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PDE based approach for segmentation of oriented patterns

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1. Introduction

Image data restoration by diffusion equation is now a well established approach since the pioneering work of Perona and Malik (Perona & Malik, 1990). Originally, image diffusion consists in a convolution by a Gaussian kernel which introduces a scale dimension related to the standard deviation of the Gaussian kernel $\sigma = \sqrt{2t}$. This convolution is equivalent to solve the following linear diffusion equation, known in physics as the heat equation:

$$\frac{\partial \psi(x, y, t)}{\partial t} = \Delta \psi(x, y, t) \quad (1)$$

with $\psi(x, y, 0) = \psi_0$

where ψ is the restored image at scale t , and ψ_0 denotes original noisy image.

Nevertheless, Eq. (1) leads to an isotropic smoothing of the original image without edge preservation. To solve this drawback, Perona and Malik (Perona & Malik, 1990) have proposed a non linear adaptive diffusion process. In order to reduce the smoothing effect near edges due to the diffusion process, the authors introduce within classical diffusion process of Eq. (1) a parameter function of the local gradient intensity. Then, they obtained the following Partial Differential Equation (PDE):

$$\frac{\partial \psi(x, y, t)}{\partial t} = \operatorname{div} \left(g(\|\nabla \psi(x, y, t)\|) \nabla \psi(x, y, t) \right) \quad (2)$$

with $\psi(x, y, 0) = \psi_0$

where g is a decreasing function of the gradient. Practical implementation of the Perona-Malik equation gives generally interesting results; Noise is almost suppressed and edges

remain stable on many scales. Nevertheless, it appears in (Catté et al, 1992) that noise can also be enhanced introducing strong oscillations, due to the non convergence of the process. Solutions were proposed in (Alvarez & Morel, 1992), (Catté et al, 1992) and (Komprobst et al, 1997) but, as the authors noticed, when diffusion is processed many times sharp edges are smoothed.

A global coherent based method was proposed by Weickert (Weickert, 1995). Author builds up a tensor driven diffusion D , imposing the way to diffuse along the smallest contrast direction and the orthogonal one and weighting the diffusion according to a measure of coherence:

$$\frac{\partial \psi(x, y, t)}{\partial t} = \text{div}(D \cdot \nabla \psi(x, y, t)) \quad (3)$$

with $\psi(x, y, 0) = \psi_0$

Many others works deal with PDE's based image restoration in corresponding literature: (Geman & Reynolds, 1992; Nitzberg & Shiota, 1992; Whitaker & Pizer, 1993; Weickert, 1995; Weickert, 1998; Terebes et al., 2002; Tschumperle & Deriche, 2002; Tschumperle and Deriche, 2005).

In (Deriche & Faugeras, 1996), authors propose a unique PDE to express the whole principle of PDE's based restoration approach:

$$\frac{\partial \psi}{\partial t} = \frac{\phi'(\|\nabla \psi\|)}{\|\nabla \psi\|} \psi_{\xi\xi} + \phi''(\|\nabla \psi\|) \psi_{\eta\eta} \quad (4)$$

with $\psi(x, y, 0) = \psi_0$

where $\eta = \nabla \psi / \|\nabla \psi\|$, $\xi \perp \eta$ (Fig. 1) and Φ is a decreasing function.

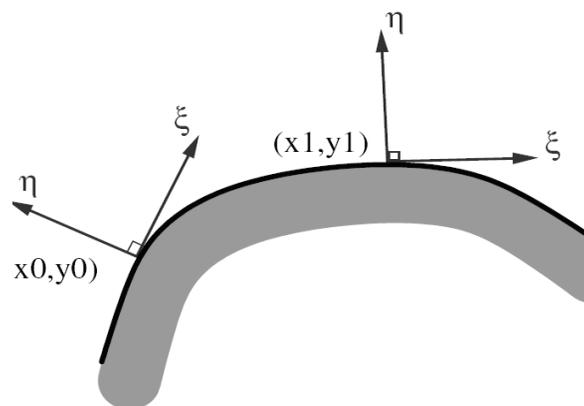


Fig. 1. An image contour and its moving vector basis (ξ, η) . Taken from (Tschumperle & Deriche, 2002).

