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Application of Media with Negative Refraction Index to Electromagnetic Imaging. Fundamental Aspects.

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1. Introduction

In the recent decade, there has been growing interest in the electrodynamics of materials characterized by negative refractive index. These materials have been introduced into consideration by V. G. Veselago (1967). The progress in the nanotechnology of composite materials made it possible to design new materials whose properties can be explained by assuming that the refractive index of these materials is negative [Smith (2000), Shelby(2001)]. Some authors [Pendry (2000), Lagarkov (2004)] suggested that these materials with negative refraction allow one to overcome the diffraction limit in optical devices. This thesis has met many objections [Williams (2001)], which are not absolutely indisputable [Pendry (2001)]. Nevertheless, the concept of a superlens (in the form of a layer of medium with negative refraction), which was proposed in [Pendry (2000)] and allows one to completely overcome the diffraction limit in the ideal case, has found many supporters (see, for example, [Cui (2005), Chen (2006), Alitalo (2007), Cheng (2005), Scalora (2007), Podolskiy (2005)] and references therein).

The superlens, as proposed by Pendry (2000), is a plane plate of material with negative refractive index. The geometry of rays originated from a point source S and focused in the point P for the superlens is shown in Fig. 1. The refraction of the rays takes place according to the Snellius law applied to the material with negative refraction index. Fig.1 shows the case $n = -1$ for which there are no geometrical aberrations. According to Pendry (2000) there is a singularity of the electromagnetic fields at the image point P of a point source S .

However, the very concept of a superlens with a singularity at the image point P seems quite strange. Indeed, it has been known since Fresnel's times that the dimensions of a focal spot are determined only by the angle between the marginal converging rays and by the radiation wavelength in the focusing region (see, for example, the textbook [Sivukhin (1980), Section 55]). The dimensions of the focal spot are a property of a converging wave and do not depend on the optical system that has formed this converging wave. In the case of a superlens, a converging wave propagates in vacuum, outside the layer with negative refractive index, and it seems reasonable that it should focus into a region of standard dimensions.

The goal of the chapter is to examine completely this knotty problem. One approach in strict formulation has been suggested in two papers of A. B. Petrin (2008). It was considered the

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Fig. 1. The geometry of the superlens and the refracted rays from a point source focused into the image point P .

propagation of an electromagnetic wave radiated by an elementary electric Hertz dipole located in the air (or vacuum) parallel to the boundary of a layer or, in a particular case, to the boundary of a half-space filled with a material with negative refractive index. It was applied a rigorous approach that dates back to Sommerfeld [Sommerfeld (1926), Wait (1998)]. The scientific discussion around the resolution of superlens has shown that there is some lack of understanding in the theory of diffraction which has a fundamental significance. The main goal of the chapter is to clear the theme of diffraction limit of lens with negative refraction from several points of view.

2. The ultimate resolution of electromagnetic imaging devices and the limits of subwavelength imaging. Fundamental aspects.

Before considering strict solutions of the problem let us investigate the ultimate resolution of electromagnetic devices from common point of view since the propagation of electromagnetic waves is obeyed the Maxwell's equations. Moreover, in the regions outside the superlens the electromagnetic waves propagate in vacuum and it is possible to apply the well-known approaches to the problem of resolution of superlens.

Let us consider the preliminary auxiliary problems.

First, consider a point source of electromagnetic radiation (in particular a point Herzian dipole, see Fig.2). Consider the image plane parallel to the dipole. The distribution of mean electric field on the plane depends on the distance from the dipole to the plane.

Fig. 2. One elementary Herzian dipole near the image plane (plane of observation)

Fig. 3. The geometry of the image spot from a dipole on the image plane.

Fig. 3 shows the image spot (shown approximately by the circle) located on the plane of observation on some level of electric field strength in the near field of the dipole. Fig. 3 shows the case when the distance d from the dipole to the plane of observation is much smaller than the wavelength of the electromagnetic wave λ . In this case of the near field zone ($d \ll \lambda$) the dimensions of the spot are of the order of d due to the fast decrease of the electric field from the dipole in the near field zone. Thus, formally, *in the near field zone there is subwavelength resolution of the dipole.*

Let's take two dipoles as shown in Fig. 4. In this case it is possible to resolve the mutual position of the two dipoles with subwavelength accuracy in the near field zone. Fig. 5 approximately shows the image spots from these two dipoles on the plane of observation. *So, in principal (when $d/\lambda \rightarrow 0$), the subwavelength resolution may be achieved without use of any lenses.* It is necessary to emphasize that such subwavelength resolution in the near field zone is well-known and trivial.

Fig. 4. The geometry of two elementary Herzian dipoles near the image plane (plane of observation).

Fig. 5. The geometry of the image spots from two dipoles on the image plane.

Let’s again consider the superlens shown in Fig. 1. And let’s show that the superresolution (that is the existence of singularity in the image spot from the point source such as dipole as in the work of Pendry (2000)) is impossible. It will be shown below that there is no superresolution neither in near field zone nor in the far field zone ($d \gg \lambda$). The existence of singularity in the image point is contrary to the Huygens principle of wave propagation and to the analogue of the principle in electromagnetism – the formulas of Stratton and Chu (see Stratton (1941)). Indeed, to the right of the plate of negatively refracted material of the superlens in Fig. 1 is the vacuum. So, the image point may be encircled with sphere C as shown in Fig. 6.

Fig. 6. The sphere surface C around the image point P situated in vacuum outside the refracted material.

Thus, for any point inside the sphere, including the image point P , it is possible to write the formulas of Stratton and Chu (see Stratton (1941)) in the following form:

$$\mathbf{E}_P = i\omega\mu\mathbf{A}_e - \frac{1}{i\omega\varepsilon} \operatorname{grad} \operatorname{div} \mathbf{A}_e - \operatorname{rot} \mathbf{A}_m, \tag{1}$$

$$\mathbf{H}_P = i\omega\varepsilon\mathbf{A}_m - \frac{1}{i\omega\mu} \operatorname{grad} \operatorname{div} \mathbf{A}_m - \operatorname{rot} \mathbf{A}_e, \tag{2}$$

where $\mathbf{A}_e = \frac{1}{4\pi} \int_C \frac{[\mathbf{n}, \mathbf{H}]}{r} e^{ikr} dC$ is the electric vector potential; $\mathbf{A}_m = \frac{1}{4\pi} \int_C \frac{[\mathbf{E}, \mathbf{n}]}{r} e^{ikr} dC$ is the magnetic vector potential; $r = |\mathbf{r}_P - \mathbf{r}_C|$ is the distance from the point of integration to the point P .

The formulas of Stratton and Chu show that the electromagnetic fields at the point P are the superposition (the vector sum) of the fields from the sources situated on the surface C . The sources are the surface electric and magnetic dipoles defined by the tangential components of electromagnetic fields. Since the surface C may be of arbitrary form it is possible to deform the surface and transform it into the plane surface as shown in Fig. 7 (the closing semisphere of the surface is depicted in Fig. 7 by dotted line). Due to the Sommerfeld's condition of radiation the integral on infinitely distant semisphere is vanished and the integral on C is reduced to the integral over the plane (see Fig. 7).

