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Control of Robotic Systems Undergoing a Non-Contact to Contact Transition

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1. Introduction

The problem of controlling a robot during a non-contact to contact transition has been a historically challenging problem that is practically motivated by applications that require a robotic system to interact with its environment. The control challenge is due, in part, to the impact effects that result in possible high stress, rapid dissipation of energy, and fast acceleration and deceleration (Tornambe, 1999). Over the last two decades, results have focused on control designs that can be applied during the non-contact to contact transition. One main theme in these results is the desire to prescribe, reduce, or control the interaction forces during or after the robot impact with the environment, because large interaction forces can damage both the robot and/or the environment or lead to degraded performance or instabilities.

In the past, researchers have used different techniques to design controllers for the contact transition task. (Hogan, 1985) treated the environment as mechanical impedance and used force feedback in an impedance control technique to stabilize the robot undergoing contact transition. (Yousef-Toumi & Gutz, 1989) used integral force compensation with velocity feedback to improve transient impact response. In recent years, two main approaches have been exploited to accommodate for the non-contact to contact transition. The first approach is to exploit kinematic redundancy of the manipulator to reduce the impact force (Walker, 1990; Gertz et al., 1991; Walker, 1994). The second mainstream approach is to exploit a discontinuous controller that switches based on the different phases of the dynamics (i.e., non-contact, robot impact transition, and in-contact coupled manipulator and environment) as in (Chiu et al., 1995; Pagilla & Yu, 2001; Lee et al., 2003). Typically, these discontinuous controllers consist of a velocity controller in the pre-contact phase that switches to a force controller during the contact phase. Motivation exists to explore alternative methods because kinematic redundancy is not always possible, and discontinuous controllers require infinite control frequency (i.e., exhibit chattering). Also, force measurements can be noisy and may lead to degraded performance. The focus of this chapter is control design and stability analysis for systems with dynamics that transition from non-contact to contact conditions. As a specific example, the development will focus on a two link planar robotic arm that transitions from free motion to contact with an unactuated mass-spring system. The objective is to control a robot from a non-contact initial condition to a desired (contact) position so that a stiff mass-spring system is regulated to a desired compressed state, while

limiting the impact force. The mass-spring system models a stiff environment for the robot, and the robot/mass-spring system collision is modeled as a linear differentiable impact, with the impact force being proportional to the mass deformation. The presence of parametric uncertainty in the robot/mass-spring system and the impact model is accounted for by using a Lyapunov based adaptive backstepping technique. The feedback elements of the controller are contained inside of hyperbolic tangent functions as a means to limit the impact force (Liang et al., 2007). A two-stage stability analysis (contact and non-contact) is included to prove that the continuous controller achieves the control objective despite parametric uncertainty throughout the system, and without the use of acceleration or force measurements.

In addition to impact with stiff environments, applications such as robot interaction with human tissue in clinical procedures and rehabilitative tasks, cell manipulation, finger manipulation, etc. (Jezernik & Morari, 2002; Li et al., 1989; Okamura et al., 2000) provide practical motivation to study robotic interaction with viscoelastic materials. (Hunt & Crossley, 1975) proposed a compliant contact model, which not only included both the stiffness and damping terms, but also eliminated the discontinuous impact forces at initial contact and separation, thus making it more suitable for robotic contact with soft environments. The model has found acceptance in the scientific community (Gilardi & Sharf, 2002; Marhefka & Orin, 1999; Lankarani & Nikravesh, 1990; Diolaiti et al., 2004), because it has been shown (Gilardi & Sharf, 2002) to better represent the physical nature of the energy transfer process at contact. In addition to the controller design using a linear spring contact model, this chapter also describes how the more general Hunt-Crossley model can be used to design a controller (Bhasin et al., 2008) for viscoelastic contact. A Neural Network (NN) feedforward term is inserted into the controller (Bhasin et al., 2008) to estimate the environment uncertainties which are not linear in the parameters. Similar to the control design in the first part of the chapter, the control objective is achieved despite uncertainties in the robot and the impact model, and without the use of acceleration or force measurements. However, the NN adaptive element enables adaptation for a broader class of uncertainty due to viscoelastic impact.

2. Control of Robotic Contact with a stiff environment

Robot applications in the industry like assembling, material handling, painting etc. have motivated the study of robot interaction with a stiff environment. We consider an academic problem of a two link robot undergoing a non-contact to contact transition with a stiff mass-spring system (Fig. 1). The contact dynamics of the stiff environment can be modeled using a simple linear spring, wherein the contact force is directly proportional to the local deformation at impact. Besides regulating the mass to a desired final position, another objective is to limit the impact force. The feedback elements for the controller are contained inside of hyperbolic tangent functions as a means to limit the impact forces resulting from large initial conditions as the robot transitions from a non-contact to a contact state. Although saturating the feedback error is an intuitive solution, several new technical challenges arise in the stability analysis. The main challenge is that the use of saturated feedback does not allow some coupling terms to be canceled in the stability analysis, resulting in the need to develop state dependent upper bounds that result in semi-global stability.

2.1 Dynamic Model

Figure 1 Robot contact with a Mass-Spring System (MSR), modeled as a linear spring

The subsequent development is motivated by the academic problem illustrated in Fig. 1. The dynamic model for the two-link planar robot depicted in Fig. 1 can be expressed in the joint-space as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + h(q) = \tau, \quad (1)$$

where $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^2$ represent the angular position, velocity, and acceleration of the robot links, respectively, $M(q) \in \mathbb{R}^{2 \times 2}$ represents the uncertain inertia matrix, $C(q, \dot{q}) \in \mathbb{R}^{2 \times 2}$ represents the uncertain Centripetal-Coriolis effects, $h(q) \triangleq [h_1(q), h_2(q)]^T \in \mathbb{R}^2$ represents uncertain conservative forces (e.g., gravity), and $\tau(t) \in \mathbb{R}^2$ represents the torque control inputs. The Euclidean position of the end-point of the second robot link is denoted by $x_r(t) \triangleq [x_{r1}(t), x_{r2}(t)]^T \in \mathbb{R}^2$, which can be related to the joint-space through the following kinematic relationship:

$$\dot{x}_r = J(q)\dot{q}, \quad (2)$$

where $J(q) \in \mathbb{R}^{2 \times 2}$ denotes the manipulator Jacobian. The unforced dynamics of the mass-spring system in Fig. 1 are

$$m\ddot{x}_m + k_s(x_m - x_0) = 0, \quad (3)$$

where $x_m(t)$ and $\ddot{x}_m(t) \in \mathbb{R}$ represent the displacement and acceleration of the unknown mass $m \in \mathbb{R}$, $x_0 \in \mathbb{R}$ represents the initial undisturbed position of the mass, and $k_s \in \mathbb{R}$ represents the unknown stiffness of the spring.

After pre-multiplying the robot dynamics by the inverse of the Jacobian transpose and utilizing (2), the dynamics in (1) and (3) can be rewritten as (Dupree et al., 2006 a; Dupree et al., 2006 b)

$$\bar{M}(x_r)\ddot{x}_r + \bar{C}(x_r, \dot{x}_r)\dot{x}_r + \bar{h}(x_r) + \begin{bmatrix} F_m \\ 0 \end{bmatrix} = F, \quad (4)$$

$$m\ddot{x}_m + k_s(x_m - x_0) = F_m, \quad (5)$$

where $F(t) \triangleq J^{-T}(q)\tau(t) \in \mathbb{R}^2$ denotes the manipulator force. In (4) and (5), $F_m(x_r, x_m) \in \mathbb{R}$ denotes the impact force acting on the mass that occurs when $x_{r1}(t) \geq x_m(t)$ (Fig. 1) that is assumed to have the following form (Tornambe, 1999; Indri & Tornambe, 2004)

$$F_m = K_I \Lambda(x_{r1} - x_m), \quad (6)$$

where $K_I \in \mathbb{R}$ represents an unknown positive stiffness constant, and $\Lambda(x_r, x_m) \in \mathbb{R}$ is defined as

$$\Lambda = \begin{cases} 1 & x_{r1} \geq x_m \\ 0 & x_{r1} < x_m. \end{cases} \quad (7)$$

The dynamic model in (4) exhibits the following properties that will be utilized in the subsequent analysis.