This PDE is characterized by an anisotropic diffusive effect in the privileged directions ξ and η allowing a denoising of scalar image.

A limitation of this diffusion process (Eq. (4)) is its high dependence to the intrinsic quality of the original image and the impossibility to integrate prior information on the pattern to be restored if it can be characterized by particular data (orientation for example). Moreover, no characterization of the uncertainty/inaccuracy compromise can be made on the studied pixel, since the scale parameter is not directly integrated in the minimisation problem in which relies the common diffusion equations (Nordström, 1990). In this article we propose an original PDE directly integrating the scale parameter and allowing the taking into account of a priori knowledge on pattern to restore. We propose more particularly, to derive this PDE, to use a recent theory known as Extreme Physical Information (EPI) recently developed by Frieden (Frieden, 1998) and applied to image processing by Courboulay et al (Courboulay et al., 2002).

The second section of this article is dealing with the presentation of EPI and with the obtaining of the particular PDE. The third one presents a direct application to the presented diffusion process which may find applicability in robotics and automation. Section 4 shows the possible application of the developed method in a particular medical application: Enhancement of tagged cardiac MRI (Magnetic Resonance Imaging). Last part is dedicated to discussion.

2. EPI and Image Diffusion

2.1 EPI

Developed by Frieden, the principle of Extreme Physical Information (EPI) is aimed at defining a new theory of measurement. The key element of this new theory is that it takes into account the effect of an observer on a measurement scenario. As stated by Frieden (Frieden, 1996; Frieden, 1998), "EPI is an observer-based theory of physics". By observing, the observer is both a collector of data and an interference that affects the physical phenomenon which produces the data. Although the EPI principle brings new concepts, it still has to rest on the definition of information. Fisher information was chosen for its ability to effectively represent the quality of a measurement. Fisher information measure was introduced by Fisher in 1922 (Fisher, 1922) in the context of statistical estimation. In the last ten years, a growing interest for this information measure has arisen in theoretical physics. In his recent book (Frieden, 1998), Frieden has characterized Fisher information measure as a versatile tool to describe the evolution laws of physical systems; one of his major results is that the classical evolution equations as the Schrödinger wave equation, the Klein-Gordon equation, the Helmholtz wave equation, or the diffusion equation, can be derived from the minimization of Fisher information measure under proper constraint. Practically speaking, EPI principle can be seen as an optimization of the information transfer from the system under measurement to the observer, each one being characterized by a Fisher Information measure denoted respectively I and J . The first one is representative of the quality of the estimation of the data, and the second one allows to take into account the effect of the subjectivity of the observer on the measure. The existence of this transfer leads to create

fluctuations on the acquired data compared to the real ones. In fact, this information channel leads to the loss of accuracy on the measure whereas the certainty is increased.

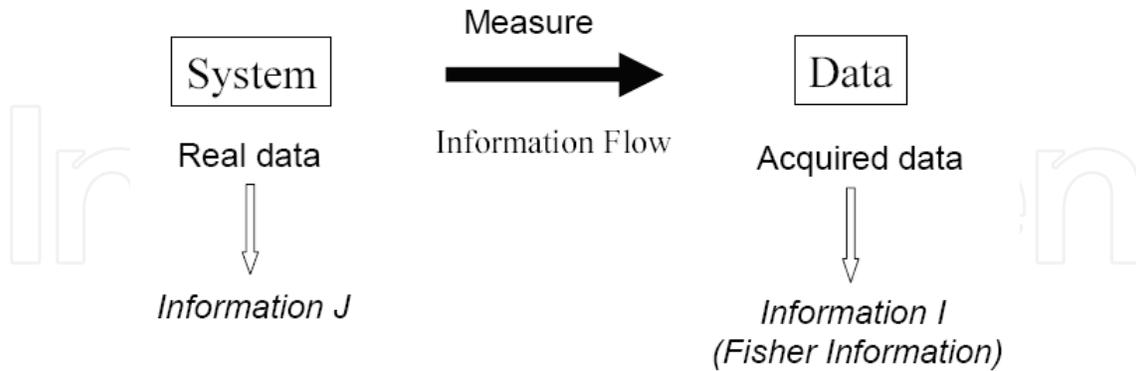


Fig.2. Fisher Information and measure

The goal of EPI is then to extremize the difference $I-J$ (i.e. the uncertainty/inaccuracy compromise) denoted K , called Physical Information of the system, in order to optimized information flow.