Thus, the elementary sources of the electromagnetic fields are situated on the plane C and give the fields at the point P by summation. So, it is obvious that the dimensions of the focus

region are defined by the two parameters: the wavelength λ and the distance d between the surface of the plate of the negatively refracted material and the image plane.

Fig. 7. The transformed surface C (the plane and the closing semisphere).

Let's prove the following: if the image plane is in the far zone ($d \gg \lambda$), the focal region dimensions are of the order of λ and if the image plane is in the near zone ($d \ll \lambda$) the focal region dimensions are of the order of d . Consider two arbitrary situated elements on the plane surface C (see Fig. 8). These elements radiate the electromagnetic fields according the formulas (1) and (2) in which the vector potential of the elements are $d\mathbf{A}_e = [\mathbf{n}, \mathbf{H}]e^{ikr}dC/4\pi r$ and $d\mathbf{A}_m = [\mathbf{E}, \mathbf{n}]e^{ikr}dC/4\pi r$. Consider how the contributions to the fields from the elements change if the point of observation moves from the point P to the point P' (see Fig. 8).

If the image plane is in the far zone ($d \gg \lambda$) and if the displacement of the point P is of the order of λ then the distance from any element dC is approximately the same (in relative sense) if the point of observation move from the point P to P' (due to $\lambda/d \rightarrow 0$). So, only phase changing defines the changing in the fields from the element dC . The phase shanges on the distance of the orser of λ . Therefore, the focal region dimensions are of the order of λ . Since the fields from any element dC change noticeably for the displacement of the order of λ the same conclusion may be made for the total or summed fields for the considering case of far zone.

If the image plane is in the near zone ($d \ll \lambda$) and if the displacement of the point P is of the order of d then the phase difference is neglectively small. So, only the terms without phase exponent give contribution to the fields at the points P and P' . Due to fast decreasing of the near fields of the elementary dipoles the focal region dimensions are of the order of d . Since

Fig. 8. The geometry of two elementary sources which give contribution to the electromagnetic fields in the points P and P' .

the fields from any element dC change noticeably for the displacement of the order of d the same conclusion may be made for the total or summed fields for the considering case of near field zone. The same dimensions of the spot will be in the case of Fig. 5, so, in this sense the lens with negative material is unnecessary for achieving better resolution.

Thus, the conclusion from the above reasoning is the following: *there is no singularity in principle neither in near field zone nor in the far or intermediate zone.* This conclusion is not depended on the lens design or the absorption of the material of the lens. So, the term superresolution has no sense. It is only possible to speak about common subwavelength resolution in the near field zone.

3. The strict electromagnetic theory of lens with negative refraction (a plane layer filled with a medium with negative refractive index) and the image of a point source of radiation. Correspondence of common media and media with negative refraction.

In this paragraph we consider the strict solution of the problem of focus spot dimension from the point source inside the semispace with negative refraction and in vacuum for the superlens (after refraction in a plane layer filled with a medium with negative refractive index).

3.1 Radiation from a Hertzian dipole parallel to a layer of medium with negative refractive index. Strict theory.

Consider (Fig. 9) an elementary horizontal dipole (a Hertzian dipole) with unit current moment

Fig. 9. Geometry of the problem. Media 2 and/or 3 are characterized by negative refractive index. The source of electromagnetic waves, an electric Hertzian dipole, is located in medium 1 (vacuum or air, $\epsilon_1 = \mu_1 = 1$).

$$\mathbf{J} = \delta(x)\delta(y)\delta(z-d)\mathbf{e}_x, \quad (3)$$

where $\delta(x)$ is the delta function, $\mathbf{e}_x$ is a unit vector directed along the x -axis, and d is the z -coordinate of the dipole. Note that in this paper we use a complex representation of all quantities with the time dependence $e^{-i\omega t}$. A detailed account of the problem of propagation of electromagnetic waves radiated from a Hertzian dipole oriented parallel to the plane boundary of a half-space filled with an absorbing medium with positive real parts of permittivity and permeability was given in King&Smith (1981). Below, we generalize the approach developed in King&Smith (1981) and apply it to solving the problem of radiation from a dipole parallel to a layer with negative refractive index.

We can write Maxwell's equations in the three domains as

$$\text{rot } \mathbf{E}_j = i\omega \mathbf{B}_j, \quad (4)$$

$$\text{rot } \mathbf{B}_j = \mu_j (-i\omega \epsilon_j \mathbf{E}_j + \mathbf{J}), \quad (5)$$

where $j = 1$ for domain 1 ($z > 0$), $j = 2$ for domain 2 ($-h < z < 0$), and $j = 3$ for domain 3 ($z < -h$), h is the layer thickness.

The boundary conditions on the surfaces of the layer require that the tangential components of the fields $\mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu$, as well as the normal components of the fields $\mathbf{D} = \epsilon\mathbf{E}$ and $\mathbf{B}$, be continuous (provided that there are no external current sources on the surfaces). However, one can show that, in the case of harmonic fields, it suffices to require that only the tangential components of $\mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu$ should be continuous.

The electric field can be represented as a Fourier expansion:

$$\mathbf{E}(x, y, z) = (2\pi)^{-2} \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta e^{i\xi x + i\eta y} \tilde{\mathbf{E}}(\xi, \eta, z). \quad (6)$$

Similar expressions can be written for $\mathbf{B}$ and $\mathbf{J}$:

$$\mathbf{B}(x, y, z) = (2\pi)^{-2} \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta e^{i\xi x + i\eta y} \tilde{\mathbf{B}}(\xi, \eta, z),$$

$$\mathbf{J}(x, y, z) = (2\pi)^{-2} \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta e^{i\xi x + i\eta y} \tilde{\mathbf{J}}(\xi, \eta, z).$$

Using the Fourier transformation, from (3) we obtain the Fourier transform of the current source:

$$\tilde{\mathbf{J}}(\xi, \eta, z) = \delta(z - d) \mathbf{e}_x. \quad (7)$$

Using the Fourier representations of the fields, we can rewrite Maxwell's equations in the three domains as

$$i\eta \tilde{E}_{jz} - \frac{\partial}{\partial z} \tilde{E}_{jy} = i\omega \tilde{B}_{jx}, \quad (8)$$

$$\frac{\partial}{\partial z} \tilde{E}_{jx} - i\xi \tilde{E}_{jz} = i\omega \tilde{B}_{jy}, \quad (9)$$

$$i\xi \tilde{E}_{jy} - i\eta \tilde{E}_{jx} = i\omega \tilde{B}_{jz}, \quad (10)$$

$$i\eta \tilde{B}_{jz} - \frac{\partial}{\partial z} \tilde{B}_{jy} = -i\omega \epsilon_j \mu_j \tilde{E}_{jx} + \delta_{1j} \mu_j \delta(z - d), \quad (11)$$

$$\frac{\partial}{\partial z} \tilde{B}_{jx} - i\xi \tilde{B}_{jz} = -i\omega \epsilon_j \mu_j \tilde{E}_{jy}, \quad (12)$$

$$i\xi \tilde{B}_{jy} - i\eta \tilde{B}_{jx} = -i\omega \epsilon_j \mu_j \tilde{E}_{jz}, \quad (13)$$

where δ_{sp} is the Kronecker delta.