Property 1: The inertia matrix $\bar{M}(x_r)$ is symmetric, positive definite, and can be lower and upper bounded as

$$a_1 \|\xi\|^2 \leq \xi^T \bar{M} \xi \leq a_2 \|\xi\|^2 \quad \forall \xi \in \mathbb{R}^2, \quad (8)$$

where $a_1, a_2 \in \mathbb{R}$ are positive constants.

Property 2: The following skew-symmetric relationship is satisfied

$$\xi^T \left(\frac{1}{2} \dot{\bar{M}}(x_r) - \bar{C}(x_r, \dot{x}_r) \right) \xi = 0 \quad \forall \xi \in \mathbb{R}^2. \quad (9)$$

Property 3: The robot dynamics given in (4) can be linearly parameterized as

$$Y(x_r, x_m, \dot{x}_r, \ddot{x}_r) \theta = \bar{M}(x_r) \ddot{x}_r + \bar{C}(x_r, \dot{x}_r) \dot{x}_r + \bar{h}(x_r) + \begin{bmatrix} F_m \\ 0 \end{bmatrix}, \quad (10)$$

where $\theta \in \mathbb{R}^p$ contains the constant unknown system parameters, and $Y(x_r, x_m, \dot{x}_r, \ddot{x}_r) \in \mathbb{R}^{2 \times p}$ denotes the known regression matrix.

Property 4: The following inequalities are valid for all $\xi = [\xi_1, \dots, \xi_n]^T \in \mathbb{R}^n$ (Dixon et al., 1999)

$$\|\xi\| \geq \|\text{Tanh}(\xi)\|$$

$$\|\xi\| + 1 \geq \frac{\|\xi\|}{\tanh(\|\xi\|)} \quad (11)$$

$$\|\xi\|^2 \geq \sum_{i=1}^n \ln(\cosh(\xi_i)) \geq \ln(\cosh(\|\xi\|)) \quad (12)$$

$$\xi^T \text{Tanh}(\xi) \geq \text{Tanh}^T(\xi) \text{Tanh}(\xi) = \|\text{Tanh}(\xi)\|^2 \geq \tanh^2(\|\xi\|). \quad (13)$$

Remark 1: To aid the subsequent control design and analysis, we define the vector $\text{Tanh}(\cdot) \in \mathbb{R}^n$ as follows

$$\text{Tanh}(\delta) = [\tanh(\delta_1), \dots, \tanh(\delta_n)]^T, \quad (14)$$

where $\delta = [\delta_1, \dots, \delta_n]^T \in \mathbb{R}^n$.

The following assumptions will be utilized in the subsequent control development.

Assumption 1: We assume that $x_{r1}(t)$ and $x_m(t)$ can be bounded as

$$\zeta_{x_r} \leq x_{r1}(t) \quad x_m(t) \leq \zeta_{x_m}, \quad (15)$$

where $\zeta_{x_r} \in \mathbb{R}$ is a known constant that is determined by the minimum coordinate of the robot along the X_1 axis, and $\zeta_{x_m} \in \mathbb{R}$ is a known positive constant. The lower bound assumption for $x_{r1}(t)$ is based on the geometry of the robot, and the upper bound assumption for $x_m(t)$ is based on the physical fact that the mass is attached by the spring to some object, and the mass will not be able to move past that object.

Assumption 2: We assume that the mass of the mass-spring system can be upper and lower bounded as

$$\underline{m} \leq m \leq \bar{m}, \quad (16)$$

where $\underline{m}$, $\bar{m} \in \mathbb{R}$ denote known positive bounding constants. The unknown stiffness constants K_I and k_s are also assumed to be bounded as

$$\underline{K_I} \leq K_I \leq \bar{K_I}, \quad (17)$$

$$\underline{k_s} \leq k_s \leq \bar{k_s}, \quad (18)$$

where $\underline{K_I}$, $\bar{K_I}$, $\underline{k_s}$, $\bar{k_s} \in \mathbb{R}$ denote known positive bounding constants.

Assumption 3: The subsequent development is based on the assumption that $q(t)$, $\dot{q}(t)$, $x_m(t)$, and $\dot{x}_m(t)$ are measurable, and that $x_r(t)$ and $\dot{x}_r(t)$ can be obtained from $q(t)$ and $\dot{q}(t)$.

Remark 2: During the subsequent control development, we assume that the minimum singular value of $J(q)$ is greater than a known, small positive constant $\delta > 0$, such that $\max\{\|J^{-1}(q)\|\}$ is known a priori, and hence, all kinematic singularities are always avoided.

2.2 Control Development

The subsequent control design is based on integrator backstepping methods. A desired trajectory is designed for the robot (i.e., a virtual control input) to ensure the robot impacts the mass and regulates it to a desired position. A force controller is developed to ensure that the robot tracks the desired trajectory despite the transition from free motion to an impact collision and despite parametric uncertainties throughout the MSR system.

2.2.1 Control Objective

The control objective is to regulate the states of the mass-spring system via a two-link planar robot that transitions from non-contact to contact with the mass-spring through an impact collision. An additional objective is to limit the impact force to prevent damage to the robot or the environment (i.e., the mass-spring system). An error signal, denoted by $e(t) \in \mathbb{R}^3$, is defined to quantify this objective as

$$e = \begin{bmatrix} e_m & e_r^T \end{bmatrix}^T,$$

where $e_r(t) \triangleq [e_{r1}(t), e_{r2}(t)]^T \in \mathbb{R}^2$ and $e_m(t) \in \mathbb{R}$ denote the errors for the end-point of the second link of the robot and mass-spring system (Fig. 1), respectively, and are defined as

$$e_r = x_{rd} - x_r \quad e_m = x_{md} - x_m. \quad (19)$$

In (19), $x_{md} \in \mathbb{R}$ denotes the constant known desired position of the mass, and $x_{rd}(t) \triangleq [x_{rd1}(t), x_{rd2}(t)]^T \in \mathbb{R}^2$ denotes the desired position of the end-point of the second link of the robot. To facilitate the subsequent control design and stability analysis, filtered tracking errors, denoted by $\eta_m(t) \in \mathbb{R}$ and $r_r(t) \in \mathbb{R}^2$, are defined as (Dixon et al., 2000)

$$\begin{aligned} \eta_m &= \dot{e}_m + \alpha_1 \tanh(e_m) + \alpha_2 \tanh(e_f) \\ r_r &= \dot{e}_r + \alpha e_r, \end{aligned} \quad (20)$$

where $\alpha, \alpha_1, \alpha_2 \in \mathbb{R}$ are positive, constant gains, and $e_f(t) \in \mathbb{R}$ is an auxiliary filter variable designed as (Dixon et al., 2000)

$$\dot{e}_f = -\alpha_3 \tanh(e_f) + \alpha_2 \tanh(e_m) - k_1 \cosh^2(e_f) \eta_m, \quad (21)$$

where $k_1 \in \mathbb{R}$ is a positive constant control gain, and $\alpha_3 \in \mathbb{R}$ is a positive constant filter gain. The filtered tracking error $r_r(t)$ is introduced to reduce the terms in the Lyapunov analysis (i.e., $r_r(t)$ can be used in lieu of including both $e_r(t)$ and $\dot{e}_r(t)$ in the stability analysis). The filtered tracking error $\eta_m(t)$ and the auxiliary signal $e_f(t)$ are introduced to eliminate a dependence on acceleration in the subsequently designed force controller (Dixon et al., 2003).

