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## Modelling of Parameter and Bound Estimation Laws for Adaptive-Robust Control of Mechanical Manipulators Using Variable Function Approach

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### 1. Introduction

Two different approaches have been actively studied to maintain performance in the presence of parametric uncertainties: adaptive control and robust control. The basic philosophy of adaptive controller is that incorporates some sort of parameter estimation and adaptive controller can learn from experiences in the sense that parameters are changed. Some of the adaptive control laws introduced by Craig et al.(1987), Middleton&Goodwin (1988), Spong&Ortega (1990) require the acceleration measurements and/or the computation of the inverse of the inertia matrix containing estimated parameters. Later, Slotine&Li (1987, 1988) Spong et.al (1990), Egeland&Godhavn (1994) have derived adaptive control algorithms without using the joint accelerations and the inverse of inertia matrix. Other adaptive control laws are proposed in references (Carelli et al 1995, Kelly et al 1989, Burkan&Uzmay 2005, Burkan 2005, Burkan, 2006). Comparative studies of adaptive control laws are given in references (Ortega&Spong 1989, Colbaugh at al 1996).

On the other hand, robust control has been successfully used to design controller with disturbance, unmodelled dynamics and other sources of uncertainty. The papers about application these techniques for the background of robotic application are given in survey papers (Abdullah at al 1991, Sage at al 1999).

Based on the approach of Corless-Leitmann (1981), Spong (1992) developed a new robust control law. In this approach (Spong 1992), the Leitmann (1981) or Corless-Leitmann (1981) approach was used to design a robust controller. Different extension of the scheme by Spong (1992) has been developed by Liu&Goldenberg (1996a, 1997), Yaz (1993), Candudas de Wit et al. (1996). An adaptive scheme of uncertainty bound is given in the papers (Koo&Kim 1994, Burkan and Uzmay 2003b, Burkan and Uzmay 2005). Similar algorithms have proposed by Dawson at. al. (1993) and Zenieh&Corless (1997). Comparative studies of robust controllers are given in the references (Liu & Goldenberg 1996b, Jaritz & Spong (1996).

In pure adaptive control laws, parameters are updated in time and there is no additional control input. However, parameters are not adaptive and fixed (or adaptive) uncertainty bound is used as an additional control input in robust control laws. In the studies (Burkan, 2002; Uzmay & Burkan 2002, Burkan & Uzmay 2003 a, Burkan & Uzmay 2006) adapts previous results on both robust and adaptive control techniques for robot manipulators in an unified scheme, so an adaptive-robust control law is proposed. As distinct from previous studies, variable functions are used in derivation, and parameter and bound estimation laws are updated using exponential and logarithmic functions depending on the robot link parameters and tracking error.

## 2. Adaptive Control Law

In the absence of friction or other disturbances, the dynamic model of an n-link manipulator can be written as (Spong & Vidyasagar, 1989)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau \quad (1)$$

where  $q$  denotes generalised coordinates,  $\tau$  is the n-dimensional vector of applied torques (or forces),  $M(q)$  is the nxn symmetric positive definite inertia matrix,  $C(q, \dot{q})\dot{q}$  is the n-dimensional vector of centripetal and Coriolis terms and  $G(q)$  is the n-dimensional vector of gravitational terms. Equation (1) can also be expressed in the following form.

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Y(q, \dot{q}, \ddot{q})\pi \quad (2)$$

where  $\pi$  is a constant ( $p \times 1$ ) dimensional vector of inertia parameters and  $Y$  is an  $n \times p$  matrix of known functions of the joint position, velocity and acceleration. For any specific trajectory consider known the desired position, velocity and acceleration vectors  $q_d$ ,  $\dot{q}_d$  and  $\ddot{q}_d$  and measured the actual position and velocity errors  $\tilde{q} = q_d - q$ , and  $\dot{\tilde{q}} = \dot{q}_d - \dot{q}$ . Using the above information a corrected desired velocity and acceleration vectors for nonlinearities and decoupling effects are proposed as:

$$\dot{q}_r = \dot{q}_d + \Lambda \tilde{q} \quad \ddot{q}_r = \ddot{q}_d + \Lambda \dot{\tilde{q}} \quad (3)$$

where  $\Lambda$  is a positive definite matrix. Then the following control law is considered.

$$\tau = M(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + G(q) + K\sigma \quad (4)$$

where  $\sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda \tilde{q}$  is a corrected velocity error and  $K\sigma$  is the vector of PD action.

Suppose that the computational model has the same structure as that of the manipulator dynamic model, but its parameters are not known exactly. The control law (4) is then modified into

$$\begin{aligned} \tau &= \hat{M}(q)\ddot{q}_r + \hat{C}(q, \dot{q})\dot{q}_r + \hat{G} + K\sigma \\ &= Y(q, \dot{q}, \ddot{q}_r)\hat{\pi} + K\sigma \end{aligned} \quad (5)$$

where  $\hat{\pi}$  represents the available estimate on the parameters, and accordingly,  $\hat{M}$ ,  $\hat{C}$ ,  $\hat{G}$  denote the estimated terms in the dynamic model. Substituting (5) into (2) gives

$$M(q)\dot{\sigma} + C(q, \dot{q})\sigma + K\sigma = -\tilde{M}(q)\ddot{q}_r - \tilde{C}(q, \dot{q})\dot{q}_r - \tilde{G} = -Y(q, \dot{q}, \ddot{q}_r)\tilde{\pi} \quad (6)$$

where  $\tilde{\pi} = \hat{\pi} - \pi$  is the property of linearity in the parameter error. Error quantities concerning system parameters are characterised by

$$\tilde{M} = \hat{M} - M, \quad \tilde{C} = \hat{C} - C, \quad \tilde{G} = \hat{G} - G \quad (7)$$

The Lyapunov function candidate is defined as

$$V(\sigma, \tilde{q}, \tilde{\pi}) = \frac{1}{2}\sigma^T M(q)\sigma + \frac{1}{2}\tilde{q}^T B\tilde{q} + \frac{1}{2}\tilde{\pi}^T K_{\pi}\tilde{\pi} > 0 \quad (8)$$

where  $\pi$  is a  $p$  dimensional vector containing the unknown manipulators and load parameters,  $\hat{\pi}$  is its estimate and  $\tilde{\pi} = \hat{\pi} - \pi$  denotes the parameter estimation error vector.  $B$  and  $K_{\pi}$  are positive definite, usually diagonal matrix. Using the property  $\sigma^T [M(q) - 2C(q, \dot{q})]\sigma = 0 \quad \forall \sigma \in \mathbb{R}^n$  and choosing  $B = 2\Lambda K$ , the time derivative of  $V$  along the trajectory of system (6) is

$$\dot{V} = -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} + \tilde{\pi}^T (K_{\pi} \dot{\tilde{\pi}} - Y^T(q, \dot{q}, \ddot{q}_r)\sigma) \quad (9)$$

If the estimate of the parameter vector is updated as the adaptive law

$$\dot{\hat{\pi}} = K_{\pi}^{-1} Y^T(q, \dot{q}, \ddot{q}_r)\sigma \quad (10)$$

Equation (9) becomes

$$\dot{V} = -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} \quad (11)$$

So,  $\dot{V}$  is negative semidefinite and Equation (6) is stable. It should be noted that  $\dot{\hat{\pi}} = \dot{\tilde{\pi}}$  ( $\pi$  is constant) (Sciavicco & Siciliano, 1996). The parameter estimation law (10) can also be written as

$$\hat{\pi} = \int K_{\pi}^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma dt + \hat{\pi}(0) \quad (12)$$

where  $\hat{\pi}(0)$  is the initial estimation of the parameters. The resulting block diagram of the adaptive control law is given in Fig. 1 (Sciavicco & Sciliano, 1996)

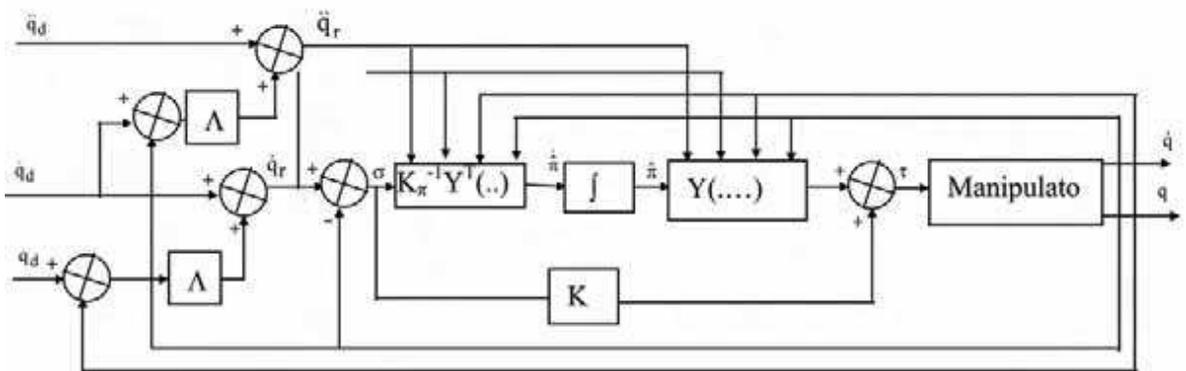


Figure 1. Implementation of the adaptive control law (10) (Sciavicco & Siciliano, 1996).