2.2 Application to image diffusion

Application to image diffusion can be illustrated by Fig. 3.

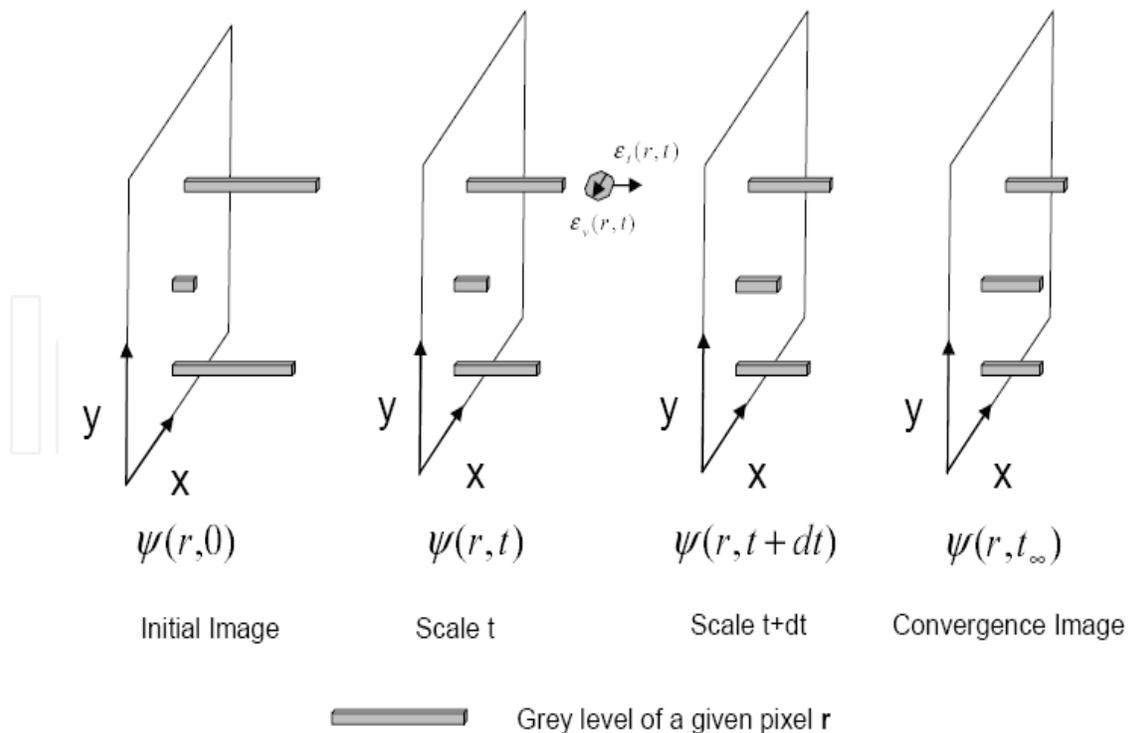


Fig. 3. Uncertainty/inaccuracy compromise and isotropic image diffusion. When parameter t tends to infinity, luminance of all pixels ($r=(x,y)$) of the corresponding image is the same and equal to the spatial average of the initial image.

As far as isotropic image diffusion is concerned, the uncertainty deals with the fluctuations of the grey level of a given pixel compared with its real value, whereas the inaccuracy deals with the fluctuations of the spatial localisation of a given pixel compared with the real one. The two different errors ($\varepsilon_r(t)$ and $\varepsilon_v(t)$) of Fig. 3 which are introduced all along the diffusion process are characterized by a measure of Fisher information. Intrinsic Fisher information J will be an integration of the diffusion constrained we impose on the processing.