2.2.2 Closed-Loop Error System

By taking the time derivative of $m\dot{\eta}_m(t)$ and utilizing (5), (6), (19), and (20), the following open-loop error system can be obtained:

$$m\dot{\eta}_m = Y_d \theta_d - K_I \Lambda (x_{r1} - x_m) + \alpha_2 m \cosh^{-2}(e_f) \dot{e}_f + \alpha_1 m \cosh^{-2}(e_m) \dot{e}_m. \quad (22)$$

In (22), $Y_d(x_m) = (x_m - x_o)$ and $\theta_d = k_s$. To facilitate the subsequent analysis, the following notation is introduced (Dixon et al., 2000):

$$Y_d \theta_d = Y_d K_I K_I^{-1} \theta_d = Y_{dk} \theta_{dk} = K_I (x_m - x_o) \begin{bmatrix} k_s \\ K_I \end{bmatrix}. \quad (23)$$

After using (20) and (21), the expression in (22) can be rewritten as

$$m\dot{\eta}_m = Y_d \theta_d + K_I (x_{rd1} - \Lambda x_{r1}) + K_I \Lambda x_m - K_I x_{rd1} + \chi - \alpha_2 m k_1 \eta_m, \quad (24)$$

where $\chi(e_m, e_f, \eta_m, t) \in \mathbb{R}$ is an auxiliary term defined as

$$\begin{aligned} \chi &= \alpha_1 m \cosh^{-2}(e_m) (\eta_m - \alpha_1 \tanh(e_m)) - \alpha_1 \alpha_2 m \cosh^{-2}(e_m) \tanh(e_f) \\ &\quad + \alpha_2 m \cosh^{-2}(e_f) (\alpha_2 \tanh(e_m) - \alpha_3 \tanh(e_f)). \end{aligned} \quad (25)$$

The auxiliary expression, $\chi(e_m, e_f, \eta_m, t)$ defined in (25) can be upper bounded as

$$|\chi| \leq \zeta_1 \|z\|, \quad (26)$$

where $\zeta_1 \in \mathbb{R}$ is a positive bounding constant, and $z(t) \in \mathbb{R}^3$ is defined as

$$z = [\eta_m \quad \tanh(e_m) \quad \tanh(e_f)]^T. \quad (27)$$

Based on (24) and the subsequent stability analysis, the desired robot link position is designed as

$$\begin{aligned} x_{rd1} &= Y_d \hat{\theta}_{dk} + x_m + k_2 \tanh(e_m) - k_1 k_2 \cosh^2(e_f) \tanh(e_f) \\ x_{rd2} &= \varepsilon. \end{aligned} \quad (28)$$

In (28), $\varepsilon \in \mathbb{R}$ is an appropriate positive constant (i.e., ε is selected so the robot will impact the mass-spring system in the vertical direction), $k_2 \in \mathbb{R}$ is a positive constant control gain, and the control gain $k_1 \in \mathbb{R}$ is defined as

$$k_1 = \frac{1}{m} (3 + k_{n1} \zeta_1^2), \quad (29)$$

where $k_{n1} \in \mathbb{R}$ is a positive constant nonlinear damping gain. The parameter estimate $\hat{\theta}_{dk}(t) \in \mathbb{R}$ in (28) is generated by the adaptive update law

$$\dot{\hat{\theta}}_{dk} = \text{proj}(\Gamma Y_d \eta_m). \quad (30)$$

In (30), $\Gamma \in \mathbb{R}$ is a positive constant, and $\text{proj}(\cdot)$ denotes a sufficiently smooth projection algorithm (Cai et al., 2006) utilized to guarantee that $\hat{\theta}_{dk}(t)$ can be bounded as

$$\underline{\theta}_{dk} \leq \hat{\theta}_{dk} \leq \bar{\theta}_{dk}, \quad (31)$$

where $\underline{\theta}_{dk}, \bar{\theta}_{dk} \in \mathbb{R}$ denote known, constant lower and upper bounds for $\theta_{dk}(t)$, respectively. After substituting (28) into (24), the closed-loop error system for $\eta_m(t)$ can be obtained as

$$\begin{aligned} m\dot{\eta}_m &= K_I (x_{rd1} - \Lambda x_{r1}) + K_I (\Lambda x_m - x_m) + K_I k_1 k_2 \cosh^2(e_f) \tanh(e_f) + Y_{dk} \tilde{\theta}_{dk} \\ &\quad - K_I k_2 \tanh(e_m) + \chi - \alpha_2 m k_1 \eta_m. \end{aligned} \quad (32)$$

In (32), the parameter estimation error $\tilde{\theta}_{dk}(t) \in \mathbb{R}$ is defined as

$$\tilde{\theta}_{dk} = \theta_{dk} - \hat{\theta}_{dk}.$$

The open-loop robot error system can be obtained by taking the time derivative of $r_r(t)$ and premultiplying by the robot inertia matrix as

$$\bar{M} \dot{r}_r = Y_r \theta_r - \bar{C} r_r - F, \quad (33)$$

where (4), (19), and (20) were utilized, and

$$Y_r \theta_r = \bar{M} \ddot{x}_{rd} + \alpha \bar{M} \dot{e}_r + \bar{h} + \bar{C} \dot{x}_{rd} + \alpha \bar{C} x_{rd} + \begin{bmatrix} K_I \Lambda(x_{r1} - x_m) \\ 0 \end{bmatrix} - \alpha \bar{C} x_r, \quad (34)$$

where $Y_r(x_r, \dot{x}_r, x_m, \dot{x}_m, e_f, \eta_m, t) \in \mathbb{R}^{2 \times P}$ denotes a known regression matrix, and $\theta_r \in \mathbb{R}^P$ denotes an unknown constant parameter vector. By making substitutions from the dynamic model and the previous error systems, $\ddot{x}_{rd}(t)$ can be expressed without a dependence on acceleration terms (Liang et al., 2007). Based on (33) and the subsequent stability analysis, the robot force control input is designed as

$$F = Y_r \hat{\theta}_r + \text{Tanh}(e_r) + k_3 \text{Tanh}(r_r), \quad (35)$$

where $k_3 \in \mathbb{R}$ is a positive constant control gain, and $\hat{\theta}_r(t) \in \mathbb{R}^P$ is an estimate for θ_r generated by the following adaptive update law

$$\dot{\hat{\theta}}_r = \text{proj}(\Gamma_r Y_r^T r_r). \quad (36)$$

In (36), $\Gamma_r \in \mathbb{R}^{P \times P}$ is a positive definite, constant, diagonal, adaptation gain matrix, and $\text{proj}(\cdot)$ denotes a projection algorithm utilized to guarantee that the i -th element of $\hat{\theta}_r(t)$ can be bounded as

$$\underline{\theta}_{ri} \leq \hat{\theta}_{ri} \leq \bar{\theta}_{ri}, \quad (37)$$

where $\underline{\theta}_{ri}, \bar{\theta}_{ri} \in \mathbb{R}$ denote known, constant lower and upper bounds for each element of $\theta_r(t)$, respectively. The closed-loop error system for $r_r(t)$ can be obtained after substituting (35) into (33) as

$$\bar{M} \dot{r}_r = Y_r \tilde{\theta}_r - \bar{C} r_r - \text{Tanh}(e_r) - k_3 \text{Tanh}(r_r). \quad (38)$$

In (38), the parameter estimation error $\tilde{\theta}_r(t) \in \mathbb{R}^P$ is defined as

$$\tilde{\theta}_r = \theta_r - \hat{\theta}_r. \quad (39)$$

2.3 Stability Analysis

Theorem: The controller given by (28), (30), (35), and (36) ensures semi-global asymptotic regulation of the MSR system in the sense that

$$|e_m(t)| \rightarrow 0 \quad \|e_r(t)\| \rightarrow 0 \quad \text{as } t \rightarrow \infty$$

provided the control gains are chosen sufficiently large (Liang et al., 2007).