Using Eqs. (9) and (13), we express $\tilde{E}_{jz}$ and $\tilde{B}_{jy}$ in terms of $\tilde{E}_{jx}$ and $\tilde{B}_{jx}$ and, using (10) and (12), express $\tilde{E}_{jy}$ and $\tilde{B}_{jz}$ in terms of $\tilde{E}_{jx}$ and $\tilde{B}_{jx}$:

$$\tilde{E}_{jy} = \frac{1}{k_j^2 - \xi^2} \left(-\eta \xi \tilde{E}_{jx} + i\omega \frac{\partial}{\partial z} \tilde{B}_{jx} \right), \quad (14)$$

$$\tilde{E}_{jz} = \frac{1}{k_j^2 - \xi^2} \left(i\xi \frac{\partial}{\partial z} \tilde{E}_{jx} + \omega \eta \tilde{B}_{jx} \right), \quad (15)$$

$$\tilde{B}_{jy} = \frac{1}{k_j^2 - \xi^2} \left(-i\omega \varepsilon_j \mu_j \frac{\partial}{\partial z} \tilde{E}_{jx} - \xi \eta \tilde{B}_{jx} \right), \quad (16)$$

$$\tilde{B}_{jz} = \frac{1}{k_j^2 - \xi^2} \left(-\eta \omega \varepsilon_j \mu_j \tilde{E}_{jx} + i\xi \frac{\partial}{\partial z} \tilde{B}_{jx} \right), \quad (17)$$

where we introduced complex wavenumbers for the domains by the formulas $k_j^2 = \omega^2 \varepsilon_j \mu_j$, $j = 1, 2, 3$.

Substituting expressions (14)–(17) into (8) and (11), we obtain

$$\left(\frac{d^2}{dz^2} + \gamma_j^2 \right) \tilde{E}_{jx} = \frac{k_j^2 - \xi^2}{i\omega \varepsilon_j} \delta(z - d), \quad (18)$$

$$\left(\frac{d^2}{dz^2} + \gamma_j^2 \right) \tilde{B}_{jx} = 0. \quad (19)$$

where $\gamma_j^2 = k_j^2 - \xi^2 - \eta^2$.

For fixed values of ξ and η , expressions (18) and (19) are ordinary differential equations in the variable z . The problem consists in solving Eqs. (18) and (19) for $\tilde{E}_{jx}$ and $\tilde{B}_{jx}$ in the three domains. Knowing the components $\tilde{E}_{jx}$ and $\tilde{B}_{jx}$, one can determine the other components of the field from Eqs. (14)–(17).

The solution must satisfy not only the equations but also the boundary conditions on the two interfaces $z = 0$ and $z = -h$ between media.

Formulas (18) and (19) contain the functions $\gamma_j^2 = k_j^2 - \xi^2 - \eta^2$. To find the solutions of these equations, we should single out analytic branches of the functions γ_j versus the complex variable λ , where $\lambda^2 = \xi^2 + \eta^2$. For real wavenumbers, we have the formula (see King & Smith (1981))

$$\gamma_j(\lambda) = \begin{cases} \sqrt{k_j^2 - \lambda^2}, & \lambda^2 \leq k_j^2 \\ i\sqrt{\lambda^2 - k_j^2}, & \lambda^2 \geq k_j^2. \end{cases} \quad (20)$$

In the general case of an absorbing medium, there exist two branching points of the function $\gamma_j(\lambda)$: the point

$$k_{j,1} = \omega \sqrt{|\varepsilon_j| |\mu_j|} \exp\left(i\left(\arg(\varepsilon_j) + \arg(\mu_j)\right)/2\right)$$

and the point $k_{j,2} = e^{i\pi} k_{j,1}$. An analytic branch of the function $\gamma_j(\lambda)$ suitable for describing materials with negative refractive index should be the same as that for ordinary materials,

either with or without absorption (with appropriate values of permeability and permittivity). This branch can be determined as

$$\gamma_j(\lambda) = \sqrt{|k_{j,1} - \lambda|} \exp\left(\frac{i \arg(k_{j,1} - \lambda)}{2}\right) \sqrt{|k_{j,2} - \lambda|} \exp\left(\frac{i \arg(\lambda - k_{j,2})}{2}\right), \quad (21)$$

where the functions $|\xi|$ and $\arg(\xi)$ are the modulus and the argument of the complex variable ξ .

Important that the considering choice of the analytic branch of the function $\gamma_j(\lambda)$ may be considered as a principle of correspondence of common media and media with negative refraction. The description of common media and media with negative refraction are based on the same analytic branch. There is no physical reason of jumping from one branch to another.

Figure 10 shows why it is the analytic branch (21) that correctly describes both an ordinary lossy medium (in the limit, with infinitely small losses) and a passive (generally speaking, lossy) medium with negative refractive index. Figure 10a represents the real and imaginary parts of the function $\gamma(\lambda)$ described by formula (21) for an ordinary lossy medium with parameters $\varepsilon = 1 + i0.01$ and $\mu = 1 + i0.01$. The imaginary parts of these parameters are chosen solely to make clear the diagrams; the wave frequency is 3 GHz. For such a function $\gamma(\lambda)$, the waves $e^{i\gamma z}$ with positive increasing z will propagate with decay along the z -axis. Figure 10b represents the real and imaginary parts of the function $\gamma(\lambda)$ described by formula (21) for a lossy medium with negative refractive index and with the parameters $\varepsilon = -1 + i0.01$ and $\mu = -1 + i0.01$. For such a function $\gamma(\lambda)$, the waves $e^{i\gamma z}$ with positive increasing z will propagate opposite to the axis z ($\text{Re} < 0$), but they decay along the z -axis; i.e., they decay while propagating away from the wave source. An incorrect choice of the analytic branch of the function $\gamma(\lambda)$ often leads to unphysical results that violate the cause-and-effect relation.