Then, we can apply EPI to image diffusion process by considering an image as a measure of characteristics (as luminance, brightness, contrast) of a particular scene, and diffusion as the observer of this measure at a given scale. Extreme Physical Information K is then defined as follows (Frieden, 1998):

$$K(\psi) = \iint d\Omega dt \times \left[(\nabla - A)(\nabla - A)\psi^2 + \left(\frac{\partial \psi}{\partial t} \right)^2 - \psi^2 \right] \quad (5)$$

where A is a potential vector representing the parameterizable constrain integrated within diffusion process. Extremizing K by Lagrangian approach leads to a particular diffusion equation given by :

$$\frac{\partial \psi}{\partial t} = (\nabla - A)(\nabla - A)\psi \quad (6)$$

with $\psi(x, y, 0) = \psi_0$

As a consequence, thanks to the possible parameterization of A , it is possible to take into account particular characterized patterns to preserve from the diffusion process.

2.3 About A

The A potential allows to control the diffusion process and introduce some prior constrains during image evolution. For instance, if no constrain are to be taken into account, we set A as vector null and then Eq. (6) becomes:

$$\frac{\partial \psi}{\partial t} = \nabla \cdot \nabla \psi = \Delta \psi \quad (7)$$

with $\psi(x, y, 0) = \psi_0$

which is the well known heat equation characterized by an isotropic smoothing of the data processed. In order to enlarge the possibility given by Eq. (6), the choice we make for g is based on the fact that Eq. (6) allows a weighting of the diffusion process with the difference of orientation between the local calculated gradient and A . More precisely, to explain the way A is practically implemented, let consider Fig. 4.

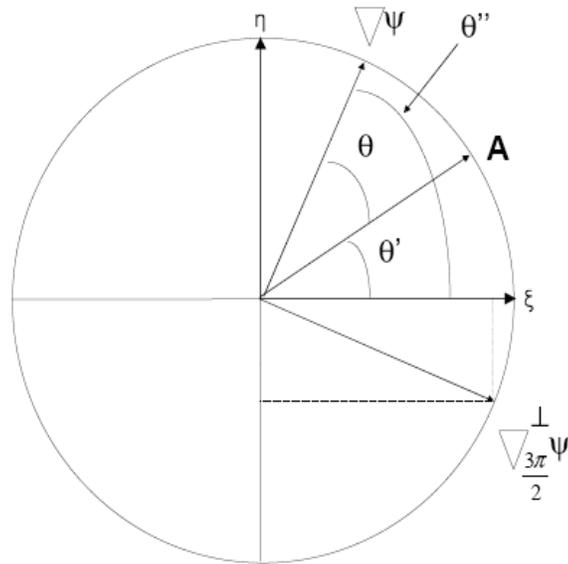


Fig. 4. Local geometrical implementation of \mathbf{A} in terms of the local gradient.

The expression of the local gradient in terms of θ'' is, considering Fig. 4 :

$$\nabla \psi = \begin{pmatrix} \|\nabla \psi\| \cos \theta'' \\ \|\nabla \psi\| \sin \theta'' \end{pmatrix} \quad (8)$$

and an expression of \mathbf{A} in terms of θ' is :

$$\mathbf{A} = \begin{pmatrix} \|\nabla \psi\| \cos \theta' \\ \|\nabla \psi\| \sin \theta' \end{pmatrix} \quad (9)$$

Norm of \mathbf{A} is imposed in order to make it possible the comparison with the gradient. To this point, the most interesting expression of \mathbf{A} would be the one in terms of θ , which represents the difference angle between \mathbf{A} and the local gradient. If we made so, using trigonometrical properties and noticing that $\theta = \theta'' - \theta'$, we obtain a new expression for \mathbf{A} :

$$\mathbf{A} = \begin{pmatrix} \|\nabla \psi\| (\cos \theta'' \cos \theta + \sin \theta'' \sin \theta) \\ \|\nabla \psi\| (\sin \theta'' \cos \theta - \cos \theta'' \sin \theta) \end{pmatrix} \quad (10)$$