Proof: Let $V(t) \in \mathbb{R}$ denote the following non-negative, radially unbounded function (i.e., a Lyapunov function candidate):

$$V = \frac{1}{2} r_r^T \bar{M} r_r + \frac{1}{2} \tilde{\theta}_r^T \Gamma_r^{-1} \tilde{\theta}_r + \frac{1}{2} \tilde{\theta}_{dk}^T K_I \Gamma^{-1} \tilde{\theta}_{dk} + k_2 K_I \left[\ln(\cosh(e_m)) + \ln(\cosh(e_f)) \right] + e_r^T e_r + \frac{1}{2} m \eta_m^2 + \ln(\cosh(e_{r1})) + \ln(\cosh(e_{r2})). \quad (40)$$

After using (9), (13), (20), (21), (29), (30), (32), (36), and (38), the time derivative of (40) can be determined as

$$\begin{aligned} \dot{V} \leq & -k_3 \tanh^2(\|r_r\|) - \alpha_1 k_2 K_I \tanh^2(e_m) - \alpha_3 k_2 K_I \tanh^2(e_f) + 2e_r^T r_r - 2\alpha e_r^T e_r - 3\alpha_2 \eta_m^2 \\ & - k_{n1} \zeta_1^2 \alpha_2 \eta_m^2 - \alpha \tanh^2(\|e_r\|) + \eta_m [K_I (x_{rd1} - \Lambda x_{r1}) + K_I (\Lambda x_m - x_m) + \chi]. \end{aligned} \quad (41)$$

The expression in (41) will now be examined under two different scenarios.

Case 1-Non-contact: For this case, the systems are not in contact ($\Lambda = 0$) and (41) can be rewritten as

$$\begin{aligned} \dot{V} \leq & -k_3 \tanh^2(\|r_r\|) - \alpha_1 k_2 K_I \tanh^2(e_m) - \alpha_3 k_2 K_I \tanh^2(e_f) + 2e_r^T r_r - 2\alpha e_r^T e_r \\ & - 3\alpha_2 \eta_m^2 - k_{n1} \zeta_1^2 \alpha_2 \eta_m^2 - \alpha \tanh^2(\|e_r\|) + \eta_m [K_I x_{rd1} - K_I x_m + \chi]. \end{aligned} \quad (42)$$

Based on the development in (Liang et al., 2007), the above expression can be reduced to

$$\dot{V} \leq -\lambda \|y\|^2 + \varepsilon_x, \quad (43)$$

where λ is a known constant, and $y(t) \in \mathbb{R}^5$ is defined as

$$y = \begin{bmatrix} z^T & \tanh(\|e_r\|) & \tanh(\|r_r\|) \end{bmatrix}^T. \quad (44)$$

Based on (40) and (43), if $\lambda \|y(t)\|^2 > \varepsilon_x$, then Barbalat's Lemma can be used to conclude that $\dot{V}(t) \rightarrow 0$ since $V(t)$ is lower bounded, $\dot{V}(t)$ is negative semi-definite, and $\dot{V}(t)$ can be shown to be uniformly continuous (UC). As $\dot{V}(t) \rightarrow 0$, eventually $\lambda \|y(t)\|^2 \leq \varepsilon_x$. While $\lambda \|y(t)\|^2 > \varepsilon_x$, (40) and (43) can be used to conclude that $V(t) \in \mathbf{L}_\infty$. When $\lambda \|y(t)\|^2 \leq \varepsilon_x$, the definitions (27) and (44) can be used to conclude that $e_m(t)$, $e_f(t)$, $e_r(t)$, $r_r(t)$, $\eta_m(t) \in \mathbf{L}_\infty$.

Since $\tilde{\theta}_r(t)$ and $\tilde{\theta}_{dk}(t) \in \mathbf{L}_\infty$ from the use of a projection algorithm, the previous facts can be used to conclude that $V(t) \in \mathbf{L}_\infty$. Signal chasing arguments can be used to prove the remaining closed-loop signals are also bounded during the non-contact case provided the control gains are chosen sufficiently large.

If the initial conditions for $V(0)$ are inside the region defined by ε_x , then $V(t)$ can grow larger until $\lambda \|y(t)\|^2 \leq \varepsilon_x$. Therefore, further development is required to determine how large $V(t)$ can grow. Sufficient gain conditions are developed in (Liang et al., 2007), which guarantee a semi-global stability result and ensure that the manipulator makes contact with the mass-spring system.

Case 2-Contact: For the case when the dynamic systems collide ($\Lambda = 1$) and the two dynamic systems become coupled, then (41) can be rewritten as

$$\begin{aligned} \dot{V} \leq & -k_3 \tanh^2(\|r_r\|) - \alpha_1 k_2 K_I \tanh^2(e_m) - \alpha_3 k_2 K_I \tanh^2(e_f) - 3\alpha_2 \eta_m^2 - \alpha \tanh^2(\|e_r\|) \\ & - [\alpha \|e_r\|^2 - \overline{K_I} \eta_m \|e_r\|] - [k_{n1} \zeta_1^2 \alpha_2 \eta_m^2 - \zeta_1 \|z\| \eta_m] - [\alpha \|e_r\|^2 - 2\|e_r\| \|r_r\|], \end{aligned}$$

where (17) was substituted for K_I and (26) was substituted for $\chi(e_m, e_f, \eta_m, t)$. Completing the square on the three bracketed terms yields

$$\dot{V} \leq -\beta \|z\|^2 - \alpha \tanh^2(\|e_r\|) - \left(\alpha_2 - \frac{\bar{K}_I}{4\alpha} \right) \eta_m^2 - \left(k_3 - \frac{\|r_r\|^2}{\alpha \tanh^2(\|r_r\|)} \right) \tanh^2(\|r_r\|). \quad (45)$$

Because (40) is non-negative, as long as the gains are picked sufficiently large (Liang et al., 2007), (45) is negative semi-definite, and $r_r(t)$, $\tilde{\theta}_r(t)$, $\tilde{\theta}_{dk}(t)$, $e_r(t)$, $e_m(t)$, $e_f(t)$ and $\eta_m(t) \in \mathbf{L}_\infty$. Based on the development in (Liang et al., 2007), Barbalat's Lemma can be used to conclude that $\tanh(e_m(t))$, $\tanh(\|r_r(t)\|)$, $|\eta_m(t)| \rightarrow 0$ as $t \rightarrow \infty$, which also implies $|e_m(t)|$, $\|r_r(t)\| \rightarrow 0$ as $t \rightarrow \infty$. Based on the fact that $\|r_r(t)\| \rightarrow 0$ as $t \rightarrow \infty$, standard linear analysis methods (see Lemma A.15 of (Dixon et al., 2003)) can then be used to prove that $\|e_r(t)\| \rightarrow 0$ as $t \rightarrow \infty$. Signal chasing arguments can be used to show that all signals remain bounded.

