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# Modelling Driving Forces of Urban Growth with Fuzzy Sets and GIS

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## Abstract

Urban growth occurs in conjunction with a series of decision-making processes and is, on the whole, not deterministic but rather is the outcome of competing local demands and uncontrolled, chaotic processes. Fuzzy sets theory is ideally suited to treat the complexity and uncertainties in the decision-making process. This chapter presented an example of how fuzzy sets can be applied to model urban growth driving forces within geographical information system environment. The mathematical models for measuring, computing and defining 10 fuzzy urban growth factors to form fuzzy driving forces of urban growth in Riyadh City, Saudi Arabia, were discussed. Four factors were considered as the driving forces for urban growth in Riyadh City: the transport support factor, urban agglomeration and attractiveness factor, topographical constraints factor, and planning policies and regulations factor. The urban growth factors were established using fuzzy set theory, which quantified the effect of distance decay using fuzziness. This approach is a transparent method of interpreting the curve of distance decay using linguistic variables. This feature does not exist in the linear, negative exponential or inverse power functions. The results indicate that fuzzy accessibility and fuzzy urban density factors are capable of mimicking and representing the uncertainty in the behaviour of the human decision-making process in land development in a very efficient manner.

**Keywords:** urban growth, fuzzy sets, GIS, driving forces, Riyadh, Saudi Arabia

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## 1. Introduction

Urban growth takes place in conjunction with a series of decision-making processes. This is a feature of the complexity and uncertainties of reality and decision-making in planning. While matters of high policy regarding urban growth are determined by senior urban planners and elected personnel, site selection is usually a subjective process which follows individuals'

preferences. This process is, on the whole, not deterministic, but is the outcome of competing local demands and uncontrolled and chaotic decision-making processes. This approach is founded on the supposition that people in general use linguistic constructs for the evaluation of environmental or social situations [1]. It is, therefore, valid to specify rules on the basis of studied judgment, as opposed to definitive attributes. Heuristic decision-making processes would thus guide the rules of urban growth [2]. As the complexity of an urban system increases, the utility of traditional mathematical methods decreases. Where urban growth decisions are noted for a high level of complexity and uncertainty, this is more relevant [1]. Proponents of this approach contend that for handling uncertainties and complexity in decision-making, fuzzy sets and fuzzy logic are ideally suited [3, 4].

Uncertainty, under the fuzzy approach, is assumed to have an inherently vague nature such that it becomes identified through 'possibility' and not 'probability', which is the norm in statistical calculations [2]. Modelling urban development, then, is made possible by integrating heuristic decision-making processes with simulation such as cellular automata (CA). 'Heuristic' is taken to mean that the transition rule is not decided on the basis of numbers but rather on linguistic variables using fuzzy sets that describe certain preconditions of a decision. Development propensities are, then, assessed not using explicit mathematical formula but by adopting indistinct and subjective natural language statements using linguistic variables [1, 2]. For example, where a house is in close proximity to a road, the accessibility can be rated as being 'Very good access'. If a system is represented by a series of 'linguistic variables' such as 'good', 'very good' or 'moderate' [2, 5–8], a concept of degree of membership is defined in a fuzzy set. This is employed in illustrating the extent of uncertainties within the fuzzy set. The processes of 'fuzzification' and 'defuzzification' make it possible to model the behaviour of human thought in decision-making in an appropriately multifaceted manner [3]. Fuzzy set theory is used in many different fields of study to address a range of imprecise problems and issues [9]. Since mathematics deals with ideal figures, it does not always provide accurate representations for modelling the behaviour of complex systems. Zadeh [10] proposed a solution through a different set of computations. The cloudy quantities or mathematics of fuzzy are not realized with respect to statistical probability distributions. While the fuzzy set theory is conceptualised to deal with mathematical problems which have only approximate values and in which events are vague, fuzzy logic is applied to show how fuzzy variables interact in a complex system [3].

Several applications of fuzzy set theory for spatial planning have been attempted in recent years. Notable amongst these have been the multi-dimensional evaluation of areas on the land market [1]. Leung [11] applied fuzzy linear programming for urban land-use planning. Fuzzy clustering methods have been employed in transport modelling [12] and in geo-demographic [13]. Davidson *et al.* [14] and Hall *et al.* [15] have made use of applications of fuzzy set theory and fuzzy logic in land suitability analysis. In Expert and Spatial Decision Support Systems, Leung and Leung [16] have proposed an expert system shell for an intelligent knowledge-based geographic information system (GIS) that is governed by a fuzzy logic inference engine. For long-term agricultural planning, Davidson *et al.* [14] developed a fuzzy logic-based land evaluation. Indeed, academic literature investigating the possible integration of fuzzy reasoning and fuzzy query into GIS has flourished since the 1990s [6, 7, 17–19].

According to Tobler's so-called first law of geography, 'everything is related to everything else but near things are more related than distant things' ([20], pp. 236). It is implicit that there is a spatial interaction between any two localities, and this interaction can be measured by a distance decay function which is based on the premise that the interaction between two localities declines as the distance increases. Most urban models quantify the distance decay effect of accessibility using either a negative exponential function or an inverse power function [21–24]. These two methods describe, however, distance decay as a rapid decline in accessibility with distance from a centre. There are some difficulties when one tries to interpret these calibrated parameters [25, 26]. However, a quantified distance decay effect of accessibility using fuzzy sets provides a more realistic set of results and an efficient manner in terms of interpretation. The calibrated parameters of fuzzy distance decay can directly be read and understood from the shape of the membership function [27]. Therefore, the meaning of the figure can be read linguistically and describes an individual's preference for selecting an urban area.

A fuzzy cellular urban growth model (FCUGM), which is capable of simulating and predicting the complexities of urban growth for the city of Riyadh, Saudi Arabia, was presented by Al-Ahmadi et al. [27–31]. The work within this chapter has reviewed the driving forces of urban growth in both the scientific literature and in the context of Riyadh, and also reviewed the principles of fuzzy set theory and presented an example of how these principles are applied for modelling urban growth driving forces. Additionally, the mathematical models for measuring, computing and defining fuzzy input variables (accessibility and urban density (UD)) and fuzzy driving forces of urban growth of Riyadh were also discussed.

## 2. Driving forces of urban growth

Urban growth indicates a transformation of a vacant land or a natural environment to the urban fabrics including residential, industrial and infrastructure developments. It mostly occurred in the firings of urban areas. Urban expansion is driven by many factors such as population growth and urbanization, the development of urban economy, the increase of investment on fixed assets such as infrastructure (utility networks and roads) and the increase of income and living standard in urban areas (better housing, health, social and recreational services) [32]. According to the scientific literatures, these factors are considered to some extent as driving forces in both western and eastern countries. However, the driving factors of urban growth might differ from one place to another, according to political, economic, demographic, social and physical factors.

In terms of the driving forces that have been used in most urban models to model urban growth of cities, the main urban growth forces mentioned already can be dealt with as spatial factors such as (1) travel time or spatial distance to road networks, railways or subways, (2) travel time or spatial distance to a town centre or a metropolitan sub-centres, (3) development density, (4) proximity to protected sources (such as drinking water, forest or wetland), (5) physical properties such as altitude, slope or soil and geology type, (6) proximity to socioeconomic services, commercial

or industrial districts, (7) environmental suitability or (8) Government's zoning regulations. Although most of the studies in the literature relate to different contexts such as countries in North America, Europe and Asia, there are very limited studies being applied to investigate the relationship between spatial patterns of urban growth and its driving forces for Middle East cities such as Riyadh.

Given that, in this research, the urban growth driving forces were selected based on three criteria: (1) characteristics of urban growth in Riyadh based on the understanding of the historical documents, master plans, and its attached studies and opinion of expert planners, (2) scientific literature and (3) some empirical structural analysis. According to these three criteria, in this research, four factors are considered as the driving forces for urban growth: (1) transport support factor (TSF), (2) urban agglomeration and attractiveness factor (UAAF), (3) topographical constraints factor (TCF) and (4) planning policies and regulations factor (PPRF). These four driving forces are formed by integrating 10 input variables (in this research). **Table 1** illustrates the input variables and its corresponding driving forces. However, the authors argue that these driving forces are potentially comprehensive and can be applied in most contexts, since they are capable of covering most of the urban growth factors which have previously been used in the study as the driving forces of urban growth. These four driving forces can be seen as general containers, where one can remove or add different input variables for a driving force according to the context (different study areas) and data availability.

This section gives an examination of the four 'driving forces' (TSF, UAAF, TCF and PPRF) and their corresponding 10 'input variables' listed in **Table 1**. Firstly, each driving force will be examined in the context of international scientific literature and, secondly, in the context of Riyadh city.

Raw data	Input files	Input variables	Driving forces
Road network maps	Road network	Accessibility to local road (ALR)	TSF
Road network maps	Road network	Accessibility to main road (AMR)	TSF
Road network maps	Road network	Accessibility to major road (AMJR)	TSF
Satellite images	Classified urban areas	Urban density (UD)	UAAF
Land-use maps	Town centre	Accessibility to town centre (ATC)	UAAF
Land-use maps	Employment centres and Socioeconomic services	Accessibility to employment centres and socioeconomic services (AECSES)	UAAF
DEM	Topographical characteristics	Slope gradient (SG)	TCF
DEM	Topographical characteristics	Altitude (A)	TCF
Master plans	Planning and zoning regulation areas	Planned areas (PA)	PPRF
Master plans	Planning and zoning regulation areas	Excluded areas (EA)	PPRF

TSF: transport support factor; UAAF: urban agglomeration & attractiveness factor; TCF: topographical constraints factor; PPRF: planning policies and regulations factor.

**Table 1.** The raw data, input files, input variables and driving forces of urban growth in Riyadh.



## 2.1. Transport support factor

The transport network has proved to be the key factor in the urban evolution process because of its influence over urban form and land price. Although the relationships between the road network and the urban form are rarely apparent, it is considered an intersecting process. Transport is considered as a significant factor in accelerating urban growth and attracting new development. It offers easy and efficient access from a certain location to other supporting activities such as employment centres, socioeconomic services and amenities. An efficient and affordable transport network increases the accessibility to land, and land with good accessibility is usually more attractive for urban development. The use of a transport network as a factor influencing urban growth can be traced back to Alonso's [33] theory of urban land markets. Before that, of course, Van Thunen's [34] theory of agricultural rent, firms and households established the basic economic relationships. These theories can be used to demonstrate that land or locations with good accessibility are more attractive and have a higher market value than less accessible locations.

One of the early attempts in United States to examine the inter-relationships between transport and urban development was by Hansen [35] in which he argued that locations with good accessibility had a higher chance of being developed and at a higher density, than less accessible or remote locations. Torrens [36] argued that developing transport systems and improving accessibility will alter the decisions of landlords, developers, firms and households about where to locate a new business or a home. For example, the private automobile along with good transport networks has made most areas across a city almost evenly appropriate as a place to live or work [37]. But according to travel-cost minimization theories and discrete choice theories, each household pursues a location that yields the greatest utility or satisfaction by minimizing the travel time or travel cost from a place to live to human activities such as working, shopping, education or leisure [36, 38].

Transport is a crucial factor influencing land use and is usually one of the chief factors in contemporary urban land-use models and urban CA models. It can be noted apparently that the accessibility of urban land to the transport network factor has been widely employed in most urban CA models [22, 24, 39–45]. With respect to Riyadh, the early development of the city was tightly connected to the development of its road networks. Since 1970, the urban area of Riyadh expanded outwards along the radial roads. Most urban construction was carried out alongside roads for easy access to reduce development costs. Therefore, transport is a vital factor that should be introduced into the model in this research. The concept of accessibility is widely used in urban and regional planning, land use and transport modelling. Accessibility has been defined 'as the ease with which activities at a given destination may be reached from an origin location using a particular mode of transport' ([36], pp. 49). Urban accessibility can be defined 'as the capability to get from place to place and the connectivity of a place with other places' ([46], pp. 1). Accessibility is the spatial constraint on urbanization and can be measured by the distance, travel cost, travel time or several other measures based on these variables. From a spatial perspective, accessibility to different human activities is a major factor in urban development [36, 46].

Thus, the transport support factor (TSF) is chosen in this research as a driving force. In the Riyadh context, it is formed by integrating three types of roads—accessibility to local road

(ALR), accessibility to main road (AMR) and accessibility to major road (AMJR). As such, integration is more appropriate in modelling practices because it reduces the number of variables that lead to a more understandable model.

