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Fuzzy Control Systems: LMI-Based Design

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1. Introduction

This chapter describes widespread methods of model-based fuzzy control systems. The subject of this chapter is a systematic framework for the stability and design of nonlinear fuzzy control systems. We are trying to build a bridge between conventional fuzzy control and classic control theory. By building this bridge, the strong well developed tools of classic control could be used in model-based fuzzy control systems

Model-based fuzzy control, with the possibility of guaranteeing the closed loop stability, is an attractive method for control of nonlinear systems. In recent years, many studies have been devoted to the stability analysis of continuous time or discrete time model based fuzzy control systems (Takagi & Sugeno, 1985; Rhee & Won, 2006; Chen et al., 1993; Wang et al., 1996; Zhao et al., 1996; Tanaka & Wang, 2001; Tanaka et al., 2001). Among such methods, the method of Takagi-Sugeno (Takagi & Sugeno, 1985) has found many applications for modelling complex nonlinear systems (Tanaka & Sano, 1994; Tanaka & Kosaki, 1997; Li et al., 1998). The concept of sector nonlinearity (Kawamoto et al., 1992) provided means for exact approximation of nonlinear systems by fuzzy blending of a few locally linearized subsystems. One important advantage of using such a method for control design is that the closed-loop stability analysis, using the Lyapunov method, becomes easier to apply. Various stability conditions have been proposed for such systems (Tanaka & Wang, 2001), (Ting, 2006), where the existence of a common solution to a set of Lyapunov equations is shown to be sufficient for guaranteeing the closed-loop stability. Some relaxed conditions are also proposed in (Kim & Lee, 2000; Ding et al, 2006; Fang et al., 2006, Tanaka & Ikeda, 1998). Parallel Distributed Compensator (PDC) is a generalization of the state feedback controller to the case of nonlinear systems, using the Takagi-Sugeno fuzzy model (Wang et al., 1996). This method is based on partitioning nonlinear system dynamics into a number of linear subsystems, for which state feedback gains are designed and blended in a fuzzy sense. Takagi-Sugeno model and parallel distributed compensation have been used in many applications successfully (Sugeno & Kang, 1986, Lee et al., 2006, Hong & Langari, 2000, Bonissone et al.,

1995). The Linear Matrix Inequality (LMI) technique offers a numerically tractable way to design a PDC controller with objectives such as stability (Wang et al., 1996; Ding et al., 2006; Fang et al., 2006; Tanaka & Sugeno 1992), H_∞ control (Lee et al., 2001), H_2 control (Lin & Lo, 2003), pole-placement (Jon et al., 1997; Kang & Lee, 1998), and others (Tanaka & Wang, 2001).

2. Takagi-Sugeno fuzzy model

The main idea of the Takagi-Sugeno fuzzy modeling method is to partition the nonlinear system dynamics into several locally linearized subsystems, so that the overall nonlinear behavior of the system can be captured by fuzzy blending of such subsystems. The fuzzy rule associated with the i -th linear subsystem for the continuous fuzzy system and the discrete fuzzy system, can then be defined as

Continuous fuzzy system

$$\begin{aligned} \text{Rule } i: & \text{IF } Z_1(t) \text{ is } M_{i1} \dots \text{ and } Z_r(t) \text{ is } M_{ir} \\ \text{THEN } & \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad i=1,2,\dots,r \end{aligned} \quad (1)$$

Discrete Fuzzy System

$$\begin{aligned} \text{Rule } i: & \text{IF } Z_1(t) \text{ is } M_{i1} \dots \text{ and } Z_r(t) \text{ is } M_{ir} \\ \text{THEN } & \begin{cases} x(t+1) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad i=1,2,\dots,r \end{aligned} \quad (2)$$

where, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $A_i \in R^{n \times n}$, $B_i \in R^{n \times m}$, $C_i \in R^{q \times n}$; $\{z_1(t), z_2(t), \dots, z_p(t)\}$ are nonlinear functions of the state variables obtained from the original nonlinear equation, and $M_{ij}(z_i)$ are the degree of membership of $z_i(t)$ in a fuzzy set M_{ij} . Whenever there is no ambiguity, the time argument in $z(t)$ is dropped. The overall output, using the fuzzy blend of the linear subsystems, will then be as follows:

Continuous fuzzy system

$$\begin{aligned} \dot{X} &= \frac{\sum_{i=1}^r w_i(z) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r w_i(z)} = \sum_{i=1}^r h_i(z) (A_i x(t) + B_i u(t)) \\ y(t) &= \frac{\sum_{i=1}^r w_i(z) C_i x(t)}{\sum_{i=1}^r w_i(z)} = \sum_{i=1}^r h_i(z) C_i x(t) \end{aligned} \quad (3)$$

Discrete Fuzzy System

$$\begin{aligned}
 x(t+1) &= \frac{\sum_{i=1}^r \omega_i(z(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^r \omega_i(z(t))} \\
 &= \sum_{i=1}^r h_i(z(t)) (A_i x(t) + B_i u(t)) \\
 y(t) &= \frac{\sum_{i=1}^r \omega_i(z(t)) C_i x(t)}{\sum_{i=1}^r \omega_i(z(t))} \\
 &= \sum_{i=1}^r h_i(z(t)) C_i x(t)
 \end{aligned} \tag{4}$$

Where

$$\begin{aligned}
 w_1(z) &= \prod_{j=1}^i M_{ij}(z_j) \\
 h_1(z) &= \frac{w_1(z)}{\sum_{i=1}^r w_1(z)}
 \end{aligned} \tag{5}$$

It is also true, for all t , that

$$\begin{cases} \sum_{i=1}^r w_1(z) > 0, \\ w_1(z) \geq 0, i = 1, 2, \dots, r \end{cases}$$

2.1. Building a fuzzy model

There are generally three approaches to build the fuzzy model: "sector nonlinearity," "local approximation," or a combination of the two.

2.1.1. Sector nonlinearity

Figure 1 illustrates the concept of global and local sector nonlinearity. Suppose the original nonlinear system satisfies the sector non-linearity condition (Kawamoto et al., 1992, as cited in Tanaka & Wang, 2001), i.e., the values of nonlinear terms in the state-space equation remain within a sector of hyper-planes passing through the origin. This model guarantees the stability of the original nonlinear system under the control law. A function $\Phi: \mathbb{R} \rightarrow \mathbb{R}$ is said to be sector $[a, c]$ if for all $x \in \mathbb{R}$, $y = \Phi(x)$ lies between $b_1 x$ and $b_2 x$.