2.4 Experimental Results

The testbed depicted in Fig. 2 and Fig. 3 was developed for experimental demonstration of the proposed controller. The testbed is composed of a mass-spring system and a two-link planar robot. The body of the mass-spring system includes a U-shaped aluminum plate (item (8) in Fig. 2) mounted on an undercarriage with porous carbon air bearings which enables the undercarriage to glide on an air cushion over a glass covered aluminum rail. A steel core spring (item (1) in Fig. 2) connects the U-shaped aluminum plate to an aluminum frame, and a linear variable displacement transducer (LVDT) (item (2) in Fig. 2) is used to measure the position of the undercarriage assembly. The impact surface consists of an aluminum plate connected to the undercarriage assembly through a stiff spring mechanism (item (7) in Fig. 2). A capacitance probe (item (3) in Fig. 2) is used to measure the deflection of the stiff spring mechanism. The two-link planar robot (items (4-6) in Fig. 2) is made of two aluminum links, mounted on 240.0 [Nm] (base link) and 20.0 [Nm] (second link) direct-drive switched reluctance motors. The motor resolvers provide rotor position measurements with a resolution of 614400 pulses/revolution, and a standard backwards difference algorithm is used to numerically determine angular velocity from the encoder readings. A Pentium 2.8 GHz PC operating under QNX hosts the control algorithm, which was implemented via a custom graphical user-interface to facilitate real-time graphing, data logging, and the ability to adjust control gains without recompiling the program. Data acquisition and control implementation were performed at a frequency of 2.0 kHz using the ServoToGo I/O board. The initial conditions for the robot coordinates and the mass-spring position were (in [m])

$$[x_{r1}(0) \ x_{r2}(0) \ x_m(0)] = [0.060 \ 0.436 \ 0.206]. \quad (46)$$

The initial velocity of the robot and mass-spring were zero, and the desired mass-spring position was (in [m])

$$x_{md} = 0.236. \quad (47)$$

That is, the tip of the second link of the robot was initially 176 [mm] from the desired setpoint and 146 [mm] from x_0 along the X_1 -axis (Fig. 2). Once the initial impact occurs, the robot is required to depress the spring (item (1) in Fig. 2) to move the mass 30 [mm] along the X_1 -axis. The control and adaptation gains were selected as in (Liang et al., 2007). The mass-spring and robot errors (i.e., $e(t)$) are shown in Fig. 4. The peak steady-state position

error of the end-point of the second link of the robot along the X_1 -axis (i.e., $|e_{r1}(t)|$) and along the X_2 -axis (i.e., $|e_{r2}(t)|$) are 0.216 [mm] and 0.737 [mm], respectively. The peak steady-state position error of the mass (i.e., $|e_m(t)|$) is 2.56 [μ m]. The relatively large $|e_{r1}(t)|$ is due to the mismatch between the estimate value $\hat{\theta}_{dk}(t)$ and the real value $\theta_{dk}(t)$ in $e_{r1}(t)$. The relatively large $|e_{r2}(t)|$ is due to the inability of the model to capture friction and slipping effects on the contact surface. In this experiment, a significant friction is present along the X_2 -axis between the robot tip and the contact surface due to a large normal spring force applied along the X_1 -axis. The input control torques (i.e., $J^T(q)F(t)$) are shown in Fig. 5. The resulting desired trajectory along the X_1 -axis (i.e., $x_{rd1}(t)$) is depicted in Fig. 6, and the desired trajectory along the X_2 -axis was chosen as $x_{rd2} = 0.358$ [m]. Fig. 7 depicts the value of $\hat{\theta}_{dk}(t)$ and Figs. 8-10 depict the values of $\hat{\theta}_r(t)$. The order of the curves in the plots is based on the relative scale of the parameter estimates rather than numerical order in $\hat{\theta}_r(t)$.

Figure 2. Top view of the experimental testbed including: (1) spring, (2) LVDT, (3) capacitance probe, (4) link1, (5) motor1, (6) link2, (7) stiff spring mechanism, (8) mass

Figure 3. Side view of the experimental testbed

Figure 4. The mass-spring and robot errors $e(t)$. Plot (a) indicates the position error of the robot tip along the X_1 -axis (i.e., $e_{r1}(t)$), (b) indicates the position error of the robot tip along the X_2 -axis (i.e., $e_{r2}(t)$), and (c) indicates the position error of the mass-spring (i.e., $e_m(t)$)

Figure 5. Applied control torques $J^T(q)F(t)$, for the (a) base motor and (b) second link motor

Figure 6. Computed desired robot trajectory, $x_{rd1}(t)$

Figure 7. Unitless parameter estimate $\hat{\theta}_{dk}(t)$ introduced in (28)

Figure 8. Estimate for the unknown constant parameter vector $\hat{\theta}_r(t)$. (a) $\hat{\theta}_{r10}(t)$ (b) $\hat{\theta}_{r4}(t)$, (c) $\hat{\theta}_{r1}(t)$, and (d) $\hat{\theta}_{r7}(t)$

Figure 9. Estimate for the unknown constant parameter vector $\hat{\theta}_r(t)$. (a) $\hat{\theta}_{r5}(t)$ (b) $\hat{\theta}_{r2}(t)$, (c) $\hat{\theta}_{r8}(t)$

Figure 10. Estimate for the unknown constant parameter vector $\hat{\theta}_r(t)$. (a) $\hat{\theta}_{r6}(t)$ (b) $\hat{\theta}_{r3}(t)$, (c) $\hat{\theta}_{r9}(t)$

3. Control of Robotic Contact with a Viscoelastic Environment

The study of robot interaction with a viscoelastic environment is motivated by the increasing applications involving human-robot interaction. Since viscoelastic materials exhibit damping, the linear spring model used for stiff environments, in the previous section, would be inadequate to accurately represent the physical phenomena during contact. A nonlinear Hunt-Crossley model used in this section includes both stiffness and damping terms to account for the energy dissipation at contact. The differences in the contact model result in differences in the control development/stability analysis with regard to the controller in the previous section for stiff environments. The control structure in this section includes a desired robot velocity as a virtual control input to the unactuated viscoelastic mass spring system, coupled with a force controller to ensure that the actual robot position tracks the desired position. A NN feedforward term is used in the controller to estimate the parametric and non-parametric uncertainties.

3.1 Dynamic Model

The dynamic model for a rigid two-link revolute robot interacting with a compliant viscoelastic environment (Fig. 11) is given as

$$\overline{M}(x_r)\ddot{x}_r + \overline{C}(x_r, \dot{x}_r)\dot{x}_r + \overline{h}(x_r) + \begin{bmatrix} F_m \\ 0 \end{bmatrix} = F \quad (48)$$

$$m\ddot{x}_m + k_s(x_m - x_0) = F_m. \quad (49)$$

The interaction force $F_m(x_{r1}, \dot{x}_{r1}, x_m, \dot{x}_m)$ between the robot and the viscoelastic mass is modeled as

$$F_m = \Lambda F_v, \quad (50)$$

where $\Lambda(x_{r1}, x_m) \in \mathbb{R}$ is defined in (7) as a function which switches at impact, and $F_v(x_{r1}, \dot{x}_{r1}, x_m, \dot{x}_m) \in \mathbb{R}$ denotes the Hunt-Crossley force defined as (Hunt & Crossley, 1975)

$$F_v \triangleq \lambda \delta^n + b \dot{\delta} \delta^n. \quad (51)$$

In (51), $\lambda \in \mathbb{R}$ is the unknown contact stiffness of the viscoelastic mass, $b \in \mathbb{R}$ is the unknown impact damping coefficient, $\delta(x_r, x_m) \in \mathbb{R}$ denotes the local deformation of the material and is defined as

$$\delta \triangleq x_{r1} - x_m. \quad (52)$$

Also, in (51), $\dot{\delta}(t)$ is the relative velocity of the contacting bodies, and n is the unknown Hertzian compliance coefficient which depends on the surface geometry of contact. The model in (50) is a continuous contact force-based model wherein the contact forces increase from zero upon impact and return to zero upon separation. Also, the energy dissipation during impact is a function of the damping constant which can be related to the impact velocity and the coefficient of restitution (Hunt & Crossley, 1975), thus making the model more consistent with the physics of contact. The contact is considered to be direct-central and quasi-static (i.e., all the stresses are transmitted at the time of contact and sliding and friction effects during contact are negligible) where plastic deformation effects are assumed to be negligible. In addition to the properties in (8) and (9), the following property will be utilized in the subsequent control development.

