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# Complex Digital Signal Processing in Telecommunications

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## 1. Introduction

### 1.1 Complex DSP versus real DSP

Digital Signal Processing (DSP) is a vital tool for scientists and engineers, as it is of fundamental importance in many areas of engineering practice and scientific research.

The “alphabet” of DSP is mathematics and although most practical DSP problems can be solved by using real number mathematics, there are many others which can only be satisfactorily resolved or adequately described by means of complex numbers.

If real number mathematics is the language of *real* DSP, then complex number mathematics is the language of *complex* DSP. In the same way that real numbers are a part of complex numbers in mathematics, *real* DSP can be regarded as a part of *complex* DSP (Smith, 1999).

Complex mathematics manipulates complex numbers - the representation of two variables as a single number - and it may appear that *complex* DSP has no obvious connection with our everyday experience, especially since many DSP problems are explained mainly by means of real number mathematics. Nonetheless, some DSP techniques are based on complex mathematics, such as Fast Fourier Transform (FFT), z-transform, representation of periodical signals and linear systems, etc. However, the imaginary part of complex transformations is usually ignored or regarded as zero due to the inability to provide a readily comprehensible physical explanation.

One well-known practical approach to the representation of an engineering problem by means of complex numbers can be referred to as the *assembling approach*: the real and imaginary parts of a complex number are real variables and individually can represent two real physical parameters. Complex math techniques are used to process this complex entity once it is assembled. The real and imaginary parts of the resulting complex variable preserve the same real physical parameters. This approach is not universally-applicable and can only be used with problems and applications which conform to the requirements of complex math techniques. Making a complex number entirely mathematically equivalent to a substantial physical problem is the real essence of *complex* DSP. Like complex Fourier transforms, complex DSP transforms show the fundamental nature of *complex* DSP and such complex techniques often increase the power of basic DSP methods. The development and application of *complex* DSP are only just beginning to increase and for this reason some researchers have named it *theoretical* DSP.

It is evident that *complex* DSP is more complicated than *real* DSP. Complex DSP transforms are highly theoretical and mathematical; to use them efficiently and professionally requires a large amount of mathematics study and practical experience.

Complex math makes the mathematical expressions used in DSP more compact and solves the problems which real math cannot deal with. Complex DSP techniques can complement our understanding of how physical systems perform but to achieve this, we are faced with the necessity of dealing with extensive sophisticated mathematics. For DSP professionals there comes a point at which they have no real choice since the study of complex number mathematics is the foundation of DSP.

### 1.2 Complex representation of signals and systems

All naturally-occurring signals are real; however in some signal processing applications it is convenient to represent a signal as a complex-valued function of an independent variable. For purely mathematical reasons, the concept of complex number representation is closely connected with many of the basics of electrical engineering theory, such as voltage, current, impedance, frequency response, transfer-function, Fourier and z-transforms, etc.

*Complex* DSP has many areas of application, one of the most important being modern telecommunications, which very often uses narrowband analytical signals; these are complex in nature (Martin, 2003). In this field, the complex representation of signals is very useful as it provides a simple interpretation and realization of complicated processing tasks, such as modulation, sampling or quantization.

It should be remembered that a complex number could be expressed in *rectangular*, *polar* and *exponential* forms:

$$a + jb = A(\cos \theta + j \sin \theta) = Ae^{j\theta} . \quad (1)$$

The third notation of the complex number in the equation (1) is referred to as *complex exponential* and is obtained after Euler's relation is applied. The exponential form of complex numbers is at the core of *complex* DSP and enables magnitude  $A$  and phase  $\theta$  components to be easily derived.

Complex numbers offer a compact representation of the most often-used waveforms in signal processing - *sine* and *cosine* waves (Proakis & Manolakis, 2006). The complex number representation of sinusoids is an elegant technique in signal and circuit analysis and synthesis, applicable when the rules of complex math techniques coincide with those of sine and cosine functions. Sinusoids are represented by complex numbers; these are then processed mathematically and the resulting complex numbers correspond to sinusoids, which match the way sine and cosine waves would perform if they were manipulated individually. The complex representation technique is possible only for sine and cosine waves of the same frequency, manipulated mathematically by linear systems.