A solution to Eq. (19) in domain 1 is given by the function

$$\tilde{B}_{1x} = \mu_1 (C'_1 e^{-i\gamma_1 z} + C_1 e^{i\gamma_1 z}). \quad (22)$$

If k_1 is real, then, for $\lambda^2 = \xi^2 + \eta^2 > k_1^2$, from (20) we obtain

$$\tilde{B}_{1x} = \mu_1 (C'_1 e^{\sqrt{\lambda^2 - k_1^2} z} + C_1 e^{-\sqrt{\lambda^2 - k_1^2} z}). \quad (23)$$

When $z > 0$, the first term exponentially increases, which contradicts common sense (the field of a source in free space increases at infinity). The integration of the first term by formula (6) yields an infinite integral. Therefore, $C'_1 = 0$, and for $z > 0$ we obtain

$$\tilde{B}_{1x} = \mu_1 C_1 e^{i\gamma_1 z}. \quad (24)$$

Similarly we can show that a solution to Eq. (19) in domain 3 for $z < -h$ is given by

$$\tilde{B}_{3x} = \mu_3 C_3 e^{-i\gamma_3 z}. \quad (25)$$

Fig. 10. Real and imaginary parts of the function $\gamma(\lambda)$ described by formula (21) (a) for an ordinary medium with losses and parameters $\varepsilon = 1 + i0.01$ and $\mu = 1 + i0.01$ and (b) for a medium with losses and negative refractive index for $\varepsilon = -1 + i0.01$ and $\mu = -1 + i0.01$.

In domain 3, Eq. (18) for the electric field is also homogeneous; therefore, applying similar arguments, for $z < -h$ we obtain

$$\tilde{E}_{3x} = S_3 e^{-i\gamma_3 z}. \tag{26}$$

In domain 2, there exist two linearly independent solutions; therefore,

$$\tilde{B}_{2x} = \mu_2 \left(C'_2 e^{-i\gamma_2 z} + C_2 e^{i\gamma_2(z+h)} \right), \quad (27)$$

$$\tilde{E}_{2x} = S'_2 e^{-i\gamma_2 z} + S_2 e^{i\gamma_2(z+h)}. \quad (28)$$

Equation (18) is inhomogeneous in domain 1. A partial solution to this equation is given by

$$\left(\tilde{E}_{1x} \right)_p = -\frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} e^{i\gamma_1 |z-d|},$$

therefore, the general solution is

$$\tilde{E}_{1x} = S'_1 e^{-i\gamma_1 z} + S_1 e^{i\gamma_1 z} - \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} e^{i\gamma_1 |z-d|}.$$

Again, repeating the arguments that a wave cannot increase as $z \rightarrow +\infty$, we find that $S'_1 = 0$. As a result, the electric field in domain 1 is represented as

$$\tilde{E}_{1x} = S_1 e^{i\gamma_1 z} - \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} e^{i\gamma_1 |z-d|}. \quad (29)$$

Let us write the continuity conditions for the tangential components of the electric and magnetic fields, $\tilde{E}_x$ and $\tilde{B}_x/\mu$, on the two boundaries $z = 0$ and $z = -h$:

$$C_1 = C'_2 + C_2 e^{i\gamma_2 h} \quad \text{at } z = 0 \quad (30)$$

$$C'_2 e^{i\gamma_2 h} + C_2 = C_3 e^{i\gamma_3 h} \quad \text{at } z = -h \quad (31)$$

for the magnetic field and

$$S_1 - \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} e^{i\gamma_1 d} = S'_2 + S_2 e^{i\gamma_2 h} \quad \text{at } z = 0 \quad (32)$$

$$S'_2 e^{i\gamma_2 h} + S_2 = S_3 e^{i\gamma_3 h} \quad \text{at } z = -h. \quad (33)$$

for the electric field. We have four equations and eight unknowns. Therefore, we should add equations for the continuity of $\tilde{E}_y$ and $\tilde{B}_y/\mu$ on the two boundaries. However, first, using relations (30)–(33), we eliminate the coefficients describing the fields in domain 2, i.e., C'_2 , C_2 , S'_2 , S_2 . From (30) and (31) we obtain

$$C_2 = \frac{1}{2i \sin(\gamma_2 h)} C_1 - \frac{e^{i\gamma_3 h - i\gamma_2 h}}{2i \sin(\gamma_2 h)} C_3, \quad (34)$$

$$C'_2 = -\frac{e^{-i\gamma_2 h}}{2i \sin(\gamma_2 h)} C_1 + \frac{e^{i\gamma_3 h}}{2i \sin(\gamma_2 h)} C_3, \quad (35)$$

and from (32) and (33) we obtain

$$S'_2 = -\frac{e^{-i\gamma_2 h}}{2i \sin(\gamma_2 h)} S_1 + \frac{e^{i\gamma_3 h}}{2i \sin(\gamma_2 h)} S_3 + \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} \frac{e^{i\gamma_1 d - i\gamma_2 h}}{2i \sin(\gamma_2 h)}, \quad (36)$$

$$S_2 = \frac{1}{2i \sin(\gamma_2 h)} S_1 - \frac{e^{i(\gamma_3 - \gamma_2)h}}{2i \sin(\gamma_2 h)} S_3 - \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} \frac{e^{i\gamma_1 d}}{2i \sin(\gamma_2 h)}. \quad (37)$$

Substituting expressions (34)–(37) into the boundary conditions for the tangential components of the fields $\tilde{E}_{jy}$ and $\tilde{B}_{jy}/\mu_j$ on the boundary $z = 0$, we obtain

$$\tilde{B}_{jy}/\mu_j = (-i\omega \varepsilon_j \partial \tilde{E}_{jx}/\partial z - \xi \eta \tilde{B}_{jx}/\mu_j) / (k_j^2 - \xi^2),$$

$$\frac{\tilde{B}_{1y}}{\mu_1} = \frac{1}{k_1^2 - \xi^2} (-\xi \eta C_1 + \omega \varepsilon_1 \gamma_1 S_1 + \omega \varepsilon_1 \gamma_1 Q),$$

$$\frac{\tilde{B}_{2y}}{\mu_2} = \frac{1}{k_2^2 - \xi^2} \left(-\xi \eta C_1 - i\omega \varepsilon_2 \gamma_2 \operatorname{ctg}(\gamma_2 h) S_1 + i\omega \varepsilon_2 \gamma_2 \frac{e^{i\gamma_3 h} S_3}{\sin(\gamma_2 h)} + i\omega \varepsilon_2 \gamma_2 \operatorname{ctg}(\gamma_2 h) Q \right),$$

where

$$Q = \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} e^{i\gamma_1 d}.$$

Then, the boundary condition for the magnetic field at $z = 0$ is expressed as

$$\begin{aligned} & \frac{1}{k_1^2 - \xi^2} (-\xi \eta C_1 + \omega \varepsilon_1 \gamma_1 S_1 + \omega \varepsilon_1 \gamma_1 Q) = \\ & \frac{1}{k_2^2 - \xi^2} \left(-\xi \eta C_1 - i\omega \varepsilon_2 \gamma_2 \operatorname{ctg}(\gamma_2 h) S_1 + i\omega \varepsilon_2 \gamma_2 \frac{e^{i\gamma_3 h}}{\sin(\gamma_2 h)} S_3 + i\omega \varepsilon_2 \gamma_2 \operatorname{ctg}(\gamma_2 h) Q \right). \end{aligned} \quad (38)$$