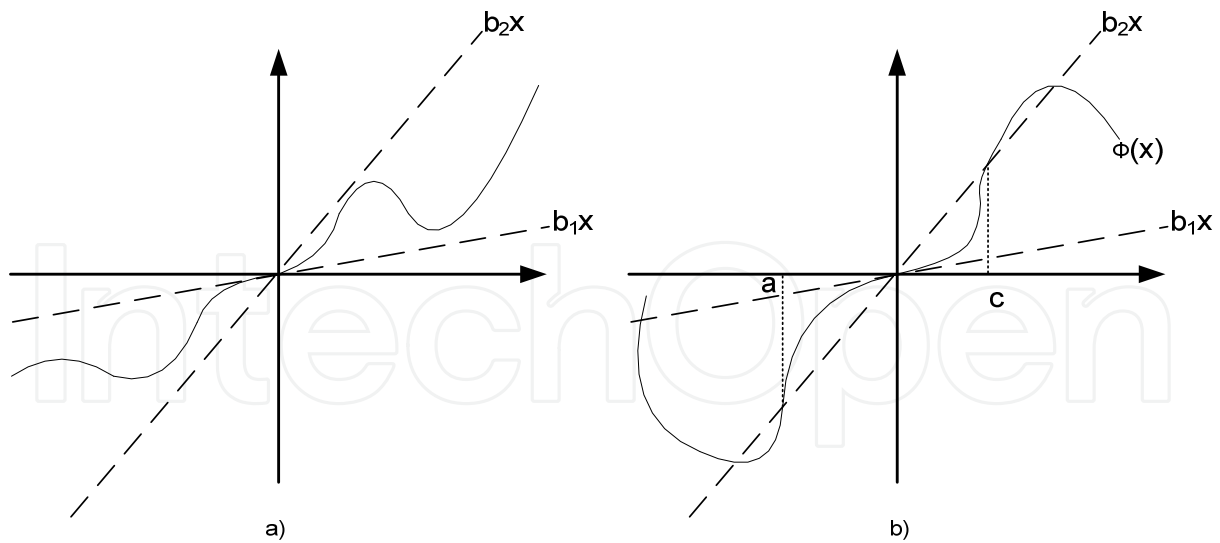


Figure 1. a) Global sector nonlinearity, b) Local sector nonlinearity

Example 1

The well-known nonlinear control benchmark, the ball-and-beam system is commonly used as an illustrative application of various control methods (Wang & Mendel, 1992) depicted in figure 2. Let $x_1(t)$ and $x_2(t)$ denote the position and the velocity of the ball and let $x_3(t)$ and $x_4(t)$ denote the angular position and the angular velocity of the beam. Then, the system dynamics can be described by the following state-space equation

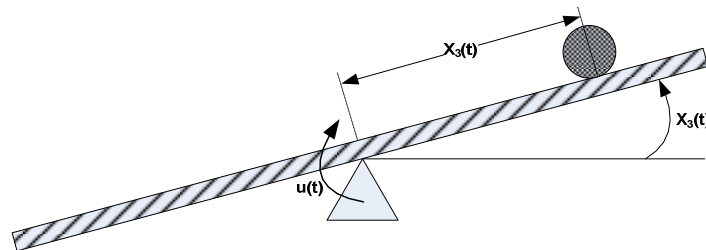


Figure 2. The ball and beam system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \quad (6)$$

Where

$$f(x) = \begin{bmatrix} x_2(t) \\ B(x_1(t)x_4^2(t) - G \sin(x_3(t))) \\ x_4(t) \\ 0 \end{bmatrix} \text{ and } g(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Where $x(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T$ and $u(t)$ is torque.

$\sin(x_3)$ and $x_1x_4^2$ are nonlinear terms in the state-space equation. We define $z_1 = \sin(x_3)$ and $z_2 = x_1x_4^2$. Assume $x_3 \in [-\pi/2 \ \pi/2]$ and $x_1x_4 \in [-d \ d]$ as the region within which the system will operate. Figure 3 shows that $z_1(t) = \sin(x_3(t))$ and its local sector operating

region. The sector $[b_1, b_2]$ consists of two lines b_1x_1 and b_2x_1 , where the slopes are $b_1 = 1$ and $b_2 = \frac{2}{\pi}$. It follows that

$$\left| \frac{2}{\pi}x \right| \leq |\sin(x)| \leq |x|, \quad (7)$$

$$-dx_4 \leq x_1x_4^2 \leq dx_4.$$

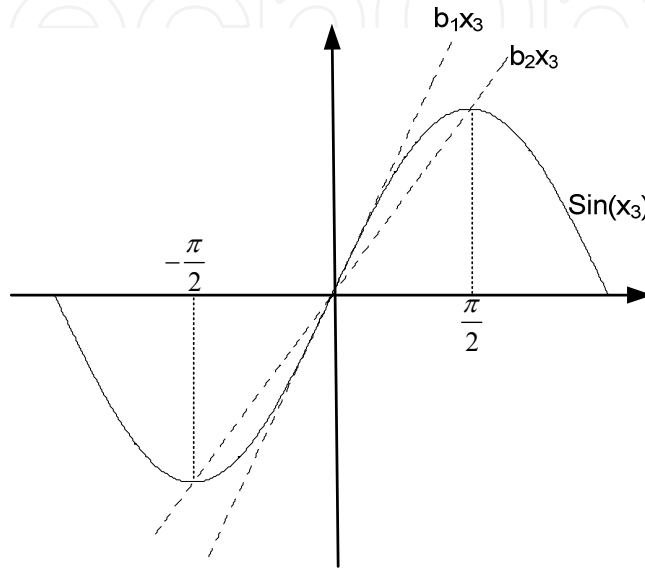


Figure 3. $\sin(x_3(t))$ and its local sector

We present $\sin(x_3(t))$ is represented as follows:

$$z_1 = \sin(x_3(t)) = \left(\sum_{i=1}^2 M_i(z_1(t)) b_i \right) x_3(t) \quad (8)$$

From the property of membership functions $[M_1(z_1(t)) + M_2(z_1(t)) = 1]$, we can obtain the membership functions

$$M_1(z_1(t)) = \begin{cases} \frac{z_1(t) - \left(\frac{2}{\pi}\right) \sin(z_1(t))}{\left(1 - \frac{2}{\pi}\right) \sin^{-1}(z_1(t))} & z_1(t) \neq 0 \\ 1, & \text{otherwise.} \end{cases} \quad (9)$$

$$M_2(z_1(t)) = \begin{cases} \frac{\sin^{-1}(z_1(t)) - z_1(t)}{\left(1 - \frac{2}{\pi}\right) \sin^{-1}(z_1(t))} & z_1(t) \neq 0 \\ 0, & \text{otherwise.} \end{cases}$$

Similarly we obtain membership functions associated with $z_2(t) = x_1(t)x_4(t)$. Assume $\max(z_2(t)) = d = \alpha_1$ and $\min(z_2(t)) = -d = \alpha_2$ we have:

$$z_2(t) = x_1(t)x_4(t) = \left(\sum_{i=1}^2 N_i(z_2(t)) b_i \right) \alpha_i \quad (10)$$

$$\begin{aligned} N_1(z_2(t)) &= \frac{z_2(t) - \alpha_2}{\alpha_1 - \alpha_2}, \\ N_2(z_2(t)) &= \frac{\alpha_1 - z_2(t)}{\alpha_1 - \alpha_2}, \end{aligned} \quad (11)$$

The exact TS-fuzzy model-based dynamic system of the ball and beam system can be obtained as following:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \sum_{i=1}^2 \sum_{j=1}^2 M_i(z_1(t)) N_j(z_2(t)) \times \left(\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Gb_i & D\alpha_j \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t) \right) \quad (12)$$

The fuzzy model has the following 4 rules:

$$\begin{aligned} \text{Rule 1: if } z_1(t) \text{ is } M_1 \text{ and } z_2(t) \text{ is } N_1 \\ \text{Then } \dot{x}(t) &= A_1 x(t) + B_1 u(t), \\ \text{Rule 2: if } z_1(t) \text{ is } M_1 \text{ and } z_2(t) \text{ is } N_2 \\ \text{Then } \dot{x}(t) &= A_2 x(t) + B_2 u(t), \\ \text{Rule 3: if } z_1(t) \text{ is } M_2 \text{ and } z_2(t) \text{ is } N_1 \\ \text{Then } \dot{x}(t) &= A_3 x(t) + B_3 u(t), \\ \text{Rule 4: if } z_1(t) \text{ is } M_2 \text{ and } z_2(t) \text{ is } N_2 \\ \text{Then } \dot{x}(t) &= A_4 x(t) + B_4 u(t) \end{aligned} \quad (13)$$

Where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Gb_1 & D\alpha_1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Gb_1 & D\alpha_2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Gb_2 & D\alpha_1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -Gb_2 & D\alpha_2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ B_1 &= B_2 = B_3 = B_4 = B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, z_1 = \sin(x_3) \text{ and } z_2 = x_1 x_4 \end{aligned}$$

2.1.2. Local approximation

The original system can be partitioned into subsystems by approximation of nonlinear terms about equilibrium points. This approach can have fewer rules and of course less complexity but it cannot guarantee the stability of the original system under the controller. Usually in this approach, construction of a fuzzy membership function requires knowledge of the behavior of the original system and of course different types of membership functions can be selected.