The use of Euler's identity results in the class of complex exponential signals:

$$x(n) = A\alpha^n = |A|e^{j\phi}e^{(\sigma_0 + j\omega_0)n} = x_R(n) + jx_I(n) . \quad (2)$$

$\alpha = e^{(\sigma_0 + j\omega_0)}$  and  $A = |A|e^{j\phi}$  are complex numbers thus obtaining:

$$x_R(n) = |A|e^{\sigma_0 n} \cos(\omega_0 n + \phi); \quad x_I(n) = |A|e^{\sigma_0 n} \sin(\omega_0 n + \phi). \quad (3)$$

Clearly,  $x_R(n)$  and  $x_I(n)$  are real discrete-time sinusoidal signals whose amplitude  $|A|e^{\sigma_0 n}$  is constant ( $\sigma_0=0$ ), increasing ( $\sigma_0>0$ ) or decreasing ( $\sigma_0<0$ ) exponents (Fig. 1).

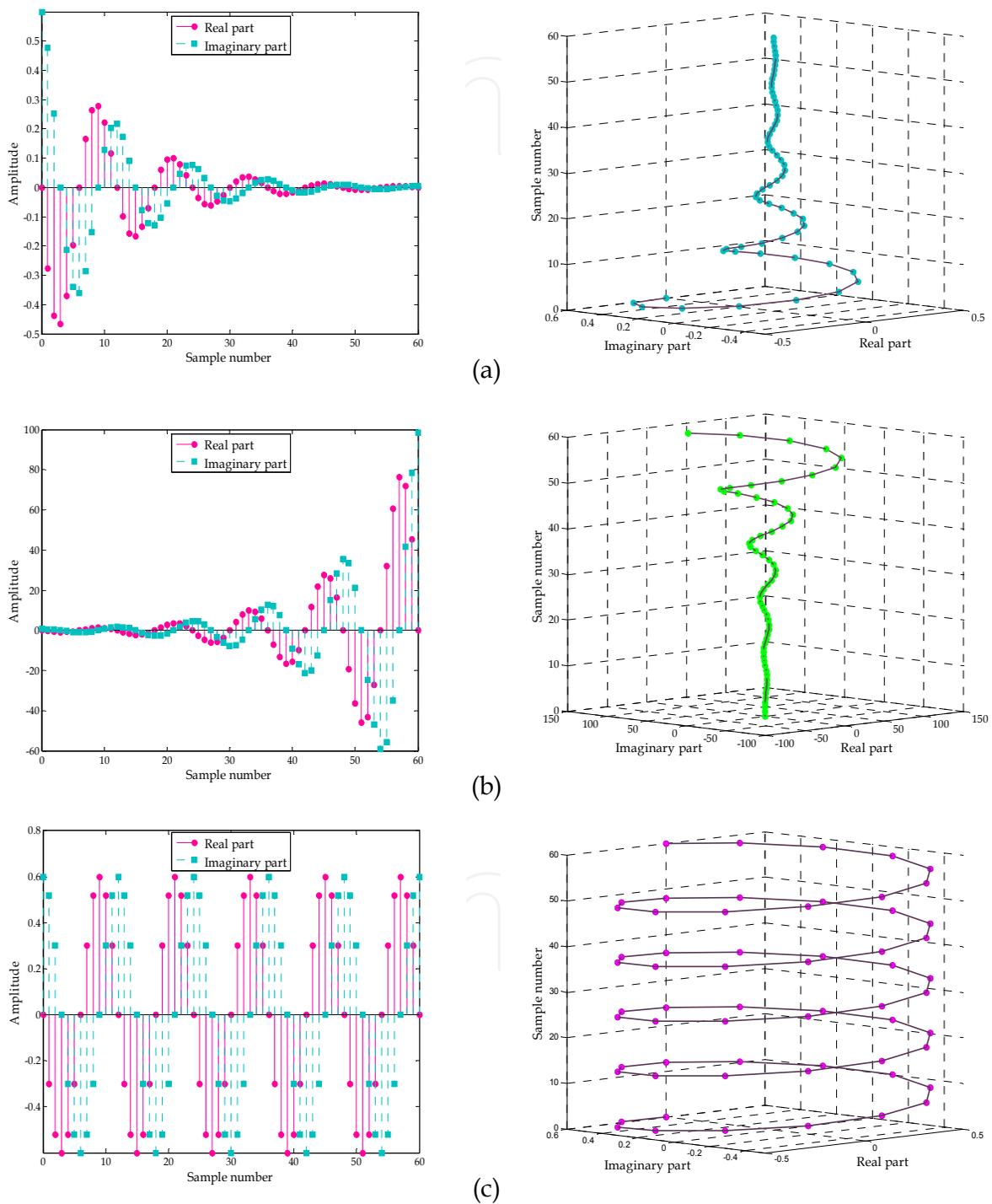


Fig. 1. Complex exponential signal  $x(n)$  and its real and imaginary components  $x_R(n)$  and  $x_I(n)$  for (a)  $\sigma_0=-0.085$ ; (b)  $\sigma_0=0.085$  and (c)  $\sigma_0=0$

The spectrum of a real discrete-time signal lies between  $-\omega_s/2$  and  $\omega_s/2$  ( $\omega_s$  is the sampling frequency in radians per sample), while the spectrum of a complex signal is twice as narrow and is located within the positive frequency range only.

Narrowband signals are of great use in telecommunications. The determination of a signal's attributes, such as frequency, envelope, amplitude and phase are of great importance for signal processing e.g. modulation, multiplexing, signal detection, frequency transformation, etc. These attributes are easier to quantify for narrowband signals than for wideband signals (Fig. 2). This makes narrowband signals much simpler to represent as complex signals.