Proceeding along the same vein, we obtain a boundary condition for the magnetic field on the second boundary $z = -h$:

$$\begin{aligned} & \frac{1}{k_2^2 - \xi^2} \left(-\xi \eta e^{i\gamma_3 h} C_3 - \frac{i\omega \varepsilon_2 \gamma_2 S_1}{\sin(\gamma_2 h)} + i\omega \varepsilon_2 \gamma_2 \operatorname{ctg}(\gamma_2 h) e^{i\gamma_3 h} S_3 + i\omega \varepsilon_2 \gamma_2 \frac{Q}{\sin(\gamma_2 h)} \right) = \\ & \frac{1}{k_3^2 - \xi^2} (-\xi \eta e^{i\gamma_3 h} C_3 - \omega \varepsilon_3 \gamma_3 e^{i\gamma_3 h} S_3) \end{aligned} \quad (39)$$

To obtain the boundary condition for the electric field on the boundary $z = 0$, we apply the expression

$$\tilde{E}_{jy} = (-\eta \xi \tilde{E}_{jx} + i\omega \partial \tilde{B}_{jx}/\partial z) / (k_j^2 - \xi^2),$$

then the boundary condition is expressed as

$$\begin{aligned} & \frac{1}{k_1^2 - \xi^2} (-\omega\gamma_1\mu_1 C_1 - \eta\xi S_1 + \eta\xi Q) = \\ & = \frac{1}{k_2^2 - \xi^2} \left(i\omega\mu_2\gamma_2 \operatorname{ctg}(\gamma_2 h) C_1 - i\omega\mu_2\gamma_2 \frac{e^{i\gamma_3 h}}{\sin(\gamma_2 h)} C_3 - \eta\xi S_1 + \eta\xi Q \right). \end{aligned} \quad (40)$$

Finally, the boundary condition for the electric field on the boundary $z = -h$ is expressed as

$$\begin{aligned} & \frac{1}{k_2^2 - \xi^2} \left(i\omega\mu_2\gamma_2 \frac{1}{\sin(\gamma_2 h)} C_1 - i\omega\mu_2\gamma_2 \operatorname{ctg}(\gamma_2 h) e^{i\gamma_3 h} C_3 - \eta\xi e^{i\gamma_3 h} S_3 \right) = \\ & = \frac{1}{k_3^2 - \xi^2} (\omega\gamma_3\mu_3 e^{i\gamma_3 h} C_3 - \eta\xi e^{i\gamma_3 h} S_3) \end{aligned} \quad (41)$$

The system of equations (38)–(41) can be represented in the matrix form as $\mathbf{A} \cdot \mathbf{T} = \mathbf{B}$, where the matrix of the problem is given by

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix},$$

where

$$\begin{aligned} A_{11} &= \xi\eta \left(\frac{1}{k_2^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right), \quad A_{21} = 0, \quad A_{31} = -\omega \left(\frac{\gamma_1\mu_1}{k_1^2 - \xi^2} + \frac{i\mu_2\gamma_2 \operatorname{ctg}(\gamma_2 h)}{k_2^2 - \xi^2} \right), \\ A_{41} &= \frac{i\omega\mu_2\gamma_2}{k_2^2 - \xi^2} \frac{1}{\sin(\gamma_2 h)}, \quad A_{12} = 0, \quad A_{22} = \xi\eta \left(\frac{1}{k_3^2 - \xi^2} - \frac{1}{k_2^2 - \xi^2} \right) e^{i\gamma_3 h}, \\ A_{32} &= \frac{i\omega\mu_2\gamma_2}{k_2^2 - \xi^2} \frac{e^{i\gamma_3 h}}{\sin(\gamma_2 h)}, \quad A_{42} = -\omega \left(\frac{\gamma_3\mu_3}{k_3^2 - \xi^2} + \frac{i\mu_2\gamma_2 \operatorname{ctg}(\gamma_2 h)}{k_2^2 - \xi^2} \right) e^{i\gamma_3 h}, \\ A_{13} &= \omega \left(\frac{\varepsilon_1\gamma_1}{k_1^2 - \xi^2} + \frac{i\varepsilon_2\gamma_2 \operatorname{ctg}(\gamma_2 h)}{k_2^2 - \xi^2} \right), \quad A_{23} = -\frac{i\omega\varepsilon_2\gamma_2}{k_2^2 - \xi^2} \frac{1}{\sin(\gamma_2 h)}, \\ A_{33} &= \xi\eta \left(\frac{1}{k_2^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right), \quad A_{43} = 0, \quad A_{14} = -\frac{i\omega\varepsilon_2\gamma_2}{k_2^2 - \xi^2} \frac{e^{i\gamma_3 h}}{\sin(\gamma_2 h)}, \\ A_{24} &= \omega \left(\frac{\varepsilon_3\gamma_3}{k_3^2 - \xi^2} + \frac{i\varepsilon_2\gamma_2 \operatorname{ctg}(\gamma_2 h)}{k_2^2 - \xi^2} \right) e^{i\gamma_3 h}, \quad A_{34} = 0, \quad A_{44} = \xi\eta \left(\frac{1}{k_3^2 - \xi^2} - \frac{1}{k_2^2 - \xi^2} \right) e^{i\gamma_3 h}. \end{aligned}$$

The right-hand side of the matrix equation and the vector of unknown coefficients are given by

$$\begin{aligned} \mathbf{B}^T &= \left[\omega \left(\frac{i\varepsilon_2\gamma_2 \operatorname{ctg}(\gamma_2 h)}{k_2^2 - \xi^2} - \frac{\varepsilon_1\gamma_1}{k_1^2 - \xi^2} \right) Q; \quad -\frac{i\omega\varepsilon_2\gamma_2}{k_2^2 - \xi^2} \frac{Q}{\sin(\gamma_2 h)}; \quad \xi\eta \left(\frac{1}{k_2^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right) Q; \quad 0 \right], \\ \mathbf{T}^T &= [C_1 \quad C_3 \quad S_1 \quad S_3]. \end{aligned}$$