#### *2.1.1. Accessibility to local road*

The road network of Saudi Arabia is classified based on the width. Roads with a width between 3 and 20 m are called local roads. Most residential uses are located along the sides of this class of roads, and the neighbourhood usually has a relatively low urban density (<10 dwellings/hectare). In addition, light commercial uses (groceries) and educational services (primary or intermediate schools) that serve one quarter of the neighbourhood, which extends over an area typically of about 4 km<sup>2</sup>, are located on local roads [47]. The availability of a paved local road is a further indicator that land adjacent to a local road is ready for development, that is, mostly covered with key infrastructure such as electricity supply, water and drainage, sewage and telephone lines.

#### *2.1.2. Accessibility to main road*

Roads with a width between 30 and 40 m are called main roads. Most mixed uses such as residential and commercial land use are located beside main roads, and the density of the population is commonly average or medium (20–40 dwellings/hectare) [47]. Commercial use includes retail, education (high schools), light industrial uses and services (administrative offices, branch of companies) that serve one neighbourhood (approximately 2 × 2 km) and are usually located on main roads.

#### *2.1.3. Accessibility to major road*

Roads with a width more than 40 m are called major roads. They are typically found in commercial areas (firms and establishments), governmental uses (ministries and main administrative offices) and some mixed uses, that is, residential/commercials are located along such roads. The urban density is generally high (>40 dwellings/hectare). Additionally, the high levels of educational services (universities and colleges) and heavy industrial uses (factories) that serve the whole city are usually located on main roads.

### **2.2. Urban agglomeration and attractiveness factor**

UAAF driving force comprises three input variables including urban density (UD), accessibility to employment centres and socioeconomic services (AECSES) and accessibility to town centre (ATC). Of course, other factors can also be included such as accessibility to sub-centres.

#### *2.2.1. Urban density*

Proximity to an existing urban area is a significant factor in land development. Non-developed land adjacent to an existing development usually has a higher possibility of imminent development than other land. Land near to urban agglomerations is more attractive for households and developers, because they can take advantage of the existing facilities, infrastructures and

commercial and social activities such as electricity, water supply, drainage and sewage, telephone lines and business opportunities. Many urban CA models include urban density or development density, which is the proportion of developed land within a specific area (neighbourhood), as a factor for urban development [22, 24, 39–45]. In this research, the number of neighbouring developed cells that surround a non-developed cell influences the transformation of the state of that non-developed cell. For instance, a central cell with a non-urban state located within a neighbourhood with a high urban density (the majority of cells are urban cells) will be forced to transform into an urban state by its urban neighbours.

### *2.2.2. Accessibility to employment centres and socioeconomic services*

The AECSES is an important factor for residential development because most people wish to minimise commuting distance, time or cost from home to work. This argument can be supported by the notion of Lowry's framework that examines the relationship between residential development and employment centres. Lowry's [48] Model of Metropolis, which is the first operational attempt to implement an urban land-use transport model, proceeds on the assumption that the place of residence is governed by the place of employment, that is, jobs decide where people live, and this idea continues to be at the heart of several of the contemporary urban models [36, 37]. With respect to Riyadh city, in this research, the AECSES factor includes most employment centres (such as administrative offices, firms and establishments) and services (such as schools, hospitals, mosques and community centres). AECSES has been considered as an urban growth factor in much of the urban CA literature [2, 40, 41, 44, 45].

### *2.2.3. Accessibility to town centre*

The ATC factor is important for urban development because the town centre of the capital of Saudi Arabia (SA), Riyadh city, functions as the key administrative and commercial centre not only for Riyadh but for all surrounding regions. According to the bid-rent theory [34], the new development will happen as near as possible to the city centre while at the same time trying to minimize the land cost which controversially may lead to a trend of finding new sites far away from the urban centre. Land development usually takes place around the city centres first, then further away. The town centre provides better facility services, more working and business opportunities, etc. In Riyadh, the town centre includes Government offices, ministries, retail centres and headquarters of major companies. Hence, the city provides opportunities that are unlikely to be found anywhere else in Riyadh city. This does not necessarily mean that living in the town centre is the most appropriate choice but the greater the distance to the town centre, the less attractive is a location for development. This factor is usually included in urban CA models [24, 42, 49].

## **2.3. Topographical constraints factor**

The physical characteristics of a site or a location can be an important factor in deciding whether or not to proceed to develop the site. The physical characteristics typically include the sub-surface and surface geology, soil, elevation, slope, vegetation and other factors. Although all of these physical characteristics could be considered for use in modelling urban growth in this



research, there is no reason to consider soil, geology or vegetation in the case of Riyadh because most of the soils are sandy, the geology is stable and there is very little vegetation for much of the year. Hence, slope and altitude or elevation are the only physical factors included in the model. But other factors could, of course, be considered and used in the model if they were felt to be important at a different geographical location.

### *2.3.1. Slope gradient*

The slope of the land surface is important because it influences the use that can be made of the land, costs of building houses, access roads and other infrastructure and safety. Urban growth can be constrained by slopes, typically the steeper a slope the greater the constraint. For example, land with steep slopes requires substantially more investment to develop. Land-owners or developers would develop a site if they judge that they can make a profit on a development site with very steep slopes. The profit margins can be calculated as a function of the compromise between input costs (the costs of developing a site) and the anticipated selling price of the development [36]. The steepness of slopes has been considered to be a significant factor influencing the attractiveness of a site and its potential for development in many scientific research papers [22, 39, 43, 49].

### *2.3.2. Altitude*

Altitude can similarly affect the attractiveness of a site for development. If a site is low-lying, it might be subject to flooding; if it is at a relatively high altitude, it might be more or less attractive depending on the local topography, scenery and views. The topography of Riyadh area includes mountains and relatively large variations in altitude from 200 to 1200 m above the sea level. Urban development can occur in areas of steep terrain and high altitude under certain circumstances, for instance, where there is great demand for building land and there is a short supply of relatively flat land, or when people with high income prefer to live in elevated areas in order to secure attractive scenic views from their windows [43]. But the cost of building in general is greater with an increase in elevation. Altitude is therefore one of the prime factors in the model of this research.

## **2.4. Planning policies and regulations factor**

Just as urban growth is generally constrained by physical or site-specific factors, it is also constrained by institutional controls and planning policies. The urban growth process is subject to local planning regulations and policies imposed by the national, regional or local authorities. Planning regulations and policies serve diverse purposes but one of the most important is to control the urban expansion and prevent unplanned urban sprawl. Wegener [50] argued that uncontrolled urban sprawl in the USA was generally regarded to be undesirable, often results in unidentified and undistinguished neighbourhoods, reduces or damages open spaces, annihilates natural habitats and leads to a significant increase in traffic volume. The PPRF has therefore been used in several urban CA models [22, 24, 43]. The PPRF is introduced to the model in this research as two variables: (1) planned area (PA) and (2) excluded area (EA).

#### 2.4.1. Planned area and excluded area

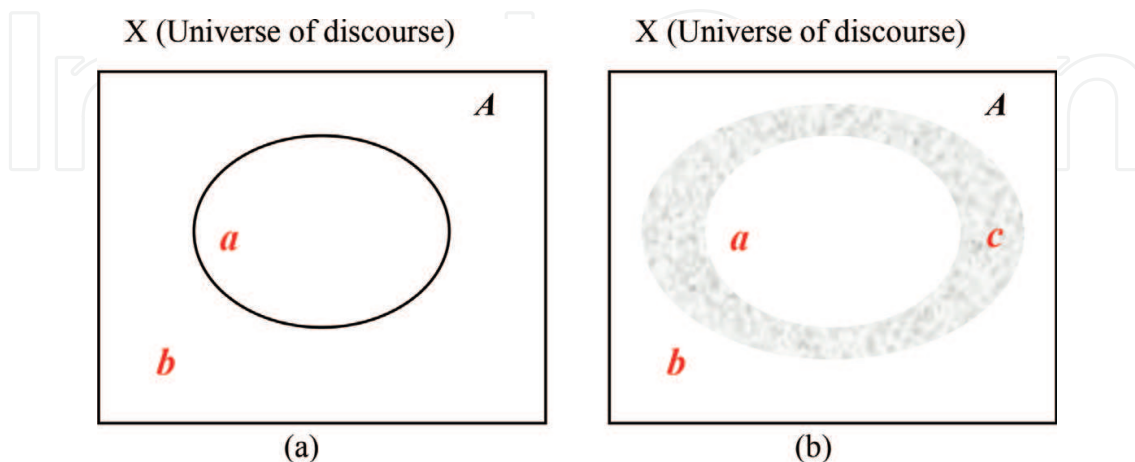
The planned area refers to areas outlined in the Master Plan of Riyadh and depicts the spatial boundary of the area planned for urban growth. The excluded area denotes the area where urban growth cannot be considered either owing to its physical characteristics such as river courses, low-lying valleys subject to seasonal flooding, or water bodies, or to cultural, environmental or governmental policies to conserve the land for cultural pursuits such as heritage places, wildlife-protected areas, groundwater areas or pollution mitigation areas. The EA cannot be developed because it is forbidden by law or by urban-zoning regulations and policies.

### 3. Fuzzy sets

In set theory, a classical set is usually defined as a collection of abstract objects or elements, which are characterized as a finite, distinct, crisp, sharp, countable object that has an unambiguous boundary. The universe of discourse or the set domain is all possible elements which a set can contain [51]. The classical set is defined by distinct or crisp boundaries, that is there is no uncertainty about the objects or location of the boundaries of the set, as shown in **Figure 1a**, where the boundary of the set  $A$  is an unambiguous line [9].

In classical set theory, a single element is assessed using Boolean logic; the element either belongs or does not belong to the set [52]. This can be expressed with the *characteristic function* for the elements of a given universe to belong to a certain subset of this universe. Mathematically, a set can be expressed as follows:

Let  $X$  be a universe of discourse (collection of objects), whose generic element is denoted as  $x$ . Thus,  $X = \{x\}$ . A crisp set  $A$  in  $X$  is described by the *characteristic function*, whereby  $\chi_A$  of  $A$  is defined as  $\chi_A: X \rightarrow \{0, 1\}$  and expressed mathematically as follows:



**Figure 1.** Diagram for (a) crisp set boundary and (b) fuzzy set boundary (after Ross, [9]).

$$XA(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases} \quad (1)$$

where the symbol,  $\in$ , denotes membership and  $\notin$ , non-membership. It maps the universe to the set of only two elements, 0 and 1. One can indicate whether an element belongs or does not belong to a distinct or a crisp set. Some degree of uncertainty can, however, be allowed as to whether or not an element belongs to a set. This can be expressed by the membership of an element to a set by its membership function using fuzzy sets. 'A fuzzy set is described by vague or ambiguous properties; hence its boundaries are ambiguous, as illustrated by the fuzzy boundary for set  $A$  in **Figure 1b**. This shows the vague, ambiguous boundary of a fuzzy set  $A$  on the discourse universe  $X$ , the shaded boundary representing the boundary region of  $A$ . In the central (unshaded) region of the fuzzy set, a point  $a$  is clearly a member of the set, while point  $b$  is clearly not a member. However, the membership of point  $c$ , which is on the boundary region, is ambiguous. If point  $a$  is represented by 1 (complete membership) and point  $b$  with 0 (no membership), then point  $c$  must have some intermediate value of membership (partial membership) in fuzzy set  $A$  on the interval  $[0, 1]$  ([9], pp. 24).

Fuzzy set theory was inspired and developed by [8, 10, 53] in order to extend classical set theory for handling soft (continuous) rather than crisp classifications. A fuzzy set can be defined as a collection of objects in which the transition from a full membership to a non-membership is gradual rather than abrupt [10]. A set can be regarded as fuzzy if an element can partly belong to it, rather than having to belong completely or not at all. Consequently, fuzzy set theory assigns membership grades to objects which are not restricted to 0 (non-membership) or 1 (full-membership), but which may locate somewhere between 0 and 1 [43]. A fuzzy set can be expressed mathematically as follows:

A fuzzy set  $A$  of a universe  $X$  is defined by a *membership function*  $\mu_A$  (vs. *characteristic function*  $XA$  in a crisp set) such that  $\mu_A: X \rightarrow [0,1]$ . A fuzzy set  $A$  in  $X$  is a set of ordered pairs,

$$A = \{(x, \mu_A(x)) \mid x \in X\} \quad (2)$$

where  $\mu_A(x)$  represent the grade of membership of  $x$  in  $A$ , which associate with each  $x$  a real number in  $[0,1]$ , and the universe  $X$  is always a crisp set.

