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Mental Models are Compatible with Logical Forms

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Abstract

The mental models theory is a current cognitive approach claiming that human intellectual activity is essentially about semantic possibilities and that syntax and logical form are not relevant. Many experimental results support this theory and its predictions. So, it appears to be justified to assume its main theses. However, in this paper, I argue that the acceptance of the mental models theory does not necessarily have to lead to a rejection of logical forms. Clear relationships between the theory and standard logic can be easily found, and I try to show this in two ways. On the one hand, I claim that the models that the theory assigns to sentences are compatible with truth tables. On the other hand, I also defend that the definitions of a connective by means of another one that can be given in standard logic hold in the mental models theory too, since the sentences that are equivalent in the former have exactly the same models in the latter.

Keywords: logical form, mental models, semantics, standard logic, syntax

1. Introduction

The mental models theory (MMT) is a very important framework explaining human reasoning. Its main theses are to be found in many works (e.g., [1–14]). However, the most important aspects of it for this paper are related to language. The theory states that what is most relevant to individuals in a sentence is its content and meaning and not its syntax or its logical form. This is so because people reason and make inferences mainly reviewing the semantic possibilities that can be attributed to sentences, and not considering their formal structure.

MMT has become a very successful theory, since many experimental results seem to support its assumptions and confirm its predictions. Indeed, it appears to be able to account for human reasoning in a way in principle impossible for standard propositional calculus. True, the natural

deduction system proposed by Gentzen [15] cannot explain certain usual responses given by people in reasoning tasks. For example, in that calculus, there is a rule that people do not often use. That rule is the disjunction introduction rule, that is, the one that enables to infer a formula such as $[p \vee q]$ (where $\vee$ stands for disjunction) from a formula such as $[p]$, and, as said, individuals do not apply it in many occasions (e.g., [13]). Another example can be the difficulties related to the principle of explosion (*ex contradictione quodlibet sequitur*). Following standard calculus, given a contradiction such as, for instance, $[p \wedge \neg p]$ (where $\wedge$ represents conjunction and ' $\neg$ ' is negation), every formula that one can imagine can be derived, and this is something that, obviously, people do not usually do (e.g., [5], p. 204). And a third case can be that of the infinite conclusions that can be built in standard calculus. In this calculus, from a formula such as, for instance, $[p \wedge q]$, an indeterminate number of formulae can be derived, some of them being, for example, $[p \wedge q \wedge q]$, $[p \wedge q \wedge q \wedge q]$, $[p \wedge q \wedge q \wedge q \wedge q]$, and so on (e.g., [9], p. 202).

Of course, more examples of problems of standard logic in explaining human reasoning can be given. However, what is important for this paper is that, as shown below, MMT proposes a framework in which those problems either disappear or make no sense. I will not challenge this fact here. My basic aim is just to argue that although MMT is assumed, that does not in general mean that logic and logical forms have to be ignored, since it is easy to find correspondences between the combinations of possibilities that MMT assigns to some of the traditional connectives in standard logic (e.g., the conditional, conjunction, and the inclusive disjunction) and the truth values and the definitions that this last logic attributes to those same connectives. Actually, this idea is not absolutely new. In papers such as, for example, those of López-Astorga [16, 17], work in a similar direction has already been done. However, in this paper, I will try to indicate the relationships between MMT and standard logic in a clearer and more obvious way.

Thus, to achieve my goals, I will begin by describing the general theses of MMT that are relevant for my argumentation, that is, its general theses on connectives such as the conditional, conjunction, the inclusive disjunction, and their negations. Then, I will show that the possibilities or models that, following MMT, correspond to those connectives and their negations are evidently compatible with the truth tables in standard logic for those very connectives and their negations. Finally, I will also propose that the models that MMT assigns to the aforementioned connectives are consistent with the definitions of them by means of other connectives that are valid in standard logic as well. So, the first section is about some general theses of MMT.

2. MMT, some logical operators, and their negations

As said, logical form is ignored by MMT [5]. According to it, what is interesting in language and reasoning is just the combinations of semantic possibilities, which are in general called 'models' by the theory, related to every sentence. However, the word 'semantic' does not have in this theory the same meaning as in standard logic. In describing a model, letters such as 'p' or 'q' can be used, but just to shorten. Based on theses such as those of Peirce [18], MMT claims that the models are 'iconic', and that they reproduce the complete structure of a situation in the world (e.g., [6], pp. 135–136; [9], p. 207).