### 3. Robust Control Law

Consider the nominal control vector for the model system described by Equations (1) and (2).

$$\begin{aligned} \tau_0 &= M_0(q)\ddot{q}_r + C_0(q, \dot{q})\dot{q}_r + G_0(q) - K\sigma \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\pi_0 - K\sigma \end{aligned} \quad (13)$$

The definition of the nominal control law  $\tau_0$  is based on the adaptive algorithm of Slotine and Li (1987). It is important to understand that the nominal control vector  $\tau_0$  in Equation (13) is defined in terms of fixed parameters which are not changed or updated in time as would be an adaptive control strategy. The control input  $\tau$  can be defined in terms of the nominal control vector  $\tau_0$  and a compensation vector for parameter variations as:

$$\tau = \tau_0 + Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)u(t) = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\pi_0 + u(t)) - K\sigma \quad (14)$$

where

$$\tilde{q} = q - q_d; \dot{\tilde{q}}_r = \dot{q}_d - \Lambda \tilde{q}; \ddot{\tilde{q}}_r = \ddot{q}_d - \Lambda \dot{\tilde{q}} \tag{15}$$

It is supposed that the parameter estimation vector  $\pi$  is uncertain and it is assumed that both  $\pi_0 \in \mathbb{R}^p$  and  $\rho \in \mathbb{R}$  are known a priori, such that

$$\|\tilde{\pi}\| = \|\pi - \pi_0\| \leq \rho \tag{16}$$

Let  $\varepsilon > 0$  and the additional control vector as defined by Spong (1992) as:

$$u(t) = \begin{cases} -\rho \frac{Y^T \sigma}{\|Y^T \sigma\|} & \text{if } \|Y^T \sigma\| > \varepsilon \\ -\rho \frac{Y^T \sigma}{\varepsilon} & \text{if } \|Y^T \sigma\| \leq \varepsilon \end{cases} \tag{17}$$

Considering adaptive control law (Sciavicco & Siciliano, 1996), the block diagram of the pure robust controller is given in Fig. 2.

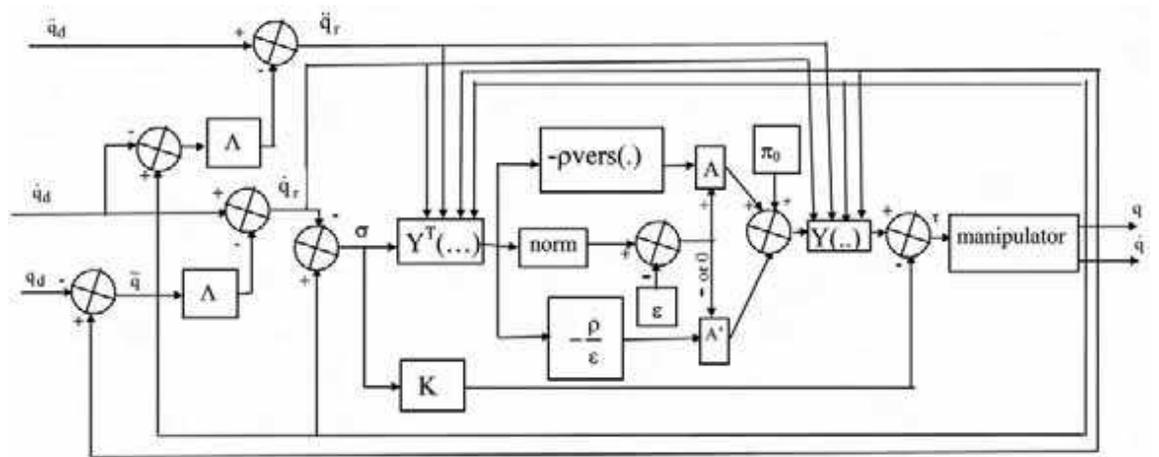


Figure 2 Block diagram of the robust control law. (Burkan & Uzmay, 2003 c)

Since the controller which is defined by Equation (17) consists of two different input depending on  $\varepsilon$ , the matrices  $A$  and  $A'$  are introduced to select appropriate control input. The  $A$  matrix is diagonal with ones and zeros on the diagonal. When  $\|Y^T \sigma\| - \varepsilon > 0$ , a one is present in  $A$ , a zero is present in  $A'$  and the first additional control input is in effect. When  $\|Y^T \sigma\| - \varepsilon \leq 0$  a zero is present in  $A$ , a one is present in  $A'$ , and so the second additional control input is in ef-

fect. Hence, the matrices  $A$  and  $A'$  are simple switches which set the mode of additional control input to be used (Burkan & Uzman, 2003 c).

As a measure of parameter uncertainty on which the additional control input is based,  $\rho$  can be defined as

$$\rho = \left( \sum_{i=1}^p \rho_i^2 \right)^{1/2} \quad (18)$$

Having a single number  $\rho$  to measure the parametric uncertainty may lead to overly conservative design, higher than necessary gains, ect. For this reason we may be interested in assigning different “weights” or gains to the components of  $\tau$ . We can do this as follows. Suppose that we have a measure of uncertainty for each parameter  $\tilde{\pi}_i$  separately as:

$$|\tilde{\pi}_i| \leq \rho_i \quad i=1,2,\dots,p \quad (19)$$

Let  $v_i$  denote the  $i$ th component of the vector  $Y^T \sigma$ ,  $\varepsilon_i = i=1,2,\dots,p_i$  represent the  $i$ th component of  $\varepsilon$ , and define the  $i$ th component of the control input  $\tau_i$  as (Spong, 1992), then

$$u(t)_i = \begin{cases} -\rho_i v_i / |v_i| & \text{if } |v_i| > \varepsilon_i \\ -(\rho_i / \varepsilon_i) v_i & \text{if } |v_i| \leq \varepsilon_i \end{cases} \quad (20)$$

#### 4. Adaptive-Robust Control Law

Considering the dynamic model of a  $n$ -link robot manipulator given by Equations (1) and (2), the control input vector that comprises the parameter estimation and the additional control input is defined as

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\hat{\pi} + \delta(t)) + K\sigma \quad (21)$$

Substituting (21) into (1) and some arrangements yield

$$M(q)\dot{\sigma} + C(q, \dot{q})\sigma + K\sigma = -Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{\pi} - Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\delta(t) \quad (22)$$

Adaptive robust parameters are identical as adaptive control law in the known parameter case such as  $\sigma$ ,  $q_r$ ,  $\Lambda$  and  $K$ . It is assumed that the parameter error is unknown such that

$$\tilde{\pi} = \hat{\pi} - \pi = \rho(t) \quad (23)$$

where  $\hat{\pi}$  is the estimate of the available parameters and updated in time. The upper bounding function  $\hat{\rho}(t)$  is assumed to be unknown, and should be determined using the estimation law to control the system properly. Finally the error  $\tilde{\rho}(t)$  shows the difference between parameter error and upper bounding function as

$$\tilde{\rho}(t) = \rho(t) - \hat{\rho}(t) = \hat{\pi} - \pi - \hat{\rho}(t) \quad (24)$$

**Theorem (Burkan & Uzmay, 2003 a):**

Let  $\alpha > 0$  be a positive number,  $\pi$  be the unloaded and lower bound of parameter, and  $\rho$  be the upper uncertainty bound of Equation (16). The three of them are supposed to be known initially. If the estimate of parameter  $\hat{\pi}$  and the additional control input  $\delta(t)$  in control law (21) are defined, respectively as

$$\hat{\pi} = -\frac{2}{\alpha} e^{-\alpha t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) + \pi; \quad \delta(t) = \rho e^{-\frac{\alpha}{2} t} \quad (25)$$

and substitute them in the control input (21) for the trajectory control of the model manipulator, then the tracking errors  $\tilde{q}$  and  $\dot{\tilde{q}}$  will converge to zero.