From the equation $\mathbf{A} \cdot \mathbf{T} = \mathbf{B}$ one can determine the column of coefficients $\mathbf{T} = \mathbf{A}^{-1} \cdot \mathbf{B}$ and, as a particular case, $C_3 = T_2$ and $S_3 = T_4$; then, the components of the electric and magnetic fields in domain 3 are determined by the formulas

$$\begin{aligned}\tilde{E}_{3x} &= S_3 e^{-i\gamma_3 z}; \\ \tilde{B}_{3x} &= \mu_3 C_3 e^{-i\gamma_3 z}; \\ \tilde{E}_{3y} &= \frac{1}{k_3^2 - \xi^2} \left(-\eta \xi \tilde{E}_{3x} + i\omega \frac{\partial}{\partial z} \tilde{B}_{3x} \right) = \frac{1}{k_3^2 - \xi^2} (-\eta \xi S_3 + \gamma_3 \omega \mu_3 C_3) e^{-i\gamma_3 z}; \\ \tilde{B}_{3y} &= \frac{1}{k_3^2 - \xi^2} \left(-i\omega \varepsilon_3 \mu_3 \frac{\partial}{\partial z} \tilde{E}_{3x} - \xi \eta \tilde{B}_{3x} \right) = \frac{\mu_3}{k_3^2 - \xi^2} (-\gamma_3 \omega \varepsilon_3 S_3 - \xi \eta C_3) e^{-i\gamma_3 z}; \\ \tilde{E}_{3z} &= \frac{1}{k_3^2 - \xi^2} \left(i\xi \frac{\partial}{\partial z} \tilde{E}_{3x} + \omega \eta \tilde{B}_{3x} \right) = \frac{1}{k_3^2 - \xi^2} (\xi \gamma_3 S_3 + \omega \eta \mu_3 C_3) e^{-i\gamma_3 z}; \\ \tilde{B}_{3z} &= \frac{1}{k_3^2 - \xi^2} \left(-\eta \omega \varepsilon_3 \mu_3 \tilde{E}_{3x} + i\xi \frac{\partial}{\partial z} \tilde{B}_{3x} \right) = \frac{\mu_3}{k_3^2 - \xi^2} (-\eta \omega \varepsilon_3 S_3 + \xi \gamma_3 C_3) e^{-i\gamma_3 z}.\end{aligned}$$

Knowing the Fourier components of the fields, one can determine the field components themselves by applying the inverse Fourier transformation. As a result, for example, the formula for the electric field component $E_{3x}(x, y, z)$ in domain 3 is given by

$$E_{3x}(x, y, z) = (2\pi)^{-2} \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta e^{i\xi x + i\eta y} [\mathbf{A}^{-1} \cdot \mathbf{B}]_4 e^{-i\gamma_3 z}. \quad (42)$$

In the special case of a Hertzian dipole parallel to the boundary of a half-space when the media 2 and 3 are filled with the same material ($\varepsilon_2 = \varepsilon_3$, $\mu_2 = \mu_3$, $k_2 = k_3$, and $\gamma_2 = \gamma_3$), the system of equations (38)–(41) is simplified and reduces to

$$\begin{cases} C_3 = C_1 \\ S_1 = S_3 + Q \\ -\left\{ \frac{\mu_3 \gamma_3}{k_3^2 - \xi^2} + \frac{\gamma_1 \mu_1}{k_1^2 - \xi^2} \right\} \omega C_1 + \left\{ \frac{1}{k_3^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right\} \eta \xi S_3 = 0 \\ \left\{ \frac{1}{k_3^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right\} \xi \eta C_1 + \left(\frac{\varepsilon_3 \gamma_3}{k_3^2 - \xi^2} + \frac{\varepsilon_1 \gamma_1}{k_1^2 - \xi^2} \right) \omega S_3 = -2 \frac{\omega \varepsilon_1 \gamma_1}{k_1^2 - \xi^2} Q \end{cases}.$$

The substitution of

$$Q = \frac{k_1^2 - \xi^2}{2\gamma_1 \omega \varepsilon_1} e^{i\gamma_1 d}, \quad k_1^2 = \omega^2 \varepsilon_1 \mu_1 \quad \text{and} \quad k_3^2 = \omega^2 \varepsilon_3 \mu_3$$

into the last two equations, we obtain precisely the system of equations (3.14) from King & Smith (1981):

$$\begin{cases} \left\{ \frac{\mu_3 \gamma_3}{k_3^2 - \xi^2} + \frac{\gamma_1 \mu_1}{k_1^2 - \xi^2} \right\} C_1 - \left\{ \frac{1}{k_3^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right\} \eta \xi \left(\frac{S_3}{\omega} \right) = 0 \\ - \left\{ \frac{1}{k_3^2 - \xi^2} - \frac{1}{k_1^2 - \xi^2} \right\} \xi \eta C_1 - \left(\frac{k_3^2 \gamma_3}{\mu_3 (k_3^2 - \xi^2)} + \frac{k_1^2 \gamma_1}{\mu_1 (k_1^2 - \xi^2)} \right) \left(\frac{S_3}{\omega} \right) = e^{i \gamma_1 d} \end{cases}.$$

It can be shown that explicit expressions for the coefficients are given by

$$\begin{cases} C_1 = - \frac{\xi \eta (\mu_1 \varepsilon_1 - \mu_3 \varepsilon_3)}{(\mu_1 \gamma_3 + \mu_3 \gamma_1) (\varepsilon_1 \gamma_3 + \varepsilon_3 \gamma_1)} e^{i \gamma_1 d} \\ S_3 = - \frac{\gamma_1 \mu_1 (k_3^2 - \xi^2) + \mu_3 \gamma_3 (k_1^2 - \xi^2)}{\omega (\mu_1 \gamma_3 + \mu_3 \gamma_1) (\varepsilon_1 \gamma_3 + \varepsilon_3 \gamma_1)} e^{i \gamma_1 d} \end{cases}.$$

Then, in the particular case considered, formula (42) reduces to the following explicit form:

$$E_{3x}(x, y, z) = -(2\pi)^{-2} \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta e^{i\xi x + i\eta y} \frac{\gamma_1 \mu_1 (k_3^2 - \xi^2) + \mu_3 \gamma_3 (k_1^2 - \xi^2)}{\omega (\mu_1 \gamma_3 + \mu_3 \gamma_1) (\varepsilon_1 \gamma_3 + \varepsilon_3 \gamma_1)} e^{i \gamma_1 d - i \gamma_3 z}. \quad (43)$$

An explicit expression for the y -component of the electric field is as follows:

$$E_{3y}(x, y, z) = -(2\pi)^{-2} \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta e^{i\xi x + i\eta y} \frac{\xi \eta (\gamma_1 \mu_1 + \mu_3 \gamma_3)}{\omega (\mu_1 \gamma_3 + \mu_3 \gamma_1) (\varepsilon_1 \gamma_3 + \varepsilon_3 \gamma_1)} e^{i \gamma_1 d - i \gamma_3 z}. \quad (44)$$

Passing to cylindrical coordinates, from expressions (43) and (44) we can determine the electric field components in cylindrical coordinates King&Smith (1981) (i.e., in the plane of image perpendicular to the z -axis; further analysis will be focused on the field distribution in this plane):

$$E_{3\rho} = - \frac{\cos \varphi}{4\pi} \int_0^\infty \left\{ \frac{\omega \mu_1 \mu_3}{M} [J_0(\lambda \rho) + J_2(\lambda \rho)] + \frac{\gamma_1 \gamma_3}{\omega N} [J_0(\lambda \rho) - J_2(\lambda \rho)] \right\} e^{i(\gamma_1 d - \gamma_3 z)} \lambda d\lambda \quad (45)$$

$$E_{3\varphi} = \frac{\sin \varphi}{4\pi} \int_0^\infty \left\{ \frac{\omega \mu_1 \mu_3}{M} [J_0(\lambda \rho) - J_2(\lambda \rho)] + \frac{\gamma_1 \gamma_3}{\omega N} [J_0(\lambda \rho) + J_2(\lambda \rho)] \right\} e^{i(\gamma_1 d - \gamma_3 z)} \lambda d\lambda \quad (46)$$

where $M = \mu_1 \gamma_3 + \mu_3 \gamma_1$, $N = \varepsilon_1 \gamma_3 + \varepsilon_3 \gamma_1$, J_n is a Bessel function of order n , and (ρ, φ, z) are cylindrical coordinates (see Fig. 9).

