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Spare Parts Forecasting Based on Reliability

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Additional information is available at the end of the chapter

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Abstract

Stochastic models for spare parts forecasting have not been widely researched in scientific literature from the aspect of their reliability. In this chapter, the authors present models which analyze standard reliability parameters of technical systems' parts/components. By analyzing system reliability and failure rate, we estimate the required number of spare parts in the moment of expected failure or when reliability falls below the predefined level. Two different approaches based on data availability are presented herewith.

Keywords: reliability, spare parts, forecasting methods, Rayleigh model, Weibull model, failure rate, inventory

1. Introduction

Technical systems, for example, aircrafts or weapon systems, are typically highly complex and composed of a large number of components and parts. Maintaining those systems adheres to strict rules and procedures for specific part or component. In order for such a system to be successfully maintained, the efficient management of spare part inventory is required, that is, a specific part should be provided in the right place at the right time [1].

Unpredictability of future events, that is, equipment and parts failure, has major impact on this problem. One way to reduce the level of unpredictability is to maintain a sufficient number of spare parts in inventory which results in the increase of warehouse costs and capital trapped in spare parts; another way is the assessment of spare parts inventory by using one of the available models for spare parts forecasting [2].

Forecasting is an essential skill on predicting future events and represents a foundation for every valid assessment. Although forecasts are often deemed expensive and time consuming, a vast number of researchers have been involved in finding novel methods with improved and more accurate assessment in recent years.

Concerning the demand for spare parts, exponential smoothing model and Croston's model are still among the most utilized due to their simplicity. The exponential smoothing model described in Refs. [3, 4] is based on pounder-moving average and can be implemented straightforwardly. In 1972, Croston presented a model based on exponential smoothing but far superior to it [5]. This method was most widely used in industry and is still a part of many software packages for spare parts forecasting. Rao [6] and then Schultz [7] studied Croston's method and proposed certain alterations but with no effect on execution results. Willemain et al. [8], and later Johnston and Boylan [9] proved that for majority of cases Croston's method gives better results than exponential smoothing method. Syntetos and Boyland [10, 11] made a considerable contribution to this scientific field by proposing a modified method based on criticism of Croston's model, calling it biased with reference to spare parts demand per time unit.

Another commonly used method is Bootstrap method—a computer statistical technique which, based on available data sample, creates a large number of new samples of same range as the original sample, by random sampling with replacement from original data set. From the aspect of inventory management, this model was examined in detail in Refs. [12, 13]. In addition, Refs. [14, 15] examine spare parts forecasting using Poisson's model. A conclusion was reached that traditional statistical methods based on analysis of time series can incorrectly assess functional form related to dependent and independent variables.

The nature of the spare parts demand is stochastic, and previously stated models do not always provide most accurate assessments [16]. For that reason, the number of models dealing with assessing the required number of spare parts, based on parameters such as spare part reliability, maintenance possibilities, life span, maintenance costs, and so on, has greatly increased during the past decade.

Due to the increase of system complexity [17], presenting reliability as a quantitative measure is commonly considered. By analyzing system's reliability and failure rate, we can estimate the required number of spare parts in the moment of expected failure or when reliability falls below the predefined level. There are numerous papers on the topic of determining the required number of spare parts, particularly as a part of logistic support [18, 19]. Refs. [20, 21] mostly deal with repairable spare parts or inventory managements with the aim of achieving previously set system reliability. On the other side, quantitative theories based on the theory of reliability were used to estimate the failure rate in order to precisely determine demand for a specific spare part [22–25]. An overview of the abovementioned models in spare parts forecasting with some new approaches proposed has been given in Ref. [26].

In this chapter, we present two models for spare parts forecasting based on the analysis of reliability parameter. Each model can be used depending on data availability.

2. Spare parts forecasting using Rayleigh model

Spare parts manufacturers provide only basic information on the part they produce. In this case, we observe the average life span of a part/component (T_{ut}) expressed in *hours*, μ , as a

stochastic process modeled with Rayleigh's distribution [27]. Probability density function (PDF) of Rayleigh's model is stated in Eq. (1)

$$p(\mu) = \frac{\mu}{\sigma^2} \exp\left(-\frac{\mu^2}{2\sigma^2}\right), \quad (1)$$

where σ is the parameter of Rayleigh's distribution determined by relation $E(\mu^2) = 2\sigma^2$. $E(\mu)$ is the mathematical expectation of Rayleigh's random variable μ . Based on that, T_{ut} can be presented as follows:

$$T_{ut} = \int_0^\infty \mu p(\mu) d\mu = \int_0^\infty \frac{\mu^2}{\sigma^2} \exp\left(-\frac{\mu^2}{2\sigma^2}\right) d\mu. \quad (2)$$