Degree of vagueness or uncertainty is usually exhibited by many phenomena. The boundaries of spatial features, for example, often are not clearly defined. Ideas and impressions like 'steep', 'close' or 'suitable' can be better articulated with degrees of membership to a fuzzy set, rather than with a binary yes/no classification. In human thinking and language, we often use uncertain or vague concepts. Our thinking and language is not binary, that is, black and white, zero or one, yes or no. In real life, we add much more variation to our judgements and classifications. These vague or uncertain concepts are understood and assumed to be fuzzy. We encounter fuzziness almost everywhere in our everyday lives [54]. With respect to this study of urban growth, we might look for a suitable site to develop land for housing. The criteria describing the area that we are looking for could be formulated as shown in **Table 2**.

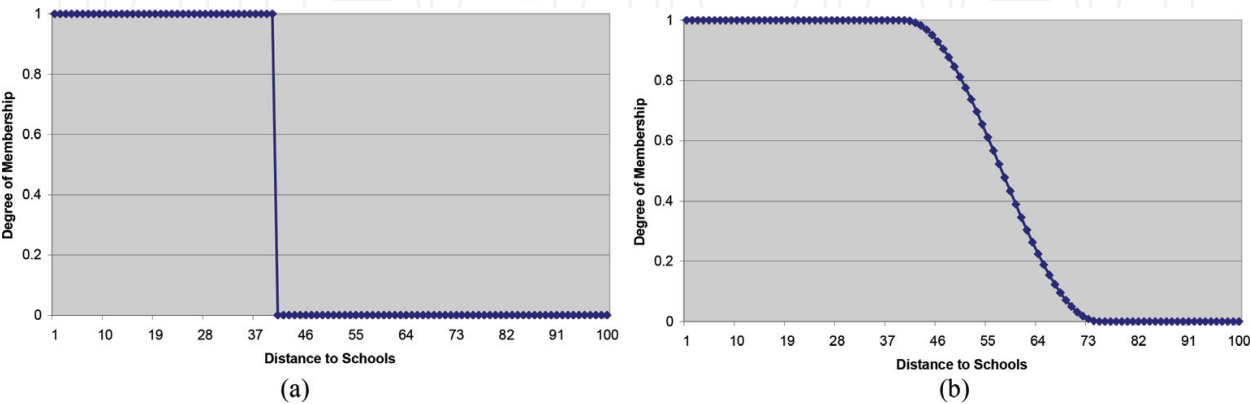
The site should:

Fuzzy	Crisp
have a <i>moderate</i> altitude	have altitude between 800 and 1000 m
have a <i>favourable</i> slope	have slope less than 8°
have a <i>favourable</i> aspect	have aspect between 175 and 250°
be <i>nearby</i> schools	be within 40 m to schools
be <i>not close</i> to an airport	not be within 1 km of an airport

**Table 2.** Example of differences between fuzzy and crisp in terms of site suitability.

All of the conditions mentioned in the fuzzy column are vague and uncertain but correspond to the way we express these conditions in our languages and thinking. Using the classical approach, the fuzzy conditions would be translated into distinct, sharp or crisp classes, such as those in the crisp column. Taking the school criteria as an example, if a location falls within the given criteria (be within 40 m of a school), we would accept it for developing, otherwise it would be excluded from analysis (**Figure 2a**). If, however, we allow degrees of membership to our classes, we can accommodate those locations that just miss a criterion by a few metres. They may get a low degree of membership, but will be included in the analysis as shown in **Figure 2b**, where locations that have distance to school within 40 m assigned full membership (1.0), and those locations above 40 and below 70 m still have membership but decrease gradually as the distance increases, and those locations with distance over 70 m assigned no-membership (0.0), that is, they are definitely not nearby schools.

The fuzzy set membership function is a crucial component of fuzzy set theory and is one of the building blocks of a fuzzy model because it is used to define the model variables, which are based upon imprecise and abstract concepts, for example, a ‘young’ or ‘old age’, or a ‘short’ or ‘long’ distance [55]. Although the shape and form of the membership function should accurately reflect the semantics of the underlying concept in order to represent the real world as closely as possible, fuzzy systems tend to be extremely tolerant of variations in fuzzy set representations, which makes the resulting models highly robust [55, 56]. Different membership functions



**Figure 2.** (a) Crisp set of school criteria and (b) fuzzy set of school criteria.

represent different fuzzy sets, even though they may have a similar context. A membership function can generally assume any shape although the surface tends to be a continuous line from the left to the right edge of the set. Engineering applications tend to use one of the following parameterized membership functions: Gaussian, triangular, trapezoidal or sinusoidal [51]. The trapezoidal and sinusoidal functions can present different shapes: symmetric, monotonically decreasing or monotonically increasing. The different shapes of membership functions are controlled by different parameters in terms of the number and place. There are several different ways of deciding where and how many membership functions to place on a variable domain. The most important requirement is that they overlap, but the optimal number will be determined by experiment and the complexity of the problem. Ross [9] outlines a series of methods which are used to decide on the placement of membership functions, including the use of intuition or expert knowledge, genetic algorithms and neural networks. For example, triangular is widely used in engineering applications, whereas symmetric sinusoidal is commonly used for spatial or geographical applications.

## 4. Results and discussion

### 4.1. Fuzzy accessibility

Accessibility to local road (ALR), accessibility to main road (AMR), accessibility to major road (AMJR), accessibility to town centre (ATC) and accessibility to employment centres and socio-economic services (AECSES) were created by computing accessibility to these variables using a Euclidean distance algorithm [27]. The FCUGM tends, however, not to use the direct result of applying this technique. This is because the Euclidean distance is based on a spatial linear function as opposed to the nature of urban development. This results in the FCUGM transforming the linear results of applying the Euclidean method into a non-linear fuzzy measure of accessibility using fuzzy set theory.

The first step for measuring accessibility is to calculate the Euclidean distance. This can be defined as the straight-line distance between two points on a plane [57]. Euclidean distance, or distance 'as the crow flies', can be calculated using Pythagoras's theorem as expressed in Eq. (3). The Euclidean distance was calculated using ESRI ArcGIS 9.2 software as follows [57]:

$$ED_{AB} = \sqrt{AC^2 + CB^2} \quad (3)$$

where

$ED_{AB}$  is the Euclidean distance between the centres of two locations A and B;

C is the hypotenuse of the right triangle;

After the Euclidean distance has been calculated for a particular input variable, the resultant layer is weighted and standardized (1–100) using the FCUGM according to Eq. (4).



$$WSED_{ijk} = \frac{ED_{ijk} - \min[ED_{ijk}]}{\max[ED_{ijk}] - \min[ED_{ijk}]} * W_k * 100 \quad (4)$$

where

$WSED_{ijk}$  is the weighted standardized score of Euclidian distance of a cell  $ij$  for input variable  $k$  (falling within the range 1–100)  $ED_{ijk}$  is the original value of Euclidian distance of a cell  $ij$  for input variable  $k$

$W_k$  is the parameter weight for input variable  $k$

After the Euclidean distance has been calculated for a particular input variable, the resultant layer is weighted and standardized. This was followed by transforming the weighted standardized accessibility layer into a fuzzy accessibility measure using the FCUGM. Although there are several types of membership functions (fuzzy sets) such as triangular, trapezoidal, Gaussian, bell-shaped or sinusoidal, the author adopted the sinusoidal function for the FCUGM. The rationale for this choice was based on consideration of three criteria: (1) non-linear parameterization, (2) shape of curve or slope that was relatively easy to interpret and (3) the function that is smoothly and gradually descending. There are different shapes of the sinusoidal function; however, the decrease monotonically has been employed in this research to calculate the accessibility of input variables. This is because according to the results of the structural analysis conducted on the input variables [31], a relatively similar shape was found as monotonically decreasing. The sinusoidal function is defined by Eq. (5) and is controlled by the two parameters  $P1$  and  $P2$  which generally control the slope of the curve as shown in **Figure 3**.  $P1$  is equivalent to the maximal elemental value of the domain that acquired a membership degree of 1.0 (definite membership to a particular set), while  $P2$  is equivalent to the minimal elemental value of the domain which acquired a membership degree of 0 (no membership). For a given elemental value,  $sed_{ijk}$ , in the domain, the membership value can be computed as follows:

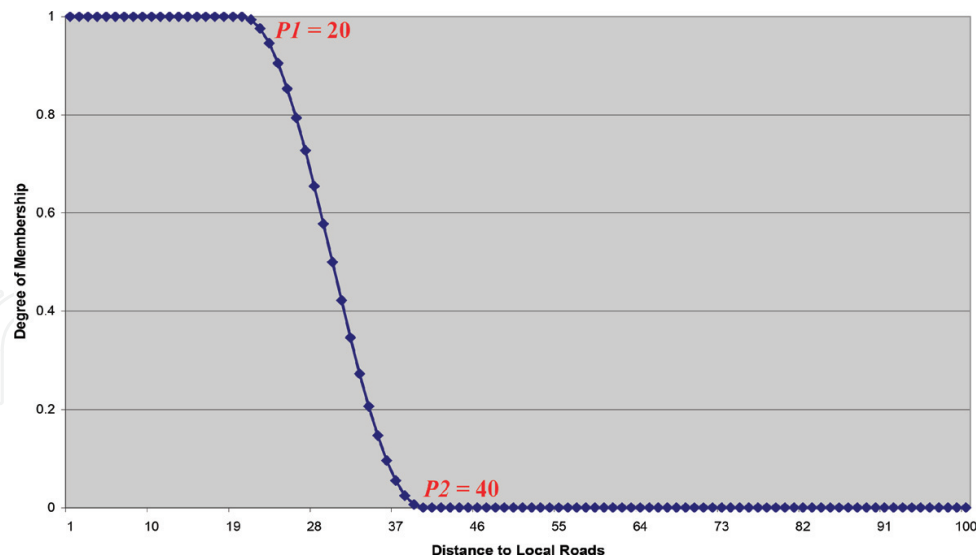
$$\mu_{Fuzzy A}(wsed_{ij}) = \begin{cases} 1 & 0 < wsed_{ijk} < P1 \\ \frac{1}{2} (1 + \cos(\pi (wsed_{ijk} - P1) / (P2 - P1))) & P1 \leq wsed_{ijk} \leq P2 \\ 0 & wsed_{ijk} > P2 \end{cases} \quad (5)$$

where

$wsed_{ijk}$  is the weighted standardized score of Euclidean distance of a cell  $ij$  for input variable  $k$ , and (falling within the range 1–100)

$P1$  and  $P2$  are the parameters that quantify the distance decay effects of accessibility from a cell  $ij$  to input variable  $k$ .

$\mu_{Fuzzy A}(wsed_{ij})$  is the fuzzy accessibility (e.g. to local roads) of a cell  $ij$  for input variable  $k$  (falling within the range 1–100);