**Proof:**

By taking into account above parameters and control algorithm, the Lyapunov function candidate is defined as

$$V(\sigma, \tilde{q}, \tilde{\rho}(t)) = \frac{1}{2} \sigma^T M(q) \sigma + \frac{1}{2} \tilde{q}^T B \tilde{q} + \frac{1}{2} \tilde{\rho}(t)^T \Gamma \tilde{\rho}(t) \quad (26)$$

Apart from similar studies,  $\Gamma$  is the positive definite diagonal matrix and change in time. The time derivative of Equation (26) is written as

$$\dot{V} = \sigma^T M(q) \dot{\sigma} + \sigma^T \frac{1}{2} \dot{M}(q) \sigma + \tilde{q}^T B \dot{\tilde{q}} + \frac{1}{2} \tilde{\rho}(t)^T \dot{\Gamma} \tilde{\rho}(t) + \tilde{\rho}(t)^T \Gamma \dot{\tilde{q}}(t) \quad (27)$$

where

$$\tilde{\rho}(t) = \rho(t) - \hat{\rho}(t) = \hat{\pi} - \pi - \hat{\rho}(t); \quad \dot{\tilde{\rho}}(t) = \dot{\rho}(t) - \dot{\hat{\rho}}(t) = \dot{\hat{\pi}} - \dot{\hat{\rho}}(t) \quad (28)$$

Let  $B = 2\Lambda K$  and use the property  $\sigma^T [\dot{M}(q) - 2C(q, \dot{q})] \sigma = 0$ ,  $\forall \sigma \in R^n$ , the time derivative of  $V$  along the system (22) is

$$\dot{V} = -\dot{\tilde{q}}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} - \sigma^T Y \delta(t) - \sigma^T Y \rho(t) + \frac{1}{2} \tilde{\rho}(t)^T \dot{\Gamma} \tilde{\rho}(t) + \tilde{\rho}(t)^T \Gamma \dot{\tilde{q}}(t) \quad (29)$$

Since  $K > 0$ , and  $\Lambda > 0$  the first terms of Equation (29) are less or equal to zero that is:

$$-\tilde{q}^T K \dot{\tilde{q}} - \tilde{q}^T \Lambda K \Lambda \tilde{q} \leq 0 \quad (30)$$

So, in order to find conditions to make  $\dot{V} \leq 0$  we concentrate on the remaining terms of the equation. If the rest of Equation (29) is equal to or less than zero, the system will be stable. Substituting Equation (24) into the remaining terms of Equation (29) the following equation is obtained:

$$-\sigma^T Y \delta(t) - \sigma^T Y \rho(t) + \frac{1}{2} [\rho(t) - \hat{\rho}(t)]^T \dot{\Gamma} [\rho(t) - \hat{\rho}(t)] + [\rho(t) - \hat{\rho}(t)]^T \Gamma [\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \quad (31)$$

Now, in considering  $\delta(t)$  as an estimated term of uncertainty bound, that is,  $\delta(t) = -\hat{\rho}(t)$  then Equation (31) is written as:

$$\sigma^T Y \hat{\rho}(t) - \sigma^T Y \rho(t) + \frac{1}{2} [\rho(t) - \hat{\rho}(t)]^T \dot{\Gamma} [\rho(t) - \hat{\rho}(t)] + [\rho(t) - \hat{\rho}(t)]^T \Gamma [\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \quad (32)$$

Taken  $[\rho(t) - \hat{\rho}(t)]$  as a common multiplier, Equation (32) is written as:

$$[[\rho(t) - \hat{\rho}(t)]^T [-Y^T \sigma + \frac{1}{2} \dot{\Gamma} [\rho(t) - \hat{\rho}(t)] + \Gamma [\dot{\rho}(t) - \dot{\hat{\rho}}(t)]] = 0 \quad (33)$$

Hence, we look for the conditions for which the equation  $-Y^T \sigma + \frac{1}{2} \dot{\Gamma} [\rho(t) - \hat{\rho}(t)] + \Gamma [\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0$  is satisfied. The terms constituting the above equation are expressed as

$$\tilde{\rho}(t) = \rho(t) - \hat{\rho}(t); \quad \tilde{\pi} = \rho(t) = \hat{\pi} - \pi; \quad \dot{\rho}(t) = \dot{\hat{\pi}}; \quad \tilde{\dot{\rho}}(t) = \dot{\rho}(t) - \dot{\hat{\rho}}(t) = \dot{\hat{\pi}} - \dot{\hat{\rho}}(t) \quad (34)$$

Substituting the parameters in Equation (34) into Equation (33) yields

$$-Y^T \sigma + \frac{1}{2} \dot{\Gamma} [\hat{\pi} - \pi - \hat{\rho}(t)] + \Gamma [\dot{\hat{\pi}} - \dot{\hat{\rho}}(t)] = 0 \quad (35)$$

Then

$$-Y^T \sigma + \frac{1}{2} \dot{\Gamma} (\hat{\pi} - \pi) + \Gamma \dot{\hat{\pi}} - [\frac{1}{2} \dot{\Gamma} \hat{\rho}(t) + \Gamma \dot{\hat{\rho}}(t)] = 0 \quad (36)$$

A solution for Equation (36) can be derived if it is divided into two equations as:

$$-Y^T \sigma + \frac{1}{2} \dot{\Gamma}(\hat{\pi} - \pi) + \Gamma \dot{\hat{\pi}} = 0 \quad (37)$$

$$-\left(\frac{1}{2} \dot{\Gamma} \hat{\rho}(t) + \Gamma \dot{\hat{\rho}}(t)\right) = 0 \quad (38)$$

Equation (37) can also be written as;

$$\frac{1}{2} \dot{\Gamma}(\hat{\pi} - \pi) + \Gamma \dot{\hat{\pi}} = Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma \quad (39)$$

For the proposed approach,  $\Gamma$  and its time derivative are chosen as a positive definite diagonal matrix of the form

$$\Gamma = e^{\alpha t} I, \quad \dot{\Gamma} = \alpha e^{\alpha t} I \quad (40)$$

where  $I$  is a  $p \times p$  dimensional matrix. Substitution of Equation (40) into Equation (39) yields;

$$e^{\alpha t} \dot{\hat{\pi}} + \frac{1}{2} \alpha e^{\alpha t} (\hat{\pi} - \pi) = Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma \quad (41)$$

Dividing Equation (41) by the factor  $e^{\frac{\alpha}{2} t}$  result in the following expression.

$$e^{\frac{\alpha}{2} t} \dot{\hat{\pi}} + \frac{1}{2} \alpha e^{\frac{\alpha}{2} t} \hat{\pi} = e^{-\frac{\alpha}{2} t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \frac{1}{2} \alpha e^{\frac{\alpha}{2} t} \pi \quad (42)$$

Equation (42) can be arranged as

$$\frac{d}{dt} (e^{\frac{\alpha}{2} t} \hat{\pi}) = e^{-\frac{\alpha}{2} t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \frac{1}{2} \alpha e^{\frac{\alpha}{2} t} \pi \quad (43)$$

For a given instant,  $Y^T$  and  $\sigma$  can be assumed to be constant. Integrating both sides of Equation (43) yields;

$$(e^{\frac{\alpha}{2} t} \hat{\pi}) = \int (e^{-\frac{\alpha}{2} t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \frac{1}{2} \alpha e^{\frac{\alpha}{2} t} \pi) dt = -\frac{2}{\alpha} e^{-\frac{\alpha}{2} t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + e^{\frac{\alpha}{2} t} \pi + C \quad (44)$$

If Equation (44) is divided by  $e^{\frac{\alpha}{2}t}$ , the result is

$$\dot{\hat{\pi}} = -\frac{2}{\alpha} e^{-\alpha t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \pi + C e^{\frac{1}{2}\alpha t} \quad (45)$$

If the initial condition is  $\hat{\pi}(0) = \pi$ , the constant  $C$  becomes zero. So, the parameter adaptation algorithm is derived as

$$\dot{\hat{\pi}} = -\frac{2}{\alpha} e^{-\alpha t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \pi \quad (46)$$

Adaptive parameter estimation law is obtained as a solution of Equation (37). As a result of Equation (38), robust parameter estimation law  $\hat{\rho}(t)$  can be also obtained. Substitution of Equation (40) into Equation (38) yields;

$$-(e^{\alpha t} \dot{\hat{\rho}}(t) + \frac{1}{2} \alpha e^{\alpha t} \hat{\rho}(t)) = 0 \quad (47)$$

By dividing Equation (47) by the factor  $e^{\frac{\alpha}{2}t}$ , the following expression is found.

$$-(e^{\frac{\alpha}{2}t} \dot{\hat{\rho}}(t) + \frac{1}{2} \alpha e^{\frac{\alpha}{2}t} \hat{\rho}(t)) = 0 \quad (48)$$

If Equation (48) is arranged according to  $\hat{\rho}(t)$

$$-\frac{d}{dt} (e^{\frac{\alpha}{2}t} \hat{\rho}(t)) = 0 \quad (49)$$

Integrating both sides of Equation (49) yields

$$-(e^{\frac{\alpha}{2}t} \hat{\rho}(t)) = C \Rightarrow -(\hat{\rho}(t)) = C e^{-\frac{\alpha}{2}t} \quad (50)$$

If  $\hat{\rho}(0) = \rho$  is taken as an initial condition, the constant  $C$  is equivalent to  $\rho$ . So, the robust parameter estimation algorithm is derived as

$$\hat{\rho}(t) = -\rho e^{-\frac{\alpha}{2}t} \quad (51)$$

Since  $\delta(t) = -\hat{\rho}(t)$ , the control vector can be written as

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \left[ -\frac{2}{\alpha} e^{-\alpha t} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma + \pi + \rho e^{-\frac{\alpha}{2} t} \right] + K \sigma \tag{52}$$

The block diagram of adaptive-robust control law is shown in Fig. 3.