With replacement $\mu^2/(2\sigma^2) = x$, Eq. (2) is transformed into

$$T_{ut} = \sqrt{2}\sigma \int_0^\infty x^{1/2} \exp(-x) dx = \sqrt{2}\sigma \Gamma\left(\frac{3}{2}\right). \quad (3)$$

$\Gamma(a)$ is Gamma function [28] and $\Gamma(3/2) = \sqrt{\pi}/2$, so the average life span can be presented as $T_{ut} = \sigma\sqrt{\pi/2}$.

Based on the aforementioned, the PDF of Rayleigh's model can now be presented as follows:

$$p(\mu) = \frac{\mu\pi}{4T_{ut}^2} \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right). \quad (4)$$

Cumulative density function (CDF) can similarly be determined as

$$F(\mu) = \int_0^\mu p(\mu) d\mu = \int_0^\mu \frac{\mu\pi}{2T_{ut}^2} \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right) d\mu \quad (5)$$

If we substitute $\mu^2\pi/(4T_{ut}^2) = u$ in the previous equation, it can be reduced to

$$F(\mu) = \int_0^{\mu^2\pi/(4T_{ut}^2)} \exp(-u) du = 1 - \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right). \quad (6)$$

The function of part's reliability can further be determined with Eq. (7)

$$R(\mu) = 1 - F(\mu) = \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right). \quad (7)$$

Finally, based on the previous equations, we can define the failure function as a probability that the examined part will cease to perform its function in a specific time interval

$$\lambda(\mu) = \frac{p(\mu)}{R(\mu)} = \frac{\mu\pi}{2T_{ut}^2}. \quad (8)$$

The essence of this model's implementation is in an approach of determining the quantity of spare parts in inventory and costs that occur due to negative level of inventory at the end of usability period of examined part (underage costs) originally presented in Ref. [29]. This approach stresses the stochastic nature of variable T_{ut} . By observing the expected number of variations of this random variable in time interval $(T_{ut} + dT_{ut})$ for a given slope \dot{T}_{ut} in its specific environment dT_{ut} for N units of stated component, we can then determine the number of components most likely to fail as follows:

$$n = \int_0^{+\infty} p(\mu|\dot{T}_{ut})p(\dot{T}_{ut})d\dot{T}_{ut} = \int_0^{+\infty} p(\mu)\dot{T}_{ut} \frac{1}{\sqrt{2\pi\dot{\sigma}}} \exp\left(-\frac{\dot{T}_{ut}^2}{2\dot{\sigma}^2}\right)d\dot{T}_{ut}. \quad (9)$$

By implementing and substituting $\dot{T}_{ut}^2/(2\dot{\sigma}^2) = u$, the previous equation is reduced to

$$n = p(\mu) \frac{\dot{\sigma}}{\sqrt{2\pi}} \int_0^{+\infty} \exp(-u)du = p(\mu) \frac{\dot{\sigma}}{\sqrt{2\pi}}, \quad (10)$$

where \dot{T}_{ut} is the Gauss random variable with variance $V(\dot{T}_{ut}) = \dot{\sigma}$.

Now, given that μ is a Rayleigh's random variable with mathematical expectation $E(\mu) = T_{ut}$ and variance $V(\mu) = 2 T_{ut}^2/\pi$, then the average number of components n , which will be subject to failure in time T_{ut} , can be determined as

$$n = \sqrt{2}T_{ut}p(\mu) = \frac{\mu\pi}{\sqrt{2}T_{ut}} \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right). \quad (11)$$

Moreover, the number of spare parts required in inventory can be determined by observing the total time when random variable μ is below T_{ut} :

$$q = \frac{F(\mu)}{n} = \frac{1 - \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right)}{\frac{\mu\pi}{\sqrt{2}T_{ut}} \exp\left(-\frac{\mu^2\pi}{4T_{ut}^2}\right)} \quad (12)$$

This model can be a part of software for spare parts forecasting as in Ref. [30].