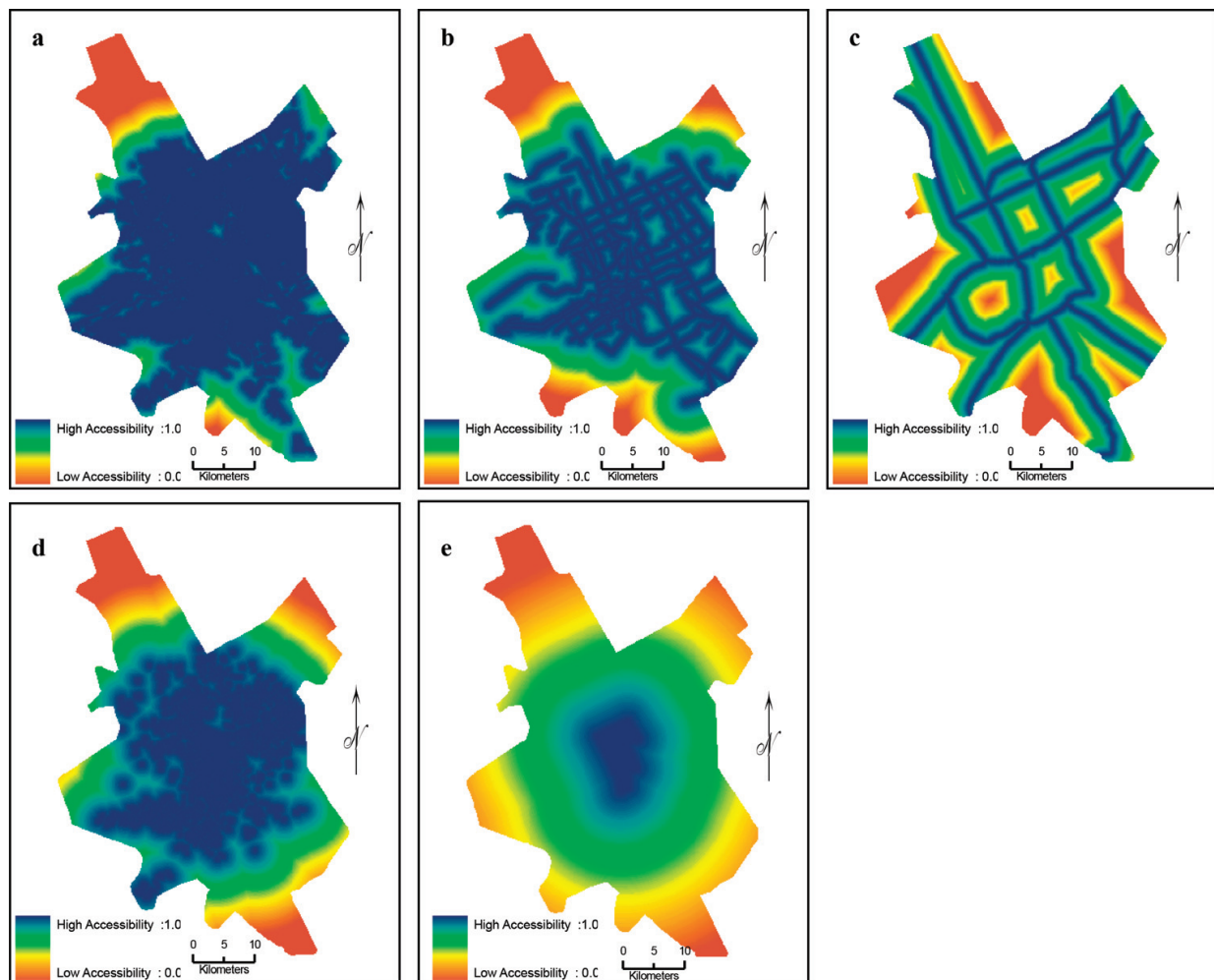


**Figure 3.** Accessibility to main road using fuzzy set.

The two parameters  $P1$  and  $P2$  are critically important for urban growth because they reflect people's behavioural inclination over decisions about living *near* or *far* from a certain variable. In other words, the two parameters  $P1$  and  $P2$  tend to quantify the distance decay effect of a particular variable, that is, to what extent people prefer to live near or far from local, main or major roads, employment centres or the town centre. In the FCUGM, as the distance between cell  $x_{ij}$  and an input variable  $k$  (such as local roads) increases, the accessibility to that variable decreases, and the quantity of that decreases is determined by the two parameters  $P1$  and  $P2$  which are calibrated [27].

Using the approach described earlier, five input variables of the FCUGM are generated through computing fuzzy accessibility to their features including local road, main road, major road, employment centres and socioeconomic services and town centre. Firstly, the linear Euclidean accessibility was generated for the five input variables; **Figure 4** shows the resultant linear accessibility of the five input variables.

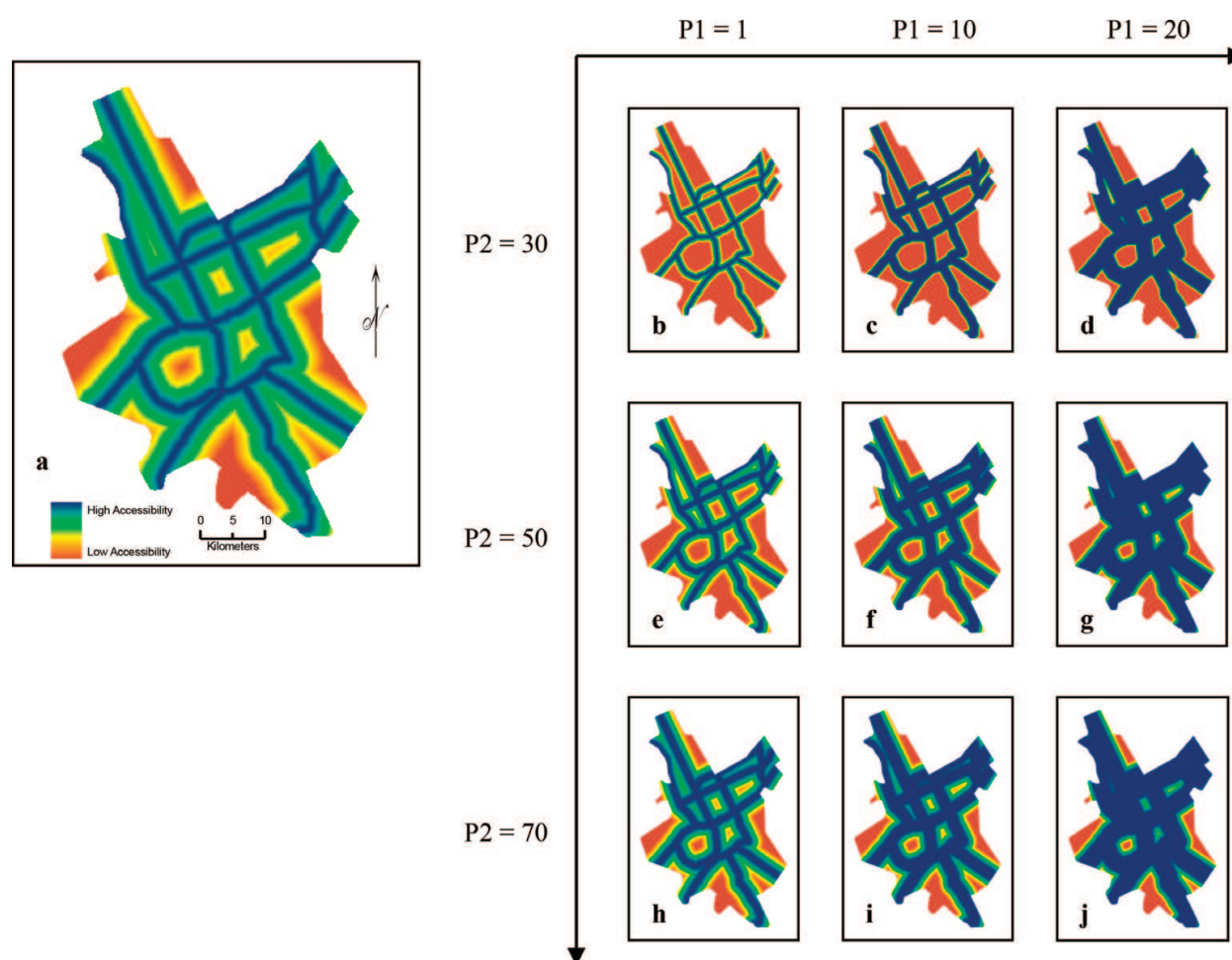
The process of measuring fuzzy accessibility (Eq. 3) is performed using the 'Fuzzifier' model in the SIM-FCUGM [27–29]. As explained previously, the fuzzy accessibility is controlled by two fuzzy distance decay parameters denoted as  $P1$  and  $P2$ . As has been argued previously,  $P1$  and  $P2$  are of great importance as they reflect people's behavioural inclination over decisions about living *near* or *far* from certain variables (e.g. local roads). Thus, in order to investigate the effects that  $P1$  and  $P2$  might play in terms of spatial accessibility, different values of these two parameters were tested on fuzzy accessibility to major roads. **Figure 5** shows an example of the original linear Euclidean accessibility to major roads along with nine fuzzy accessibility maps in accordance with three different values of  $P1$  and three different values of  $P2$ . It can be seen from the original linear Euclidean (**Figure 5a**) that the accessibility has a linear effect at a distance. This means that as the distance to the major roads gets larger, the accessibility gets lower. This has the knock-on effect of meaning that land with low accessibility will have a low potential of being developed, which is not reflecting the behaviour of decision-makers in terms



**Figure 4.** Euclidean linear accessibility to input variables: local roads (ALR), main roads (AMR), major roads (AMJR), employment centres and socioeconomic services (AECSSES) and town centre (TC) of Riyadh city in 2005 (a–e, respectively).

of developing lands. In contrast, it can be noted from the nine fuzzy maps that both high and low accessibility areas might have a non-linear development potentiality by trading off values of  $P1$  and  $P2$  parameters.

By examining both maps and figures in **Figures 5** and **6**, it can be inferred that the fuzzy accessibility to several features can be characterized through three parts of membership including ‘full’, ‘part’ and ‘no’ memberships as illustrated in **Figure 5a** and **6a**. Areas with ‘full’ (dark blue) and ‘no’ (brown) membership denote the areas with the highest and lowest accessibilities, respectively, so they are crisp (0 or 1) and *compact* to either class. In contrast, ‘part’ membership areas are those in which the accessibility decreases gradually from ‘full’ to ‘no’ membership, and these areas are *smooth*. The value of  $P1$  controls the amount of areas that have ‘full’ membership, while  $P2$  controls the areas of ‘no’ membership. As the value of  $P1$  increases, the areas of high accessibility increase; in contrast, as the value of  $P2$  decreases, the areas of low accessibility increase. The three maps b, c and d of **Figure 5** and the three figures b, c and d of **Figure 6**, for example, show that the size of areas with high accessibility (blue)

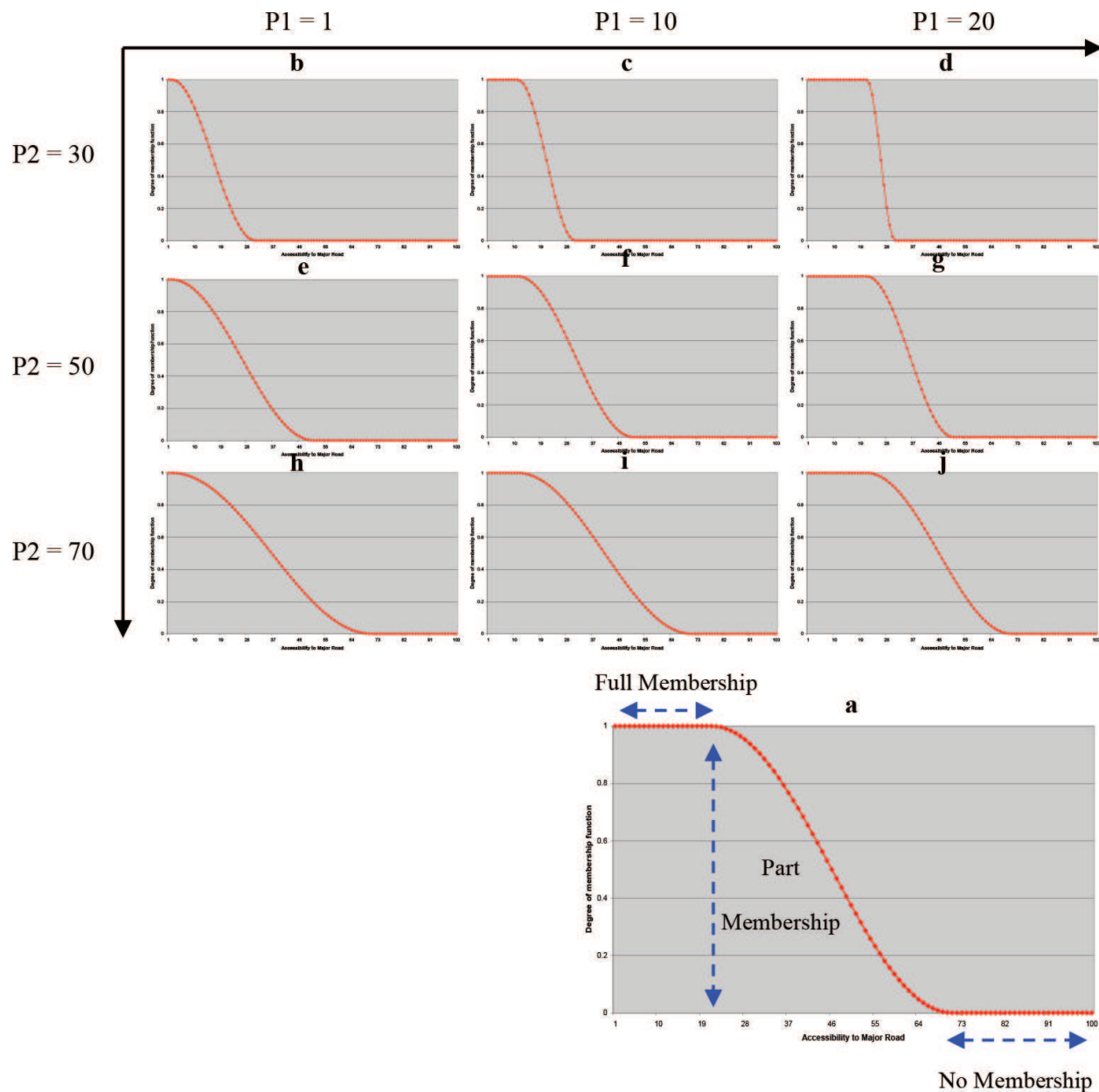


**Figure 5.** Original linear Euclidean accessibility (a) and fuzzy accessibility to major roads (b–j).

getting larger as the values of  $P1$  increase. The three maps and figures b, e and h, on the contrary, show that the size of areas with low accessibility (brown) gets larger as the values of  $P2$  decrease. Interestingly, the diagonal from the top-left to bottom-right exhibits the levels to which both parameters are traded off against each other. Additionally, the difference between  $P1$  and  $P2$  values controls the size of areas in which the accessibility surface decreases smoothly, which represents the area of ‘part’ membership.