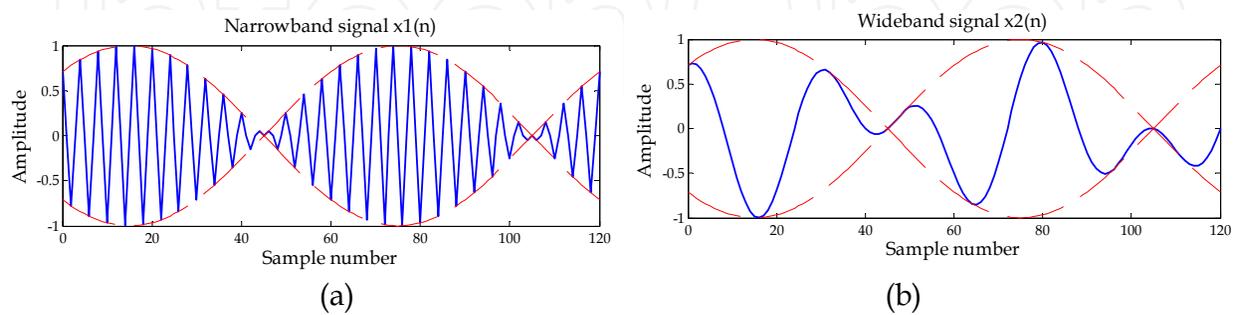


Fig. 2. Narrowband signal (a)  $x_1(n) = \sin(\pi/60n + \pi/4)\cos(\pi/2n)$ ;  
wideband signal (b)  $x_2(n) = \sin(\pi/60n + \pi/4)\cos(\pi/16n)$

Over the years different techniques of describing narrowband complex signals have been developed. These techniques differ from each other in the way the imaginary component is derived; the real component of the complex representation is the real signal itself.

Some authors (Fink, 1984) suggest that the imaginary part of a complex narrowband signal can be obtained from the first  $x'_R(n)$  and second  $x''_R(n)$  derivatives of the real signal:

$$x_I(n) = -x'_R(n) \sqrt{\frac{-x_R(n)}{x''_R(n)}}. \quad (4)$$

One disadvantage of the representation in equation (4) is that insignificant changes in the real signal  $x_R(n)$  can alter the imaginary part  $x_I(n)$  significantly; furthermore the second derivative can change its sign, thus removing the sense of the square root.

Another approach to deriving the imaginary component of a complex signal representation, applicable to harmonic signals, is as follows (Gallagher, 1968):

$$x_I(n) = \frac{-x_R(n)}{\omega_0}, \quad (5)$$

where  $\omega_0$  is the frequency of the real harmonic signal.