3. Spare parts forecasting using Weibull's model

For spare parts forecasting in cases where the total unit time of a product is unavailable but data on previous failure rates are available, Weibull's model can be used [31]. This model is most often used when reliability of a technical system is being determined. The PDF of

Weibull's model with two parameters, shape parameter β and characteristic life η , has the following form:

$$f(t) = \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp\left(-\frac{t}{\eta}\right)^{\beta}, \quad t \geq 0, \eta > 0, \beta > 0. \quad (13)$$

The function of reliability can be expressed by the following formula:

$$R(t) = \exp\left(-\frac{t}{\eta}\right)^{\beta}, \quad (14)$$

while CDF is

$$F(t) = 1 - R(t) = 1 - \exp\left(-\frac{t}{\eta}\right)^{\beta}. \quad (15)$$

The failure rate function of Weibull's model is

$$\lambda(t) = \left(\frac{\beta}{\eta}\right) \left(\frac{t}{\eta}\right)^{\beta-1}. \quad (16)$$

In relation to Weibull's model, a problem can arise while estimating the distribution parameters. There are multiple ways for parameter estimation but based on Ref. [32] if the available data sample is less than 15, then linear regression for parameter estimation is used, and in the opposite case maximum likelihood estimator (MLE) gives best results.

If linear regression is used, we start from CDF function of Weibull's model expressed as follows:

$$F(t) = 1 - \exp\left(-\frac{t}{\eta}\right)^{\beta} \Rightarrow 1 - F(t) = \exp\left(-\frac{t}{\eta}\right)^{\beta}. \quad (17)$$

Taking the logarithm of previous formula, we get

$$\ln[1 - F(t)] = \ln\left[\exp\left(-\frac{t}{\eta}\right)^{\beta}\right] = \ln\left[\frac{1}{\ln[1 - F(t)]}\right] = \left(\frac{t}{\eta}\right)^{\beta}. \quad (18)$$

If we take the logarithm of the previous expression once again

$$\ln\left[\ln\left[\frac{1}{\ln[1 - F(t)]}\right]\right] = \beta \ln t - \beta \ln \eta \quad (19)$$

and implement

$$y = \ln[-\ln[1 - F(t)]] \quad (20)$$

and

$x = \ln t$, $a = \beta$ and $b = -\beta \ln \eta$, then Eq. (19) can be expressed in the following linear form:

$$y = ax + b \quad (21)$$

If t_1, t_2, \dots, t_n are observed values of random variable t , the procedure for parameter estimation is as follows [33]:

1. Data should be arranged in an ascending order: $t_{(1)} \leq t_{(2)} \leq \dots \leq t_{(n)}$.
2. Value of empirical function of distribution is then calculated: $\hat{F}(t_{(i)})$, $i = 1, 2, \dots, n$.
3. Calculating $y_i = \ln[-\ln[1 - \hat{F}(t_{(i)})]]$, $i = 1, 2, \dots, n$.
4. Calculating $x_i = \ln t_{(i)}$, $i = 1, 2, \dots, n$.
5. Drawing points $T_i = (x_i, y_i)$ in the coordinate system and mathematically determine direction $y = \hat{\beta}x + \hat{b}$ which best approximates obtained data set. The most frequently used is the least-square method or linear regression which minimizes the vertical deviation of observed points from the given direction, in the following way:

$$\sum_{i=1}^N (\hat{a} + \hat{b}x_i - y_i)^2 = \min \sum_{i=1}^N (\hat{a} + \hat{b}x_i - y_i)^2 \quad (22)$$

where \hat{a} and \hat{b} are the least-squares estimates of a and b , and can be determined with the following formulae:

$$\hat{b} = \frac{\sum_{i=1}^N x_i y_i - \frac{\sum_{i=1}^N x_i \sum_{i=1}^N y_i}{N}}{\sum_{i=1}^N x_i^2 - \frac{(\sum_{i=1}^N x_i)^2}{N}} \quad (23)$$

and

$$\hat{a} = \frac{\sum_{i=1}^N y_i}{N} - \hat{b} \frac{\sum_{i=1}^N x_i}{N} = \bar{y} - \hat{b}\bar{x} \quad (24)$$

6. Based on the equations, it is now simple to determine parameters β and η .

In cases where data sample is greater than 15, parameters are determined by maximum likelihood estimator method [33].