Eq. (10) could be simplified by integrating the vectorial expression of the local gradient (Eq. (8)) :

$$A = \nabla \psi \cos \theta + \nabla^{\perp} \frac{3\pi}{2} \psi \sin \theta \quad (11)$$

From Eq. (11), we could then derive a general expression for \mathbf{A} considering it as a vectorial operator :

$$A = \nabla \cdot \cos \theta + \nabla^{\perp} \frac{3\pi}{2} \cdot \sin \theta \quad (12)$$

with θ the relative angle between \mathbf{A} and $\nabla \psi$ for a given pixel and ∇^{\perp} the local vector orthogonal to ∇ (Fig. 4). This expression only represents the way it is possible to reexpress \mathbf{A} by an orthogonal projection in the local base (ξ, η) (Fig. 1) . Considering it, Eq. (6) becomes:

$$\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial \eta^2} (1 - \cos \theta) + \frac{\partial^2 \psi}{\partial \xi^2} (1 - \cos \theta) \quad (13)$$

One can notice on Eq. (13) that when angle $\theta = 0$ (i.e. \mathbf{A} and $\nabla \psi$ are colinear), the studied pixel will not be diffused for $\frac{\partial \psi}{\partial t} = 0$. On the contrary, a nonzero value of θ will lead to a weighted diffusion of the considered neighbourhood of the pixel (Eq. (13)). As a consequence, by imposing local θ values, it is possible to preserve particular patterns from the diffusive effect within the processed image.

3. Application to oriented pattern extraction

In this section, we present results obtained on simple images in order to show the restoration and denoising potential of the method. For practical numerical implementation, the process of Eq. (13) is discretized with a time step τ . The images $\psi(t_n)$ are calculated, with Eq. (13), at discrete instant $t_n = n\tau$ with n the number of iterations of the process.

Let first consider an image showing vertical, horizontal, and $45^\circ \pm$ -oriented dark stripes on a uniform background (Fig. 5).

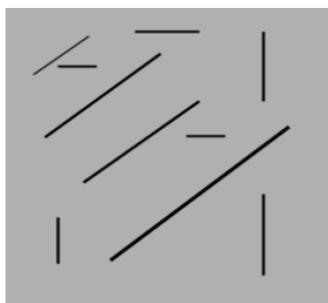


Fig. 5. Image 1: Dark stripes with various orientations on a uniform background.

Considering Eq. (13), by imposing two possible orientations for \mathbf{A} (135° , 325°) which corresponds to the gradient orientations of the diagonal stripes, one could expect to preserve them from isotropic diffusion. Diffusion results are presented Fig. 6.

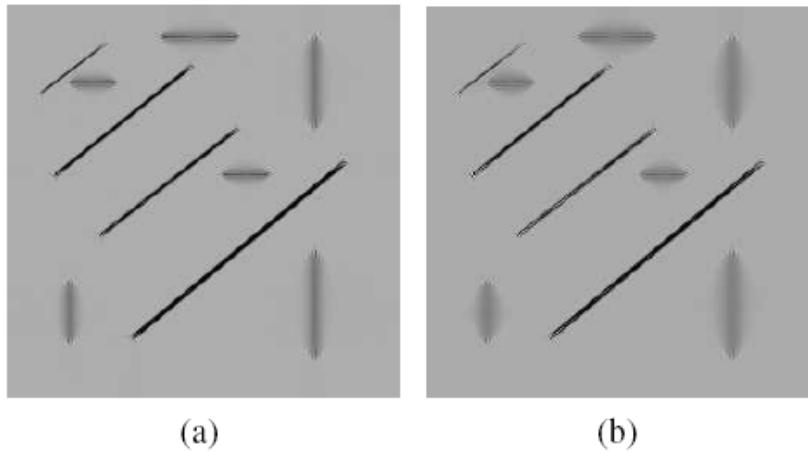


Fig. 6. Diffusion of "Image 1" (Fig. 5) for (a) $n=100$ and (b) $n=200$. \mathbf{A} is chosen in order to preserve only diagonal stripes from isotropic diffusion process. Time step τ is fixed to 0.2.