3. Parallel distributed compensation

Parallel distributed compensation (PDC) is a model-based design procedure introduced in (Wang et al., 1995). Using the Takagi-Sugeno fuzzy model, a fuzzy combination of the stabilizing state feedback gains, $F_i, i = 1, 2, \dots, r$, associated with every linear subsystem is used as the overall state feedback controller. The general structure of the controller is then as

$$\text{If } z_1(t) \text{ is } M_{i1}, \text{ and } z_2(t) \text{ is } M_{i2}, \dots, m, \text{ and } z_p(t) \text{ is } M_{ip} \text{ then } u = -F_i x(t), i = 1, 2, \dots, r \quad (14)$$

The output of the controller is represented by

$$u = -\frac{\sum_{i=1}^r \omega_i(z) F_i x(t)}{\sum_{i=1}^r \omega_i} = -\sum_{i=1}^r h_i(z) F_i x(t). \quad (15)$$

The Takagi-Sugeno model and the Parallel Distributed Compensation have the same number of fuzzy rules and use the same membership functions.

4. Stability conditions and control design

4.1. LMI

A variety of problems arising in system and control theory can be reduced to a few standard convex or quasi-convex optimization problems involving linear matrix inequalities (LMIs).

Lyapunov published his theory in 1890 and showed that $\frac{d}{dt}x(t) = Ax(t)$ is stable if and only

if there exists a positive-definite matrix P such that $A^T P + PA < 0$. The Lyapunov inequality, $P > 0$ and $A^T P + PA < 0$ is a form of an LMI.

An LMI has the form

$$F(x) \triangleq F_0 + \sum_{i=1}^m x_i F_i > 0, \quad (16)$$

Where $F_i \in R^{n \times n}, i = 0, \dots, m$ are the given symmetric matrices and $x \in R^m$ is the variable and the inequality symbol shows that $F(x)$ is positive definite (Boyd, 1994).

4.2. Stability conditions

There are a large number of works on stability conditions and control design of fuzzy systems in the literature. A sufficient stability condition for ensuring stability of PDC was derived by Tanaka and Sugeno (Tanaka & Sugeno, 1990; 1992).

By substituting the controller output (15) into the TS model for the continuous fuzzy control (4), we have:

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \{A_i - B_i F_j\} x(t) \quad (17)$$

or

$$\begin{aligned} \dot{x}(t) = & \sum_{j=1}^r h_i(z(t)) h_i(z(t)) G_{ii} x(t) \\ & + 2 \sum_{i=1}^r \sum_{i < j} h_i(z(t)) h_j(z(t)) \left\{ \frac{G_{ij} + G_{ji}}{2} \right\} x(t) \end{aligned} \quad (18)$$

where $G_{ij} = A_i - B_i F_j$, Similarly for the discrete fuzzy system we have

$$x(t+1) = \sum_{i=1}^r \sum_{j=1}^r h_i(z(t)) h_j(z(t)) \{A_i - B_i F_j\} x(t) \quad (19)$$

or

$$\begin{aligned} \dot{x}(t+1) = & \sum_{j=1}^r h_i(z(t)) h_i(z(t)) G_{ii} x(t) \\ & + 2 \sum_{i=1}^r \sum_{i < j} h_i(z(t)) h_j(z(t)) \left\{ \frac{G_{ij} + G_{ji}}{2} \right\} x(t) \end{aligned} \quad (20)$$

Theorem 1: The equilibrium of the continuous fuzzy system (3) with $u(t) = 0$ is globally asymptotically stable if there exists a common positive definite matrix P such that

$$A_i^T P + P A_i < 0, \quad i = 1, 2, \dots, r \quad (21)$$

that is, a common P has to exist for all subsystems.

Theorem 2: The equilibrium of the discrete fuzzy system (4) with $u(t) = 0$ is globally asymptotically stable if there exists a common positive definite matrix P such that

$$A_i^T P A_i - P < 0, \quad i = 1, 2, \dots, r \quad (22)$$

that is, a common P has to exist for all subsystems.

The stability of the closed loop system can be derived by using theorem 1 and 2.

Theorem 3: The equilibrium of the continuous fuzzy control system described by (18) is globally asymptotically stable if there exists a common positive definite matrix P such that

$$G_{ii}^T P + P G_{ii} < 0, \quad (23)$$

$$\left(\frac{G_{ij} + G_{ji}}{2} \right)^T P + P \left(\frac{G_{ij} + G_{ji}}{2} \right) \leq 0,$$

$$i < j \text{ s.t. } h_i \cap h_j \neq \emptyset \quad (24)$$

Theorem 4: The equilibrium of the discrete fuzzy control system described by (20) is globally asymptotically stable if there exists a common positive definite matrix P such that

$$G_{ii}^T P G_{ii} - P < 0, \quad (25)$$

$$\left(\frac{G_{ij} + G_{ji}}{2} \right)^T P \left(\frac{G_{ij} + G_{ji}}{2} \right) - P \leq 0,$$

$$i < j \text{ s.t. } h_i \cap h_j \neq \emptyset \quad (26)$$

4.3. Stable controller design

By using the following conditions, the solution of the LMI problem for continuous and discrete fuzzy systems gives us the state feedback gains F_i and the matrix P (if the problem is solvable).

Consider a new variable $X = P^{-1}$ then the stable fuzzy controller design problem is:

Continuous fuzzy system

Find $X > 0$ and M_i , $i = 1, 2, \dots, r$

$$\begin{aligned} -XA_i^T - A_i X + M_i^T B_i^T + B_i M_i &> 0, \\ -XA_i^T - A_i X - XA_j^T - A_j X & \\ + M_j^T B_i^T + B_i M_j + M_i^T B_j^T + B_j M_i &\geq 0. \end{aligned} \quad (27)$$

$$X = P^{-1} \quad i < j \text{ s.t. } h_i \cap h_j \neq \emptyset \quad (28)$$

The conditions (27) and (28) gives us a positive definite matrix X and M_i (or that there is no solution). From the solution X and M_i , a common P and the feedback gains can be found as:

$$P = X^{-1}, F_i = M_i X^{-1} \quad (29)$$

Similarly for a discrete fuzzy system the design problem is

Find $X > 0$ and M_i , $i = 1, 2, \dots, r$

$$\begin{aligned} & X - (A_i X - B_i M_i)^T X^{-1} (A_i X - B_i M_i) > 0, \\ & X - \frac{1}{4} X (A_i X - B_i M_i + A_j X - B_j M_j)^T X^{-1} \\ & \times (A_i X - B_i M_i + A_j X - B_j M_j) X \geq 0. \end{aligned} \quad (30)$$

4.4. Decay rate

Decay rate is associated with the speed of response. The decay rate fuzzy controller design helps to find feedback gains that provide better settling time (Tanaka et al., 1996; 1998a; 1998b).