By observing $t_1, t_2, t_3, \dots, t_n$ data sample of size n population with given PDF equation (13), then the joint density of likelihood function can be determined as a product of the densities of each point

$$L(t) = \prod_{i=1}^n f(t_i) = \left(\frac{\beta}{\eta}\right) \left(\frac{t_i}{\eta}\right)^{\beta-1} \exp\left(-\left(\frac{t_i}{\eta}\right)^{\beta}\right) = \left(\frac{\beta}{\eta}\right) \left(\frac{t_i}{\eta}\right)^{n\beta-n} \sum_{i=1}^n t_i^{\beta-1} \exp\left(-\sum_{i=1}^n \left(\frac{t_i}{\eta}\right)^{\beta}\right). \quad (25)$$

Taking natural logarithm of both sides gives

$$\ln L = n \ln\left(\frac{\beta}{\eta}\right) + (\beta - 1) \sum_{i=1}^n \ln t_i - \ln(\eta^{\beta-1}) - \sum_{i=1}^n \left(\frac{t_i}{\eta}\right)^{\beta}. \quad (26)$$

By differentiating partially previous equation, we obtain

$$\frac{\partial}{\partial \beta} \ln L = \frac{n}{\beta} + \sum_{i=1}^n \ln t_i - \frac{1}{\eta} \sum_{i=1}^n t_i^{\beta} \ln t_i = 0 \quad (27)$$

and

$$\frac{\partial}{\partial \eta} \ln L = \frac{n}{\eta} + \frac{1}{\eta^2} \sum_{i=1}^n t_i^{\beta} = 0. \quad (28)$$

From the previous equation, we can estimate η as

$$\hat{\eta} = \frac{1}{n} \sum_{i=1}^n t_i^{\hat{\beta}}. \quad (29)$$

By replacing Eq. (29) into Eq. (27), we get

$$\frac{1}{\hat{\beta}} + \frac{1}{n} \sum_{i=1}^n \ln t_i - \frac{\sum_{i=1}^n t_i^{\hat{\beta}} \ln t_i}{\sum_{i=1}^n t_i^{\hat{\beta}}} = 0. \quad (30)$$

Eq. (30) does not have a closed-form solution, so we can estimate shape parameter β by using Newton-Raphson's method or any other numeric procedure. After we determined $\hat{\beta}$, by replacing it into Eq. (29) we can calculate $\hat{\eta}$.

Once the parameters of Weibull's model are determined, we can estimate the required number of spare parts that should be in inventory in the given time interval. In order to achieve that, we use the approach presented in Ref. [34]. The PDF of Weibull's distributed failure time is given by Eq. (13), while the PDF of Rayleigh's distribute failure time is given by Eq. (1). Based on those two equations, we can conclude that $\sigma = \frac{\eta}{\sqrt{2}}$, while $\mu = t_2^{\frac{\beta}{2}}$, wherein μ is Rayleigh's random and t is Weibull's random variable.

$$p_{\dot{t}\dot{t}}(t, \dot{t}) = p_{\mu\dot{\mu}}\left(\frac{\beta}{t^2}, \dot{t}\frac{\beta}{2t^2-1}\right)|J|. \quad (31)$$

$|J|$ represents Jacobian transformation expressed with Eq. (32)

$$|J| = \begin{vmatrix} \frac{d\mu}{dt} & \frac{d\mu}{d\dot{t}} \\ \frac{d\dot{\mu}}{dt} & \frac{d\dot{\mu}}{d\dot{t}} \end{vmatrix} = \frac{\beta^2}{4} t^{\beta-2}. \quad (32)$$

By substituting Eq. (32) in Eq. (31), we obtain Eq. (33)

$$p_{\dot{t}\dot{t}}(t, \dot{t}) = \frac{\beta^2}{4} t^{\beta-2} p_{\mu\dot{\mu}}(\mu, \dot{\mu}) \quad (33)$$

If we stress the stochastic nature of a specific part's failure rate by observing the expected number of variations of Rayleigh's random variable in interval $(\mu, \mu+d\mu)$ for a given slope $\dot{\mu}$ in specific environment $d\mu$, then the number of spare parts that are most likely to fail can be determined as follows:

$$n = \int_0^{+\infty} \dot{\mu} p_{\mu\dot{\mu}}(\mu, \dot{\mu}) d\mu = \int_0^{+\infty} \dot{\mu} \frac{\mu}{\sigma^2} e^{-\frac{\mu^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\dot{\mu}^2}{2\sigma^2}} d\mu = \int_0^{+\infty} \dot{t} p_{\dot{t}\dot{t}}(t, \dot{t}) dt. \quad (34)$$