It can be seen from both **Figures 5** and **6** that fuzzy accessibility has a high capability to mimic and represent the uncertainty in the behaviour of the human decision-making process in land development in a very efficient way. For example, one can represent the behaviour of people in three aspects ‘full membership’, ‘part membership’ and ‘no membership’ for those people who prefer to live ‘near’, ‘fairly near’ and ‘far’ from a certain feature, respectively. Thus, the numerical distances using fuzzy sets are converted to linguistic words and can be modified easily by parameters  $P1$  and  $P2$ . This feature does not exist in the linear, negative exponential or inverse power functions. While the latter two functions can generate non-linear accessibility, the meanings of their parameters are somewhat difficult to interpret. It can be seen that as the difference between the two parameters gets larger, the size of areas with ‘part’ membership





**Figure 6.** Fuzzy membership functions of the nine maps shown in **Figure 5**.

gets greater. The three maps and figures b, e and h, for example, show the gradual increase of the smooth areas, represented by green to yellow areas in maps and areas under the curve in figures. In particular, the b and h maps represent the lowest ( $30-1 = 29$ ) and highest ( $70-1 = 69$ ) differences between  $P1$  and  $P2$ , respectively, which as a result decrease and increase the smooth accessibility areas.

The lower value of  $P1$  results in compact areas with high accessibility but only for those cells situated close to the feature of interest. This implies that only areas located very near to major roads have a high potential for development compared to other areas (see maps b, e and h in **Figure 5** and figures b, e and h in **Figure 6**).



In contrast, the higher value of P1 expands the size of areas with high accessibility further outward from the feature of interest (see maps d, g and i in **Figure 5** and figures d, g and i in **Figure 6**). The value of P2 decreases, however, as the size of areas with low accessibility increases. This results in more cells possessing a low potential of being developed. Therefore, the lower the value of P2, the less the areas of land for development and vice versa (see maps b, e and h in **Figure 5**). It can be deduced that P1 and P2 parameters attempt to represent the behaviour of people when deciding whether to develop land near or far from an input variable (in the above example of major roads). The diagonal from the top-left to bottom-right of **Figures 5** and **6** shows three different behaviours of people, those who prefer to develop land *very close*, *moderately close* and *far* to major roads. The optimal values of P1 and P2 parameters were calibrated [27].

#### 4.2. Fuzzy urban density

The urban density variable is broadly calculated based on the proportion of developed cells within a neighbourhood (kernel matrix). The configuration of a neighbourhood, such as its shape and size might, however, have an influence on the resulting urban density layer. It has been argued that the circular shape of a neighbourhood has no bias in any direction as opposed to rectangular or square neighbourhoods which generate significant distortions [58]. As a result, the FCUGM adopts a circular shape of neighbourhood (kernel matrix). In terms of the size of the neighbourhood, according to [59], there are no rules of thumb or theoretical basis for specifying the neighbourhood size when computing the urban agglomeration variable (urban density or development density). In most urban CA models, the neighbourhood size is decided arbitrarily based on the view of the modeller or the objective of the study. In contrast, in the FCUGM, the neighbourhood size is considered as a parameter that is calibrated and optimized along with other parameters during the calibration process. The size of a neighbourhood is calibrated [27] to generate the urban density variable, and its optimized size value reflects the degree of pressure resulting from the developed urban density within a neighbourhood of a non-developed cell. The urban density input variable is computed using Eq. (6).

$$UD_{ij} = \frac{\sum D_{ijk}}{\pi\zeta^2} \quad (6)$$

where

$UD_{ij}$  is the development urban density of a cell  $ij$ ;

$D_{ijk}$  is the number of urban developed cells in the neighbourhood  $k$  of the cell  $ij$  and  $\zeta$  is the radius parameter of the circular neighbourhood  $k$ .

Once the urban density has been calculated, the resultant layer is weighted and standardized (1–100) by the FCUGM using Eq. (7).

$$WSUD_{ij} = \frac{UD_{ij} - \min[UD_{ij}]}{\max[UD_{ij}] - \min[UD_{ij}]} * W_k * 100 \quad (7)$$

where

$WSUD_{ij}$  is the weighted standardized score of the urban density of a cell  $ij$ ;

$UD_{ij}$  is the original value of urban density of a cell  $ij$  and

$W_k$  is the parameter weight for input variable  $k$ .

This is followed by transforming the weighed standardized urban density layer into a fuzzy urban density using the FCUGM. The membership function type, shape and parameters that are used for measuring the fuzzy accessibility are applied for the fuzzy urban density as well and calculated as shown in Eq. (8).

$$\mu_{Fuzzy\ UD}(wsud_{ij}) = \begin{cases} 0 & wsud_{ij} < P1 \\ \frac{1}{2} (1 - \cos(\pi (wsud_{ij} - P1) / (P2 - P1))) & P1 \leq wsud_{ij} \leq P2 \\ 1 & wsud_{ij} > P2 \end{cases} \quad (8)$$

where

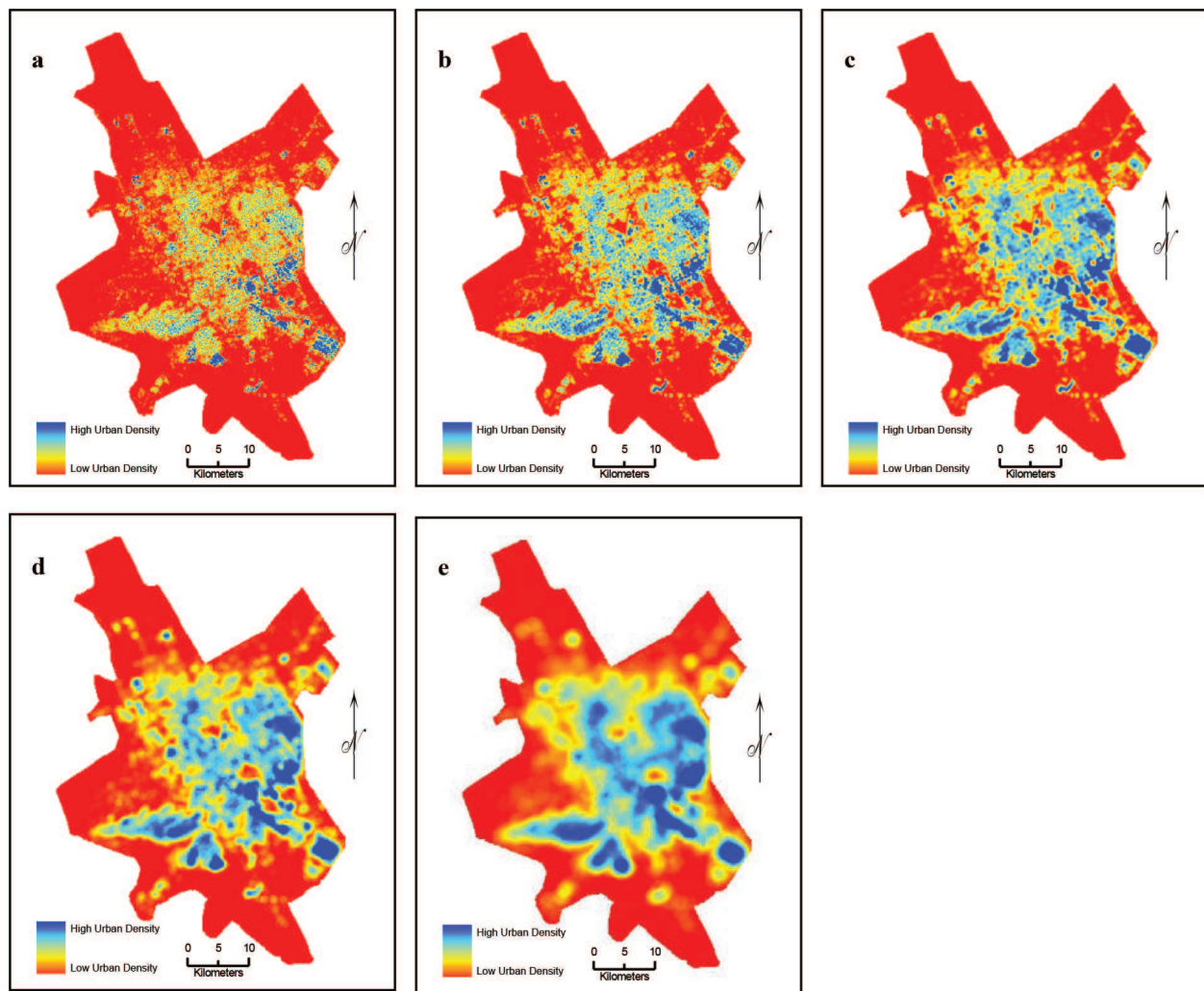
*Fuzzy UD* is the standardized score of urban density of cell  $ij$  for input variable  $k$  (falling within the range 1–100);

$wsud_{ij}$  is the weighted standardized score of the urban density of a cell  $ij$  and

$P1$  and  $P2$  are the parameters that quantify the distance decay effects of urban density for a cell  $ij$ .

The only difference is that urban density is calculated using monotonically increasing sinusoidal function, rather than decreasing one as used for calculating the accessibility, because increasing shape could better fit the effect of urban density on urban expansion as found in the structural analysis [31]. By the monotonically increasing function, those high values of the domain (in this case urban density values) acquired a membership degree of 1.0, and because urban expansion took place in relatively high density urban areas, such shape is employed. As previously discussed, developers prefer to build near to existing developed land to take advantage of available services, facilities and infrastructure. The urban density input variable is computed by calculating the proportion of developed lands within a neighbourhood. This process is performed using the 'Fuzzifier' in the SIM-FCUGM [27–29]. Urban density is a smooth surface controlled by the size of a neighbourhood along with two distance decay parameters  $P1$  and  $P2$ . In the FCUGM, the size of a neighbourhood ranges between 2 and 40 cells, with a cell size of 20 m.