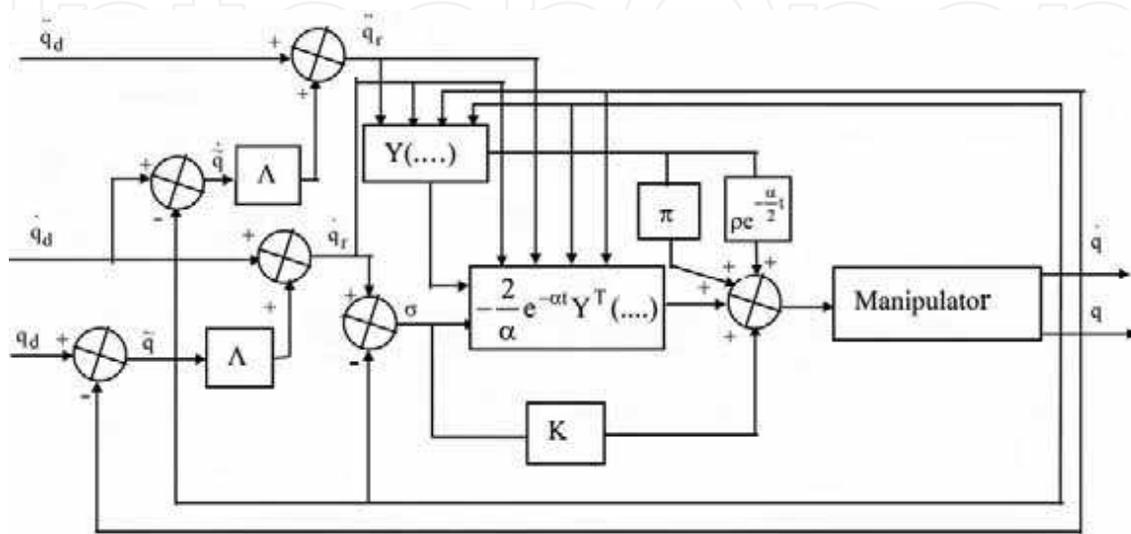


Figure 3. Block diagram of the adaptive-robust control law (52) (Burkan & Uzmay, 2003a)

If Equation (46) and (51) are substituted in Equation (29) it will become a negative semidefinite function of the form of Equation (30). So, the system (22) will be stable under the conditions assumed in the theorem.

At this point, it is very important to choose the variable function  $\Gamma$  in order to solve the Equations (38) and (39), and there is no a certain rule for selection of  $\Gamma$  for this systems. We use system state parameters and mathematical insight to search for appropriate function of  $\Gamma$  as a solution of the first order differential in Equations (38) and (39). For the second derivation, we choose variable function  $\Gamma$  and its derivative such that (Uzmay & Burkan, 2002).

$$\Gamma = e^{\int Y^T \sigma dt} ; \dot{\Gamma} = Y^T \sigma e^{\int Y^T \sigma dt} \tag{53}$$

where  $\Gamma$  is a  $p \times p$  dimensional identity matrix. Substitution of (53) into (39) yields

$$e^{\int Y^T \sigma dt} \dot{\hat{\pi}} + \frac{1}{2} Y^T \sigma e^{\int Y^T \sigma dt} (\hat{\pi} - \pi) = Y^T \sigma \tag{54}$$

Remembering that  $\dot{\hat{\pi}} = \dot{\hat{\pi}}$  ( $\pi$  is constant). Dividing Equation (54) by  $e^{\int Y^T \sigma dt}$  yields

$$\dot{\hat{\pi}} + \frac{1}{2} Y^T \sigma \hat{\pi} = e^{-\int Y^T \sigma dt} Y^T \sigma + \frac{1}{2} Y^T \sigma \pi \quad (55)$$

Multiplying Equation (55) by the factor  $e^{\frac{1}{2} \int Y^T \sigma dt}$  results

$$e^{\frac{1}{2} \int Y^T \sigma dt} \dot{\hat{\pi}} + \frac{1}{2} Y^T \sigma e^{\frac{1}{2} \int Y^T \sigma dt} \hat{\pi} = e^{\frac{1}{2} \int Y^T \sigma dt} e^{-\int Y^T \sigma dt} Y^T \sigma + \frac{1}{2} Y^T \sigma e^{\frac{1}{2} \int Y^T \sigma dt} \pi \quad (56)$$

Equation (56) can be arranged as

$$\frac{d}{dt} (e^{\frac{1}{2} \int Y^T \sigma dt} \hat{\pi}) = e^{-\frac{1}{2} \int Y^T \sigma dt} Y^T \sigma + \frac{1}{2} Y^T \sigma e^{\frac{1}{2} \int Y^T \sigma dt} \pi \quad (57)$$

Integrating both sides of Equation (57) yields

$$e^{\frac{1}{2} \int Y^T \sigma dt} \hat{\pi} = \int e^{-\frac{1}{2} \int Y^T \sigma dt} Y^T \sigma dt + \frac{1}{2} \int \pi Y^T \sigma e^{\frac{1}{2} \int Y^T \sigma dt} dt \quad (58)$$

$$e^{\frac{1}{2} \int Y^T \sigma dt} \hat{\pi} = -2e^{-\frac{1}{2} \int Y^T \sigma dt} + \pi e^{\frac{1}{2} \int Y^T \sigma dt} + C \quad (59)$$

By dividing both sides of Equation (59) by  $e^{\frac{1}{2} \int Y^T \sigma dt}$ , the following result is obtained.

$$\hat{\pi} = -2e^{-\int Y^T \sigma dt} + \pi + Ce^{-\frac{1}{2} \int Y^T \sigma dt} \quad (60)$$

If the condition of  $\hat{\pi}(0) = \pi$  is taken as an initial condition, the constant  $C$  is equivalent to 2. Hence, the parameter adaptation law is derived as

$$\hat{\pi} = -2e^{-\int Y^T \sigma dt} + \pi + 2e^{-\frac{1}{2} \int Y^T \sigma dt} = 2(e^{-\frac{1}{2} \int Y^T \sigma dt} - e^{-\int Y^T \sigma dt}) + \pi \quad (61)$$

In order to drive  $\hat{\rho}(t)$ , Equation (53) is substituted into (38) yields

$$-e^{\frac{1}{2} \int Y^T \sigma dt} \dot{\hat{\rho}}(t) + \frac{1}{2} Y^T \sigma e^{-\int Y^T \sigma dt} \hat{\rho}(t) = 0 \quad (62)$$

By dividing  $e^{\frac{1}{2} \int Y^T \sigma dt}$  Equation (62), the following expression is found

$$-\left(e^{\frac{1}{2}\int Y^T \sigma dt} \dot{\hat{\rho}}(t) + \frac{1}{2} Y^T \sigma e^{\frac{1}{2}\int Y^T \sigma dt} \hat{\rho}(t)\right) = 0 \tag{63}$$

Equation (63) is arranged according to

$$-\frac{d}{dt} \left( e^{\frac{1}{2}\int Y^T \sigma dt} \hat{\rho}(t) \right) = 0 \tag{64}$$

Integrate both side of Equation (64) yields

$$\left( e^{\frac{1}{2}\int Y^T \sigma dt} \hat{\rho}(t) \right) = -C \tag{65}$$

Then

$$\hat{\rho}(t) = -C e^{-\frac{1}{2}\int Y^T \sigma dt} \tag{66}$$

If  $\hat{\rho}(0) = \rho$  is taken as an initial condition, the constant C will be equivalent to  $\rho$ . Hence the bound estimation law is derived as

$$\hat{\rho}(t) = -\rho e^{-\frac{1}{2}\int Y^T \sigma dt} \tag{67}$$

As a result, the adaptive-robust control law is obtained as (Uzmay & Burkan, 2002).

$$\tau = Y(q, \dot{q}, \ddot{q}_r, \ddot{q}_r) \left[ 2 \left( e^{-\frac{1}{2}\int Y^T \sigma dt} - e^{-\int Y^T \sigma dt} \right) + \pi + \rho e^{-\frac{1}{2}\int Y^T \sigma dt} \right] + K\sigma \tag{68}$$

The block diagram of adaptive-robust control law is shown in Fig. 4.