Another important point of the theory is that it states that people do not always identify all the models that can be linked to a sentence. To detect all of them, it is necessary to make certain effort, and individuals do not often make enough effort (see, e.g., [6], p. 138, Table 9.2). Nevertheless, for simplicity, I will work here only with ideal circumstances in which individuals identify all of the possible models. Likewise, I will use a form to express the models different from the usual form of MMT. The reason for this is just that the form that I will use here enables to make the equivalences to standard logic explicit in a clearer way. Usually, the theory describes the different models or semantic possibilities as follows:

- (I) $p \ q$
- (II) $p \ \neg q$
- (III) $\neg p \ q$
- (IV) $\neg p \ \neg q$

Obviously, [I] represents a possibility in which both [p] and [q] happen. In [II], [p] happens but [q] does not. Exactly the opposite occurs in [III]: [p] does not happen and [q] does. Finally, in [IV], neither [p] nor [q] happens. Thus, for example, if [p] means that 'Johns is smart' and [q] that 'John studies', [I] refers to a scenario in which John is smart and studies, [II] denotes a situation in which John is smart but he does not study, [III] is the case in which John is not smart but he studies, and [IV] shows the circumstance in which neither John is smart nor he studies.

But, as stated, I will refer to these possibilities in a way that will allow a smoother writing. That will be this one:

- (I) (p, q)
- (II) $(p, \neg q)$
- (III) $(\neg p, q)$
- (IV) $(\neg p, \neg q)$

There is no doubt that the use of brackets and commas will make it possible that the writing is not continually interrupted and the sentences are not constantly cut. And, in this way, the complete sets of possibilities that, based, for example, in Table 9.2 in Ref. [6], p. 138, are attributed by the theory to the mentioned connectives that can be expressed in this simpler manner:

Conjunction, that is, 'and' in sentences of the type 'p and q', for example, 'John is smart and he studies': (p, q) .

Conditional, that is, 'if... then...' in sentences of the type 'if p then q', for example, 'if John is smart then he studies': (p, q) ; $(\neg p, q)$; $(\neg p, \neg q)$.

Inclusive disjunction, that is, 'either... or... or both of them' in sentences of the type 'either p or q or both of them', for example, either John is smart or he studies or both of them: (p, q) ; $(p, \neg q)$; $(\neg p, q)$.

As far as the denied sentences are concerned, MMT provides a method to detect its models: taking into account the set consisting of [I]–[IV], the missing model(s) in the positive form of

the sentence is (are) the model(s) of the negative form of that very sentence (see, e.g., [10, 11]). Accordingly, the complete sets of possibilities corresponding to the denials of the previous connectives are these ones:

Denied conjunction, that is, the case of the sentences of the kind 'it is not the case that p and q', for example, 'it is not the case that John is smart and he studies': $(p, \neg q)$; $(\neg p, q)$; $(\neg p, \neg q)$.

Denied conditional, that is, the case of the sentences of the kind 'it is not the case that if p then q', for example, 'it is not the case that if John is smart then he studies': $(p, \neg q)$.

Denied inclusive disjunction, that is, the case of the sentences of the kind 'it is not the case that either p or q or both of them', for example, 'it is not the case that either John is smart or he studies or both of them': $(\neg p, \neg q)$.

In this way, with all this machinery, MMT can explain the habitual response that people give in most of the reasoning tasks used in the cognitive science literature. For example, given an inference with the premises 'if p then q' (e.g., 'if John is smart then he studies') and 'not-q' (e.g., 'John does not study'), if all of the models of the conditional are detected, that is, (p, q) , $(\neg p, q)$, and $(\neg p, \neg q)$, it is only possible to infer 'not-p' ('John is not smart'), since 'not-q' appears only in the third model, that is, in [IV], and, in that model, 'not-p' appears too and 'p' does not appear (see, e.g., [1], p. 283).