Based on the previous equations, the number of spare parts that are subject to failure in time period of $[0, t]$ is

$$n = \frac{4\sqrt{2}t^{\frac{\beta}{2}}}{\eta} e^{-\frac{t^{\frac{\beta}{2}}}{\eta^2}} \quad (35)$$

As in the previous section, it is now necessary to determine the number of spare parts required to be kept in inventory in time interval $[0, t]$. We will use the approach presented in paper [29] but in this case the quantity will be calculated as a quotient of CDF function of Weibull's random variable and size n . More accurately, as parameter η marks the time in which 63.2% of units will fail and is approximately equal to MTTF [35], we estimate the time interval in which t is below η as

$$q = \frac{F(t)}{n} \quad (36)$$

When we determine the quantity of spare parts that need to be kept in inventory, then in case when we know the price per unit of product based on Newsvendor model [36], we can determine the underage costs, that is, cost per unit of unsatisfied demand as

$$q = \Phi^{-1}\left(\frac{c_u}{c_u + c_o}\right) \quad (37)$$

where Φ^{-1} presents inverse distribution function (complementary error function), c_u are underage costs, and c_o are the overage costs, which in our case is spare part price.

4. Case study

Depending on data availability, one of the two recommended approaches will be implemented. If the total unit time (T_{ut}) is available for a specific part, we opt to use model for spare parts

Time	Failure/ censored	Time	Failure/ censored	Time	Failure/ censored	Time	Failure/ censored	Time	Failure/ censored
40	F	2089	F	3166	F	280	S	2137	S
301	F	2097	F	3344	F	313	S	2141	S
309	F	2135	F	3376	F	389	S	2163	S
943	F	2154	F	3385	F	487	S	2183	S
1070	F	2190	F	3443	F	622	S	2240	S
1124	F	2194	F	3467	F	900	S	2314	S
1248	F	2223	F	3478	F	952	S	2435	S
1281	F	2224	F	3578	F	996	S	2464	S
1303	F	2229	F	3595	F	1003	S	2543	S
1432	F	2300	F	3699	F	1101	S	2560	S
1480	F	2324	F	3779	F	1085	S	2592	S
1505	F	2349	F	3924	F	1092	S	2600	S
1506	F	2385	F	4035	F	1152	S	2670	S
1568	F	2481	F	4121	F	1183	S	2717	S
1615	F	2610	F	4167	F	1244	S	2819	S
1619	F	2625	F	4240	F	1249	S	2820	S
1652	F	2632	F	4255	F	1262	S	2878	S
1652	F	2646	F	4278	F	1360	S	2950	S
1757	F	2661	F	4305	F	1436	S	3003	S
1795	F	2688	F	4376	F	1492	S	3102	S
1866	F	2823	F	4449	F	1580	S	3304	S
1876	F	2890	F	4485	F	1719	S	3483	S
1899	F	2902	F	4570	F	1794	S	3500	S
1911	F	2934	F	4602	F	1915	S	3622	S
1912	F	2962	F	4663	F	1920	S	3665	S
1914	F	2964	F	4694	F	1963	S	3695	S
1918	F	3000	F	46	S	1978	S	4015	S
2010	F	3103	F	140	S	2053	S	4628	S
2038	F	3114	F	150	S	2065	S	4806	S
2085	F	3117	F	248	S	2117	S	4881	S
								5140	S

Table 1. Windshield failure time.

forecasting based on Rayleigh’s distribution. When historic failure/censored data are available, we use model based on Weibull’s distribution.

Table 1 provides data on specific aircraft model’s Windshield, taken from paper [37]. **Table 1** consists of 88 records on the part’s failure time and 65 records on censored time expressed in