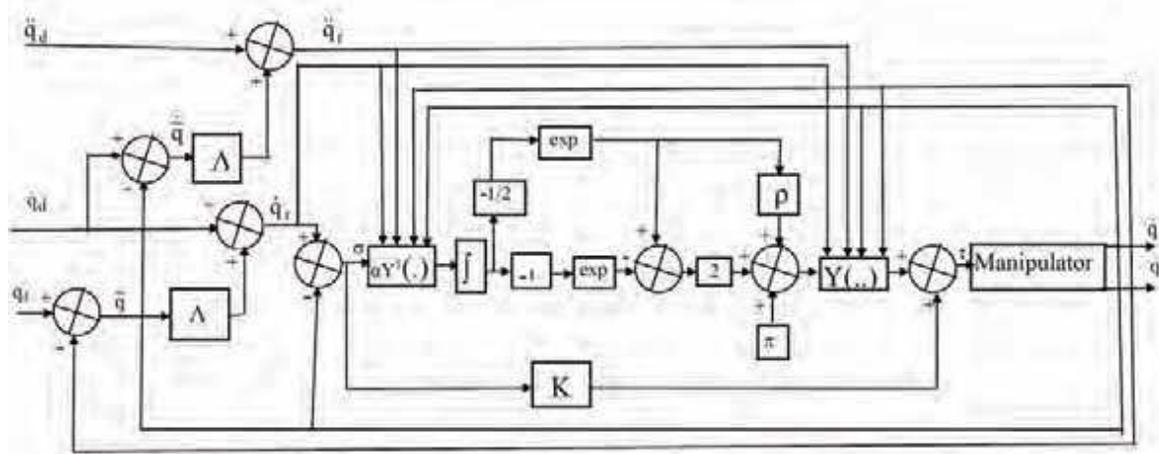


Figure 4. Block diagram of the adaptive-robust control law (68)

**Theorem 2: (Burkan & Uzmay, 2006):**

Let  $\alpha \in \mathbb{R}^+$ ,  $(\alpha \int Y^T \sigma dt)_i \geq 0$ ,  $i=1,2,\dots,p$ ,  $\rho_i$   $i=1,2,\dots,p$  be the initial estimation of the upper bounding function  $\hat{\rho}(t)$  and it is assumed to be known initially. If the estimation of parameter  $\hat{\pi}$ , estimation of the upper bounding function  $\hat{\rho}(t)$  and the additional control input  $\delta(t)$  are defined respectively as

$$\hat{\pi} = (1/\alpha) \begin{bmatrix} \frac{\ln((\alpha \int Y^T \sigma dt)_1 + 1)}{\int (\alpha Y^T \sigma dt)_1 + 1} \\ \frac{\ln((\alpha \int Y^T \sigma dt)_2 + 1)}{\int (\alpha Y^T \sigma dt)_2 + 1} \\ \dots \\ \frac{\ln((\alpha \int Y^T \sigma dt)_p + 1)}{\int (\alpha Y^T \sigma dt)_p + 1} \end{bmatrix} + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix}; \hat{\rho}(t) = - \begin{bmatrix} \frac{\rho_1}{\int (\alpha Y^T \sigma)_1 dt + 1} \\ \frac{\rho_2}{\int (\alpha Y^T \sigma)_2 dt + 1} \\ \dots \\ \frac{\rho_p}{\int (\alpha Y^T \sigma)_p dt + 1} \end{bmatrix}; \delta(t) = \begin{bmatrix} \frac{\rho_1}{\int (\alpha Y^T \sigma)_1 dt + 1} \\ \frac{\rho_2}{\int (\alpha Y^T \sigma)_2 dt + 1} \\ \dots \\ \frac{\rho_p}{\int (\alpha Y^T \sigma)_p dt + 1} \end{bmatrix} \quad (69)$$

where  $\delta(t) = -\hat{\rho}(t)$ . Substitute  $\hat{\pi}$  and  $\delta(t)$  into the control input (21) for the trajectory control of the model manipulator, then the tracking errors  $\tilde{q}$  and  $\dot{\tilde{q}}$  will converge to zero.

Proof:

In the previous approaches, it is difficult to derive another parameter and bound estimation law because selection of appropriate variable function  $\Gamma$  and solution of the differential equation are not simple. However, the selection of the  $\Gamma$  and solution of the differential equation are simplified in the studies (Burkan 2005, Burkan & Uzmay, 2006) In order to simplify selection of the variable function  $\Gamma$  and simplify the solution of the differential equation, the following Lyapunov function is developed (Burkan & Uzmay, 2006).

$$V(\sigma, \tilde{q}, \tilde{\rho}(t)) = \frac{1}{2} \sigma^T M(q) \sigma + \frac{1}{2} \tilde{q}^T B \tilde{q} + \frac{1}{2} \tilde{\rho}(t)^T \Gamma^2 \tilde{\rho}(t) \quad (70)$$

where  $\Gamma$  is a  $p \times p$  dimensional diagonal matrix and change in time. The time derivative of Equation (70) is written as

$$\dot{V} = \sigma^T M(q) \dot{\sigma} + \sigma^T \frac{1}{2} \dot{M}(q) \sigma + \tilde{q}^T B \dot{\tilde{q}} + \tilde{\rho}(t)^T \Gamma \dot{\tilde{\rho}}(t) + \tilde{\rho}(t)^T \Gamma^2 \dot{\tilde{\rho}}(t) \quad (71)$$

Let  $B = 2\Lambda K$  and use the property  $\sigma^T [\dot{M}(q) - 2C(q, \dot{q})] \sigma = 0$ ,  $\forall \sigma \in \mathbb{R}^n$ , the time derivative of  $V$  along the system (22) is determined as

$$\dot{V} = -\tilde{q}^T K \tilde{q} - \tilde{q}^T \Lambda K \Lambda \tilde{q} - \sigma^T Y \delta(t) - \sigma^T Y \rho(t) + \tilde{\rho}(t)^T \Gamma \dot{\tilde{\rho}}(t) + \tilde{\rho}(t)^T \Gamma^2 \dot{\tilde{p}}(t) \quad (72)$$

Substituting Equation (24) into Equation (72) yields the following equation.

$$-\sigma^T Y \delta(t) - \sigma^T Y \rho(t) + [\rho(t) - \hat{\rho}(t)]^T \Gamma \dot{[\rho(t) - \hat{\rho}(t)]} + [\rho(t) - \hat{\rho}(t)]^T \Gamma^2 [\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \quad (73)$$

Now, let's consider  $\delta(t) = -\hat{\rho}(t)$ , then Equation (73) is written as:

$$\sigma^T Y \hat{\rho}(t) - \sigma^T Y \rho(t) + [\rho(t) - \hat{\rho}(t)]^T \Gamma \dot{[\rho(t) - \hat{\rho}(t)]} + [\rho(t) - \hat{\rho}(t)]^T \Gamma^2 [\dot{\rho}(t) - \dot{\hat{\rho}}(t)] = 0 \quad (74)$$

Taking  $[\rho(t) - \hat{\rho}(t)]$  as a common multiplier, Equation (74) is arranged as:

$$[[\rho(t) - \hat{\rho}(t)]^T [-Y^T \sigma + \Gamma \dot{(\rho(t) - \hat{\rho}(t))} + \Gamma^2 (\dot{\rho}(t) - \dot{\hat{\rho}}(t))]] = 0 \quad (75)$$

Substituting the parameters in Equation (34) into (75) yields

$$-Y^T \sigma + \Gamma \dot{[\hat{\pi} - \pi - \hat{\rho}(t)]} + \Gamma^2 [\dot{\hat{\pi}} - \dot{\hat{\rho}}(t)] = 0 \quad (76)$$

Then

$$-Y^T \sigma + \Gamma \dot{(\hat{\pi} - \pi)} + \Gamma^2 \dot{\hat{\pi}} - [\Gamma \dot{\hat{\rho}}(t) + \Gamma^2 \dot{\hat{\rho}}] = 0 \quad (77)$$

As a result, two different equations can be obtained from Equation (77) as follows.