Likewise, MMT can also account for the reasons why the disjunction introduction rule is not usually applied. People do not deduce 'either p or q or both of them' (e.g., 'either John is smart or he studies or both of them') from a premise such as 'p' (e.g., 'John is smart') because, as indicated, the former refers to the models (p, q) , $(p, \neg q)$, and $(\neg p, q)$, and, as it can be noticed, in the third model, that is, in [III], the premise 'p' is false ('not-p' appears). So, contrary to what standard propositional calculus provides, from a premise, in principle, it is not possible to draw a disjunction in which that same premise is one of the disjuncts, since the disjunction is related to a possible scenario in which the premise is false (see, e.g., [13]).

Furthermore, it is clear that problems such as that of the principle of explosion or that of the possibility to derive infinite conclusions, as said, make no sense in this framework, as, simply, those kinds of deductions are not possible given the semantic machinery of MMT. However, another relevant concept of the theory further stresses the important role that semantics plays in the human intellectual activity. That is the concept of modulation. Modulation is 'the process in the construction of models in which content, context, or knowledge can prevent the construction of a model and can add information to a model' ([9], p. 202). This certainly means that semantics or pragmatics can remove some models, make other models hard to identify, and better describe other models. An example can be useful to show how modulation works. Let us consider this conditional sentence:

'If the workers settle for lower wages then the company may still go bankrupt' ([8], p. 663, Table 4).

Because the structure of this sentence is 'if p then q', one might think that, given 'not-q', that is, in this case, 'the company does not go bankrupt', 'not-p', that is, 'the workers do not settle for lower wages', should be concluded. Nevertheless, this is not so for this sentence, since modulation

modifies the models corresponding to it. Now, the models are not the previous ones mentioned for the conditional in general, but (p, q) , $(p, \neg q)$, and $(\neg p, q)$. The reason is evident: $(\neg p, \neg q)$ is not a possibility for this conditional because it is not possible that 'the workers do not settle for lower wages', that is, 'not- p ', and, at the same time, 'the company does not go bankrupt', that is, 'not- q .' On the other hand, $(p, \neg q)$ must be added because it is possible that 'the workers settle for lower wages', that is, ' p ', and that, at the same time, 'the company does not go bankrupt', that is, 'not- q .' In this way, the final result is that, given the conditional and the information that 'not- q ', 'not- p ' cannot be derived, since, in this particular case, the only model in which 'not- q ' appears is a model in which ' p ' appears too (see, e.g., [8], p. 663, Table 4; [19], pp. 287–288).

Obviously, all this demonstrates that MMT has a great potential and that, as stated, can account for most of the answers that people often give in reasoning tasks. And, as also indicated, these facts will not be challenged in this paper. My intention is only to show that this theory does not lead necessarily to a rejection of syntax and logical forms as important parts of human communication and thought. In this way, in the next section, I will argue that the models assigned by MMT to the connectives reviewed are clearly consistent with the truth tables of standard logic. As also said, to make the explanation clearer and simpler, I will only consider ideal situations in which all the models of a sentence are detected. In the same way, and for the same reason, I will ignore the situations in which the models change because of modulation as well.

3. The combinations of possibilities of MMT and the truth tables

Really, it is almost trivial to claim that the models of conjunction, the conditional, the inclusive disjunction, and their denials are related to the truth tables of standard logic, since it is evident that such models correspond to the cases in which the logical structures to which they are attributed are true in a truth table. Nonetheless, it can be interesting to make this explicit in order to show that, indeed, it is necessary to acknowledge the role played by logical form even though the main theses of MMT are correct.

A first important point in this regard is that, following papers such as, for example, those of López-Astorga [16, 17], the different combinations of possibilities can be transformed into well-formed formulae of standard logic by means of conjunction. It is enough to link their elements with a conjunction and consider them to be conjuncts of that conjunction. Thus, [I]–[IV] can be transformed into these formulae:

- (I) $p \wedge q$
- (II) $p \wedge \neg q$
- (III) $\neg p \wedge q$
- (IV) $\neg p \wedge \neg q$

This clearly means that it can be thought that there are logical forms related to the models of MMT. However, even one more step is possible. Following, in the same way, the general theses provided in papers such as those cited, given that the models are really possibilities, they can

be linked in turn by means of disjunction. For instance, if the models of the conditional correspond, according to what has just been said, to the formulae $[p \wedge q]$, $[\neg p \wedge q]$, and $[\neg p \wedge \neg q]$, it can be stated that its models set can be expressed by means of this more complex formula: $[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)]$. This clearly makes sense, since, because the models are, as indicated, possibilities, it is not hard to consider them to be disjuncts in a disjunctive formula (remember that, in a disjunction, only one disjunct needs to be true).