As one was expected it, the vertical and horizontal dark stripes in diffused images tend to disappear whereas the diagonal stripes are preserved all along the diffusion process.

Let now consider a noisy simple grid diagonally oriented corrupted by a Gaussian noise of standard deviation set to 0.3.

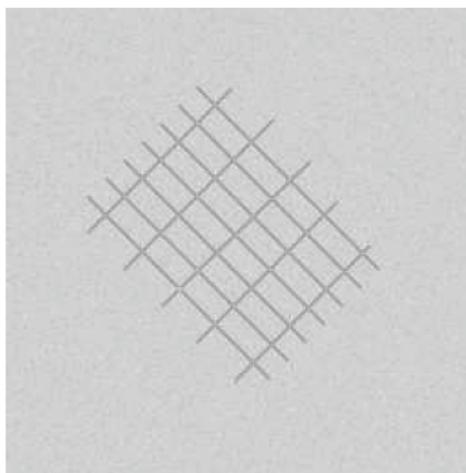


Fig. 7. Image 2: Noisy diagonally oriented grid (Gaussian noise). PSNR (calculated with the non corrupted version of the grid as reference) is equal to 68 dB.

If we apply the same diffusion process of Eq. (10) to this noisy simple grid imposing this time four possible orientations for \mathbf{A} corresponding to the four possible gradient orientations of the grid, it is then possible to show the denoising effect of the diffusion process (Fig. 8).

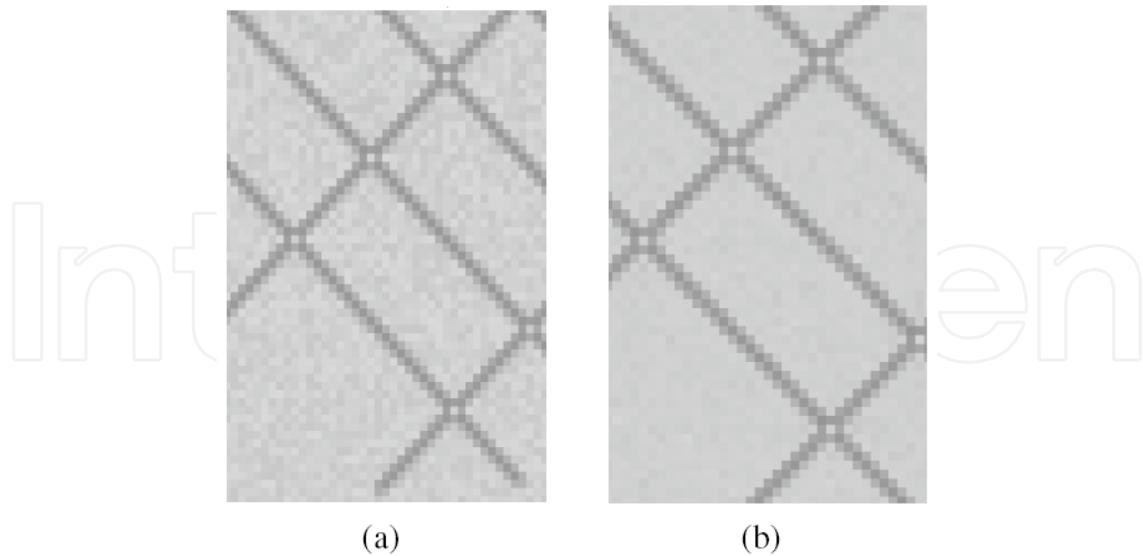


Fig. 8. Diffusion of "Image 2" (a) (Fig. 7) for (b) $n=50$. As one can notice, the grid itself is preserved from the diffusive effect of Eq. (3) whereas noise is iteration after iteration removed. Time step τ is fixed to 0.2.

As intended, the grid itself is not diffused at all and the increase of the Peak Signal to Noise Ratio (PSNR) from 68 dB to 84 dB, shows that the added Gaussian noise is removed iteration after iteration.