Continuous fuzzy system: The condition that $\dot{V}(x(t)) \leq -2\alpha V(x(t))$ (Ichikawa et al, 1993, as cited in Tanaka & Wang, 2001) for all $x(t)$ can be written as

$$\begin{aligned} & G_{ii}^T P + P G_{ii} + 2\alpha P < 0 \\ & \left(\frac{G_{ij} + G_{ji}}{2} \right)^T P + P \left(\frac{G_{ij} + G_{ji}}{2} \right) + 2\alpha P \leq 0 \end{aligned} \quad (31)$$

Where

$$G_{ij} = A_i - B_i F_i, \alpha > 0 \text{ and } i < j \text{ s.t. } h_i \cap h_j \neq \emptyset \quad (32)$$

Therefore, by solving the following generalized eigenvalue minimization problem in X , the largest lower bound on the decay rate that can be found by using a quadratic Lyapunov function:

maximize α subject to

$$\begin{aligned} & X > 0, \\ & -X A_i^T + A_i X + M_i^T B_i^T + B_i M_i - 2\alpha X > 0, \\ & -X A_i^T - A_i X - X A_j^T - A_j X + M_j^T B_j^T + B_i M_j \\ & \quad + M_i^T B_j^T + B_j M_i - 4\alpha X > 0, \end{aligned} \quad (33)$$

$$i < j \text{ s.t. } h_i \cap h_j \neq \emptyset, \text{ where } X = P^{-1}, \quad M_i = F_i X. \quad (34)$$

Similarly for a discrete fuzzy system:

The condition that $\Delta V(x(t)) \leq (\alpha^2 - 1)V(x(t))$ (Ichikawa et al, 1993, as cited in Tanaka & Wang, 2001) for all $x(t)$ can be written as

$$G_{ii}^T P G_{ii} - \alpha^2 P < 0, \\ \left(\frac{G_{ij} + G_{ji}}{2} \right)^T P \left(\frac{G_{ij} + G_{ji}}{2} \right) - \alpha^2 P \leq 0 \quad (35)$$

$$i < j \text{ s.t. } h_i \cap h_j \neq \emptyset \text{ and } \alpha < 1 \quad (36)$$

The generalized eigenvalue minimization can be found in (Tanaka & Wang, 2001).

4.5. Constraint on control

Theorem 5: Assume that the initial condition $x(0)$ is known. The constraint $\|u(t)\|_2 \leq \mu$ is satisfied at all times $t \geq 0$ if the LMIs

$$\begin{bmatrix} 1 & x(0)^T \\ x(0) & X \end{bmatrix} \geq 0 \\ \begin{bmatrix} X & M_i^T \\ M_i & \mu^2 I \end{bmatrix} \geq 0 \quad (37)$$

Hold, where $X = P^{-1}$ and $M_i = F_i X$.

The above LMI design conditions depend on the initial states. Thus, if the initial states $x(0)$ change, this means that the feedback gains F_i must be again determined. To overcome this disadvantage, modified LMI constraints on the control input have been developed, where $x(0)$ is unknown but the upper bound ϕ of $\|x(t)\|$ is known, i.e., $\|x(t)\| \leq \phi$.

Theorem 6: Assume that $\|x(t)\| \leq \phi$, where $x(0)$ is unknown but the upper bound ϕ is known. Then,

$$x^T(0) X^{-1} x(0) \leq 1 \text{ if } \phi^2 I \leq X, \quad (38)$$

Where $X = P^{-1}$

Proofs of theorem 1 and 2 are given in (Tanaka & Wang, 2001)

4.6. Performance-oriented parallel distributed compensation

In the modified PDC proposed in (Seidi & Markazi, 2011), unlike the conventional PDC, state feedback gains associated with every linear subsystem, are not assumed fixed. Instead, based on some pre-specified performance criteria, several feedback gains are designed and

used for every subsystem. The overall gain associated with each of the subsystems, is then determined by a fuzzy blending of such gains, so that a better closed-loop performance can be achieved. The required membership functions are chosen based on some pre-specified performance indices, for example, a faster response or a smaller control input. In general, the rest of the method for calculating the overall state feedback gain remains similar to the conventional PDC method, as in (14) and (15). Figure 4, depicts the general framework for the proposed method, through which and depending on various performance criteria, different characteristics for the controller can be specified. For example, two different feedback gains could be designed for a typical subsystem; one providing a lower control input with a longer settling time response, and the other a faster response but with a larger control input. The idea is then to select the overall feedback gain for this subsystem as a weighted sum of such gains, where the weights are appropriately adjusted, in a fuzzy sense, during the time evolution of the system response, so that as a whole, a faster response with a lower control input can be achieved. For this purpose, when the magnitude of the control input becomes large, the relative weight of the first feedback gain is increased, so that the magnitude of the control input is kept within the permissible limits. On the other hand, when the control input is well below the permissible limit, the weight of the second feedback gain is increased, for a faster response. The dynamics of the resulting closed-loop control system can be analyzed as follows:

Consider the following Takagi–Sugeno model of the plant

$$\dot{x} = \sum_{i=1}^r h_i(z) \{A_i x(t) + B_i u(t)\} \quad (39)$$

The following structure is proposed for the fuzzy controller rules

$$\begin{aligned} & i \text{ th rule : If } Z_1(t) \text{ is } M_{i1} \text{ and } Z_2(t) \text{ is } M_{i2}, \dots, Z_p(t) \text{ is } M_{ip}, J(t) \text{ is } H_{i1}, \dots \text{ and } J(t) \text{ is } H_{iq} \\ & \text{then } u_i(t) = \left\{ \sum_{n=1}^q m_{in}(J(t)) K_{in} \right\} x(t) \end{aligned} \quad (40)$$

Where $i = 1, 2, \dots, r$, q_i is the number of gain coefficients in the i th subsystem, m_{in} is the relevant membership degree for $J(t)$, K_{in} is the n th state feedback gain associated with the i th subsystem, H_{iq} is the n th membership function for $J(t)$, defined in the i th rule. Here $J(t)$ is a term depicting a selected performance index, for instance, if one wants to limit the magnitude of the control signal $u(t)$, then $J(t) \equiv |u(t)|$. Where the control input generated by the PDC controller is in the form of

$$\begin{aligned} u(t) &= \sum_{i=1}^r h_i(z) u_i(t) = - \left\{ \sum_{n=1}^q h_i(z) K_i \right\} x(t) \\ K_i &= \sum_{n=1}^q m_{in}(J(t)) K_{in} \end{aligned} \quad (41)$$

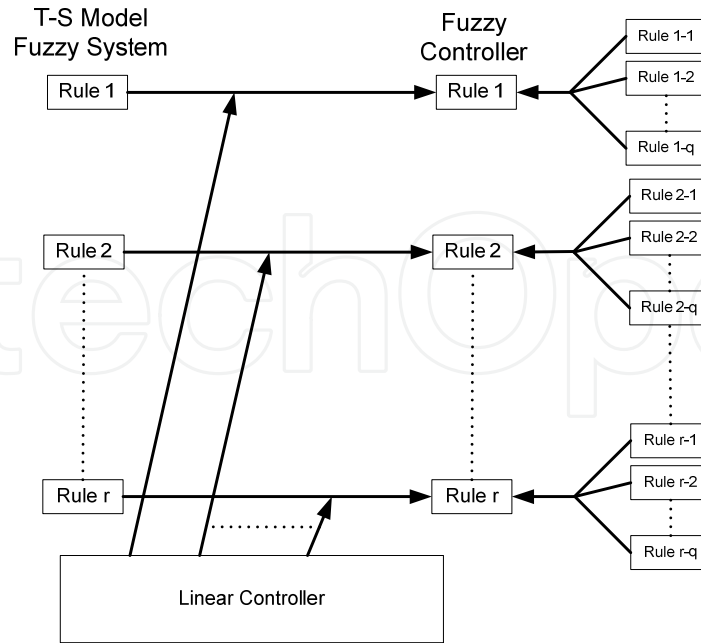


Figure 4. General methodology in the proposed PDC method

Lemma: The fuzzy control system (39), with the control strategy (41) is globally, asymptotically stable, if there exists a common positive definite matrix P such that

$$\begin{aligned} G_{iin}^T P + P G_{iin} &< 0 \\ \left(\frac{G_{ijn} + G_{jin}}{2} \right)^T P + P \left(\frac{G_{ijn} + G_{jin}}{2} \right) &\leq 0 \end{aligned} \quad (42)$$

where $i < j$, $h_i \cap h_j \neq \emptyset$, $G_{ijn} = A_i - B_i K_{jn}$.