But, if this is so, the correspondences to standard logic are obvious. As mentioned, the formula that can be assigned to the conditional is $[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)]$, and this last formula and $[p \rightarrow q]$ (where $\rightarrow$ stands for conditional relationship) are true in exactly the same cases in standard logic. Indeed, if, as, for example, in López-Astorga [16, 17], we assume that ' $v(\alpha)$ ' refers to the truth value of $[\alpha]$, it can be claimed that

$$v(p \rightarrow q) = v[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)] \quad (1)$$

And this is evident because $v(p \rightarrow q) = 0$ if and only if $v(p) = 1$ and $v(q) = 0$ (where '0' denotes that the formula is false and '1' represents the case in which the formula is true), and, in the same way, $v[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)] = 0$ if and only if $v(p) = 1$ and $v(q) = 0$ as well.

On the other hand, if we think about conjunction, all of this is even clearer. The reason is that, if the model of conjunction is transformed into a well-formed formula of standard logic in accordance with what has been indicated, the result is a formula that exactly matches the way conjunction is expressed in this last logic: $[p \wedge q]$. Thus, there is no doubt that

$$v(p \wedge q) = v(p \wedge q) \quad (2)$$

Furthermore, as it is well known, $v(p \wedge q) = 1$ if and only if $v(p) = 1$ and $v(q) = 1$, and this seems to apply to both the formula in standard logic and the model in MMT.

In connection with disjunction, its formula would be, obviously, $[(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)]$, and, again, it is clear that

$$v(p \vee q) = v[(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)] \quad (3)$$

In both cases, the value of the formula is 0 if and only if $v(p) = 0$ and $v(q) = 0$.

As far as denials are concerned, the situation is not different. Based on the accounts and arguments above, it can be said that the negated conjunction can be related to a formula such as $[(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)]$, and equivalence is evident here too, since

$$v[\neg(p \wedge q)] = v[(p \wedge \neg q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)] \quad (4)$$

Indeed, these two formulae are false in just a circumstance: if and only if $v(p) = 1$ and $v(q) = 1$.

With regard to the denied inclusive disjunction, the formula would be hence $[\neg p \wedge \neg q]$, and once again

$$v[\neg(p \vee q)] = v(\neg p \wedge \neg q) \quad (5)$$

Both $v[\neg(p \vee q)] = 1$ and $v(\neg p \wedge \neg q) = 1$ if and only if $v(p) = 0$ and $v(q) = 0$.

Finally, the case of the negated conditional is not difficult either. Its formula is clearly $[p \wedge \neg q]$ and it is also clear that

$$v[\neg(p \rightarrow q)] = v(p \wedge \neg q) \quad (6)$$

There is only one scenario in which these two last formulae can be true: if and only if $v(p) = 1$ and $v(q) = 0$.

So, given these arguments, it is hard to hold that the models of MMT have nothing to do with standard logic and its logical forms. It is true that, from critical perspectives, it has already been said that MMT reproduces the truth tables of standard logic (e.g., [20]). However, the difference between this paper and such perspectives is that this paper is not so critical of MMT. It assumes its main theses and only tries to show that those theses do not actually lead, as its proponents often claim, to a rejection of logical forms. In this section, I have shown that, true, very simple logical forms (in which only negations, conjunctions, and disjunctions are included) can be attributed to the models of MMT. Nevertheless, this idea is further supported in the next section, in which I argue that the definitions of the logical operators by means of other different logical operators that are correct in standard logic are also valid in MMT.

4. The definitions of the logical operators and MMT

Certainly, standard logic enables to define each logical connective linking two clauses by means of any other connective along with the negation. Thus, in the case of the conditional, these definitions hold:

$$p \rightarrow q \stackrel{\text{df}}{=} \neg(p \wedge \neg q) \stackrel{\text{df}}{=} \neg p \vee q \quad (7)$$

(e.g., ‘if John is smart then he studies’ is equivalent to both ‘it is not the case that John is smart and he does not study’ and ‘either John is not smart or he studies or both of them’).