Figure 11. Robot contact with a viscoelastic mass

Property 5: The expression for the interaction force $F_m(x_r, \dot{x}_r, x_m, \dot{x}_m)$ in (50) can be written, using (7) and (51), as

$$F_m = \begin{cases} 0 & \delta < 0 \\ \lambda \delta^n + b \dot{\delta} \delta^n & \delta \geq 0. \end{cases} \quad (53)$$

Based on the fact that

$$\lim_{\delta \rightarrow 0^-} F_m = 0 \quad \lim_{\delta \rightarrow 0^+} F_m = 0, \quad (54)$$

the interaction force $F_m(t)$ is continuous.

3.2 Neural Network Control

In the subsequent control development, the desired robot velocity is designed as a virtual control input to the unactuated viscoelastic mass. The desired velocity is designed to ensure that the robot impacts and then regulates the mass to a desired position. A force controller is developed to ensure that the robot tracks the desired trajectory despite the non-contact to contact transition and parametric uncertainties in the robot and the viscoelastic mass-spring system. The viscoelastic model requires that the backstepping error be developed in terms of the desired robot velocity. To overcome this limitation, a Neural Network (NN) feedforward term is used in the controller to estimate the environmental uncertainties which are not linear in the parameters (such as the Hertzian compliance coefficient in (51)). In addition to the assumptions made in (15) and (16), the following assumptions are made

Assumption 4: The local deformation of the viscoelastic material during contact, $\delta(x_r, x_m)$, defined in (52), is assumed to be upper bounded, and hence δ'' can be upper bounded as

$$\delta'' \leq \mu, \quad (55)$$

where $\mu \in \mathbb{R}$ is a positive bounding constant.

Assumption 5: The damping constant, b , in (51), is assumed to be upper bounded as

$$b \leq \bar{b}, \quad (56)$$

where $\bar{b} \in \mathbb{R}$ denotes a known positive bounding constant.

3.2.1 Control Objective

The control objective of regulating the position of a viscoelastic mass attached to a spring via a robotic contact is quantified as in (19). To facilitate the subsequent control design and stability analysis, filtered tracking errors for the robot and the mass-spring, denoted by $r_r(t) \in \mathbb{R}^2$ and $r_m(t) \in \mathbb{R}$ respectively, are redefined as

$$\begin{aligned} r_r &\triangleq \dot{e}_r + \alpha e_r \\ r_m &\triangleq \dot{e}_m + \gamma_1 e_m + \gamma_2 e_f, \end{aligned} \quad (57)$$

where $\alpha \in \mathbb{R}^{2 \times 2}$ is a positive, diagonal, constant gain matrix $\gamma_1, \gamma_2 \in \mathbb{R}$ are positive constant gains, and $e_f(t) \in \mathbb{R}$ is an auxiliary filter variable designed

$$\dot{e}_f = -\gamma_3 e_f + \gamma_2 e_m - k_1 r_m, \quad (58)$$

where $k_1, \gamma_3 \in \mathbb{R}$ are positive constant control gains.

3.2.2 NN Feedforward Estimation

NN-based estimation methods are well suited for control systems where the dynamic model contains uncertainties as in (48), (49) and (51). The universal approximation property of the Neural Network lends itself nicely to control system design. Multilayer Neural Networks have been shown to be capable of approximating generic nonlinear continuous functions. Let S be a compact simply connected set of $\mathbb{R}^{N_1+1}$. With map $f : S \rightarrow \mathbb{R}^n$, define $C^n(S)$ as the space where f is continuous. There exist weights and thresholds such that some function $f(x) \in C^n(S)$ can be represented by a three-layer NN as (Lewis, 1999; Lewis et al., 2002)

$$f(x) = W^T \sigma(V^T x) + \varepsilon(x), \quad (59)$$

for some given input $x(t) \in \mathbb{R}^{N_1+1}$. In (59), $V \in \mathbb{R}^{(N_1+1) \times N_2}$ and $W \in \mathbb{R}^{(N_2+1) \times n}$ are bounded constant ideal weight matrices for the first-to-second and second-to-third layers respectively, where N_1 is the number of neurons in the input layer, N_2 is the number of neurons in the hidden layer, and n is the number of neurons in the third layer. The activation function in (59) is denoted by $\sigma(\cdot) \in \mathbb{R}^{N_2+1}$, and $\varepsilon(x) \in \mathbb{R}^n$ is the functional reconstruction error. Note that, augmenting the input vector $x(t)$ and activation function $\sigma(\cdot)$ by 1 allows the thresholds as the first columns of the weight matrices (Lewis, 1999; Lewis et al., 2002). Thus, any tuning of W and V then includes tuning of thresholds as well. The computing power of the NN comes from the fact that the activation function $\sigma(\cdot)$ is nonlinear and the weights W and V can be modified or tuned through some learning procedure (Lewis et al., 2002). Based on (59), the typical three-layer NN approximation for $f(x)$ is given as (Lewis, 1999; Lewis et al., 2002)

$$\hat{f}(x) \triangleq \hat{W}^T \sigma(\hat{V}^T x), \quad (60)$$

where $\hat{V}(t) \in \mathbb{R}^{(N_1+1) \times N_2}$ and $\hat{W}(t) \in \mathbb{R}^{(N_2+1) \times n}$ are subsequently designed estimates of the ideal weight matrices. The estimate mismatch for the ideal weight matrices, denoted by $\tilde{V}(t) \in \mathbb{R}^{(N_1+1) \times N_2}$ and $\tilde{W}(t) \in \mathbb{R}^{(N_2+1) \times n}$, are defined as

$$\tilde{V} \triangleq V - \hat{V}, \quad \tilde{W} \triangleq W - \hat{W},$$

and the mismatch for the hidden-layer output error for a given $x(t)$, denoted by $\tilde{\sigma}(x) \in \mathbb{R}^{N_2+1}$, is defined as

$$\tilde{\sigma} \triangleq \sigma - \hat{\sigma} = \sigma(V^T x) - \sigma(\hat{V}^T x). \quad (61)$$

The NN estimate has several properties that facilitate the subsequent development. These properties are described as follows.

Property 6: (*Taylor Series Approximation*) The Taylor series expansion for $\sigma(V^T x)$ for a given x may be written as (Lewis, 1999; Lewis et al., 2002)

$$\sigma(V^T x) = \sigma(\hat{V}^T x) + \sigma'(\hat{V}^T x) \tilde{V}^T x + O(\tilde{V}^T x)^2, \quad (62)$$

where $\sigma'(\hat{V}^T x) \triangleq d\sigma(V^T x)/d(V^T x)|_{V^T x = \hat{V}^T x}$, and $O(\hat{V}^T x)^2$ denotes the higher order terms. After substituting (62) into (61) the following expression can be obtained:

$$\tilde{\sigma} = \hat{\sigma}' \tilde{V}^T x + O(\tilde{V}^T x)^2, \quad (63)$$

where $\hat{\sigma}' \triangleq \sigma'(\hat{V}^T x)$.

Property 7: (*Boundedness of the Ideal Weights*) The ideal weights are assumed to exist and be bounded by known positive values so that

$$\|V\|_F^2 \leq \bar{V}_B \quad (64)$$

$$\|W\|_F^2 \leq \bar{W}_B, \quad (65)$$

where $\|\cdot\|_F$ is the Frobenius norm of a matrix, and $tr(\cdot)$ weights, $\hat{W}(t)$ and $\hat{V}(t)$, can be bounded using the projection algorithm as in (Patre et al., 2008).

Property 9: (Boundedness of activation function σ and $\dot{\sigma}$) The typical choice of activation function is the sigmoid function

$$\sigma(z) = \frac{1}{1 + e^{\kappa z}}, \quad (66)$$

where

$$\|\sigma\| < 1 \text{ and } \|\dot{\sigma}\| \leq \sigma_n,$$

where $\sigma_n \in \mathbb{R}$ is a known positive constant.