Analytical representation is another well-known approach used to obtain the imaginary part of a complex signal, named the *analytic* signal. An analytic complex signal is represented by its *inphase* (the real component) and *quadrature* (the imaginary component). The approach includes a low-frequency envelope modulation using a complex carrier signal – a complex exponent  $e^{j\omega_0 n}$  named *cisoid* (Crystal & Ehrman, 1968) or *complexoid* (Martin, 2003):

$$x_R(n) \otimes e^{j\omega_0 n} \Rightarrow x(n) = x_R(n) e^{j\omega_0 n} = x_R(n) [\cos \omega_0 n + j \sin \omega_0 n] = x_R(n) + jx_I(n). \quad (6)$$

In the frequency domain an analytic complex signal is:

$$X_C(e^{j\omega n}) = X_R(e^{j\omega n}) + jX_I(e^{j\omega n}). \quad (7)$$

The real signal and its Hilbert transform are respectively the real and imaginary parts of the analytic signal; these have the same amplitude and  $\pi/2$  phase-shift (Fig. 3).

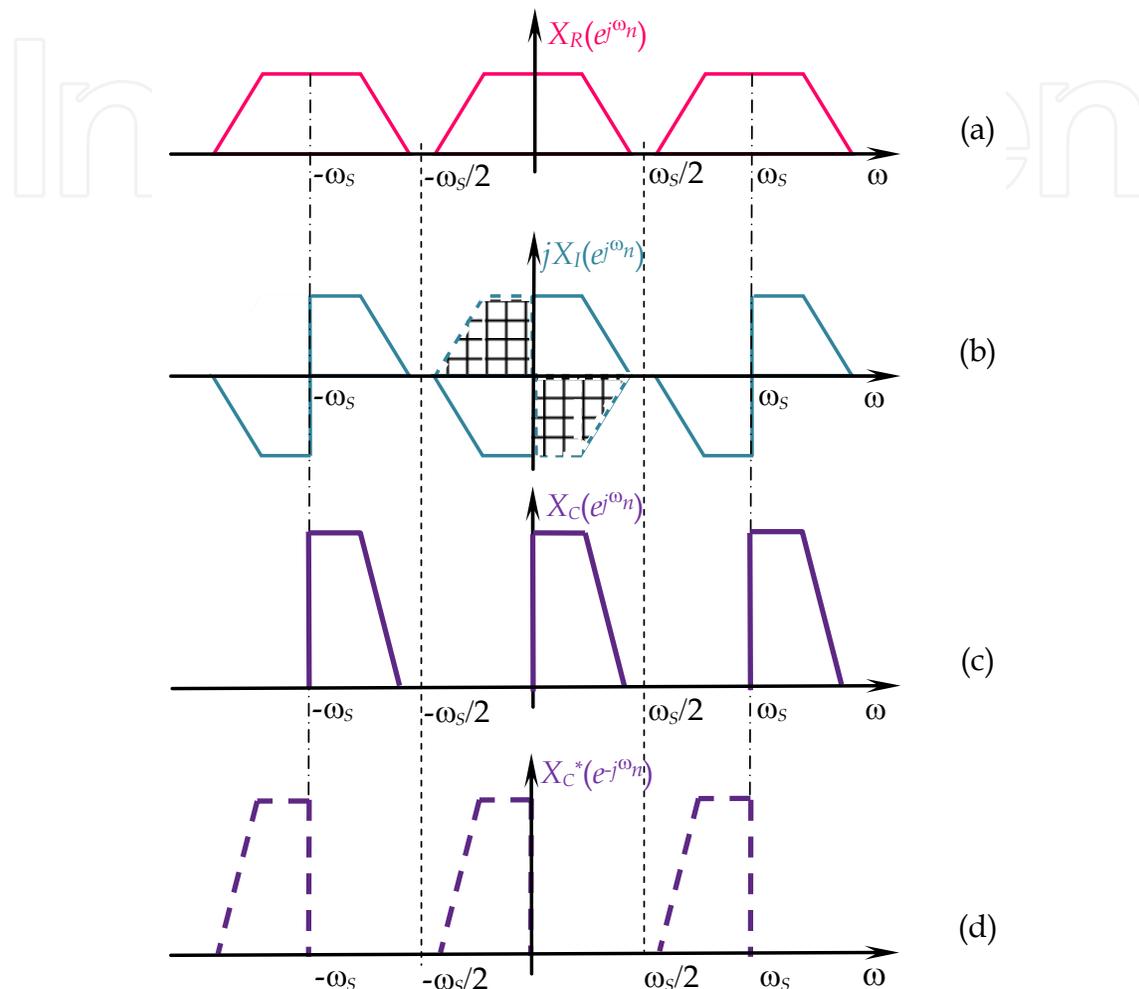


Fig. 3. Complex signal derivation using the Hilbert transformation

According to the Hilbert transformation, the components of the  $X_R(e^{j\omega n})$  spectrum are shifted by  $\pi/2$  for positive frequencies and by  $-\pi/2$  for negative frequencies, thus the pattern areas in Fig. 3b are obtained. The real signal  $X_R(e^{j\omega n})$  and the imaginary one  $X_I(e^{j\omega n})$  multiplied by  $j$  (square root of  $-1$ ), are identical for positive frequencies and  $-\pi/2$  phase shifted for negative frequencies - the solid blue line (Fig. 3b). The complex signal  $X_C(e^{j\omega n})$  occupies half of the real signal frequency band; its amplitude is the sum of the  $X_R(e^{j\omega n})$  and  $jX_I(e^{j\omega n})$  amplitudes (Fig. 3c). The spectrum of the complex conjugate analytic signal  $X_C^*(e^{-j\omega n})$  is depicted in Fig. 3d.

In the frequency domain the analytic complex signal, its complex conjugate signal, real and imaginary components are related as follows:

$$\begin{aligned} X_R(e^{j\omega n}) &= \frac{1}{2} \{ X(e^{j\omega n}) + X^*(e^{-j\omega n}) \} \\ jX_I(e^{j\omega n}) &= \frac{1}{2} \{ X(e^{j\omega n}) - X^*(e^{-j\omega n}) \} \\ X(e^{j\omega n}) &= \begin{cases} 2X_R(e^{j\omega n}) = 2jX_I(e^{j\omega n}), & 0 < \omega < \omega_s/2 \\ 0, & -\omega_s/2 \leq \omega < 0 \end{cases} \end{aligned} \quad (8)$$

Discrete-time complex signals are easily processed by digital complex circuits, whose transfer functions contain complex coefficients (Márquez, 2011).

An output complex signal  $Y_C(z)$  is the response of a complex system with transfer function  $H_C(z)$ , when complex signal  $X_C(z)$  is applied as an input. Being complex functions,  $X_C(z)$ ,  $Y_C(z)$  and  $H_C(z)$ , can be represented by their real and imaginary parts:

$$\begin{aligned} \underbrace{Y_C(z)}_{\downarrow} &= \underbrace{H_C(z)}_{\downarrow} \underbrace{X_C(z)}_{\downarrow} \\ [Y_R(z) + jY_I(z)] &= [H_R(z) + jH_I(z)] [X_R(z) + jX_I(z)] \end{aligned} \quad (9)$$

After mathematical operations are applied, the complex output signal and its real and imaginary parts become:

$$\begin{aligned} Y_C(z) &= [H_R(z) + jH_I(z)] [X_R(z) + jX_I(z)] = \\ &= \underbrace{[H_R(z)X_R(z) - H_I(z)X_I(z)]}_{\downarrow Y_R(z)} + j \underbrace{[H_I(z)X_R(z) + H_R(z)X_I(z)]}_{\downarrow Y_I(z)} \end{aligned} \quad (10)$$

According to equation (10), the block-diagram of a complex system will be as shown in Fig. 4.

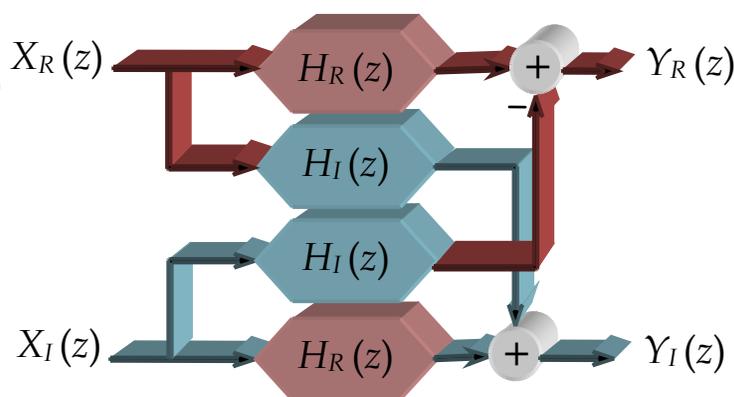


Fig. 4. Block-diagram of a complex system

### 1.3 Complex digital processing techniques - complex Fourier transforms

Digital systems and signals can be represented in three domains – time domain, z-domain and frequency domain. To cross from one domain to another, the Fourier and z-transforms are employed (Fig. 5). Both transforms are fundamental building-blocks of signal processing theory and exist in two formats - *forward* and *inverse* (Smith, 1999).