The reason is, of course, that, according to the truth tables in this logic,

$$v(p \rightarrow q) = v[\neg(p \wedge \neg q)] = v(\neg p \vee q) \quad (8)$$

But this leads us to another important point about MMT, since the model sets that can be assigned to the expressions in natural language with these three logical structures are, in principle, the same. As said, the models of $[p \rightarrow q]$ are (p, q) , $(\neg p, q)$, and $(\neg p, \neg q)$, and these are the same as those of $[\neg(p \wedge \neg q)]$ and $[\neg p \vee q]$. Indeed, if the only model of $[p \wedge \neg q]$ is $(p, \neg q)$, the models of $[\neg(p \wedge \neg q)]$ must be, as explained, the remaining ones, that is, (p, q) , $(\neg p, q)$, and

$(\neg p, \neg q)$. Likewise, the models of an expression such as $[\neg p \vee q]$ are clearly $(\neg p, q)$, $(\neg p, \neg q)$, and (p, q) , that is, in different order, again, the same as those of the conditional.

But something similar happens to the definitions corresponding to conjunction. Those are the following:

$$p \wedge q =_{\text{df}} \neg(p \rightarrow \neg q) =_{\text{df}} \neg(\neg p \vee \neg q) \quad (9)$$

(e.g., ‘John is smart and he studies’ is equivalent to both ‘it is not the case that if John is smart then he does not study’ and ‘it is not the case that either John is not smart or he does not study or both of them’).

Here, it is true too that

$$v(p \wedge q) = v[\neg(p \rightarrow \neg q)] = v[\neg(\neg p \vee \neg q)] \quad (10)$$

Nonetheless, what is actually interesting about this for this paper is that while $[p \wedge q]$ has just a model, (p, q) , that is, exactly the only model of both $[\neg(p \rightarrow \neg q)]$ and $[\neg(\neg p \vee \neg q)]$ as well. As stated, a conditional such as $[p \rightarrow q]$ has three models: (p, q) , $(\neg p, q)$, and $(\neg p, \neg q)$. Therefore, a conditional such as $[p \rightarrow \neg q]$ has to have these models: $(p, \neg q)$, $(\neg p, \neg q)$, and $(\neg p, q)$. And the only model of a denied conditional such as $[\neg(p \rightarrow \neg q)]$ can only be (p, q) . In the same way, if the models set of $[\neg p \vee \neg q]$ are $(\neg p, \neg q)$, $(\neg p, q)$, and $(p, \neg q)$, only a model is possible for $[\neg(\neg p \vee \neg q)]$: (p, q) again.

Furthermore, the case of the inclusive disjunction is not different. Its definitions are these ones:

$$p \vee q =_{\text{df}} \neg p \rightarrow q =_{\text{df}} \neg(\neg p \wedge \neg q) \quad (11)$$

(e.g., ‘either John is smart or he studies or both of them’ is equivalent to both ‘if John is not smart then he studies’ and ‘it is not the case that John is not smart and he does not study’).

And here, it is also correct that

$$v(p \vee q) = v(\neg p \rightarrow q) = v[\neg(\neg p \wedge \neg q)] \quad (12)$$

Nevertheless, as far as my aims in this paper are concerned, the most relevant point is that the models of the three formulae match in this case as well. As said, the models of $[p \vee q]$ are (p, q) , $(p, \neg q)$, and $(\neg p, q)$, and it is evident that those of $[\neg p \rightarrow q]$ are $(\neg p, q)$, $(p, \neg q)$, and (p, q) , that is, exactly the same models. On the other hand, if the model of $[\neg p \wedge \neg q]$ is $(\neg p, \neg q)$, it is clear that those of $[\neg(\neg p \wedge \neg q)]$ are also (p, q) , $(p, \neg q)$, and $(\neg p, q)$.

Thus, it is hard to question that certain correspondences and equivalences related to logical forms in standard logic are present in MMT too. As stated, this does not mean that this last theory is wrong. It only implies that, although MMT wants to ignore logical form, it cannot do that absolutely. All of its other theses can be correct and it is very possible that it describes the real mental processes why the human mind reasons and interprets language.

However, as shown, this does not necessarily remove the role that the formal structures and syntax can play.