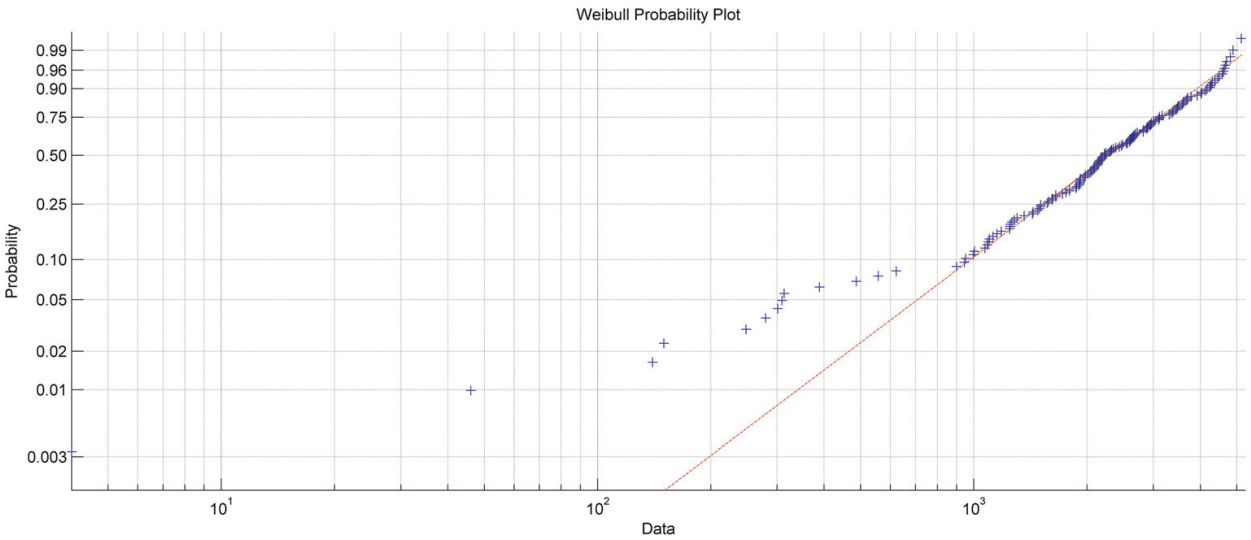


Figure 1. Weibull probability plot for Table 1 data.

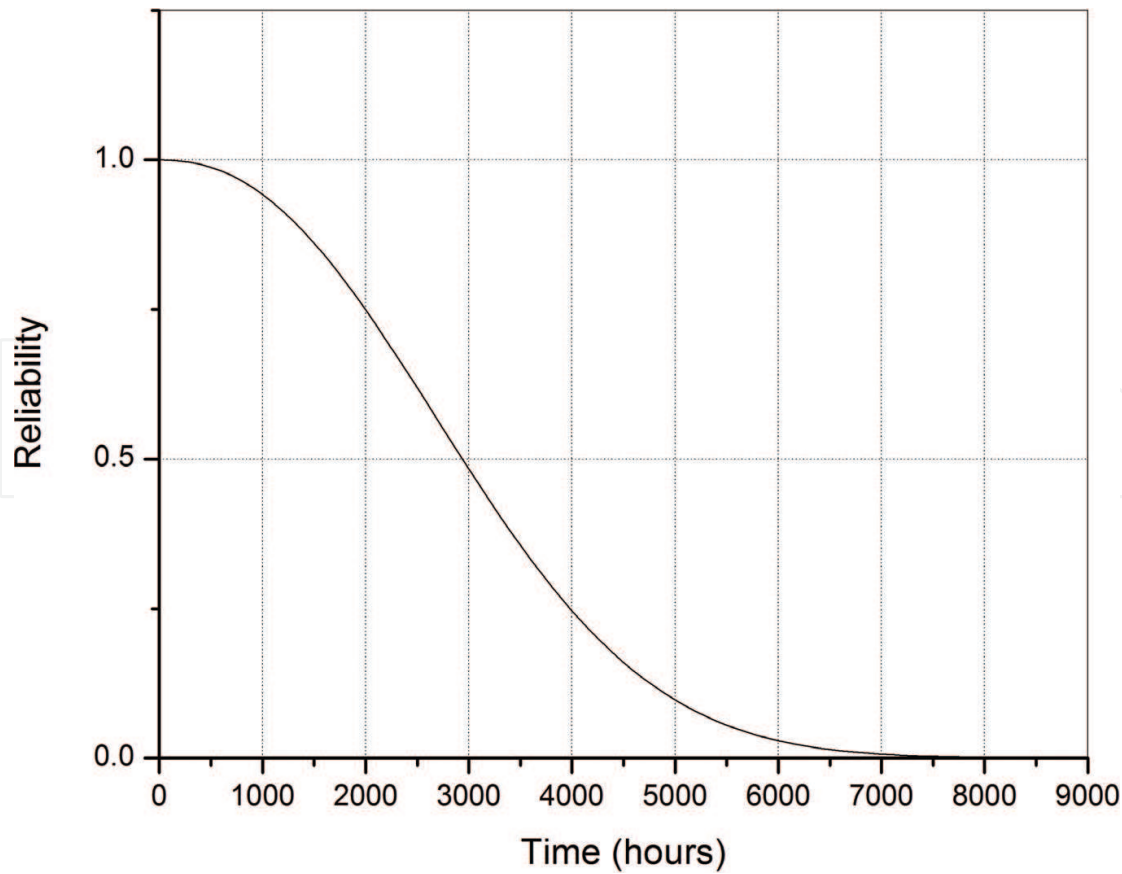


Figure 2. Windshield reliability.