$$-Y^T \sigma + \Gamma \dot{(\hat{\pi} - \pi)} + \Gamma^2 \dot{\hat{\pi}} = 0 \quad (78)$$

$$-(\Gamma \dot{\hat{\rho}}(t) + \Gamma^2 \dot{\hat{\rho}}(t)) = 0 \quad (79)$$

Equation (79) can also be written as

$$\Gamma \dot{\hat{\pi}} + \dot{\hat{\pi}} = \Gamma^{-1} Y^T \sigma + \dot{\hat{\pi}} \quad (80)$$

since  $\dot{\hat{\pi}} = \dot{\hat{\pi}}$  ( $\pi$  is a constant). Equation (80) is arranged as

$$\frac{d}{dt}(\Gamma \hat{\pi}) = \Gamma^{-1} Y^T \sigma + \dot{\hat{\pi}} \quad (81)$$

Integration both sides of Equation (81) yields

$$\Gamma \hat{\pi} = \int \Gamma^{-1} Y^T \sigma dt + \int \dot{\hat{\pi}} dt \quad (82)$$

Then, Equation (82) is arranged as

$$\Gamma \hat{\pi} = \int \Gamma^{-1} Y^T \sigma dt + \Gamma \pi + C \quad (83)$$

In Equation (83),  $\hat{\pi}$  and  $\Gamma$  are unknown and in order to derive  $\hat{\pi}$ ,  $\Gamma$  must be defined. There is no a certain rule for definition of  $\Gamma$  for this systems. We use system state parameters and mathematical insight to search for appropriate function of  $\Gamma$  as a derivation of the  $\hat{\pi}$ . For the third derivation, we choose  $\Gamma$  and  $\Gamma^{-1}$ , such that (Burkan & Uzmay, 2006).

$$\Gamma = \begin{bmatrix} (\int \alpha Y^T \sigma dt)_1 + 1 & 0 & \dots & 0 \\ 0 & (\int \alpha Y^T \sigma dt)_2 + 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & (\int \alpha Y^T \sigma dt)_p + 1 \end{bmatrix};$$

$$\Gamma^{-1} = \begin{bmatrix} \frac{1}{(\int \alpha Y^T \sigma dt)_1 + 1} & 0 & \dots & 0 \\ 0 & \frac{1}{(\int \alpha Y^T \sigma dt)_2 + 1} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \frac{1}{(\int \alpha Y^T \sigma dt)_p + 1} \end{bmatrix} \quad (84)$$

where  $\Gamma$  and  $\Gamma^{-1}$  are  $p \times p$  dimensional diagonal matrices. Substitution of Equation (84) into Equation (83) yields

$$\begin{bmatrix} (\int \alpha Y^T \sigma dt)_1 + 1 & 0 & \dots & 0 \\ 0 & (\int \alpha Y^T \sigma dt)_2 + 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & (\int \alpha Y^T \sigma dt)_p + 1 \end{bmatrix} \times \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \dots \\ \hat{\pi}_p \end{bmatrix} = \int \begin{bmatrix} \frac{(Y^T \sigma)_1}{(\int \alpha Y^T \sigma dt)_1 + 1} \\ \frac{(Y^T \sigma)_2}{(\int \alpha Y^T \sigma dt)_2 + 1} \\ \dots \\ \frac{(Y^T \sigma)_p}{(\int \alpha Y^T \sigma dt)_p + 1} \end{bmatrix} dt \quad (85)$$

$$+ \begin{bmatrix} (\int \alpha Y^T \sigma dt)_1 + 1 & 0 & \dots & 0 \\ 0 & (\int \alpha Y^T \sigma dt)_2 + 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & (\int \alpha Y^T \sigma dt)_p + 1 \end{bmatrix} \times \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} + C \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

After integration, the result is

$$\begin{bmatrix} (\int \alpha Y^T \sigma dt)_1 + 1 & 0 & \dots & 0 \\ 0 & (\int \alpha Y^T \sigma dt)_2 + 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & (\int \alpha Y^T \sigma dt)_p + 1 & \dots \end{bmatrix} \times \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \dots \\ \hat{\pi}_p \end{bmatrix} = (1/\alpha) \begin{bmatrix} \ln((\int \alpha Y^T \sigma dt)_1 + 1) \\ \ln((\int \alpha Y^T \sigma dt)_2 + 1) \\ \dots \\ \ln((\int \alpha Y^T \sigma dt)_p + 1) \end{bmatrix} \quad (86)$$

$$+ \begin{bmatrix} (\int \alpha Y^T \sigma dt)_1 + 1 & 0 & \dots & 0 \\ 0 & (\int \alpha Y^T \sigma dt)_2 + 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & (\int \alpha Y^T \sigma dt)_p + 1 \end{bmatrix} \times \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} + C \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

Multiplying both sides of Equation (86) by  $\Gamma^{-1}$  and taken initial condition as  $\hat{\pi}(0)=\pi$ , the constant C will be equivalent to zero. Hence, the parameter adaptation law is derived as

$$\hat{\pi} = (1/\alpha) \begin{bmatrix} \frac{\ln((\alpha \int Y^T \sigma dt)_1 + 1)}{\int (\alpha Y^T \sigma dt)_1 + 1} \\ \frac{\ln((\alpha \int Y^T \sigma dt)_2 + 1)}{\int (\alpha Y^T \sigma dt)_2 + 1} \\ \dots \\ \frac{\ln((\alpha \int Y^T \sigma dt)_p + 1)}{\int (\alpha Y^T \sigma dt)_p + 1} \end{bmatrix} + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} \quad (87)$$

Adaptive parameter estimation law is obtained as a solution of Equation (83). As a result of Equation (78), robust parameter estimation law  $\hat{\rho}(t)$  can be also obtained. Equation (78) is arranged as

$$-(\dot{\Gamma} \hat{\rho}(t) + \Gamma \dot{\hat{\rho}}(t)) = 0 \quad (88)$$

If Equation (88) is arranged according to  $\hat{\rho}(t)$

$$-(\frac{d}{dt} \Gamma \hat{\rho}(t)) = 0 \quad (89)$$

Integrating both sides of Equation (89) yields

$$-(\Gamma \hat{\rho}(t)) = C \Rightarrow \hat{\rho}(t) = -\Gamma^{-1}C \quad (90)$$

If  $\hat{\rho}(0) = \rho$  is taken as an initial condition as would be defined in Equation (90), the constant  $C$  will be equivalent to  $\rho$ . So, the robust parameter estimation algorithm is derived as

$$\hat{\rho}(t) = -\Gamma^{-1}\rho = - \begin{bmatrix} \frac{\rho_1}{\int(\alpha Y^T \sigma)_1 dt + 1} \\ \frac{\rho_2}{\int(\alpha Y^T \sigma)_2 dt + 1} \\ \dots \\ \frac{\rho_p}{\int(\alpha Y^T \sigma)_p dt + 1} \end{bmatrix} \quad (91)$$

Since  $\delta(t) = -\hat{\rho}(t)$ , the control vector in Equation (21) can be written as

$$\tau = Y(q, \dot{q}, \ddot{q}_r, \ddot{q}_r) \left[ \frac{\ln((\alpha \int Y^T \sigma dt)_1 + 1)}{\int(\alpha Y^T \sigma dt)_1 + 1} \right. \\ \left. \frac{\ln((\alpha \int Y^T \sigma dt)_2 + 1)}{\int(\alpha Y^T \sigma dt)_2 + 1} \right. \\ \dots \\ \left. \frac{\ln((\alpha \int Y^T \sigma dt)_p + 1)}{\int(\alpha Y^T \sigma dt)_p + 1} \right] + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} + \begin{bmatrix} \frac{\rho_1}{\int(\alpha Y^T \sigma)_1 dt + 1} \\ \frac{\rho_2}{\int(\alpha Y^T \sigma)_2 dt + 1} \\ \dots \\ \frac{\rho_p}{\int(\alpha Y^T \sigma)_p dt + 1} \end{bmatrix} + K\sigma \quad (92)$$

The resulting block diagram of the proposed adaptive-robust control law is given in Fig. 5.

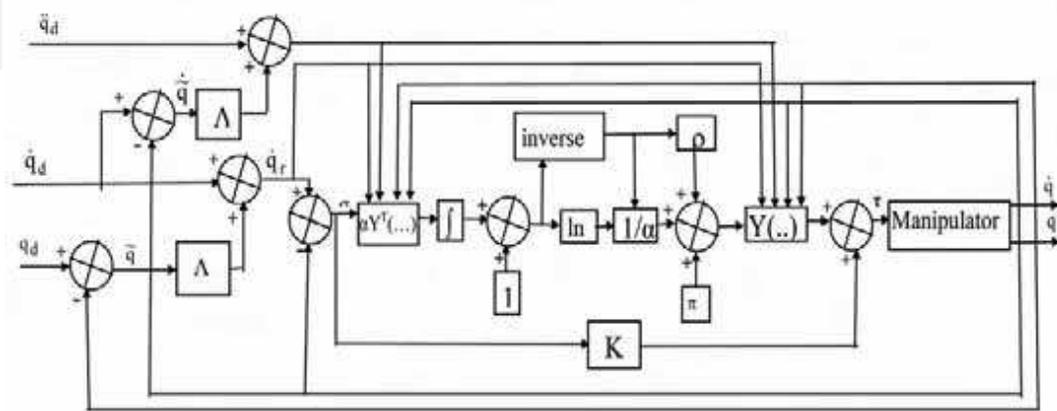


Figure 5. Implementation of the adaptive-robust control law (92) (Burkan & Uzmay, 2006).