5. Conclusions

However, what does appear to be true is that the human intellectual activity does not follow Gentzen's calculus [15]. Examples such as the ones mentioned above (that of the disjunction introduction rule, that of the principle of explosion, or that of the possible infinite conclusions) are clear proofs in this regard. But this does not prove, at the same time, that no kind of logic or syntactic forms can be related to the human thought. This paper has shown that, even accepting a purely semantic theory such as MMT, it is possible to continue to speak about logical forms and to find links between the mental activity and such forms. In this way, given that standard logic is about more than just Gentzen's system [15], the rejection of the latter does not have to lead to the rejection of the former. Thus, while standard propositional calculus does not work to account for the human mind, maybe some aspects of standard logic linked to its truth tables can do that.

In fact, following arguments such as, for example, those of López-Astorga [16, 17], it can be said that, by trying to avoid logical forms, MMT really gives a procedure to recover them. Based on this idea, it can also be stated that MMT reveals that the true forms of conjunction, the conditional, and the inclusive disjunction are, in principle, $[p \wedge q]$, $[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)]$, and $[(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)]$, respectively, and that, therefore, the human mind does use logical forms, albeit such forms are simple enough so that they only include conjunctions, disjunctions, and denials.

An objection against this idea can be that MMT assumes that, as explained, in certain circumstances, many people do not identify all the models corresponding to a sentence, this being what allows the theory to account for the reasoning mistakes. Following an example indicated above, it can be claimed that, according to MMT, the inferences that have as premises sentences such as, for instance, $[p \rightarrow q]$ and $[\neg q]$ are sometimes difficult for individuals because, as indicated, to make them and conclude $[\neg p]$, it is necessary to detect the third model of the conditional $(\neg p, \neg q)$, which does not always happen (e.g., [1], p. 283). In this way, one might think that this idea refers to processes and facts that actually have no relationship to logical forms. However, from a syntactic perspective, it can also be thought that an individual that does not detect the third model of the conditional is just an individual that fails to note that its logical form is $[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)]$, and that, because he (or she) has not identified all of its possibilities, attributes to a logical form such as, for example, $[p \wedge q]$ (if the usual arguments given by the proponents of the theory are taken into account, this would be the most probable case) or $[(p \wedge q) \vee (\neg p \wedge q)]$. So, this aspect of the theory is not really in conflict with a possible role of logical form either.

Likewise, modulation would not be a problem. If we consider the example taken from Johnson-Laird and Byrne ([8], p. 663, Table 4) again, that is, 'If the workers settle for lower wages then the company may still go bankrupt', we have to acknowledge that its logical form is not actually the

one of the conditionals, but, following that indicated above, $[(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)]$. But, as said, this is not a difficulty either, since it can be interpreted that what it truly shows is that MMT has theoretical mechanisms such as that of modulation that help detect the real logical forms of sentences. In fact, an old problem in logic is, as it is known, the one of the translation of sentences in natural language into well-formed formulae, as there is no exact correspondence between the expressions in natural language (e.g., 'if... then...', 'either... or...', or '...and...') and the logical operators (see, e.g., [16, 17]). And, obviously, from this point of view, it can be claimed that what modulation really seems to reveal is the way the true logical forms of the sentences can be found, and not that logical form is not necessary (see, e.g., [16]). Thus, in the particular case of the aforementioned example, what modulation appears to indicate is that, in spite of the fact that the sentence is expressed by means of the words 'if' and 'then', it is actually an inclusive disjunction. This is so because the formula $[(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)]$ is the one corresponding to this last operator. Hence perhaps it would be better to express the sentence as follows: 'either the workers settle for lower wages or the company will go bankrupt.'

So, that this paper has been mainly focused on ideal situations in which all of the models are identified and modulation plays no role does not seem to be a clear limitation of it. Thus, the paper appears to demonstrate that, although, as stated, Gentzen's calculus [15] is not the criterion for the human mind, it is worth continuing to carry out studies in the same direction as, for example, those of López-Astorga [16, 17], that is, studies on the relationships that can exist between the models of MMT and logical forms.

