15] Katayama H. Nonlinear sampled-data stabilization of dynamically positioned ships. IEEE Transactions on Control Systems Technology. 2010; 18(2):463-468
- [16] Nishimura M, Nagahio T, Kida T. Discrete-time tracking control system

design based on a practical stability for spacecraft. In: Proceedings of 53rd the Japan Joint Automatic Control Conference (in Japanese); 2010. pp. 895-900

[17] Ikeda Y. Nonlinear sampled-data control of spacecraft by sliding mode control. In: Proceedings of the 27th Guidance and Control Symposium (in Japanese); 2010. pp. 97-100

[18] Jiang ZP, Wang Y. Input-to-state stability for discrete-time nonlinear system. *Automatica*. 2001;**35**:857-869

[19] Laila DS, Nesic D, Astolfi A. Sampled-Data Control of Nonlinear Systems, Advanced Topics in Control Systems Theory. In: *Lecture Notes in Control and Information Sciences*. Springer; 2005. pp. 91-137