3.2 Results of calculating the radiation from a dipole parallel to the boundary of a half-space with negative refractive index and discussion of these results.

We calculated radiation from a dipole parallel to the boundary of a half-space filled with a medium with negative refractive index (medium 3; the layer with medium 2 is missing). We investigated radiation transmitted into medium 3 in order to study the focusing properties of the medium, i.e., in order to find the dipole field near the point of the geometric-optics image of the dipole. The electric field components parallel to the plane of image were calculated by formulas (45) and (46). We verified that formulas (43) and (44) give identical

results for appropriate components; however, because of the double integration, these calculations take much more time.

The electric field components were calculated for a dipole located at the point with coordinates $x = 0, y = 0, z = d = 1$ m. The radiation frequency is $f = \omega/2\pi = 3$ GHz. In free space, this frequency corresponds to a wavelength of $\lambda_0 = 0.1$ m. All the calculations were carried out for materials with negative refractive index and small absorption ($\varepsilon_3'' = 10^{-3}$ and $\mu_3'' = 10^{-3}$), because the case of low absorption is of primary importance for possible applications.

First, we calculated the electric field for a material with $\varepsilon_3' = -1$ and $\mu_3' = -1$. According to the laws of geometrical optics [see Veselago (2003)], after refraction, the dipole radiation must be focused at a mirror symmetric point with coordinates $x = 0, y = 0, z_{im} = -d$. Note that if we substitute the relative permittivity and permeability into the equations for the fields, we should multiply these equations by the permittivity ε_0 and permeability μ_0 of vacuum.

The distributions of $|E_{3\rho}|_{\varphi=0}$ in the xz -plane (i.e., in the E plane of the dipole) and of $|E_{3\varphi}|_{\varphi=\pi/2}$ in the yz -plane (i.e., in the H plane of the dipole) are shown in Fig. 11. One can see that the radiation is indeed focused to a spot centered at the point of the geometric-optics image. The dimensions of the spot at the level of 0.707 of amplitude (at the half power level) are $w_E \approx 0.0589$ m in the E plane and $w_H \approx 0.0422$ m in the H plane.

Next, we carried out calculations for a material with $\varepsilon_3' = -2$ and $\mu_3' = -2$. It is reasonable to assume that the focal spot moves to a point with coordinates $x = 0, y = 0, z_{im} = -2d = -2$ m according to the laws of geometrical optics. Since the wavelength in the medium is half that in vacuum, we can assume that the dimensions of the spot are also reduced by a factor of two. Calculations have shown that this is not the case. We have found that, although the position of the image spot on the z -axis, determined by the maximum of the field, approximately corresponds to the geometric-optics position $z_{im} \approx -2.45$ m, the dimensions of the spot in the E and H planes at the level of 0.707 of amplitude are greater than the expected values and are equal to $w_E \approx 0.0675$ m and $w_H \approx 0.0621$ m, respectively.

After that, we carried out calculations for a material with $\varepsilon_3' = -10$ and $\mu_3' = -10$. Again, according to the laws of geometrical optics, it is reasonable to assume that the image point has coordinates $x = 0, y = 0, z_{im} = -10d = -10$ m, and the spot is ten times smaller than that in the first case. Again, calculations have shown that the image plane moves to the point $z_{im} \approx -12.4$ m, and the dimensions of the spot in the E and H planes of the dipole at the level of 0.707 of amplitude are much greater than the expected values, namely, $w_E \approx 0.0701$ m and $w_H \approx 0.0662$ m. In this case ($\varepsilon_3' = -10$ and $\mu_3' = -10$), the distributions of $|E_{3\rho}|_{\varphi=0}$ in the xz -plane (in the E plane of the dipole) and $|E_{3\varphi}|_{\varphi=\pi/2}$ in the yz -plane (in the H plane of the dipole) are shown in Fig. 12. Note that, when the absorption of a medium with negative refractive index is small, as in the case considered, and the modulus $|n|$ of the refractive index is large, the image spot depends not on the relation between $|\varepsilon_3|$ and $|\mu_3|$ but only on the product $|\varepsilon_3\mu_3|$. This fact is illustrated by Eqs. (45) and (46), in which the properties of medium 3 are represented by the product $\varepsilon_3\mu_3$. This property was verified directly by numerical calculation: the dimensions of the image in the case of $\varepsilon_3' = -10$ and $\mu_3' = -10$ are almost the same as in the case of $\varepsilon_3' = -100$ and $\mu_3' = -1$.

Fig. 11. Distributions of normalized (to maximum) moduli (a) $|E_{3\rho}|_{\varphi=0}$ in xz -plane and (b) $|E_{3\rho}|_{\varphi=\pi/2}$ in the yz -plane in a half-space with negative refraction in the case of $\varepsilon'_3 = -1$ and $\mu'_3 = -1$.

a)

b)

Fig. 12. Same as in Fig. 11 for the case of $\varepsilon'_3 = -10$ and $\mu'_3 = -10$.

We determined the distance between the boundary and the image plane (where the electric field of the wave attains its maximum) as a function of the modulus of the complex refractive index $|n| \approx \sqrt{|\varepsilon_3||\mu_3|}$ (see Fig. 13). For $|n| \geq 4$, the curve can be approximated by the straight line $z_{im} = -1.243|n|d$.

Fig. 13. The distance from the plane boundary to the image (i.e., to the maximum-signal point) as a function of the modulus of the refractive index $|n| \approx \sqrt{\epsilon_3 |\mu_3|}$. The dashed line represents the function $z_{im} = -1.243|n|d$, which approximates the above function for large values of $|n|$.

Based on the function $z_{im} = z_{im}(|n|)$, we calculated the dimensions w_E and w_H of the spots, normalized by the wavelength λ_0 in free space, as a function of $|n|$. The curves are shown in Fig. 14. The functions $w_E = w_E(|n|)$ and $w_H = w_H(|n|)$ have a minimum at $|n|=1$ and tend to constant values greater than $\lambda_0/2$ as $|n| \rightarrow \infty$. This result is very important. It implies that the use of the plane boundary of a homogeneous material with negative refractive index as an elementary lens does not improve the quality of the image; therefore, such a lens cannot overcome the diffraction limit.