For the fourth derivation,  $\Gamma$  and  $\Gamma^{-1}$  are chosen such that

$$\Gamma = \begin{bmatrix} \frac{1}{(\alpha_1 Y^T \sigma)_1 + \beta_1} & 0 & \dots & 0 \\ 0 & \frac{1}{(\alpha_2 Y^T \sigma)_2 + \beta_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \frac{1}{(\alpha_p Y^T \sigma)_p + \beta_p} \end{bmatrix} \tag{93}$$

$$\Gamma^{-1} = \begin{bmatrix} (\alpha_1 Y^T \sigma)_1 + \beta_1 & 0 & \dots & 0 \\ 0 & (\alpha_2 Y^T \sigma)_2 + \beta_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & (\alpha_p Y^T \sigma)_p + \beta_p \end{bmatrix}$$

Substitution of Equation (93) into Equation (83) yields

$$\begin{bmatrix} \frac{1}{(a_1 Y^T \sigma)_1 + \beta_1} & 0 & \dots & 0 \\ 0 & \frac{1}{(a_2 Y^T \sigma)_2 + \beta_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \frac{1}{(a_p Y^T \sigma)_p + \beta_p} \end{bmatrix} x \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \dots \\ \hat{\pi}_p \end{bmatrix} = \int \begin{bmatrix} ((a_1 Y^T \sigma)_1 + \beta_1)(Y^T \sigma) \\ ((a_2 Y^T \sigma)_2 + \beta_2)(Y^T \sigma) \\ \dots \\ ((a_p Y^T \sigma)_p + \beta_p)(Y^T \sigma) \end{bmatrix} dt \tag{94}$$

$$+ \begin{bmatrix} \frac{1}{(a_1 Y^T \sigma)_1 + \beta_1} & 0 & \dots & 0 \\ 0 & \frac{1}{(a_2 Y^T \sigma)_2 + \beta_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \frac{1}{(a_p Y^T \sigma)_p + \beta_p} \end{bmatrix} x \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} + C \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

After integration, the result is

$$\begin{bmatrix} \frac{1}{(a_1 Y^T \sigma)_1 + \beta_1} & 0 & \dots & 0 \\ 0 & \frac{1}{(a_2 Y^T \sigma)_2 + \beta_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \frac{1}{(a_p Y^T \sigma)_p + \beta_p} \end{bmatrix} x \begin{bmatrix} \hat{\pi}_1 \\ \hat{\pi}_2 \\ \dots \\ \hat{\pi}_p \end{bmatrix} = \begin{bmatrix} 0.5a_1 (\int Y^T \sigma dt)_1^2 + \beta_1 \int (Y^T \sigma)_1 \\ 0.5a_2 (\int Y^T \sigma dt)_2^2 + \beta_2 \int (Y^T \sigma)_2 \\ \dots \\ 0.5a_p (\int Y^T \sigma dt)_p^2 + \beta_p \int (Y^T \sigma)_p \end{bmatrix} \quad (95)$$

$$\begin{bmatrix} \frac{1}{(a_1 Y^T \sigma)_1 + \beta_1} & 0 & \dots & 0 \\ 0 & \frac{1}{(a_2 Y^T \sigma)_2 + \beta_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \frac{1}{(a_p Y^T \sigma)_p + \beta_p} \end{bmatrix} x \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} + C \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

Multiplying both sides of Equation (95) by  $\Gamma^{-1}$  and taken initial condition as  $\hat{\pi}(0) = \pi$ , the constant C will be equivalent to zero. Hence, the parameter adaptation law is derived as

$$\hat{\pi} = \begin{bmatrix} (\alpha_1 Y^T \sigma)_1 + \beta_1 (0.5\alpha_1 (\int Y^T \sigma dt)_1^2 + \beta_1 \int (Y^T \sigma)_1 dt) \\ (\alpha_2 Y^T \sigma)_2 + \beta_2 (0.5\alpha_2 (\int Y^T \sigma dt)_2^2 + \beta_2 \int (Y^T \sigma)_2 dt) \\ \dots \\ (\alpha_p Y^T \sigma)_p + \beta_p (0.5\alpha_p (\int Y^T \sigma dt)_p^2 + \beta_p \int (Y^T \sigma)_p dt) \end{bmatrix} + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} \quad (96)$$

If  $\hat{\rho}(0) = \rho$  is taken as an initial condition as would be defined in Equation (90), the constant C will be equivalent to  $\rho$ . So, the upper bounding function is derived as

$$\hat{\rho}(t) = \begin{bmatrix} (\alpha_1 Y^T \sigma)_1 + \beta_1 \rho_1 \\ (\alpha_2 Y^T \sigma)_2 + \beta_2 \rho_2 \\ \dots \\ (\alpha_p Y^T \sigma)_p + \beta_p \rho_p \end{bmatrix} \quad (97)$$

As a result, the fourth adaptive-robust control law is derived as

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \left[ \begin{array}{c} (\alpha_1 Y^T \sigma)_1 + \beta_1 (0.5 \alpha_1 (\int Y^T \sigma dt)_1^2 + \beta_1 \int (Y^T \sigma)_1 dt) \\ (\alpha_2 Y^T \sigma)_2 + \beta_2 (0.5 \alpha_2 (\int Y^T \sigma dt)_2^2 + \beta_1 \int (Y^T \sigma)_2 dt) \\ \dots\dots\dots \\ (\alpha_p Y^T \sigma)_p + \beta_p (0.5 \alpha_p (\int Y^T \sigma dt)_p^2 + \beta_1 \int (Y^T \sigma)_p dt) \end{array} \right] + \begin{bmatrix} \pi_1 \\ \pi_2 \\ \dots \\ \pi_p \end{bmatrix} \tag{98}$$

$$+ \begin{bmatrix} (\alpha_1 Y^T \sigma)_1 + \beta_1 \rho_1 \\ (\alpha_2 Y^T \sigma)_2 + \beta_2 \rho_2 \\ \dots\dots\dots \\ (\alpha_p Y^T \sigma)_p + \beta_p \rho_p \end{bmatrix} + K \sigma$$

The resulting block diagram of the proposed adaptive-robust control law is given in Fig. 6.

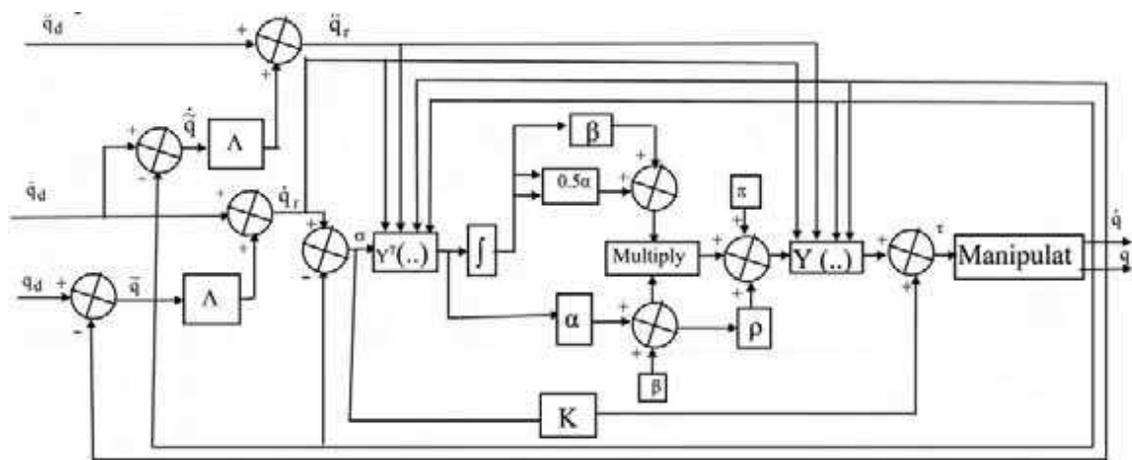


Figure 6. Implementation of the adaptive-robust control law (98)

## 5. Dynamic Model and Parametric Uncertainties

As an illustrations, a two-link robot arm manipulators shown in Fig. 7. The robot link parameters are

$$\begin{aligned} \pi_1 &= m_1 l_{c1}^2 + m_2 l_1^2 + I_1 & \pi_2 &= m_2 l_{c2}^2 + I_2 & \pi_3 &= m_2 l_1 l_{c2} \\ \pi_4 &= m_1 l_{c1} & \pi_5 &= m_2 l_1 & \pi_6 &= m_2 l_{c2} \end{aligned} \quad (99)$$

