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Load – Resistance Interference Method

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Abstract

Reliability and safety of a load carrying structure needs that its resistance R must be higher than the load effect S . So-called reliability margin $G = R - S$ and reliability index are used for reliability assessment and determination of failure probability if R and S are random quantities. This chapter explains the determination of parameters of the reliability margin and shows its use on examples, including the finding of suitable dimensions for achieving the demanded reliability.

Keywords: Reliability, safety, load, resistance, reliability margin, reliability index, failure, interference, probability

Many situations exist, which can be characterized as the conflict “load-resistance” or “action-barrier”. The reliability is ensured if the load effect is smaller than the resistance against it. An example is a load-carrying structure, such as a road bridge or a mast of a TV transmitter exposed to wind. If the instantaneous load acting on the structure is higher than its load-carrying capacity, the structure can collapse or its deformations will be larger than allowed. Several examples follow. If the voltage at the input of a device is higher than its electric strength, a breakdown of insulation will follow. If the amount of water, flowing into a reservoir during rain period, is higher than its capacity at that time, the water overflows the upper edge. The strength of a shrink-fitted connection depends on the overlap of the bolt in the hole (i.e. on the difference between the diameter of the bolt and the diameter of the hole). If this overlap is too small, the strength of the joint is insufficient. If the bus arrives at the train station later than at the time of the train departure, the passengers miss the journey. The consequences of these failures can range from negligible to fatal.

In all these cases, a tool is needed that can quantify the reliability. The object is reliable if its **resistance R** to a certain “load” is higher than the **load effect** or **stress S** . (The meaning of the

terms “load” and “resistance” can be very broad depending on the context.) For the quantification of reliability, the so-called **reliability margin** G is introduced, defined as

$$G = R - S. \quad (1)$$

This reliability margin shows, for example, how much the load-carrying capacity is higher than the load or how many minutes remain between the arrival and departure. The reliability condition can be written as

$$G = R - S > 0. \quad (2)$$

The case $G < 0$ corresponds to the resistance lower than load, which means failure.

Often, only the load varies (the wind force, for example), whereas the resistance R of a structure is constant. In such case, only the stress S is a random quantity, and the probability of failure is determined as the probability that S exceeds the value of R ,

$$P_f = P(S \geq R). \quad (3)$$

If the distribution function of the wind-caused stress in the structure, $F(S)$, is known, then its value corresponding to the known value R gives the probability that the stress will be lower than the strength, and the structure is safe. The probability of failure is the complement,

$$P_f = 1 - F(S = R). \quad (4)$$

Note that here the letter F denotes distribution function and the probability of failure is P_f . Vice versa, it is possible to determine the necessary strength R of the structure such that the probability of failure will not exceed the allowable value $P_{f,a}$.

Often, also the random variability of the resistance R must be considered. It is especially during the design stage that the actual parameters of the structure are not known yet: for example, strength or Young's modulus of the material, characteristic stiffness of rubber bearings in a bridge, thickness of the flanges of rolled steel beams, etc. Only their nominal values are known in advance. The actual values vary randomly less or more around them and can be determined accurately only after the components have been purchased or manufactured.

In such cases, both quantities R and S must be considered as random. They can be characterized by probability distribution or simply by the average value and standard deviation. The situation is depicted in Figure 1. If the distributions of both quantities do not overlap at all, no failure can occur. This can be achieved if the average resistance is sufficiently higher than the average load. However, the effort to ensure that the distributions R and S never overlap can be uneconomical, especially if the consequences of failure are not critical. Sometimes, it is

reasonable to admit a reasonably low probability of failure (e.g. small and short-time exceeding that of the allowable deformation). In this case, both distributions overlap (Fig. 1) and the probability of failure is proportional to the area below the overlapping part of both curves (see further). It is thus useful to know the failure probability for certain combinations of S and R or, vice versa, to determine in advance what cross-section dimensions should be used if the failure probability must not exceed some allowable value. The pertinent procedure, called the “load – resistance” or “stress – strength interference method” [1, 2], is explained further.

If the stress S and resistance R are random quantities, the reliability margin G is also a random quantity (Fig. 1), with its own probability distribution. Its mean μ and standard deviation σ can be calculated as

$$\begin{aligned} \mu_G &= \mu_R - \mu_S, & (a) \\ \sigma_G &= (\sigma_R^2 + \sigma_S^2)^{1/2}; & (b) \end{aligned} \quad (5)$$

the subscripts denote the corresponding quantities. When working with empirical data, μ and σ are replaced by the sample mean m and sample standard deviation s .

Figure 1. Stress – strength interference method (a schematic).

Failure occurs if the load effect is higher than the resistance, i.e. if the reliability margin G in Eq. (1) is negative (Fig. 1). The corresponding critical value of G is 0, and the probability of failure P_f can be determined as

$$P_f = P(R \leq S) = P(G \leq 0). \quad (6)$$

Reliability margin can be standardized to a nondimensional form in the following way:

$$u = (G - m_G) / s_G. \quad (7)$$

Its value for the transition between reliable state and failure ($G = 0$) equals

$$u = (0 - m_G) / s_G = -m_G / s_G. \quad (8)$$

The ratio

$$m_G / s_G = (m_R - m_S) / (s_R^2 + s_S^2)^{1/2} = \beta \quad (9)$$

is called the **reliability index** β and corresponds to the distance of the mean value of reliability margin G from 0, which is expressed as a multiple of standard deviation of G . The reliability index gives a simple measure of reliability, as it shows how far is the average value of reliability margin from the critical point. The situation is simple if the reliability margin G has normal distribution: in this case, an unambiguous relationship exists between β and the probability of failure: P_f equals the value of distribution function of standard normal distribution for $u = -\beta$. For example, $P_f = 0.02275$ for $\beta = 2$, $P_f = 0.00135$ for $\beta = 3$, and $P_f = 0.0000317$ for $\beta = 4$. Some standards for civil engineering constructions admit the reliability evaluation using the reliability index and give the characteristic values of β for various degrees of safety [3].