Example 2

Consider a single link robot with flexible joint as in Figure 5. This benchmark problem is introduced in (Spong et al., 1987).

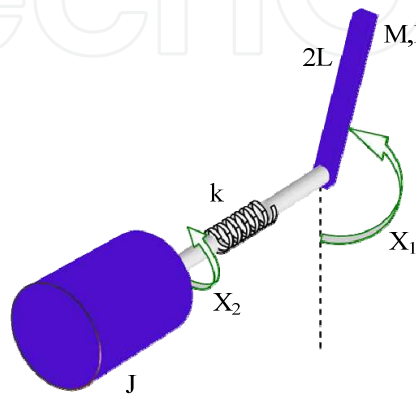


Figure 5. A single link robot with a flexible joint

The state space equations for the system of Figure 4 are

$$\begin{cases} \dot{x}_1 = x_3(t) \\ \dot{x}_2 = x_4(t) \\ \dot{x}_3 = \frac{1}{I}(k(x_2(t) - x_1(t)) - mgL \sin(x_1(t))) \\ \dot{x}_4 = \frac{1}{J}(u(t) - k(x_2(t) - x_1(t))) \end{cases} \quad (43)$$

In order to apply the PDC methodology, the fuzzy Takagi-Sugeno Model is developed first (Seidi & Markazi, 2008). The nonlinear expression $Z = \sin(x_1(t))$, for $x_1(t) \in [-\pi, \pi]$, can be expressed as

$$z = \sin(x_1(t)) = \left(\sum_{i=1}^2 M_i(z) b_i \right) x_1(t) \quad (44)$$

Where, $b_1 = 1, b_2 = 0$ and, hence, the membership functions for z are obtained as

$$\begin{aligned} M_1(z) &= \begin{cases} \frac{z}{\sin^{-1}z}, & z(t) \neq 0 \\ 1, & \text{Otherwise} \end{cases} \\ M_2(z) &= \begin{cases} \frac{\sin^{-1}z - z}{\sin^{-1}z}, & z(t) \neq 0 \\ 1, & \text{Otherwise} \end{cases} \end{aligned} \quad (45)$$

The resulting fuzzy model would then have the following fuzzy rules:

$$\begin{aligned} \text{Rule 1: If } z(t) \text{ is } M_1(z), \text{ then } \dot{x}(t) &= A_1 x(t) + B_1 u(t) \\ \text{Rule 2: If } z(t) \text{ is } M_2(z), \text{ then } \dot{x}(t) &= A_2 x(t) + B_2 u(t) \end{aligned} \quad (45)$$

Where,

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k - mgLb_1}{I} & \frac{k}{I} & 0 & 0 \\ \frac{k}{J} & -\frac{k}{J} & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k - mgLb_2}{I} & \frac{k}{I} & 0 & 0 \\ \frac{k}{J} & -\frac{k}{J} & 0 & 0 \end{bmatrix}, \quad (46)$$

and

$$B_1 = B_2 = B = [0, 0, 0, 1]^T. \quad (47)$$

Assume $k = 100 \text{ Nm/rad}$, $g = 9.8 \text{ m/s}^2$ and other parameters are assumed unity then we have

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -109.8 & 100 & 0 & 0 \\ 100 & -100 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 100 & 0 & 0 \\ 100 & -100 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

Control Rule 1:

If $z(t)$ is $M_1(z)$, then $u(t) = -F_1 x(t)$

Control Rule 2:

If $z(t)$ is $M_2(z)$, then $u(t) = -F_2 x(t)$

(48)

The final output of the controller is

$$u(t) = -\sum_{i=1}^2 h_i F_i x(t) = h_1 F_1 x(t) + h_2 F_2 x(t) \quad (49)$$

Case 1: Stable controller design

Using conditions (27) and (28) the stable controller can be obtained by solving below conditions

$$X > 0$$

$$\begin{bmatrix} -X A_1 - A_1^T X + M_1^T B^T + B M_1 \end{bmatrix} > 0, \quad (50)$$

$$\begin{bmatrix} -X A_2 - A_2^T X + M_2^T B^T + B M_2 \end{bmatrix} > 0,$$

$$\begin{bmatrix} -X A_1^T - A_1 X - X A_2^T - A_2 X + M_2^T B^T + B M_2 + M_1^T B^T + B M_1 \end{bmatrix} > 0$$

Using the MATLAB LMI Control Toolbox we obtain

$$\begin{aligned} F_1 &= [-495.76 \quad 668.96 \quad 14.112 \quad 47.388] \\ F_2 &= [-497.23 \quad 671.34 \quad 14.356 \quad 47.552] \end{aligned} \quad (51)$$

$$P = \begin{bmatrix} 42.1464 & -50.7108 & -1.5337 & -3.2007 \\ -50.7108 & 68.9721 & 2.4898 & 4.3456 \\ -1.5337 & 2.4898 & 0.2554 & 0.1719 \\ -3.2007 & 4.3456 & 0.1719 & 0.3527 \end{bmatrix}$$

Figures 6 and 7 show the response of the system and control effort, respectively.

Case 2: The decay rate

Using conditions (31) and (32) the stable controller can be obtained by solving the conditions:

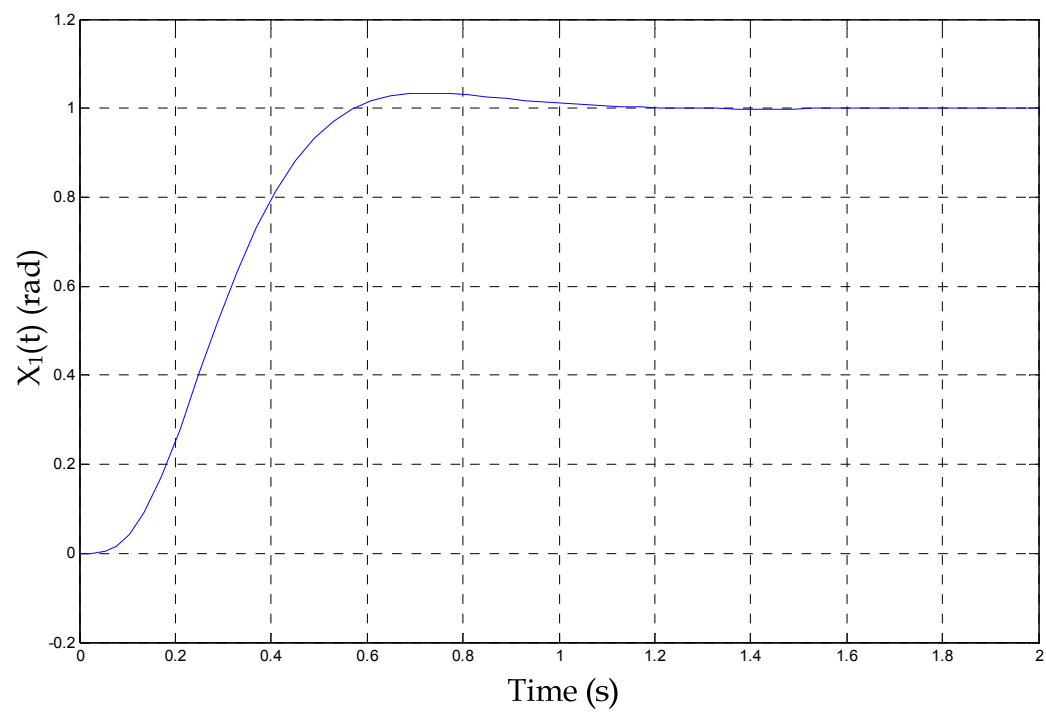


Figure 6. Response of flexible joint robots $x_1(t)$, case 1.