flight hours. Censored time (or service time) means that the Windshields have not failed at the time of observation.

Weibull probability plot for data from previous table is presented in **Figure 1**.

As noted in **Figure 1**, records from **Table 1** follow Weibull's distribution, and as their number is greater than 15, based on the previous research, we use maximum likelihood estimation method for the estimation of parameters in Weibull's distribution, described extensively in the previous section.

Implementation of this method results in the value of shape parameter $\beta = 2.28$, and characteristic life parameter $\eta = 3450.54$.

Now, when values of these parameters are known we can determine the reliability of part windshield based on Eq. (15), while its failure function can be determined based on Eq. (16). A graphical representation of these functions is presented in **Figures 2 and 3**.

Finally, we can determine the quantity of spare parts required to be kept in inventory in a given time interval based on Eq. (36). A graphical representation is presented in **Figure 4**.

Based on **Figure 4**, it can be concluded that it is necessary to have one spare part in inventory after 2000 flight hours. The results of this analysis, that is, data on spare part reliability, failure time, and required quantity in inventory can be of great value to decision makers, on questions related to what to keep in inventory and in what quantity, and when to plan maintenance activities in order to prevent the occurrence of failure.

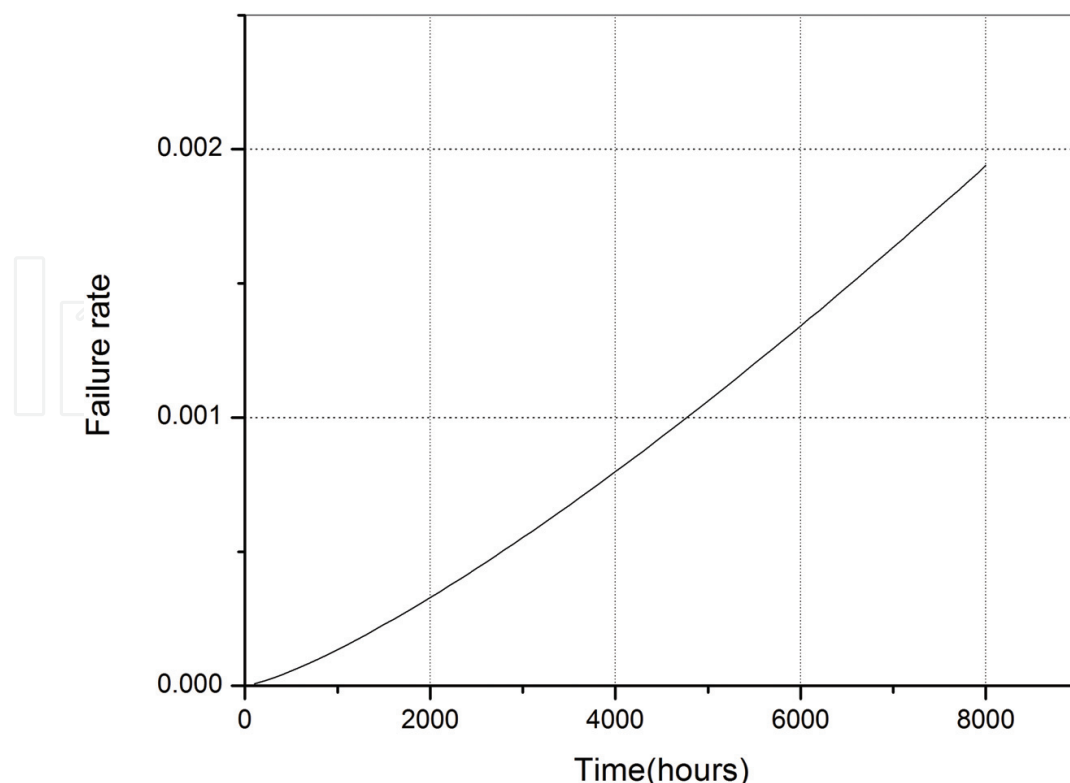


Figure 3. Failure rate function of windshield.