**Figure 7** shows the urban density of Riyadh city in 2005, with five different sizes of neighbourhood including 2.5, 5, 10, 20 and 40. It can be seen from **Figure 7** that as the neighbourhood increases in size, the urban density surface becomes smoother and the size of areas with high urban density also increases. In contrast, as the size of the neighbourhood decreases, the surface becomes more compact and areas with low urban density increase. This is mainly because a large neighbourhood size has a higher potential for urban cells to be located within it. One might assume that the small size of the neighbourhood restricts high urban density



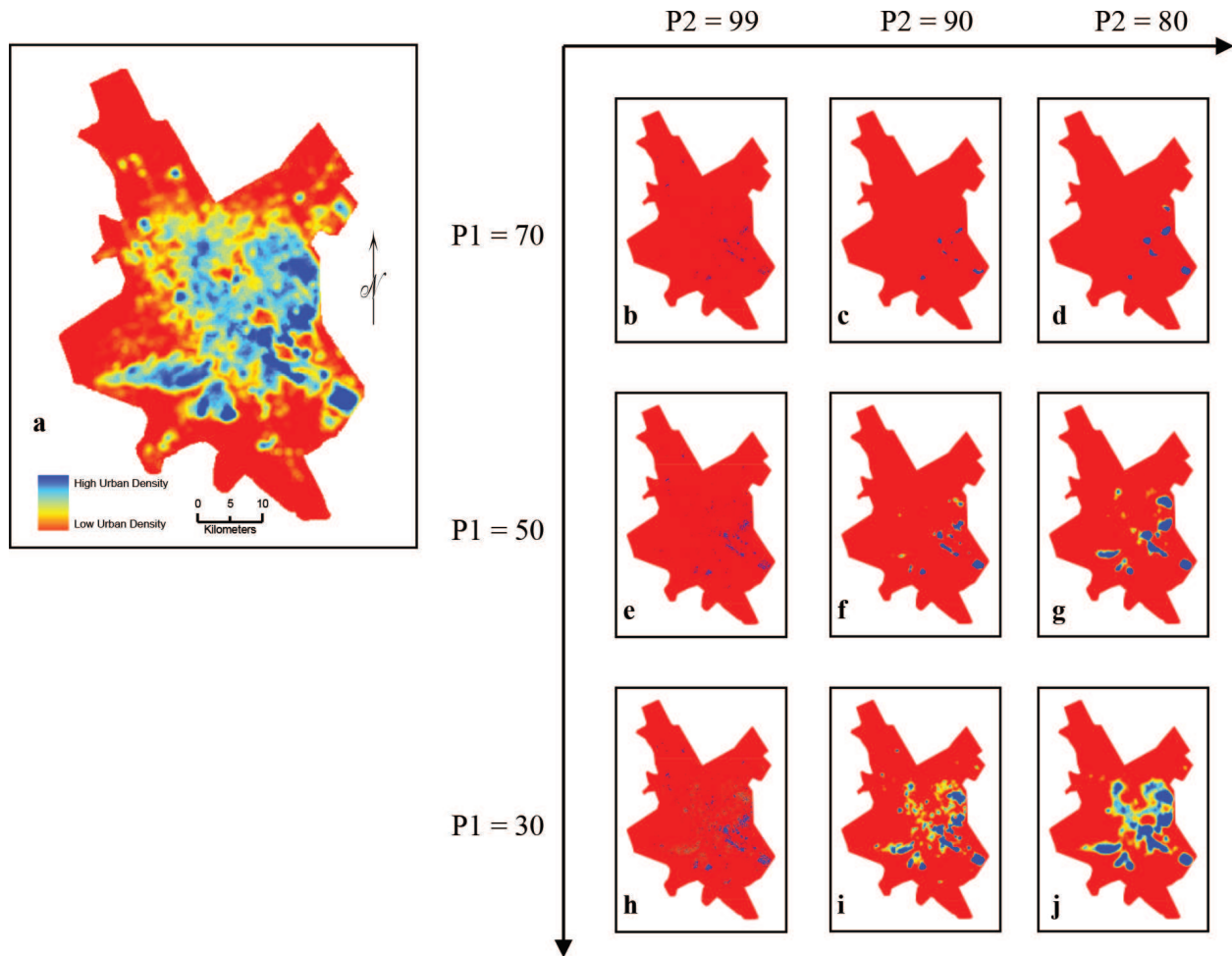
**Figure 7.** Urban density of Riyadh city in 2005 with five different neighbourhood sizes 2.5, 5, 10, 20 and 40 (a–e), respectively.

areas to being close to existing built-up areas. On the contrary, a large neighbourhood results in high urban density areas expanding outwards from the existing built-up areas.

In terms of the distance decay parameters P1 and P2, the two parameters for *urban density* have the same general effect as accessibility with little difference in interpretation of the parameters' effects. According to **Figure 8**, as the value of P2 decreases, only areas in close proximity to the very high urban density increase (dark blue areas in maps b, c and d decrease with a decrease of P2 value and become more compact). As the value of P1 decreases, areas with a very high density increase all over the city (dark blue areas in maps b, e and h increase with a decrease of P1 value and become more scattered). The difference between P1 and P2 values controls the size of areas in which the urban density surface decreases smoothly (sky blue to yellow).

#### 4.3. Fuzzy constraints factors

The slope and altitude input variables are derived directly from digital elevation model (DEM) using ESRI ArcGIS 9.2 software [60]. This step is followed by three procedures for weighting,

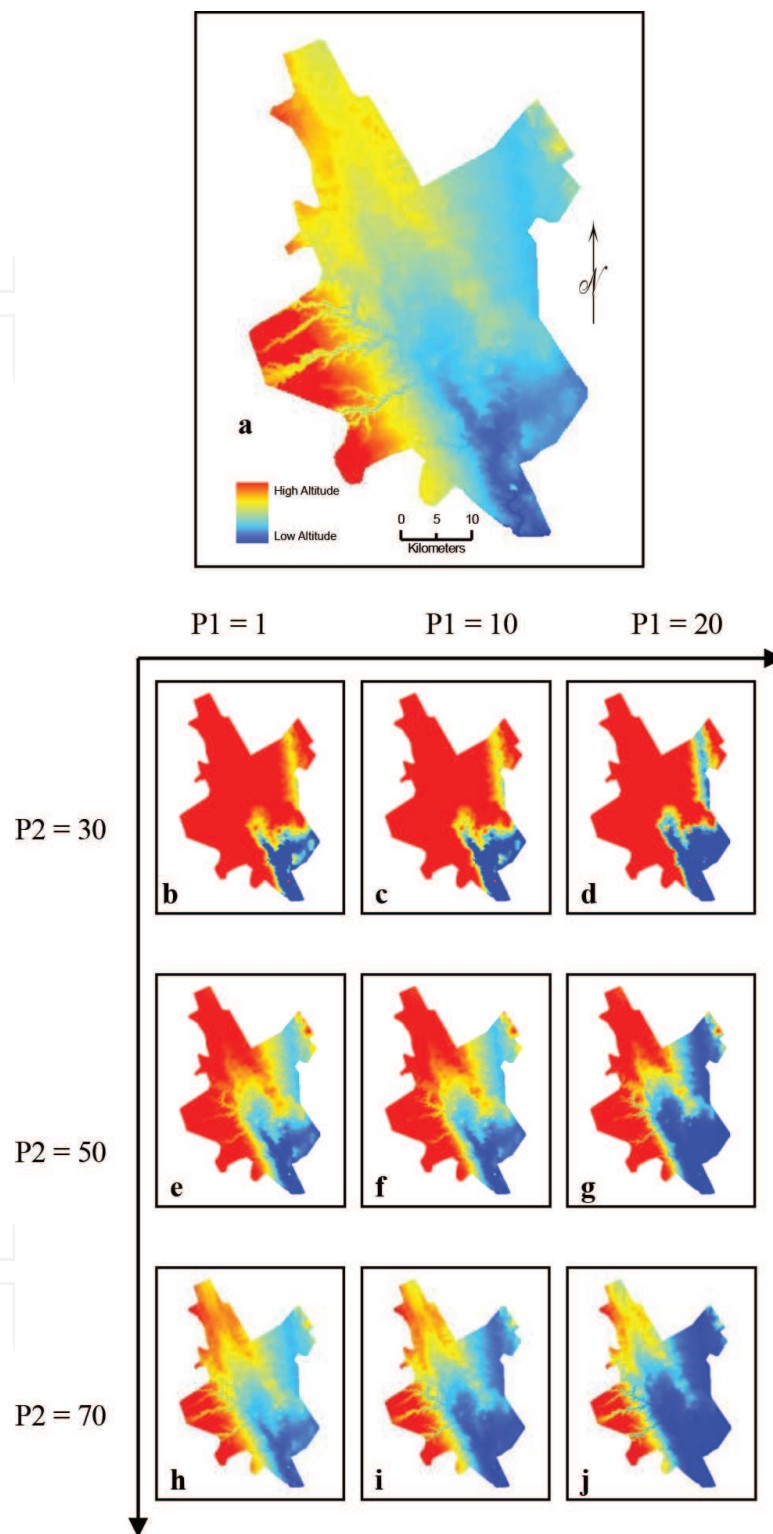


**Figure 8.** (a) Original and (b–j) fuzzy urban density.

standardization and fuzziness using Eq. (5). The physical topography of Riyadh confines to some extent the direction of urban growth. This is because the majority of urban development takes place in areas with a relatively moderate altitude and flat slope, while those areas with a steep slope and a high altitude are left without development. However, the range of slope and altitude variables varies according to the values of distance decay parameters  $P1$  and  $P2$ , which together identify three areas including the preferable range of altitude and slope (highly suitable areas), the non-preferable (non-suitable areas) and the partly preferable (moderately suitable areas). These classifications reflect the behaviour of people developing land. As can be seen from **Figures 9** and **10**, as the value of  $P2$  decreases, those non-suitable areas with a high altitude or a steep slope (red) increase. The highly suitable areas with a low altitude or a flat slope (blue), on the contrary, increase as the values of  $P1$  increase. The size of partly suitable areas increases as the difference between  $P1$  and  $P2$  values increases. The d, i and b maps of **Figures 9** and **10** show the largest areas of highly non-suitable areas, highly suitable areas and the partly suitable areas, respectively. The optimal values of  $P1$  and  $P2$  parameters were calibrated [27].

The planning area (PA) layer was generated by on-screen digitizing of boundaries of the new areas where planning applications were approved and urban development was allowed. The sources of planning areas are the Master Plans 1, 2, 3 of Riyadh city which were established by





**Figure 9.** Original altitude (a) and fuzzy altitude (b–j).

the Arriyadh Development Authority (ADA). The PA layer has a binary value of 1 for 'Planned Area' and 0 for 'non-planned area'. The excluded area (EA) layer was created from several sources such as slope images, altitude images, classified remote sensing images to derive



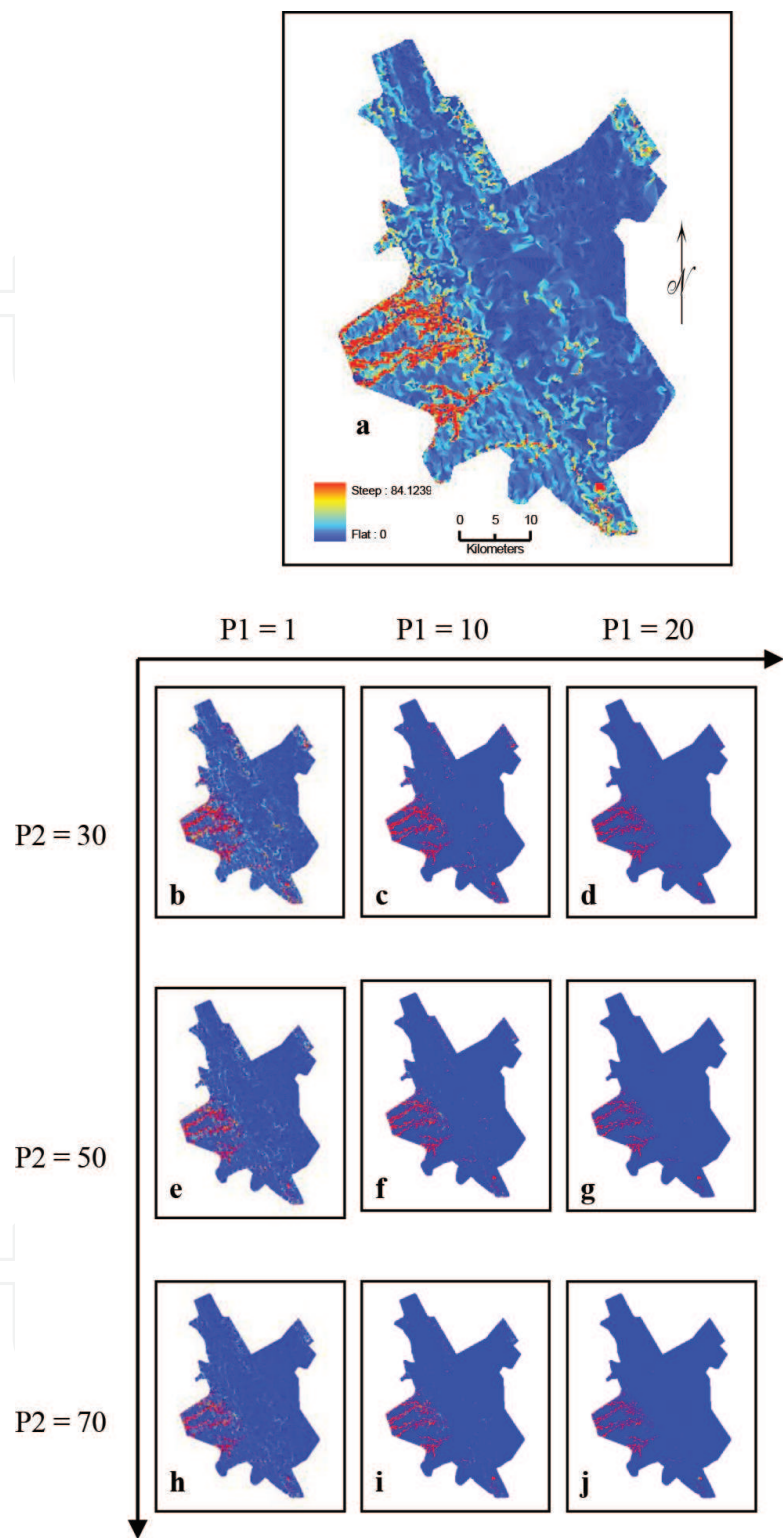


Figure 10. Original slope (a) and fuzzy slope (b–j).

agricultural areas and zoning regulation areas to reflect those in the master plan. Then, all of these layers were integrated and overlaid into one layer called EA, which has a binary value 1 for 'non-excluded area' and 0 for 'excluded area'.