Finally, maybe it is also important to mention the relevance that this problem of logical forms and their semantic possibilities can have in very different fields. On the one hand, given that MMT is a cognitive theory, it is absolutely clear that it is relevant in Psychology of Reasoning. On the other hand, speaking about syntax and semantics is always speaking about linguistics. Thus, the arguments provided in this paper can be interesting in several kinds of studies on language, including, for example, Philosophy of Language. As far as this last point is concerned, it can be stated, in addition, that the identification of both the semantic possibilities to which sentences refer and their logical forms can make it possible that a computer program or a software tool can work more easily from sentences in natural language (which could be translated into logical forms and the program or software could work considering just these last forms). Furthermore, it is obvious that identifying logical forms is identifying deep forms in linguistic messages. Therefore, the possibilities of researches in this direction are diverse and, as said, it seems that the analyses about the connections between semantic models and syntactic forms must be continued and taken into account.

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References

- [1] Byrne R M J, Johnson-Laird P N. "If" and the problems of conditional reasoning. *Trends in Cognitive Science*. 2009;**13**(7):282–287.
- [2] Hinterecker T, Knauff M, Johnson-Laird P N. Modality, probability, and mental models. *Journal of Experimental Psychology: Learning, Memory, and Cognition*. 2016;**42**(10): 1606–1620.
- [3] Johnson-Laird P N. The history of the mental models. In: Manktelow K, Chung M C, editors. *Psychology and Reasoning: Theoretical and Historical Perspectives*. New York, NY: Psychology Press; 2004. pp. 179–212.
- [4] Johnson-Laird P N. *How We Reason*. Oxford: Oxford University Press; 2006. 575 p.
- [5] Johnson-Laird P N. Against logical form. *Psychologica Belgica*. 2010;**5**(3/4):193–221.
- [6] Johnson-Laird P N. Inference with mental models. In: Holyoak K J, Morrison R G, editors. *The Oxford Handbook of Thinking and Reasoning*. New York, NY: Oxford University Press; 2012. pp. 134–145.
- [7] Johnson-Laird P N. How to improve thinking. In: Wegerif R, Li L, Kaufman J C, editors. *The Routledge International Handbook of Research on Teaching Thinking*. Abingdon & New York: Routledge; 2015. pp. 80–91.
- [8] Johnson-Laird P N, Byrne R M J. Conditionals: A theory of meaning, pragmatics, and inference. *Psychological Review*. 2002;**109**(4):646–678.
- [9] Johnson-Laird P N, Khemlani S, Goodwin G P. Logic, probability, and human reasoning. *Trends in Cognitive Sciences*. 2015;**19**(4):201–214.
- [10] Khemlani S, Orenes I, Johnson-Laird P N. Negation: A theory of its meaning, representation, and inference. *Psychological Review*. 2012;**109**(4):646–678.
- [11] Khemlani S, Orenes I, Johnson-Laird P N. The negation of conjunctions, conditionals, and disjunctions. *Acta Psychologica*. 2014;**151**:1–7.
- [12] Oakhill J, Garnham A, editors. *Mental Models in Cognitive Science. Essays in Honour of Phil Johnson-Laird*. Hove: Psychology Press; 1996. 336 p.
- [13] Orenes I, Johnson-Laird P N. Logic, models, and paradoxical inferences. *Mind & Language*. 2012;**27**(4):357–377.
- [14] Ragni M, Sonntag T, Johnson-Laird P N. Spatial conditionals and illusory inferences. *Journal of Cognitive Psychology*. 2016;**28**(3):348–365.
- [15] Gentzen G. Untersuchungen über das logische Schließen I. *Mathematische Zeitschrift*. 1935;**39**:176–210.
- [16] López-Astorga M. The disjunction introduction rule: Syntactic and semantic considerations. *Pragmalingüística*. 2015;**23**:141–149.

- [17] López-Astorga M. Evolved mechanisms versus underlying conditional relations. *Studies in Logic, Grammar and Rhetoric*. 2015;**40**(53):241–253.
- [18] Peirce C S. *Collected papers of Charles Sanders Peirce*. C. Hartshorne, P. Weiss, A. Burks, editors. Cambridge: Harvard University Press; 1931–1958. 944 p.
- [19] López-Astorga M. Logic, pragmatics, and types of conditionals. *Frontiers of Philosophy in China*. 2016;**11**(2):279–297.
- [20] Braine M D S, O'Brien D P, editors. *Mental Logic*. Mahwah: Lawrence Erlbaum Associates, Inc., Publishers; 1998. 481 p.