Property 10: (Boundedness of functional reconstruction error $\varepsilon(x)$) On a given compact set S , the net reconstruction error $\varepsilon(x)$ is bounded as

$$\|\varepsilon(x)\| \leq \varepsilon_n,$$

where $\varepsilon_n \in \mathbb{R}$ is a known positive constant.

Property 11: (Property of trace) If $A \in \mathbb{R}^{n \times m}$ and $B \in \mathbb{R}^{m \times n}$, then

$$tr(AB) = tr(BA).$$

3.2.3 Closed-Loop Error System

The open-loop error system for the mass can be obtained by multiplying (57) by m and then taking its time derivative as

$$m\dot{r}_m = k_s(x_m - x_0) - \Lambda\lambda\delta^n - \Lambda b\dot{\delta}^n + \chi - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m, \quad (67)$$

where $\chi(e_m, r_m, e_f, t) \in \mathbb{R}$ is an auxiliary term defined as

$$\chi \triangleq m\gamma_1 r_m + m\gamma_2^2 e_m - (m\gamma_1\gamma_2 + m\gamma_2\gamma_3)e_f. \quad (68)$$

The auxiliary expression $\chi(e_m, r_m, e_f, t) \in \mathbb{R}$ defined in (68) can be upper bounded as

$$|\chi| \leq \zeta_2 \|z\|, \quad (69)$$

where $\zeta_2 \in \mathbb{R}$ is a known positive constant, and $z(t) \in \mathbb{R}^3$ is defined as

$$z \triangleq \begin{bmatrix} r_m & e_m & e_f \end{bmatrix}. \quad (70)$$

The expression in (67) can be written as

$$m\dot{r}_m = f_1 - \Lambda b\dot{\delta}^n + \chi - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m, \quad (71)$$

where the function $f_1(t) \in \mathbb{R}$ containing the uncertain spring and damping constants, is defined as

$$f_1 \triangleq k_s(x_m - x_0) - \Lambda\lambda\delta^n. \quad (72)$$

The auxiliary function in (72) can be represented by a three layer NN as

$$f_1 = W_1^T \sigma_1(V_1^T x_1) + \varepsilon_m(x_1), \quad (73)$$

where the NN input $x_1(t) \in \mathbb{R}^3$ is defined as $x_1 \triangleq [1, x_m, \Lambda\delta]^T$, $W_1 \in \mathbb{R}^{(N_{m2}+1) \times 1}$, and $V_1 \in \mathbb{R}^{3 \times N_{m2}}$ are ideal NN weights, and $N_{m2} \in \mathbb{R}$ denotes the number of hidden layer neurons of the NN. Since the open loop error expression for the mass in (71) does not have an actual control input, a virtual control input, $\dot{x}_{rd1}(t)$, is introduced by adding and subtracting $(1 - \Lambda b\delta^n)\dot{x}_{rd1}$ to (71) as

$$m\dot{r}_m = f_1 - \Lambda b\delta\dot{\delta}^n + \chi - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m + (1 - \Lambda b\delta^n)\dot{x}_{rd1} - (1 - \Lambda b\delta^n)\dot{x}_{rd1}. \quad (74)$$

To facilitate the subsequent backstepping-based design, the virtual control input to the unactuated mass-spring system, $\dot{x}_{rd1}(t)$ is designed as

$$\dot{x}_{rd1} = \hat{f}_1. \quad (75)$$

Also, $x_{rd2} = \rho$ where $\rho \in \mathbb{R}$ is an appropriate positive constant, selected so the robot will impact the mass-spring system. In (75), $\hat{f}_1(t) \in \mathbb{R}$ is the estimate for $f_1(t)$ and is defined as

$$\hat{f}_1 \triangleq \hat{W}_1^T \sigma_1(\hat{V}_1^T x_1), \quad (76)$$

where $\hat{W}_1(t) \in \mathbb{R}^{(N_{m2}+1) \times 1}$ and $\hat{V}_1(t) \in \mathbb{R}^{3 \times N_{m2}}$ are the estimates of the ideal weights, which are updated based on the subsequent stability analysis as

$$\dot{\hat{W}}_1 = \Gamma_{w1} \hat{\sigma}_1 r_m - \Gamma_{w1} \hat{\sigma}_1 \hat{V}_1^T x_1 r_m \quad \dot{\hat{V}}_1 = \Gamma_{v1} x_1 \hat{W}_1^T \hat{\sigma}_1 r_m \quad (77)$$

where $\Gamma_{w1} \in \mathbb{R}^{(N_{m2}+1) \times (N_{m2}+1)}$, $\Gamma_{v1} \in \mathbb{R}^{3 \times 3}$ are constant, positive definite, symmetric gain matrices. The estimates for the NN weights in (77) are generated on-line (there is no off-line learning phase). The closed loop error system for the mass can be developed by substituting (75) into (74) and using (19) as

$$m\dot{r}_m = f_1 - \hat{f}_1 + \Lambda b\delta^n \dot{e}_{r1} - \Lambda b\delta^n \dot{e}_m + \chi - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m + (1 - \Lambda b\delta^n)\dot{x}_{rd1}. \quad (78)$$

Using (73) and (76), the expression in (78) can be written as

$$m\dot{r}_m = W_1^T \sigma_1(V_1^T x_1) - \hat{W}_1^T \sigma_1(\hat{V}_1^T x_1) + \varepsilon_m(x_1) + \Lambda b\delta^n \dot{e}_{r1} - \Lambda b\delta^n \dot{e}_m + \chi - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m + (1 - \Lambda b\delta^n)\dot{x}_{rd1}. \quad (79)$$

Adding and subtracting $W_1^T \hat{\sigma}_1 + \hat{W}_1^T \tilde{\sigma}_1$ to (79), and then using the Taylor series approximation in (63), the following expression for the closed loop mass error system can be obtained

$$m\dot{r}_m = \tilde{W}_1^T \hat{\sigma}_1 - \tilde{W}_1^T \hat{\sigma}_1 \hat{V}_1^T x_1 + \hat{W}_1^T \tilde{\sigma}_1 \hat{V}_1^T x_1 + w_1 - m\gamma_1^2 e_m - m\gamma_2 k_1 r_m, \quad (80)$$

where the notations $\hat{\sigma}_1$ and $\tilde{\sigma}_1$ were introduced in (61), and $w_1(t) \in \mathbb{R}$ is defined as

$$w_1 = \tilde{W}_1^T \hat{\sigma}_1' V_1^T x_1 + W_1^T O(\tilde{V}_1^T x_1)^2 + \varepsilon_m(x_1) + \Lambda b \delta^n \dot{e}_{r1} - \Lambda b \delta^n \dot{e}_m + \chi + (1 - \Lambda b \delta^n) \dot{x}_{rd1}.$$

It can be shown from Property 7, Property 8, and (Lewis et al., 1996) that $w_1(t)$ can be bounded as

$$|w_1| \leq c_{m1} + c_{m2} \|z\| + c_{m3} \|e_r\| + c_{m4} \|r_r\|, \quad (81)$$

where $c_{mi} \in \mathbb{R}$, ($i = 1, 2, \dots, 4$) are computable known positive constants. The open-loop robot error system can be obtained by taking the time derivative of $r_r(t)$, premultiplying by the robot inertia matrix $\bar{M}(x_r)$, and utilizing (19), (48), and (57) as

$$\bar{M} \dot{r}_r = f_2 - \bar{C} r_r - F, \quad (82)$$

where the function $f_2(t) \in \mathbb{R}^2$, contains the uncertain robot and Hunt-Crossley model parameters, and is defined as

$$f_2 \triangleq \bar{M} \ddot{x}_{rd} + \alpha \bar{M} \dot{e}_r + \bar{h} + \bar{C} \dot{x}_{rd} + \alpha \bar{C} x_{rd} + \begin{bmatrix} \Lambda(\lambda \delta^n + b \dot{\delta} \delta^n) \\ 0 \end{bmatrix} - \alpha \bar{C} x_r.$$