#### 4.4. Urban growth driving forces as fuzzy variables

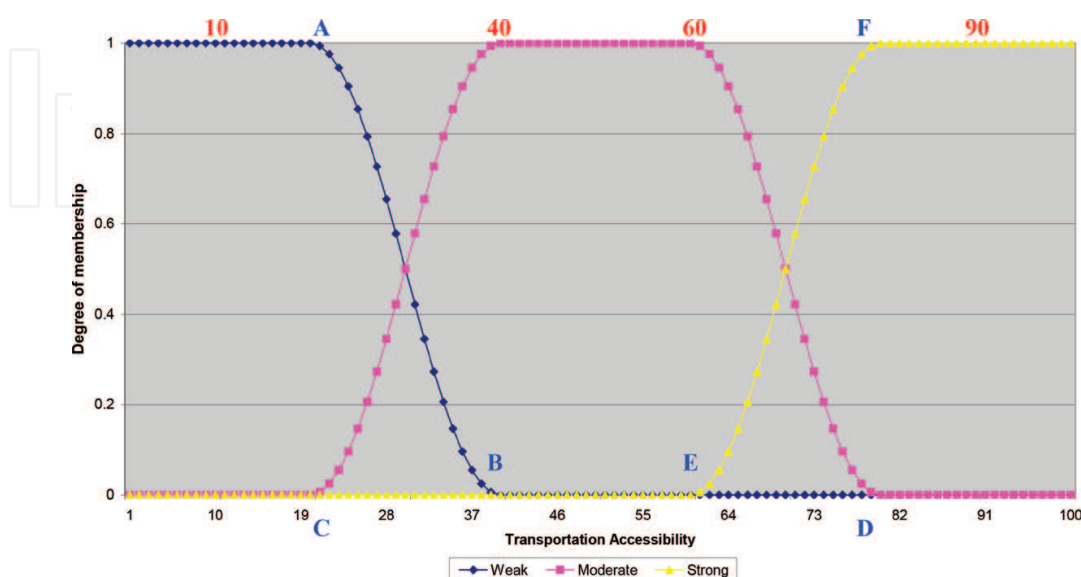
This section demonstrates and discusses the results of computing the input variables that form the four driving forces of urban growth. The four driving forces, (1) transport support factor (TSF), (2) urban agglomeration and attractiveness factor (UAAF), (3) topographical constraints factor (TCF) and (4) planning policies and regulations factor (PPRF), will be converted into linguistic variables (**Figure 11**) and then entered into the FCUGM [28, 29] for modelling the spatial pattern of urban growth for Riyadh.

##### 4.4.1. Transport support factor (TSF)

The TSF is based on three *input variables*: (1) weighted fuzzy local road accessibility (WFLRA), (2) weighted fuzzy main road accessibility (WFMRA) and (3) weighted fuzzy major road accessibility (WFMJRA). The TSF is generated in two stages: (1) integration of input variables and (2) definition of fuzzy sets. Under (1), the three *input variables* are integrated into one layer, called Transport  $T_{ij}$ , using a linear combination approach as shown in Eq. (9).

$$T_{ij} = WFLRA_{ij} + WFMRA_{ij} + WFMJRA_{ij} \quad (9)$$

Under (2), the Transport layer is used to create the TSF by transforming this layer into linguistic (fuzzy) variable by defining a series of three overlapping fuzzy sets. Although there is no rule for specifying the number of fuzzy sets for a linguistic (fuzzy) variable, most of the fuzzy modelling studies used three or five sets depending on the complexity of the problem. Since the TSF includes three input variables, which have the same spatial characteristics (all of them related to road networks), three fuzzy sets were selected. The TSF is composed of three fuzzy sets: Weak, Moderate and Strong. Each set describes a different condition of TSF. **Figure 11** depicts the placement and shape of these three fuzzy sets (membership functions) and Eqs. (10)–(12) provide the mathematical models used to calculate them. The shape of the



**Figure 11.** Membership functions of TSF driving force.

fuzzy sets follows a sinusoidal function. The fuzzy sets Weak, Moderate and Strong show a monotonically decreasing, symmetric and monotonically increasing functions, respectively. Although the shape of these fuzzy sets is usually decided by intuition or expert knowledge, in the FCUGM, they are optimized [27]. The authors believe that deciding the shape of the fuzzy sets in a subjective manner would result in restricting interpretations of the results.

The FCUGM optimizes the positions of a fuzzy set by fixing the central value of that set and calibrating the width or the range. This approach can be regarded as a hybrid and has two advantages: (1) it integrates the automatic self-tuned and expert knowledge techniques which could minimize the complexity of the problem (calibration urban CA model) and (2) decreases the number of parameters to be calibrated which reduces the complexity of the calibration process itself and the computation time required. According to **Figure 11**, the letters a–f are parameters that control the width of the fuzzy sets to be calibrated, while the numbers 10, 40, 60 and 90 are the fixed central values.

<i>Linguistic Expression</i>	<i>Membership Function</i>
$\mu_{Weak}(T_{ij}) =$	$\begin{cases} 1 & T_{ij} < A \\ \frac{1}{2} (1 - \cos(\pi (T_{ij} - A) / (B - A))) & A \leq T_{ij} \leq B \\ 0 & T_{ij} > B \end{cases} \quad (10)$

$\mu_{Moderate}(T_{ij}) =$	$\begin{cases} 0 & T_{ij} < C \\ \frac{1}{2} (1 + \cos(\pi (T_{ij} - C) / (40 - C))) & C \leq T_{ij} \leq 40 \\ 1 & 40 < T_{ij} < 60 \\ \frac{1}{2} (1 - \cos(\pi (T_{ij} - 60) / (D - 60))) & 60 \leq T_{ij} \leq D \\ 0 & T_{ij} > D \end{cases} \quad (11)$
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$\mu_{Strong}(T_{ij}) =$	$\begin{cases} 0 & T_{ij} < E \\ \frac{1}{2} (1 - \cos(\pi (T_{ij} - E) / (F - E))) & E \leq T_{ij} \leq F \\ 1 & T_{ij} > F \end{cases} \quad (12)$
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#### 4.4.2. Urban agglomeration and attractiveness factor (UAAF)

The urban agglomeration and attractiveness factor (UAAF) is formed using three *input variables* including weighted fuzzy urban density (WFUD), weighted fuzzy town centre accessibility (WFTCA) and weighted fuzzy employment centres and socioeconomic services accessibility (WFECSEA). The UAAF is generated in two stages: (1) integration of input variables and (2)

definition of fuzzy sets. Under (1), the three *input variables* are integrated into one layer, called Urban Agglomeration and Attractiveness  $UAA_{ij}$ , using a linear combination approach as shown in Eq. (13).

$$UAA_{ij} = WFUD_{ij} + WFTCA_{ij} + WFECSESA_{ij} \quad (13)$$

Under (2), the urban agglomeration and attractiveness layer is used to create the UAAF by transforming this layer into linguistic (fuzzy) variable, UAAF, by defining a series of five overlapping fuzzy sets. The motivation of selecting five sets for UAAF, rather than three as TSF, is that UAAF is formed by integrating three input variables that have different characteristics (urban density, employment centres and socioeconomic services and town centre). Thus, the degree of vagueness or uncertainty of this driving force is higher than TSF, meaning that it could be more appropriate for the UAAF driving force to be expressed with a higher number of fuzzy sets. Thus, the UAAF is composed of five fuzzy sets namely Very Low, Low, Medium, High and Very High. Each set describes a different condition of UAAF. **Figure 12** depicts the placement and shape of these five fuzzy sets (membership functions) and Eqs. (14)–(18) provide the mathematical models used to calculate those sets. In **Figure 12**, the letters A–J are parameters that control the width of the fuzzy sets which are to be calibrated [27], while the numbers 25.5, 27.5, 47.5, 52.5, 72.5 and 77.5 are the fixed central values.

*Linguistic Expression*

*Membership Function*

$$\mu_{Very\ Low}(UAA_{ij}) = \begin{cases} 0 & UAA_{ij} \leq 0 \\ 1 & 0 < UAA_{ij} < A \\ \frac{1}{2} (1 + \cos(\pi (UAA_{ij} - A) / (B - A))) & A \leq UAA_{ij} \leq B \\ 0 & UAA_{ij} > B \end{cases} \quad (14)$$

$$\mu_{Low}(UAA_{ij}) = \begin{cases} 0 & UAA_{ij} < C \\ \frac{1}{2} (1 + \cos(\pi (UAA_{ij} - C) / (25.5 - C))) & C \leq UAA_{ij} \leq 25.5 \\ 1 & 25.5 < UAA_{ij} < 27.5 \\ \frac{1}{2} (1 - \cos(\pi (UAA_{ij} - 27.5) / (D - 27.5))) & 27.5 \leq UAA_{ij} \leq D \\ 0 & UAA_{ij} > D \end{cases} \quad (15)$$

$$\mu_{Medium}(UAA_{ij}) = \begin{cases} 0 & UAA_{ij} < E \\ \frac{1}{2} (1 + \cos(\pi (UAA_{ij} - E) / (47.5 - E))) & E \leq UAA_{ij} \leq 47.5 \\ 1 & 47.5 < UAA_{ij} < 52.5 \\ \frac{1}{2} (1 - \cos(\pi (UAA_{ij} - 52.5) / (F - 52.5))) & 52.5 \leq UAA_{ij} \leq F \\ 0 & UAA_{ij} > F \end{cases} \quad (16)$$

$$\mu_{High}(UAA_{ij}) = \begin{cases} 0 & UAA_{ij} < G \\ \frac{1}{2} (1 + \cos(\pi (UAA_{ij} - G) / (72.5 - G))) & G \leq UAA_{ij} \leq 72.5 \\ 1 & 72.5 < UAA_{ij} < 77.5 \\ \frac{1}{2} (1 - \cos(\pi (UAA_{ij} - 77.5) / (H - 77.5))) & 77.5 \leq UAA_{ij} \leq H \\ 0 & UAA_{ij} > H \end{cases} \quad (17)$$

$$\mu_{Very\ High}(UAA_{ij}) = \begin{cases} 0 & UAA_{ij} \leq I \\ \frac{1}{2} (1 + \cos(\pi (UAA_{ij} - I) / (J - I))) & I \leq UAA_{ij} \leq J \\ 1 & UAA_{ij} > J \end{cases} \quad (18)$$

#### 4.4.3. Topographical constraints factor (TCF)

The TCF was based on two *input variables*: (1) weighted slope (WSG) and (2) weighted altitude (WA). The TCF is generated in two stages: (1) input variables integration and (2) definition of fuzzy sets. Under (1), the two *input variables* are integrated into one layer, called Topographical Constraints  $TC_{ij}$ , using a linear combination approach as expressed in Eq. (19).

$$TC_{ij} = WSG_{ij} + WA_{ij} \quad (19)$$

The topographical constraints layer was used to create the TCF by transforming this layer into a linguistic (fuzzy) variable by defining a series of three overlapping fuzzy sets. TCF is composed of three fuzzy sets namely slightly favourable (SM), moderately favourable (MF) and considerably

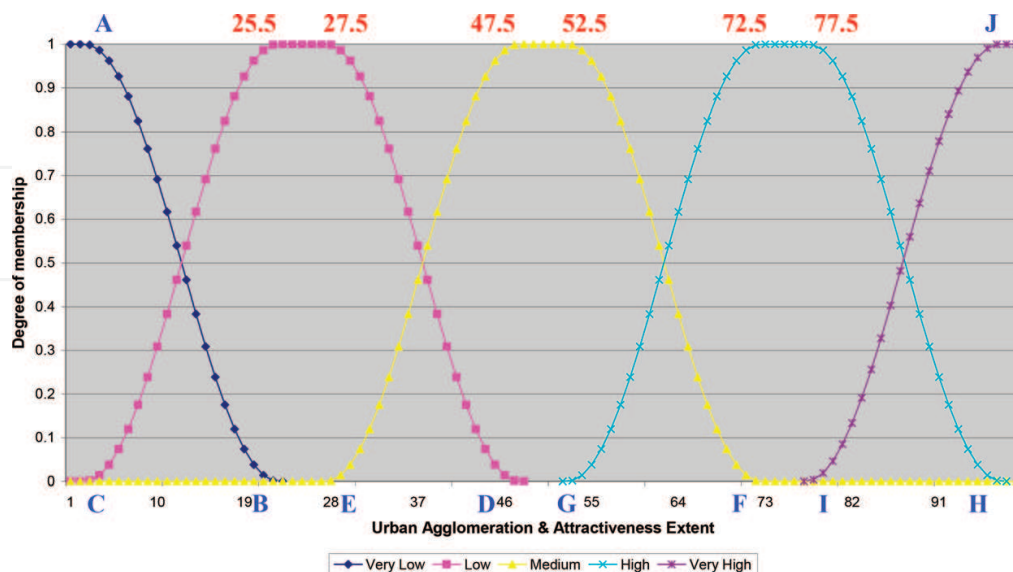


Figure 12. Membership functions of UAAF driving force.