After these first experimental results obtained on ad hoc images, we now propose a practical medical application which main objective is automation of detection and tracking of a particular pattern within image sequences.

4. Application to enhancement of tagged cardiac MRI

4.1 Presentation of the problematic

The non invasive assessment of the cardiac function is of major interest for the diagnosis and the treatment of cardiovascular pathologies. Whereas classical cardiac MRI only enables radiologists to measure anatomical and functional parameters of the myocardium (mass, volume...), tagged cardiac MRI makes it possible to evaluate local intra-myocardial displacements. For instance, this type of information can lead to a precise characterization of the myocardium viability after an infarction. Moreover, data concerning myocardium viability makes it possible to decide of the therapeutic : medical treatment, angiopathy, or coronary surgery and following of the amelioration of the ventricular function after reperfusion.

The SPAMM (Space Modulation of Magnetization) acquisition protocol (Zerhouni et al,1988) we used for the tagging of MRI data, displays a deformable 45-degrees oriented dark grid which describes the contraction of myocardium (Fig. 9) on the images of temporal Short-Axis (SA) sequences. The temporal tracking of this grid can enable radiologists to evaluate the local intramyocardial displacement. Nevertheless, tagged cardiac images present peculiar characteristics which make it difficult to analyze. More precisely, images

are of low contrast compared with classical MRI, and their resolution is only of approximately one centimeter. Numerous studies were carried out concerning the analysis of the deformations of the grid of tag on SA sequences (see (Petitjean et al, 2005; Axel et al, 2007) for a complete overview) but all have in common the necessary enhancement of tagged cardiac images. As far as no technic as allowed to develop a "Gold Standard" method, we then propose to use the presented selective diffusion scheme to enhance the oriented grid of tags (Fig. 9) in order to make easier its temporal segmentation.



Fig. 9. SA tagged MRI of the Left Ventricle extracted from a sequence acquired between end diastole and end systole.

4.2 Processing and results

Considering Fig. 9 and Eq. (10), a solution to the problem of enhancement of the grid of tags is to impose particular orientations for \mathbf{A} , considering the fact that the gradients to be preserved within tagged cardiac MRI are well known and correspond to the orientations of the grid-of-tag ones (i.e. 45° , 135° , 225° , 315°). Others solutions making a better computation of \mathbf{A} can be imagined and are at the moment under development. Nevertheless, whereas this rough solution is not the best, it is sufficient for what we want to demonstrate in this chapter.

Considering this parameterization of Eq. (10), Fig. 10 shows results obtained on the first image of a classical tagged cardiac MRI sequences (Fig. 9). For a better appreciation of the proposed results, only one direction (45°) of the grid has been taken into account during the selective diffusion process of Eq. (10)

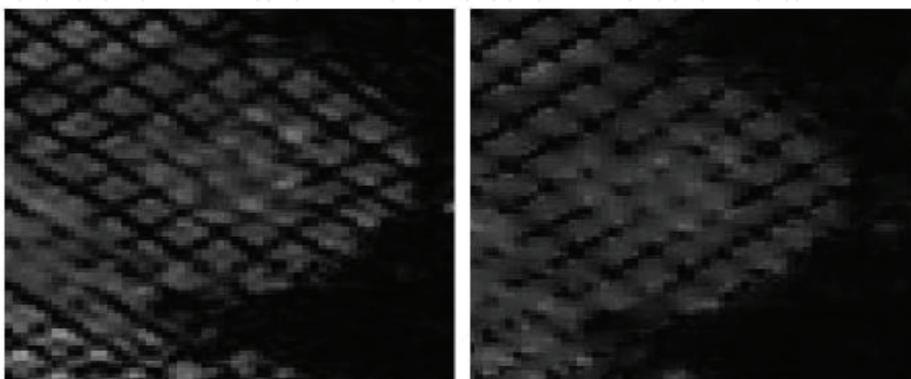


Fig. 10. Preservation of the 45° -oriented tag (right) on the initial image (left) of a tagged sequence.

As we can see in Fig.10, the diffusion process makes possible the fading of noisy artifacts, and non-45°-oriented lines. This result can be generalized to other direction of the grid.