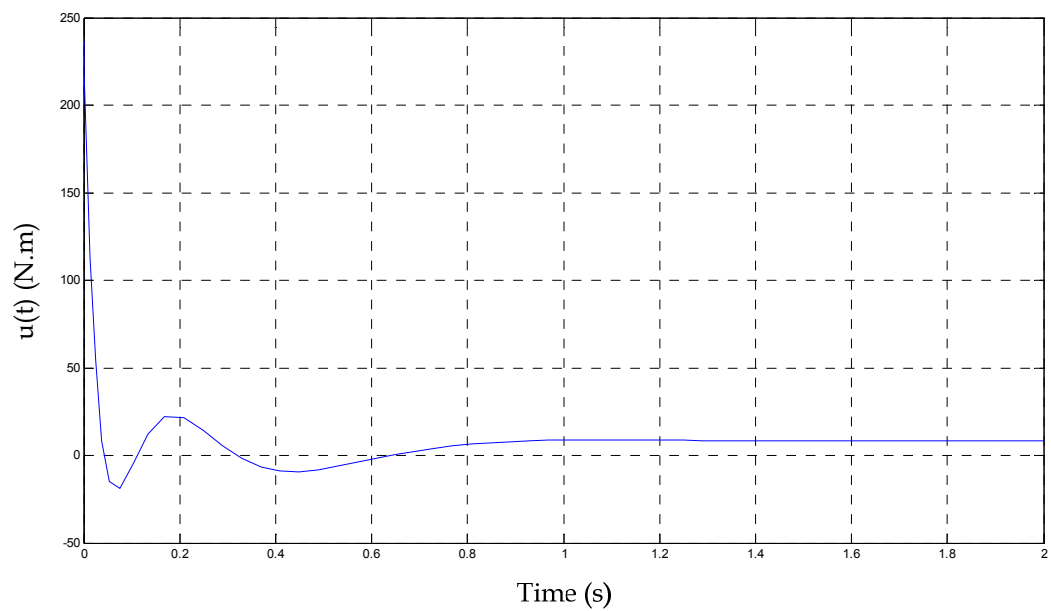


Figure 7. Control input for flexible joint robots, case 1.

$$\begin{aligned}
& [-X A_1^T - A_1 X + M_1^T B^T + B M_1 - 2\alpha X] > 0 \\
& [-X A_2^T - A_2 X + M_2^T B^T + B M_2 - 2\alpha X] > 0 \\
& [-X A_1^T - A_1 X - X A_2^T - A_2 X + M_2^T B^T \\
& \quad + B M_2 + M_1^T B^T + B M_1 - 4\alpha X] > 0
\end{aligned} \tag{52}$$

Considering $\alpha = 10$ and by using the MATLAB LMI Control Toolbox we obtain:

$$\begin{aligned}
F_1 &= [4108.8 \quad 6545.2 \quad 1271.3 \quad 127.77] \\
F_2 &= [4066.9 \quad 6502.6 \quad 1261.7 \quad 127.1] \\
P &= \begin{bmatrix} 36.5087 & 24.0140 & 6.2135 & 0.3352 \\ 24.0140 & 30.1341 & 6.3223 & 0.5013 \\ 6.2135 & 6.3223 & 1.4260 & 0.0995 \\ 0.3352 & 0.5013 & 0.0995 & 0.0099 \end{bmatrix}
\end{aligned} \tag{53}$$

Figures 8 and 9 show the response of the system and control effort, respectively.

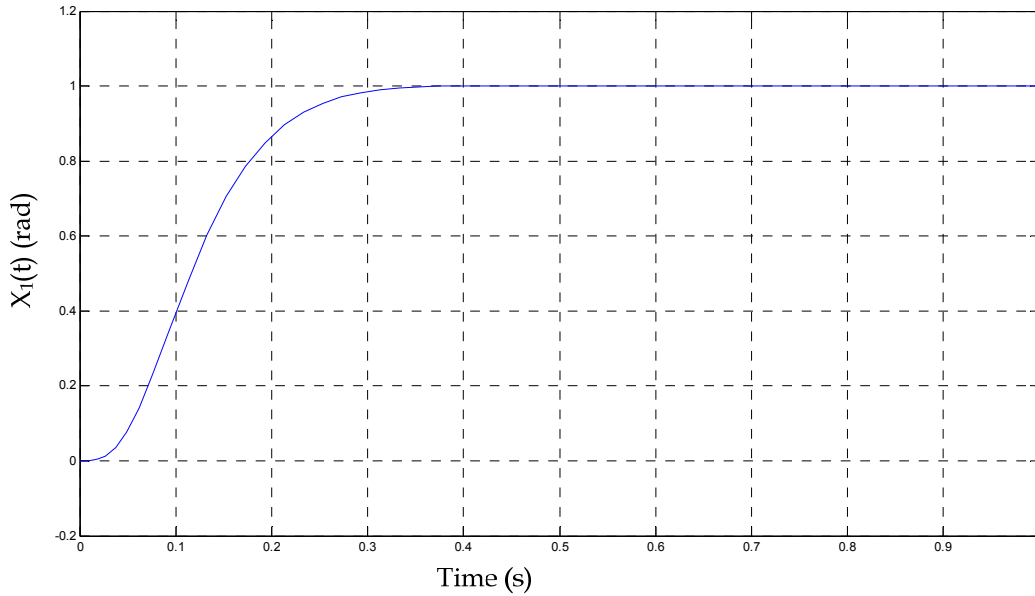


Figure 8. Response of flexible joint robots $x_1(t)$, case 2.

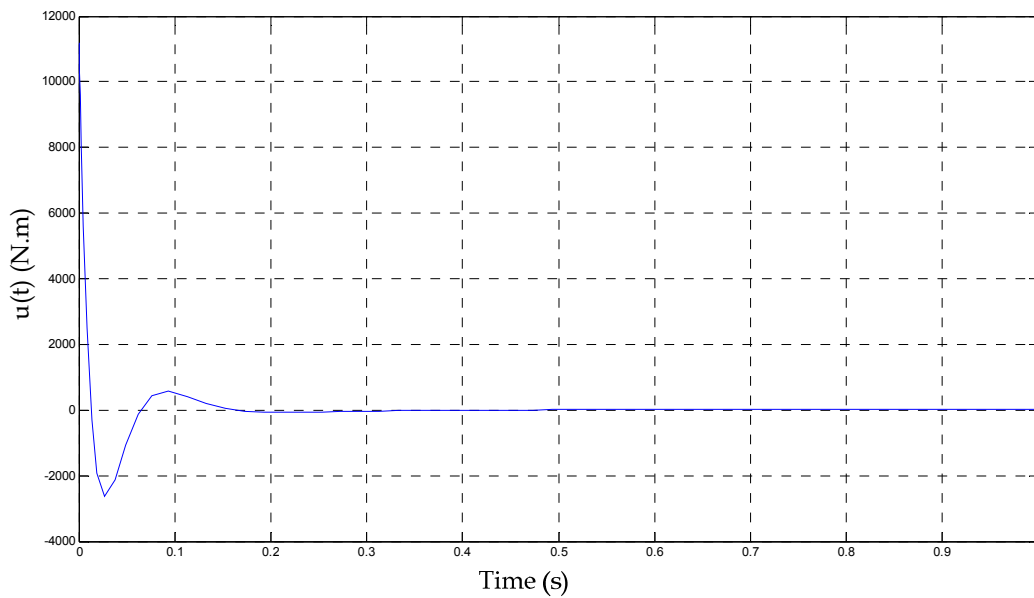


Figure 9. Control input for flexible joint robots, case 2.