The advantage of the reliability index is that it is simple and its values are of the order of units, which is near to our way of thinking. A normal distribution of G may be assumed if the coefficient of asymmetry $\alpha_G < 0.3$. Otherwise, no simple relation between β and P_f exists, and other methods for the determination of failure probability are more appropriate (see further).

As the S and R curves can overlap or interfere (Fig. 1), the term **interference method** is used for this way of reliability assessment. Its use will be illustrated on two simple examples.

Example 1

Determine the probability of failure of a pull rod loaded by tensile force. The force magnitude is normally distributed with the mean $m_F = 140,000$ N and standard deviation $s_F = 14,000$ N. The diameter D of the rod is 20 mm. The stress is determined as $\sigma = F/A$, where the cross-section $A = \pi D^2/4 = 314.16$ mm². The mean and standard deviation of the stress are: $m_s = 140,000/314.16 = 445.6$ MPa and $s_s = 14,000/314.16 = 44.6$ MPa. The strength parameters of the used steel are: $m_R = 500$ MPa and $s_R = 50$ MPa. One can assume a normal distribution of R and S as well as of G .

Solution. The mean and standard deviation of reliability margin are $m_G = m_R - m_S = 500.0 - 445.6 = 54.4$ MPa and $s_G = (s_R^2 + s_S^2)^{1/2} = (50.0^2 + 44.6^2)^{1/2} = 67.0$ MPa. The reliability index is $\beta = m_G/s_G = 54.4/67 = 0.8117$. The probability of failure is $P_f = F(-\beta) = F(-0.8117) = 0.20848 = 20.85\%$. (Various programs can be used for finding the values of standard normal distribution function F ; the

appropriate command in Excel is "norm.s.dist" or "normsdist". The statistical tables for the distribution function of standard normal distribution give the same result.)

Example 2

The failure probability from Example 1 is too high and must be reduced to $P_f = 0.0001$. Find the appropriate diameter of the rod.

Solution. The reliability index for $P_f = 0.0001$ is $\beta = 3.719$. The material parameters m_R and s_R are the same as above, so that it is necessary to determine only the stress parameters. In fact, there are two unknown parameters, m_S and s_S . However, we can assume that the coefficient of variation v of the slightly larger cross-section will be the same as in the first variant (i.e. 10%; cf. the m_S and s_S values above), so that $s_S = vm_S = 0.1m_S$. Thus, only the mean stress m_S (and the corresponding rod diameter) are to be determined. Several possible methods for finding m_S exist. The first one, exact, is based on the solution of Equation (5) for given β , m_R , s_R , and unknown m_S . This approach leads to a quadratic equation and could be preferred by those who like mathematical analysis. The second approach uses the formula for the calculation of β , varies step-by-step the value m_S or rod diameter, and calculates repeatedly β or the failure probability until the target value of β or P_f is found. This solution can be facilitated using a suitable solver: if the formula for the calculation of failure probability as a function of shaft diameter was created [using relationships $P_f = F(-\beta)$, Eq.(7), and $A = \pi D^2/4$, $\sigma = F/A$], it is possible to "ask" the solver to change the diameter D until P_f attains the demanded value. In this way, the solver in Excel has given the (accurate) value $m_S = 285.81$ MPa (for $\beta = 3.719$). The corresponding cross-section area (for the load 140,000 N) is $A = F/m_S = 140,000/285.81 = 489.84$ mm², and the rod diameter is $D = 24.97 = 25.0$ mm. (The reader is encouraged to make the check by calculating the cross-section area A , mean stress m_S , reliability index β , and failure probability P_f for this diameter.) Note how dramatically the failure probability has decreased (from 0.2048 to 0.0001) by increasing the rod diameter from 20 to 25 mm.

These computations can be done even for the values of D or m_S chosen "by hand", without a special algorithm. This "primitive" approach, which does not need analytical abilities or solver, also leads quickly to an acceptable solution, the more so that some quantities (e.g. dimensions of standard rolled steel profiles) are not continuous, but graded.

Other distributions and approaches

If the stress and resistance have asymmetrical distributions that can be approximated by log-normal functions, the above approach may be used if the reliability condition is defined not as the difference of the strength and stress, Equation (1), but as their ratio:

$$G = R / S. \quad (10)$$

Taking the logarithms of Equation (8) gives an expression similar to Equation (1):

$$\log G = \log R - \log S. \quad (11)$$

Both transformed quantities, $\log R$ and $\log S$, have normal distribution and Equation (9) resembles Equation (1) in transformed coordinates ($\log R$ corresponding to G , etc.). Thus, Equation (9) can be treated by the procedures of interference method described above.

If the distributions of the resistance and stress are only known in the form of histograms, the probability of failure can be determined by numerical integration:

$$P_f = \int_{-\infty}^{\infty} F_R(S) f_S(S) dS, \quad (12)$$

and the probability that failure does not occur:

$$P_r = \int_{-\infty}^{\infty} F_S(R) f_R(R) dR \quad (13)$$

depending on what functions are available. The differentials dS and dR are replaced by finite intervals ΔS and ΔR . For more, see [2].

The probability of failure can also be determined by numerical simulation methods, such as Monte Carlo, which will be explained in the following chapter.

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