By representing the function $f_2(t)$ by a NN, the expression in (82) can be written as

$$\bar{M} \dot{r}_r = W_2^T \sigma_2(V_2^T x_2) + \varepsilon_r(x_2) - \bar{C} r_r - F, \quad (83)$$

where the NN input $x_2(t) \in \mathbb{R}^{13}$ is defined as $x_2(t) \triangleq [1, \Lambda \delta, x_r^T, x_{rd}^T, \dot{\delta}, \dot{x}_r^T, \dot{x}_{rd}^T, \ddot{x}_{rd}^T]^T$, $W_2 \in \mathbb{R}^{(N_{r2}+1) \times 2}$ and $V_2 \in \mathbb{R}^{13 \times N_{r2}}$ are ideal NN weights and $N_{r2} \in \mathbb{R}$ denotes the number of hidden layer neurons of the NN. An expression for $\ddot{x}_{rd1}(t)$ can be developed to illustrate that the second derivative of the desired trajectory is continuous and does not require acceleration measurements. Based on (83) and the subsequent stability analysis, the robot force control input is designed as

$$F = \hat{W}_2^T \sigma_2(\hat{V}_2^T x_2) + k_2 r_r + e_r, \quad (84)$$

where $k_2 \in \mathbb{R}$ is a constant positive control gain, and $\hat{W}_2(t) \in \mathbb{R}^{(N_{r2}+1) \times 2}$ and $\hat{V}_2(t) \in \mathbb{R}^{13 \times N_{r2}}$ are the estimates of the ideal weights, which are designed based on the subsequent stability analysis as

$$\hat{W}_2 = \Gamma_{w2} \hat{\sigma}_2 r_r^T - \Gamma_{w2} \hat{\sigma}_2' \hat{V}_2^T x_2 r_r^T \quad \hat{V}_2 = \Gamma_{v2} x_2 r_r^T \hat{W}_2^T \hat{\sigma}_2' \quad (85)$$

where $\Gamma_{w2} \in \mathbb{R}^{(N_{r2}+1) \times (N_{r2}+1)}$, $\Gamma_{v2} \in \mathbb{R}^{13 \times 13}$ are constant, positive definite, symmetric gain matrices. Substituting (84) into (83) and following a similar approach as in the mass error system in (78)-(80), the closed loop error system for the robot is obtained as

$$\bar{M} \dot{r}_r = \tilde{W}_2^T \hat{\sigma}_2 - \tilde{W}_2^T \hat{\sigma}_2' \hat{V}_2^T x_2 + \hat{W}_2^T \hat{\sigma}_2' \tilde{V}_2^T x_2 + w_2 - \bar{C} r_r - k_2 r_r - e_r, \quad (86)$$

where $w_2(t) \in \mathbb{R}^2$ is defined as

$$w_2 = \tilde{W}_2^T \hat{\sigma}_2 V_2^T x_2 + W_2^T O(\tilde{V}_2^T x_2)^2 + \varepsilon_r(x_2). \quad (87)$$

It can be shown from Property 7, Property 8 and (Lewis et al., 1996) that $w_2(t)$ can be bounded as

$$\|w_2\| \leq c_{r1} + c_{r2}\|z\| + c_{r3}\|e_r\| + c_{r4}\|r_r\|, \quad (88)$$

where $c_{ri} \in \mathbb{R}$, ($i = 1, 2, \dots, 4$) are computable known positive constants.

3.2.4 Stability Analysis

Theorem: The controller given by (75), (77), (84), and (85) ensures uniformly ultimately bounded regulation of the MSR system in the sense that

$$|e_m(t)|, \|e_r(t)\| \rightarrow \varepsilon_0 \exp(-\varepsilon_1 t) + \varepsilon_2 \quad (89)$$

provided the control gains are chosen sufficiently large (Bhasin et al., 2008).

Proof: Let $V(t) \in \mathbb{R}$ denote a non-negative, radially unbounded function (i.e., a Lyapunov function candidate) defined as

$$\begin{aligned} V = & \frac{1}{2} r_r^T \overline{M} r_r + \frac{1}{2} e_r^T e_r + \frac{1}{2} \text{tr}(\tilde{W}_1^T \Gamma_{w1}^{-1} \tilde{W}_1) + \frac{1}{2} \text{tr}(\tilde{V}_1^T \Gamma_{v1}^{-1} \tilde{V}_1) + \frac{1}{2} \text{tr}(\tilde{W}_2^T \Gamma_{w2}^{-1} \tilde{W}_2) + \frac{1}{2} \text{tr}(\tilde{V}_2^T \Gamma_{v2}^{-1} \tilde{V}_2) \\ & + \frac{1}{2} m r_m^2 + \frac{1}{2} \gamma_1^2 m e_f^2 + \frac{1}{2} \gamma_1^2 m e_m^2. \end{aligned} \quad (90)$$

It follows directly from the bounds given in (8), Property 8, (64) and (65), that $V(t)$ can be upper and lower bounded as

$$\lambda_3 \|y\|^2 \leq V(t) \leq \lambda_4 \|y\|^2 + \theta, \quad (91)$$

where $\lambda_3, \lambda_4, \theta \in \mathbb{R}$ are known positive bounding constants, and $y(t) \in \mathbb{R}^7$ is defined as

$$y = \begin{bmatrix} r_r^T & e_r^T & z^T \end{bmatrix}^T. \quad (92)$$

The time derivative of $V(t)$ in (90) can be upper bounded (Bhasin et al., 2008) as

$$\dot{V}(t) \leq -\frac{\beta}{\lambda_4} V(t) + \varepsilon_x, \quad (93)$$

where $\varepsilon_x, \beta \in \mathbb{R}$ are positive constants which can be adjusted through the control gains (Bhasin et al., 2008). Provided the gains are chosen sufficiently large (Bhasin et al., 2008), the definitions in (70) and (92), and the expressions in (90) and (93) can be used to prove that $r_r(t), e_r(t), r_m(t), e_m(t), e_f(t) \in \mathbf{L}_\infty$. In a similar approach to the one developed in the first

section, it can be shown that all other signals remain bounded and the controller given by (75), (77), (84), and (85) is implementable.

4. Conclusion

In this chapter, we consider a two link planar robotic system that transitions from free motion to contact with an unactuated mass-spring system. In the first half of the chapter, an adaptive nonlinear Lyapunov-based controller with bounded torque input amplitudes is designed for robotic contact with a stiff environment. The feedback elements for the controller are contained inside of hyperbolic tangent functions as a means to limit the impact forces resulting from large initial conditions as the robot transitions from non-contact to contact. The continuous controller in (35) yields semi-global asymptotic regulation of the spring-mass and robot links. Experimental results are provided to illustrate the successful performance of the controller. In the second half of the chapter, a Neural Network controller is designed for a robotic system interacting with an uncertain Hunt-Crossley viscoelastic environment. This result extends our previous result in this area to include a more general contact model, which not only accounts for stiffness but also damping at contact. The use of NN-based estimation in (Bhasin et al., 2008) provides a method to adapt for uncertainties in the robot and impact model.

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In this book we have grouped contributions in 28 chapters from several authors all around the world on the several aspects and challenges of research and applications of robots with the aim to show the recent advances and problems that still need to be considered for future improvements of robot success in worldwide frames. Each chapter addresses a specific area of modeling, design, and application of robots but with an eye to give an integrated view of what make a robot a unique modern system for many different uses and future potential applications. Main attention has been focused on design issues as thought challenging for improving capabilities and further possibilities of robots for new and old applications, as seen from today technologies and research programs. Thus, great attention has been addressed to control aspects that are strongly evolving also as function of the improvements in robot modeling, sensors, servo-power systems, and informatics. But even other aspects are considered as of fundamental challenge both in design and use of robots with improved performance and capabilities, like for example kinematic design, dynamics, vision integration.

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