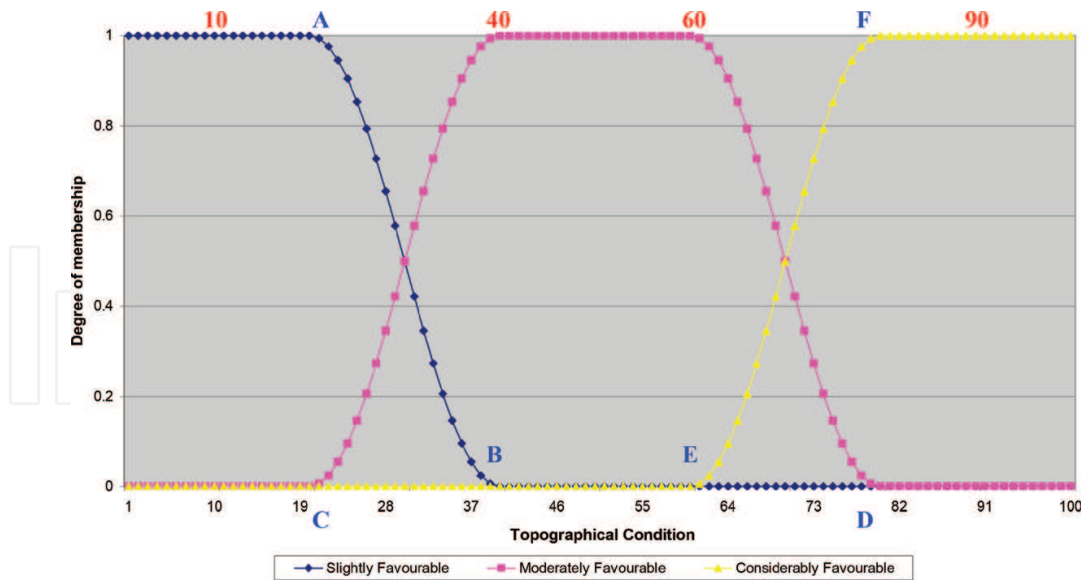


Figure 13. Membership functions of the TCF driving force.

favourable (CF). Each set describes a different condition of the TCF. **Figure 13** depicts the placement and shape of these three fuzzy sets (membership functions). Eqs. (20)–(22) provide the mathematical models for calculating these sets. According to **Figure 13**, the letters A–F are parameters which control the width of the fuzzy sets and are to be calibrated, while the numbers 10, 40, 60 and 90 are the fixed central values.

*Linguistic Expression*

*Membership Function*

$$\mu_{\text{Slightly Favorable}}(TC_{ij}) = \begin{cases} 1 & TC_{ij} < A \\ \frac{1}{2} (1 - \cos(\pi (TC_{ij} - A) / (B - A))) & A \leq TC_{ij} \leq B \\ 0 & TC_{ij} > B \end{cases} \quad (20)$$

$$\mu_{\text{Moderately Favorable}}(TC_{ij}) = \begin{cases} 0 & TC_{ij} < C \\ \frac{1}{2} (1 + \cos(\pi (TC_{ij} - C) / (40 - C))) & C \leq TC_{ij} \leq 40 \\ 1 & 40 < TC_{ij} < 60 \\ \frac{1}{2} (1 - \cos(\pi (TC_{ij} - 60) / (D - 60))) & 60 \leq TC_{ij} \leq D \\ 0 & TC_{ij} > D \end{cases} \quad (21)$$

$$\mu_{\text{Considerably Favorable}}(TC_{ij}) = \begin{cases} 0 & TC_{ij} < E \\ \frac{1}{2} (1 - \cos(\pi (TC_{ij} - E) / (F - E))) & E \leq TC_{ij} \leq F \\ 1 & TC_{ij} > F \end{cases} \quad (22)$$

#### 4.4.4. Planning policies and regulations factor (PPRF)

The PPRF is generated based on two *input variables*: (1) planning area (PA) and (2) excluded area (EA). These input variables are binary variables. The PPR is computed by combining the two variables as shown in Eq. (23).

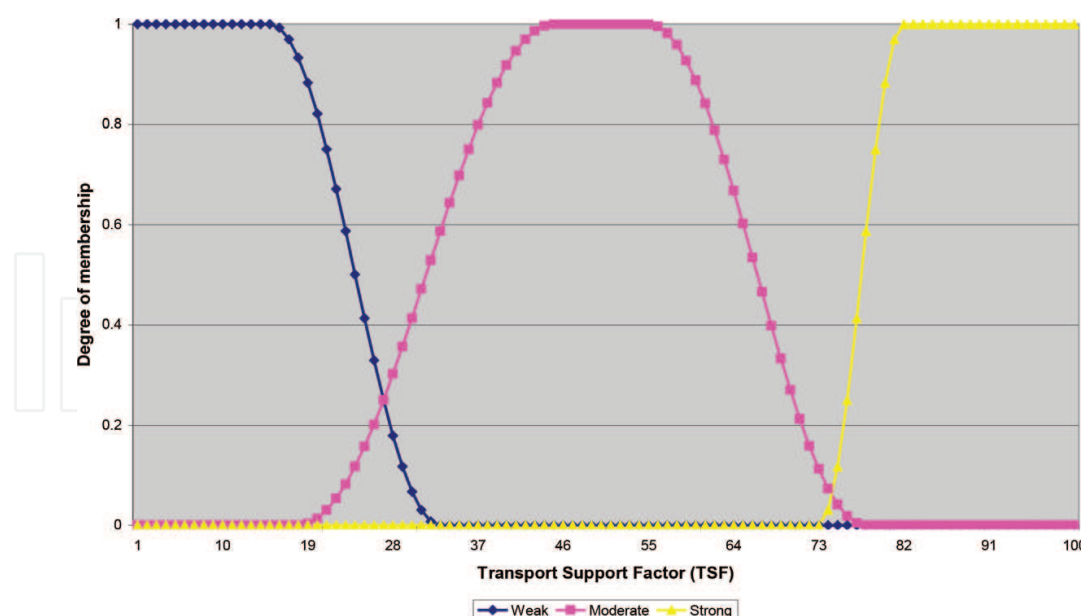
$$PPR_{ij}^t = EA_{ij}^t * PA_{ij}^t \quad (23)$$

<i>Linguistic Expression</i>	<i>Membership Function</i>		
$\mu_{Suitable}(PPR_{ij}) =$	{	<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: center; width: 10%;">0</div> <div style="text-align: left; width: 80%;"> <math>PPR_{ij} = 1</math> </div> <div style="width: 10%;"></div> </div> <div style="display: flex; justify-content: space-between; align-items: center; margin-top: 20px;"> <div style="text-align: center; width: 10%;">1</div> <div style="text-align: left; width: 80%;"> <math>PPR_{ij} = 0</math> </div> <div style="width: 10%;"></div> </div>	(24)

$\mu_{Non-Suitable}(PPR_{ij}) =$	{	<div style="display: flex; justify-content: space-between; align-items: center;"> <div style="text-align: center; width: 10%;">0</div> <div style="text-align: left; width: 80%;"> <math>PPR_{ij} = 0</math> </div> <div style="width: 10%;"></div> </div> <div style="display: flex; justify-content: space-between; align-items: center; margin-top: 20px;"> <div style="text-align: center; width: 10%;">1</div> <div style="text-align: left; width: 80%;"> <math>PPR_{ij} = 1</math> </div> <div style="width: 10%;"></div> </div>	(25)
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PPRF is then transformed into linguistic variables comprising two fuzzy sets namely Suitable and Non-Suitable. Each set describes a different condition of the TCF. Eqs. (24) and (25) provide the mathematical models for calculating these sets.

The three fuzzy input variables, TSF, UAAF and TCF, which drive the urban growth in the FCUGM, consist of different membership functions. Al-Ahmadi et al. [27] showed and discussed the calibrated fuzzy input variable parameters using genetic algorithm (GA), parallel simulated annealing (PSA) and expert knowledge (EK). The calibrated fuzzy input parameters have similar patterns for each of the three algorithms (GA, PSA and EK), scenarios and over all periods with low variations. As an example, in relation to the TSF, the average of the calibrated A,B,C,D,E and F parameters for the three fuzzy sets weak (W), moderate (M) and strong (S) that form the TSF fuzzy input variable accounts for 15, 33, 18, 77, 74 and 81, respectively. **Figure 14** depicts the optimized shape and placement of the three fuzzy sets for TSF. These optimized values reflect the area that is covered by each fuzzy set, that is, the quantity of TSF that belongs to each set. The A-B values (15–33) for the weak set indicate that locations with TSF values between 1 and 15 have a weak transport support (crisp set of Weak, i.e. full membership function). Those locations above 15 and below 33 are partly weak, that is, they might gain both weak and moderate support according to their degree of membership (fuzzy set of weak). And those with TSF values above 33 do not belong to the weak set. As the range of the calibrated values between the B parameter of *weak* set and the C parameter of *moderate* set increase, the overlap between the two sets and the quantity of fuzzy values



**Figure 14.** The mean optimized fuzzy sets for transportation support factor (TSF).

increases. Accordingly, as can be seen in **Figure 14**, the overlap between the weak and moderate sets (accounting for 15 as a range) is larger than the overlap between the moderate and strong sets (accounting for 3 as a range).

In terms of fuzzy modelling, it is better for the two adjacent sets to be intersected at 0.5 (crossover point) degree of membership, which provides a wider overlap area. In this sense, the larger the overlap areas between two sets, the better the results. This is because it allows different sets to use the same TSF values but with a different membership function according to the degree of ambiguity. The same interpretation can be applied to the other two factors UAAF and TCF.

## 5. Conclusions

This chapter presented an example of how fuzzy set theory can be applied to model urban growth factors and driving forces. The mathematical models for measuring, computing and addressing fuzzy input variables (accessibility and urban density) and fuzzy driving forces of the urban growth of Riyadh were discussed. The urban growth factors were established using fuzzy set theory, which quantified the effect of a distance decay using fuzziness. The driving forces of urban growth were designed for use with linguistics variables, which closely reflect human behaviours and attitudes regarding real-world phenomena. Such an approach provides a transparent method for interpreting the curve of a distance decay using linguistic variables.

The results indicate that fuzzy accessibility and fuzzy urban density factors are capable of mimicking and representing the uncertainty in the behaviour of the human decision-making

process in land development in a very efficient manner. The behaviour of people was represented as one of three classes: 'full membership', 'partial membership' and 'no membership', corresponding to people who prefer to live 'near', 'fairly near' and 'far' from a certain feature, respectively. The driving forces of urban growth—the transportation support factor (TSF), urban agglomeration and attractiveness factor (UAAF) and topographical constraints factor (TCF)—were established by integrating fuzzy urban growth factors and were designed for use with linguistics variables, which closely reflect human behaviours and attitudes regarding real-world phenomena. These fuzzy driving forces were designed in an attempt to model the spatial patterns of urban growth using natural language statements, which made the modelling process more realistic and transparent than other approaches.

The author concluded that instead of defining the factors and driving forces of urban growth using deterministic equations, the FCUGM would apply fuzzy logic control to generate fuzzy transition rules. Such an approach is more transparent and more similar to the real processes of urban growth while also representing the uncertainty of various constraints and driving forces underlying urban growth. In conclusion, the chapter demonstrated that it was possible to treat the complexity and uncertainty inherent in decision-making processes in terms of site selection, which is usually a subjective process that follows individuals' preferences in which people generally use linguistic constructs for the evaluation of environmental or social situations.

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