There is a simple explanation for this fact. The dipole radiation excites a certain area of size $D \sim 2d$ on the interface between media. In a medium with negative refraction, this excited surface area works as a lens (antenna) with effective aperture of about D . The wavelength in a material with negative refractive index is $\lambda_{NR} = \lambda_0/|n|$. The distance from this lens to the plane of the image is $|z_{im}| \sim |n|d$ (as found for large values of $|n|$). However, it is well known that the dimensions of the image can be estimated as

$$w \sim |z_{im}| \lambda_{NR} / D \sim (|n| d \lambda_0 / |n|) / D \sim d \lambda_0 / D \sim \lambda_0 / 2 = const .$$

Thus, for large values of the modulus $|n|$ of the refractive index, the dimensions of the image are nearly constant and do not depend on $|n|$.

The minimum on the curve in the region where $\epsilon'_2 \approx -1$ and $\mu'_2 \approx -1$ may be explained by the absence of achromatic aberration of this elementary lens for the particular case.

Fig. 14. Normalized half-power widths (or widths at an amplitude level of 0.707) of the image in the E and H planes of the dipole as a function of $|n|$ (λ_0 is the radiation wavelength in free space).

In [Petrin A. B. (2008)], it was shown that, as $\varepsilon_2'' \rightarrow 0$ and $\mu_2'' \rightarrow 0$, the dimensions of the focusing region vary by at most 30% and this variation occurs only in the domain where $\varepsilon_2' \approx -1$ and $\mu_2' \approx -1$; i.e., our conclusions about the absence of superresolution remain valid even in the limit case of a lossless material with negative refractive index.

3.3 Results of calculating the dipole radiation transmitted through a layer with negative refractive index and discussion of these results.

The results of the previous section show that the focusing of radiation from a point source of electromagnetic radiation due to refraction on the interface between air and a half-space with negative refractive index does not provide a superresolution predicted in [Pendry J. B. (2000)]. To close the question completely, we considered focusing of radiation of a point source transmitted through a layer of medium with negative refractive index. According to [Pendry J. B. (2000)], such a layer represents a lens.

We calculated the electric field for a layer of medium of thickness $h = 2\text{m}$ with parameters $\varepsilon_2' = -1$, $\varepsilon_2'' = 10^{-3}$; $\mu_2' = -1$, $\mu_2'' = 10^{-3}$ (see Fig. 9). Media 1 and 3 are air; i.e., $\varepsilon_1' = \varepsilon_3' = 1$, $\varepsilon_1'' = \varepsilon_3'' = 0$; $\mu_1' = \mu_3' = 1$ and $\mu_1'' = \mu_3'' = 0$. The radiation frequency is the same as in the previous section (3 GHz). According to the laws of geometric optics [see Pendry (2000) and Veselago (2003)], after refraction, the dipole radiation should be focused at a point with coordinates $x = 0$, $y = 0$, and $z_{im} = -(d + h) = -3\text{ m}$. The electric field in domain 3 was calculated by formula (42). Figure 15 shows the distribution of the modulus of the electric

field $|E_{3x}(0, 0, z)|$ along the z -axis near the point $z_{im} = -3$ m of the geometric-optics image. One can see that the field attains its maximum at the point $z_{im} = -3$ m. However, the size of the distribution is on the order of λ_0 . After that, we determined the distributions of the modulus of the electric field in the plane of the image (in the plane passing through the point $z_{im} = -3$ m perpendicular to z -axis) on the intersection lines with the E and H planes of the dipole.

Fig. 15. Normalized (to maximum) distribution of the electric field modulus $|E_{3x}(0, 0, z)|$ along the z -axis near the point of the geometric-optics image $z_{im} = -3\text{m} = -30\lambda$.

The distribution of the modulus of the electric field $|E_3| = |E_{3x}(x, 0, z_{im})|$ along the intersection line of the xz -plane (i.e., the E plane of the dipole) and the plane of image is shown in Fig. 16a. A similar distribution $|E_3| = |E_{3x}(0, y, z_{im})|$ along the intersection line of the yz -plane (i.e., the H plane of the dipole) and the plane of image is shown in Fig. 16b. Figures 16 show that radiation is indeed focused to a spot centered at the point of the geometric-optics image. The dimensions of the spot at the amplitude level of 0.707 (at the half-power level) are $w_E \approx 0.0594$ m in the E plane and $w_H \approx 0.0432$ m in the H plane. These results allow us to draw an important conclusion: the distribution of the electric field modulus is slightly expanded after refraction on the second boundary. In other words, superresolution in such a system is not observed.

Thus, the results of the investigation lead to a unanimous conclusion that the use of materials with negative refractive index does not allow one to reach a superresolution of noncoherent objects (to overcome the diffraction limit in far zone).

Fig. 16. Normalized (to maximum) distributions of the electric field moduli (a) $|E_3| = |E_{3x}(x, 0, z_{im})|$ and (b) $|E_3| = |E_{3x}(0, y, z_{im})|$ along the intersection lines of the xz - and yz -planes, respectively, with a plane passing through the image point $z_{im} = -3m = -30\lambda$.

4. Conclusion

In this chapter it was proved the fundamental result of diffraction theory: *for any focusing system the existence of singularity at image point of a point source is impossible in principle*. The result was illustrated for the important focusing system consisted of a layer of material with negative refraction (the superlens).

5. References

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In the recent decades, there has been a growing interest in micro- and nanotechnology. The advances in nanotechnology give rise to new applications and new types of materials with unique electromagnetic and mechanical properties. This book is devoted to the modern methods in electrodynamics and acoustics, which have been developed to describe wave propagation in these modern materials and nanodevices. The book consists of original works of leading scientists in the field of wave propagation who produced new theoretical and experimental methods in the research field and obtained new and important results. The first part of the book consists of chapters with general mathematical methods and approaches to the problem of wave propagation. A special attention is attracted to the advanced numerical methods fruitfully applied in the field of wave propagation. The second part of the book is devoted to the problems of wave propagation in newly developed metamaterials, micro- and nanostructures and porous media. In this part the interested reader will find important and fundamental results on electromagnetic wave propagation in media with negative refraction index and electromagnetic imaging in devices based on the materials. The third part of the book is devoted to the problems of wave propagation in elastic and piezoelectric media. In the fourth part, the works on the problems of wave propagation in plasma are collected. The fifth, sixth and seventh parts are devoted to the problems of wave propagation in media with chemical reactions, in nonlinear and disperse media, respectively. And finally, in the eighth part of the book some experimental methods in wave propagations are considered. It is necessary to emphasize that this book is not a textbook. It is important that the results combined in it are taken “from the desks of researchers”. Therefore, I am sure that in this book the interested and actively working readers (scientists, engineers and students) will find many interesting results and new ideas.

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