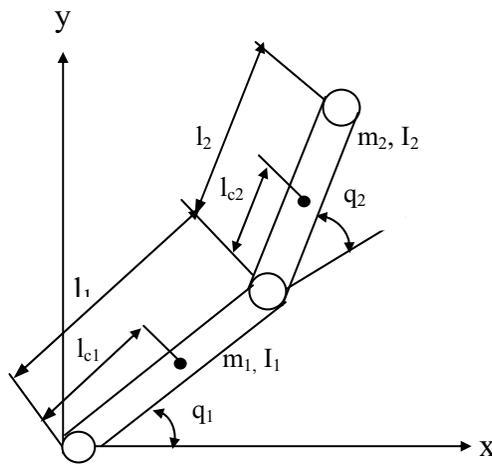


Figure 7. Two-link planar robot (Spong, 1992)

With this parameterization, the dynamic model in Equation (1) can be written as

$$Y(q, \dot{q}, \ddot{q})\pi = \tau \quad (100)$$

The component  $y_{ij}$  of  $Y(q, \dot{q}, \ddot{q})$  are given as

$$\begin{aligned} y_{11} &= \ddot{q}_1; & y_{12} &= \ddot{q}_1 + \ddot{q}_2; \\ y_{13} &= \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) - \sin(q_2)(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2); \\ y_{14} &= g_c \cos(q_1); \\ y_{15} &= g_c \cos(q_1); \\ y_{16} &= g_c \cos(q_1 + q_2); \end{aligned} \quad (101)$$

$$\begin{aligned} y_{21} &= 0; \\ y_{22} &= \ddot{q}_1 + \ddot{q}_2; \\ y_{23} &= \cos(q_2)\ddot{q}_1 + \sin(q_2)(\dot{q}_1^2); \\ y_{24} &= 0; \end{aligned}$$

$$y_{25}=0 ;$$

$$y_{26}= g_c \cos(q_1+q_2).$$

$Y(q, \dot{q}, \ddot{q}_r)$  has the component

$$y_{11} = \ddot{q}_{r1} ;$$

$$y_{12} = \ddot{q}_{r1} + \ddot{q}_{r2} ;$$

$$y_{13} = \cos(q_2)(2\ddot{q}_{r1} + \ddot{q}_{r2}) - \sin(q_2)(\dot{q}_1\dot{q}_{r2} + \dot{q}_1\dot{q}_{r2} + \dot{q}_2\dot{q}_{r2});$$

$$y_{14}=g_c \cos(q_1);$$

$$y_{15}= g_c \cos(q_1) ;$$

$$y_{16}= g_c \cos(q_1+q_2)$$
(102)

$$y_{21}=0;$$

$$y_{22} = \ddot{q}_{r1} + \ddot{q}_{r2} ;$$

$$y_{23} = \cos(q_2)\ddot{q}_{r1} + \sin(q_2)(\dot{q}_1\dot{q}_{r1}) ;$$

$$y_{24}=0 ;$$

$$y_{25}=0 ;$$

$$y_{26}= g_c \cos(q_1+q_2).$$

For illustrated purposes, let us assume that the parameters of the unloaded manipulator are known and the chosen values of the link parameters are given by Table 1. Using these values in Table 1, the *i*th component of  $\pi$  obtained by means of Equation (99) are given in Table 2. These parametric values also show lower and unloaded robot parameters.

$m_1$	$m_2$	$l_1$	$l_2$	$l_{c1}$	$l_{c2}$	$I_1$	$I_2$
10	5	1	1	0.5	0.5	10/12	5/12

Table 1. Parameters of the unloaded arm (Spong, 1992)

$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$
8.33	1.67	2.5	5	5	2.5

Table 2.  $\pi_i$  for the unloaded arm (Spong, 1992)

If an unknown load carried by the robot is regarded as part of the second link, then the parameters  $m_2$ ,  $l_{c2}$ , and  $I_2$  will change  $m_2+\Delta m_2$ ,  $l_{c2}+\Delta l_{c2}$  and  $I_2+\Delta I_2$ , respectively. A controller will be designed that provides robustness in the intervals

$$0 \leq \Delta m_2 \leq 10 ; \quad 0 \leq \Delta l_{c2} \leq 0.5 ; \quad 0 \leq I_2 \leq \frac{15}{12} \quad (103)$$

$\pi_0$  is chosen as a vector of nominal parameters and it also has the loaded arm parameters and their upper bounds. The computed values for  $i$ th component of  $\pi_0$  are given in Table 3.

$\pi_{01}$	$\pi_{02}$	$\pi_{03}$	$\pi_{04}$	$\pi_{05}$	$\pi_{06}$
13.33	8.96	8.75	5	10	8.75

Table 3. Nominal parameter vector  $\pi_0$  (Spong, 1992)

With this choice of nominal parameter vector  $\pi_0$  and uncertainty range given by (103), it is an easy matter to calculate the uncertainty bound  $\rho$  as follows:

$$\|\tilde{\pi}\|^2 = \sum_{i=1}^6 (\pi_{i0} - \pi_i)^2 \leq 181.26 \quad (104)$$

and thus  $\rho = \sqrt{181.26} = 13.46$ . Since extended algorithm (20) is used, the uncertainty bounds for each parameter separately are shown in Table 4. The uncertainty bounds  $\rho_i$  in Table 4 are simply the difference between values given in Table 3 and Table 2 and that the value of  $\rho$  is the Euclidean norm of the vector with components  $\rho_i$  (Spong, 1992).

$\rho_1$	$\rho_2$	$\rho_3$	$\rho_4$	$\rho_5$	$\rho_6$
5	7.29	6.25	0	5	6.25

Table 4. Uncertainty bound (Spong, 1992)

## 7. Conclusion

In the studies (Burkan, 2002; Uzmay & Burkan 2002, Burkan & Uzmay 2003 a), it is very difficult to use different variable functions for other derivation, and derivation of parameter and bound estimation laws are also not simple. However, in the recent studies (Burkan, 2005; Burkan & Uzmay 2006), first of all, a new method is developed in order to derive new parameter and bound estimation laws based on the Lyapunov function that guarantees stability of the uncertain system and the studies (Burkan, 2002; Uzmay&Burkan, 2002; Burkan&Uzmay, 2003a) provides basis of this study. In this new method, deriva-

tion of the parameter and bound estimation laws are simplified and it is not only possible to derive a single parameter and bound estimation laws, but also it is possible to derive various parameters and bound estimation laws using variable functions.

Parameters and bound estimation laws can be derived depending the variable function  $\Gamma$ , and if another appropriate variable function  $\Gamma$  is chosen, it will be possible to derive other adaptive-robust control laws. In derivation, other integration techniques are also possible to use in derivation for the new parameter and bound estimation laws.

$\hat{\pi}$  and  $\hat{\rho}(t)$  are error-derived estimation rules act as a compensators, that is, estimates the most appropriate parameters and upper bounding function to reduce tracking error. The aim of this approach is to solve for finding a control law that ensures limited tracking error, and not to determine the actual parameters and upper bounding function.  $\hat{\pi}$  is considered as an adaptive compensator,  $\hat{\rho}(t)$  is implemented as a robust controller and both of them are employed during the control process. This has the advantages that the employed adaptive controller increases the learning, while the employed robust controller offers the ability to reject disturbance and ensures desired transient behaviour. This improvement is achieved by computation of the upper bounding function.

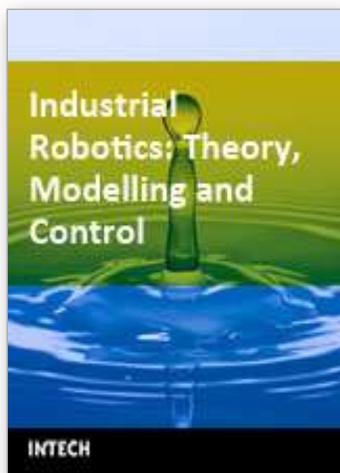
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This book covers a wide range of topics relating to advanced industrial robotics, sensors and automation technologies. Although being highly technical and complex in nature, the papers presented in this book represent some of the latest cutting edge technologies and advancements in industrial robotics technology. This book covers topics such as networking, properties of manipulators, forward and inverse robot arm kinematics, motion path-planning, machine vision and many other practical topics too numerous to list here. The authors and editor of this book wish to inspire people, especially young ones, to get involved with robotic and mechatronic engineering technology and to develop new and exciting practical applications, perhaps using the ideas and concepts presented herein.

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