Case 3: The decay rate with the constraint on the input

We design a stable fuzzy controller by considering the decay rate and the constraint on the control input. The design problem of the FJR is defined as follows:

Maximize α

$$\begin{aligned}
 & X > 0 \\
 & \begin{bmatrix} -XA_1^T - A_1X + M_1^T B^T + BM_1 - 2\alpha X \end{bmatrix} > 0 \\
 & \begin{bmatrix} -XA_2^T - A_2X + M_2^T B^T + BM_2 - 2\alpha X \end{bmatrix} > 0 \\
 & \begin{bmatrix} -XA_1^T - A_1X - XA_2^T - A_2X + M_2^T B^T \\ \quad + BM_2 + M_1^T B^T + BM_1 - 4\alpha X \end{bmatrix} > 0 \\
 & \begin{bmatrix} X & M_1^T \\ M_1 & \mu^2 I \end{bmatrix} > 0 \\
 & \begin{bmatrix} X & M_2^T \\ M_2 & \mu^2 I \end{bmatrix} > 0 \\
 & \begin{bmatrix} X - \phi^2 I \end{bmatrix} > 0
 \end{aligned} \tag{54}$$

Where $X = P^{-1}$, $M_i = F_i X$, $\mu = 4600$, $\phi = 1$.

Using the MATLAB LMI toolbox to solve the LMI conditions (50), we can get the positive definite matrix and a set of gains (51), that make the system stable.

$$\alpha = 0.072401$$

$$\begin{aligned}
 P &= \begin{bmatrix} 0.7301 & 0.32486 & 0.096794 & 0.0034552 \\ 0.32486 & 0.55483 & 0.10616 & 0.010209 \\ 0.096794 & 0.10616 & 0.023049 & 0.0017139 \\ 0.0034552 & 0.010209 & 0.0017139 & 0.00023565 \end{bmatrix} \\
 F_1 &= [327.57 \quad 1745 \quad 261.86 \quad 57.475] \\
 F_2 &= [356.05 \quad 1739.2 \quad 259.77 \quad 57.5]
 \end{aligned} \tag{55}$$

Figures 10 and 11 show the response of the system and control effort, respectively.

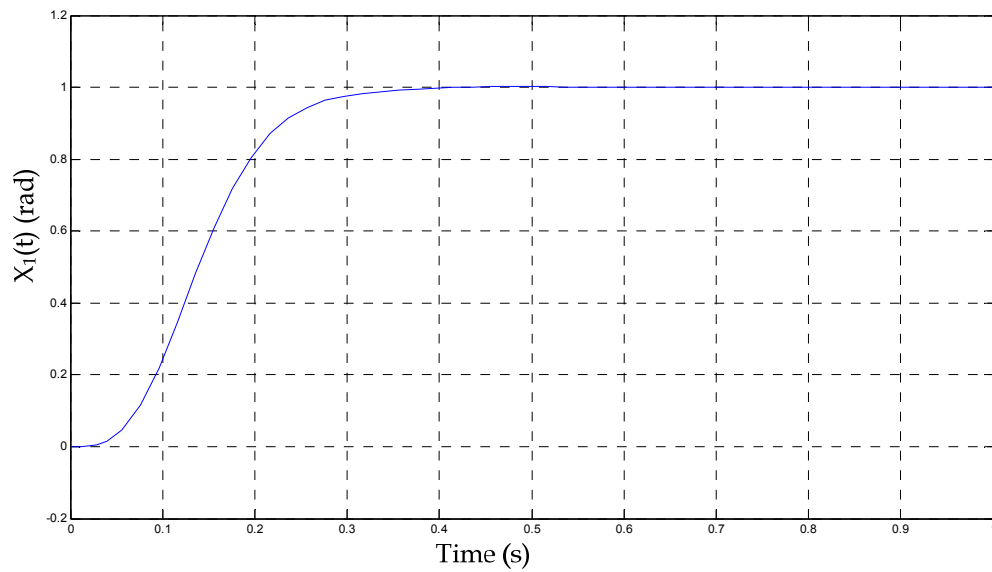


Figure 10. System responses of the single-link flexible joint, case 3.

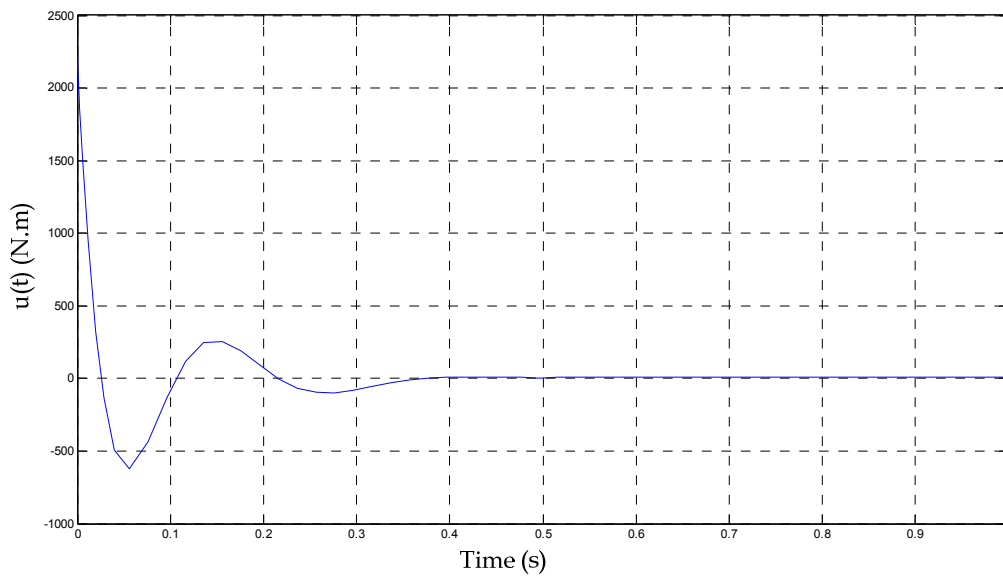


Figure 11. Control input for flexible joint robots, case 3.

Case 4: Performance-oriented parallel distributed compensation

The following stabilizing feedback gains are chosen using the pole placement method, so that K_{11} and K_{21} produce large magnitude inputs for subsystems 1 and 2, respectively, and K_{22} and K_{21} induce low magnitude inputs for those subsystems. In particular,

$$\begin{aligned} K_{11} &= [6667.2 \quad 4411.9 \quad 1052.4 \quad 92.6] \\ K_{12} &= [-33.321 \quad 1413.7 \quad 191.63 \quad 51.2] \\ K_{21} &= [6658.7 \quad 4332.4 \quad 1025.4 \quad 91.1] \\ K_{22} &= [72.3 \quad 1389.8 \quad 189.6 \quad 50.6] \end{aligned} \quad (56)$$

The required simple membership functions are selected as in Figure 12, so that, with a decrease in the corresponding plant input, in subsystems 1 and 2 respectively, the overall feedback gains come closer to K_{11} and K_{21} , and with an increase in the corresponding control input respectively, the overall feedback gains come closer to K_{21} and K_{22} . Now, the fuzzy rules for the controller are constructed as follows:

Rule 1: If $z(t)$ is $M_1(z)$ and $|u(t)|$ is "small" then $u(t) = K_{11}x(t)$

Rule 2: If $z(t)$ is $M_1(z)$ and $|u(t)|$ is "large" then $u(t) = K_{12}x(t)$

Rule 3: If $z(t)$ is $M_2(z)$ and $|u(t)|$ is "small" then $u(t) = K_{21}x(t)$

Rule 4: If $z(t)$ is $M_2(z)$ and $|u(t)|$ is "large" then $u(t) = K_{22}x(t)$

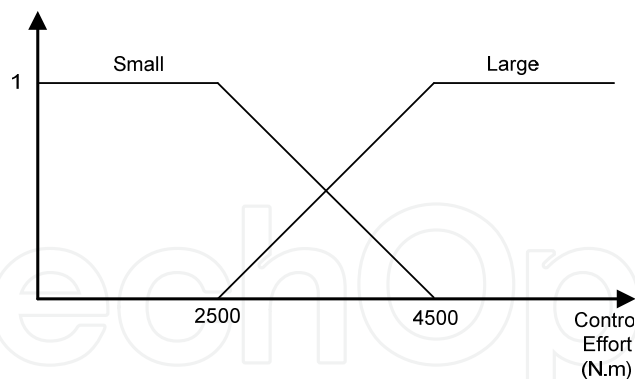


Figure 12. Membership functions for the control effort in the flexible joint robots.