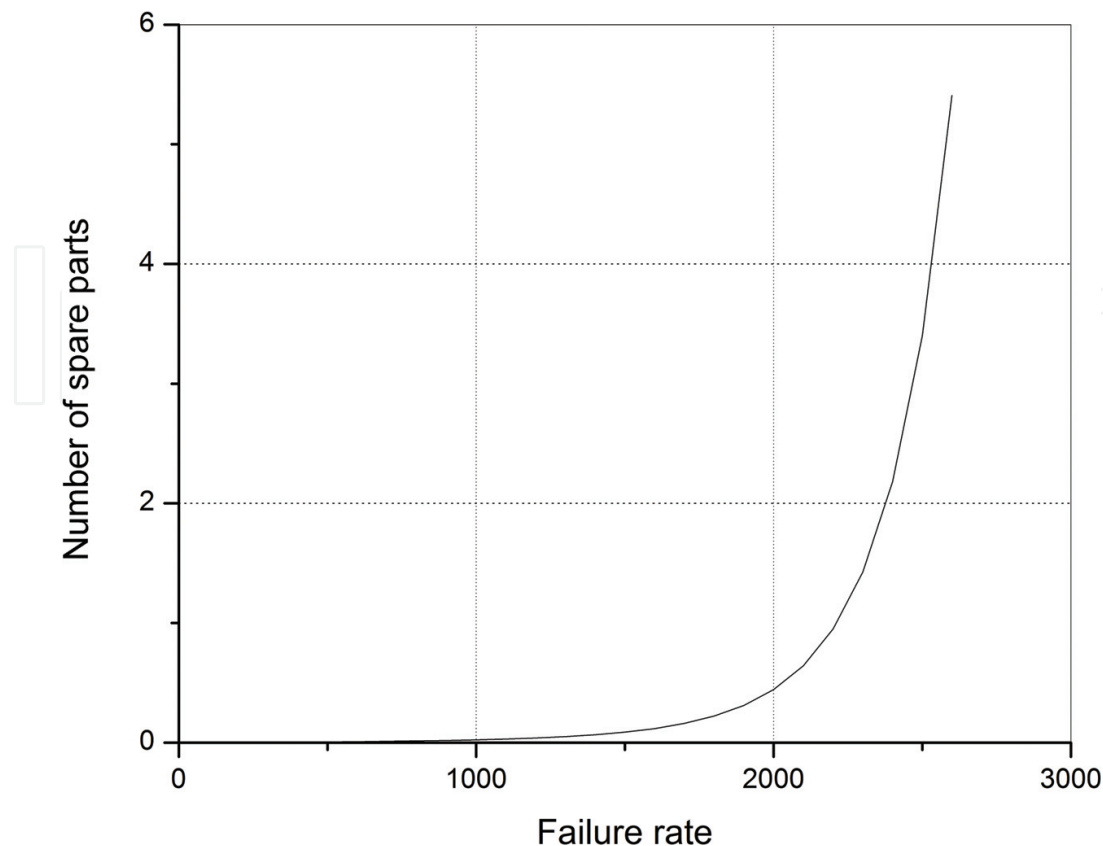


Figure 4. Quantity of spare parts in function of time.

The aforementioned models provide possibility for taking into consideration underage costs during decision-making process by using Eq. (37), for those cases when price of the part in issue is available. These costs are difficult to determine objectively. Some consequences of lack of spare parts, for example, damage to company's reputation due to delays, are difficult to express quantitatively.

5. Conclusions

This section elaborates on determining spare parts required to be kept in inventory from the aspect of reliability analysis of that part. Depending on data availability, either Rayleigh's or Weibull's method can be used. In case of Weibull's method, two approaches for the assessment of parameters are given, depending on data sample size. Information obtained with this analysis can have a major role in the process of supply management. Based on them, it is possible to reduce costs that occur due to delays, unplanned cancellations, and so on. These models can serve as a solid foundation for the creation of software for spare parts forecasting. Although the emphasis was placed on planning maintenance activities and avoiding delays, all the aforementioned leads to limiting the consequences of suboptimal supply management, that is, minimize the spare parts overstocking. In the case when we are dealing with repairable systems, researches should focus on the determination or minimization of the repair rate of such system.

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