At last, the enhancement effect of the selective diffusive process can only be seen on the evolution of a particular intensity profile extracted from original and enhanced tagged cardiac image. Indeed, Fig. 11 shows that selectiveness of Eq. (10) makes it possible the smoothing of all data except tag profiles.

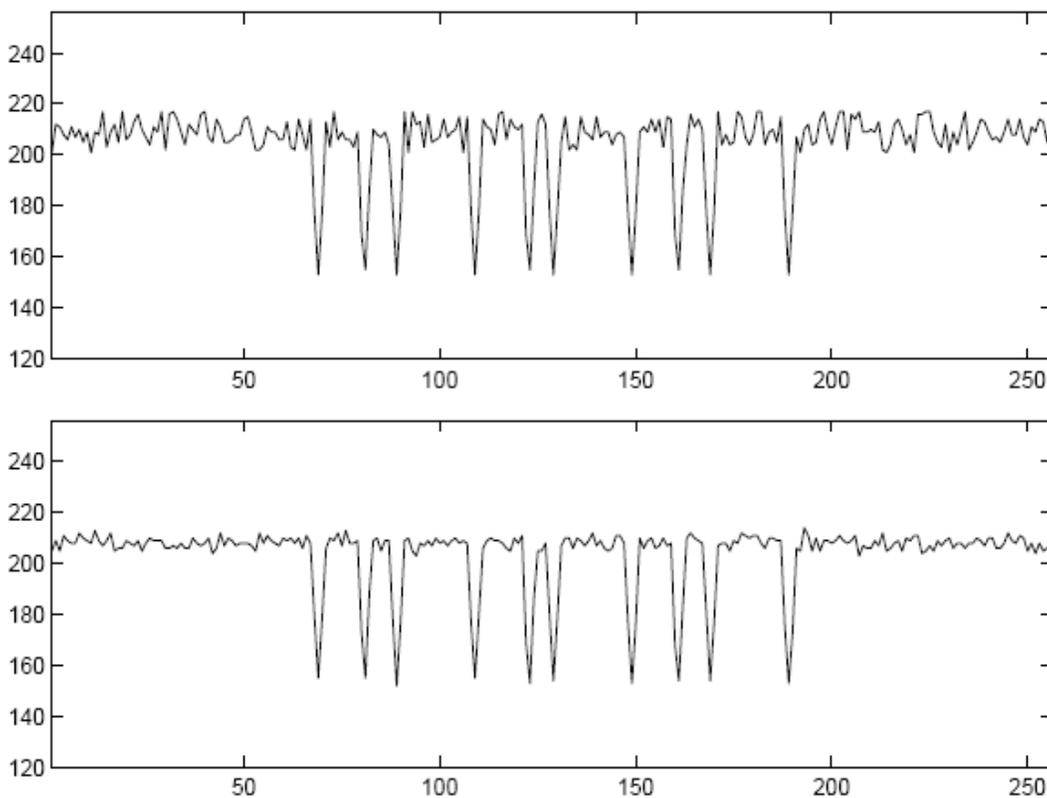


Fig. 11. Up: Original intensity profile extracted from original tagged cardiac MRI (orthogonally to orientation of the grid of tags). Bottom: Enhanced image thanks process of Eq. (10) with parametrization presented in previous subsection. As one can notice, selectiveness of Eq. (7) makes it possible the smoothing of all data except tag profiles.

5. Conclusion

In this chapter an original diffusion method, based on the use of a particular PDE (Eq. (10)) derived from EPI theory, has been presented as an alternative to classical PDE's based approach. It has been shown that the integration of the potential vector \mathbf{A} within the formulation of this PDE makes the integration, within the diffusion scheme, of particular constrains possible. This has been assimilated to integration of selectiveness within classical isotropic diffusion process. Examples on ad hoc images have been presented to show the potential of the presented method in the areas of denoising and extraction of oriented patterns. A particular application has also been proposed in the case of enhancement of tagged cardiac MRI. This method may find applicability in others areas like vision in robotics for instance.

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