A common positive definite matrix, P , satisfying the stability conditions (42) is obtained by solving the LMI problems:

$$P = 10^4 \times \begin{bmatrix} 121710 & 15858 & 2558.5 & 63.525 \\ 15858 & 8624.4 & 1458.4 & 105.36 \\ 2558.5 & 1458.4 & 702.24 & 42.529 \\ 63.525 & 105.36 & 42.529 & 5.0962 \end{bmatrix}$$

Applying a unit step reference signal for $x_1(t)$, the response history and the corresponding control input are shown in Figures (13) and (14), respectively. Simulation results are investigated for the following three controllers:

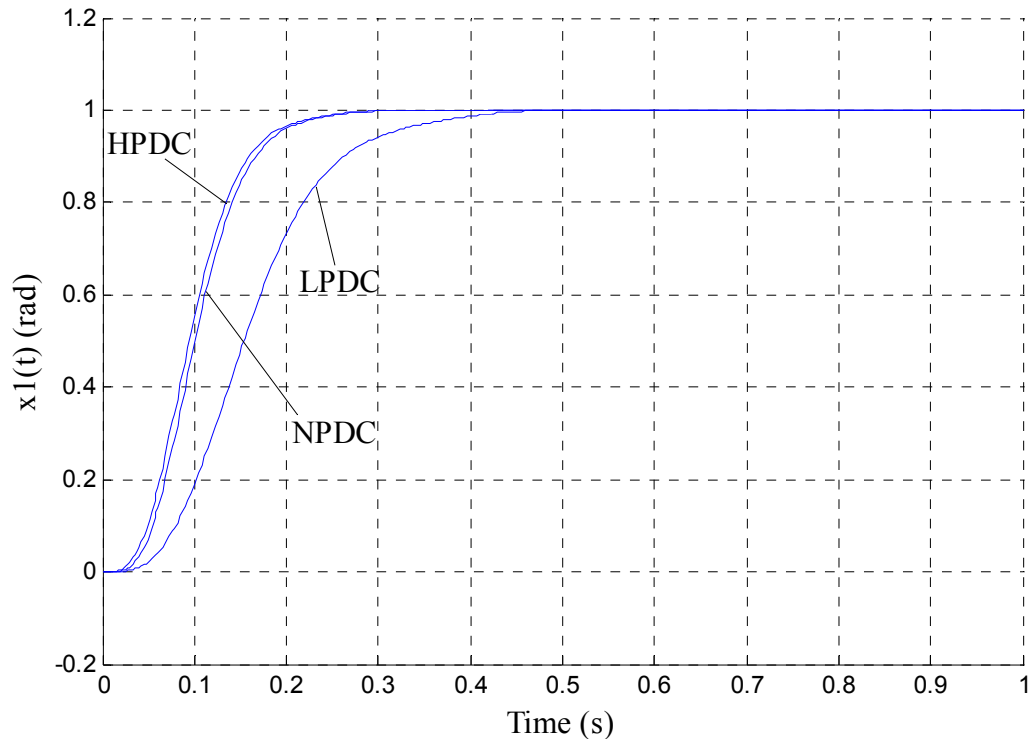


Figure 13. Response of flexible joint robot $x_1(t)$, case 4.

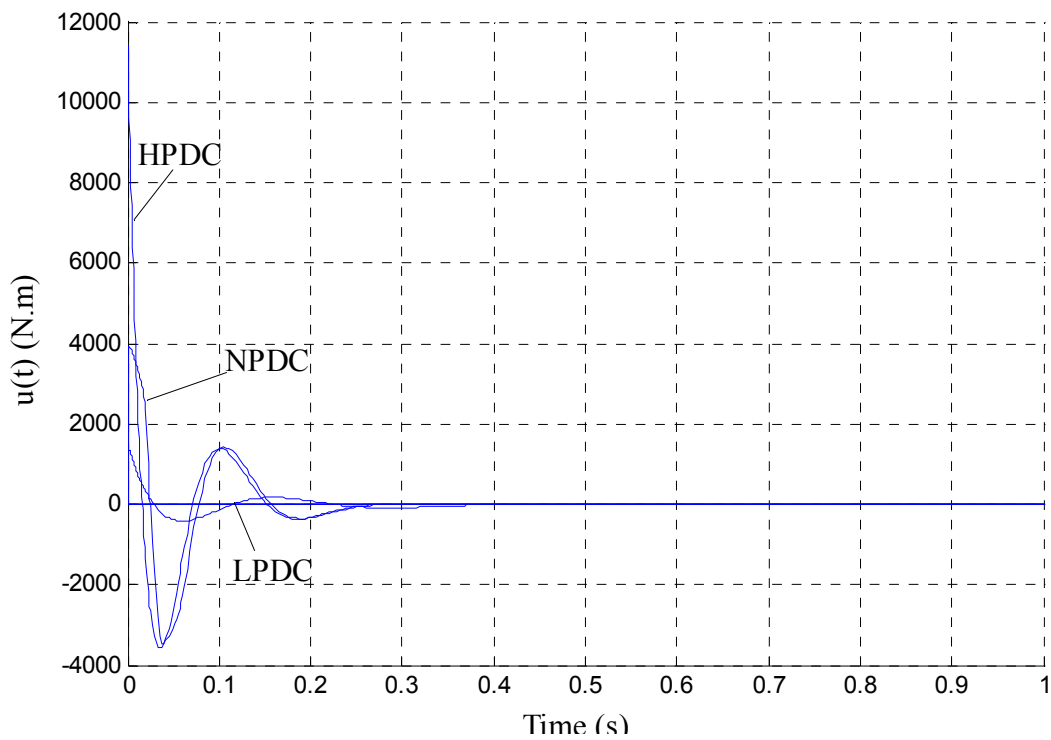


Figure 14. Control input for flexible joint robot, case 4.

1. A PDC controller with feedback gains K_{11} and K_{21} providing a high speed response, and with possible high control inputs (HPDC controller).
2. A PDC controller with feedback gains K_{22} and K_{21} providing a low speed response, and with a lower control input, as compared with the HPDC case (LPDC controller).
3. Proposed modified PDC controller, providing a fast response, yet with an acceptable level of control input (NPDC controller).

It is observed that the new controller provides a settling time similar to the HPDC case, with a much lower magnitude for the control input.

5. Conclusion

This chapter deals with approximation of the nonlinear system using Takagi-Sugeno (T-S) models with linear models as rule consequences and a construction procedure of T-S models. Also, the stability conditions and stabilizing control design of parallel distributed compensation (PDC) are discussed. It is seen that PDC a linear control method can be used to control the nonlinear system. Moreover, the stability analysis and control design problems for both continuous and discrete fuzz control systems can be transformed to linear matrix inequality (LMI) problems and they can be solved efficiently by convex programming techniques for LMIs. Design examples demonstrate the effectiveness of the LMI-based designs.

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