Fig. 5. Relationships between frequency, time, and z- domains

The Fourier transforms group contains four families, which differ from one another in the type of time-domain signal which they process - *periodic* or *aperiodic* and *discrete* or *continuous*. Discrete Fourier Transform (DFT) deals with *discrete periodic* signals, Discrete Time Fourier Transform (DTFT) with *discrete aperiodic* signals, and Fourier Series and Fourier Transform with *periodic* and *aperiodic continuous* signals respectively. In addition to having forward and inverse versions, each of these four Fourier families exists in two forms - *real* and *complex*, depending on whether real or complex number math is used. All four Fourier transform families decompose signals into sine and cosine waves; when these are expressed by complex number equations, using Euler's identity, the *complex* versions of the Fourier transforms are introduced.

DFT is the most often-used Fourier transform in DSP. The DFT family is a basic mathematical tool in various processing techniques performed in the frequency domain, for instance frequency analysis of digital systems and spectral representation of discrete signals. In this chapter, the focus is on *complex* DFT. This is more sophisticated and wide-ranging than real DFT, but is based on the more complicated complex number math. However, numerous digital signal processing techniques, such as convolution, modulation, compression, aliasing, etc. can be better described and appreciated via this extended math. (Sklar, 2001)

*Complex* DFT equations are shown in Table 1. The *forward complex* DFT equation is also called *analysis* equation. This calculates the frequency domain values of the discrete periodic signal, whereas the *inverse (synthesis)* equation computes the values in the time domain.

<b>Complex Discrete Fourier Transform</b>	
<b>Forward (analysis) equation</b>  $X(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cos \frac{2\pi kn}{N} - j \sin \frac{2\pi kn}{N}$	<b>Inverse (synthesis) equation</b>  $x(n) = \sum_{k=0}^{N-1} \text{Re } X(k) \cos \frac{2\pi kn}{N} + j \sin \frac{2\pi kn}{N} - \sum_{k=0}^{N-1} \text{Im } X(k) \sin \frac{2\pi kn}{N} - j \cos \frac{2\pi kn}{N}$

Table 1. Complex DFT transforms in rectangular form

The time domain signal  $x(n)$  is a complex discrete periodic signal; only an  $N$ -point unique discrete sequence from this signal, situated in a single time-interval ( $0 \leq n < N$ ,  $-N/2 \leq n < N/2$ , etc.) is

considered. The forward equation multiplies the periodic time domain number series from  $x(0)$  to  $x(N-1)$  by a sinusoid and sums the results over the complete time-period.

The frequency domain signal  $X(k)$  is an  $N$ -point complex periodic signal in a single frequency interval, such as  $[0 \div 0.5\omega_s]$ ,  $[-0.5\omega_s \div 0]$ ,  $[-0.25\omega_s \div 0.25\omega_s]$ , etc. (the sampling frequency  $\omega_s$  is often used in its normalized value). The inverse equation employs all the  $N$  points in the frequency domain to calculate a particular discrete value of the time domain signal. It is clear that *complex* DFT works with finite-length data.

Both the time domain  $x(n)$  and the frequency domain  $X(k)$  signals are complex numbers, i.e. *complex* DFT also recognizes negative time and negative frequencies. Complex mathematics accommodates these concepts, although imaginary time and frequency have only a theoretical existence so far. *Complex* DFT is a symmetrical and mathematically comprehensive processing technology because it doesn't discriminate between negative and positive frequencies.

Fig. 6 shows how the forward *complex* DFT algorithm works in the case of a complex time-domain signal.  $x_R(n)$  is a real time domain signal whose frequency spectrum has an even real part and an odd imaginary part; conversely, the frequency spectrum of the imaginary part of the time domain signal  $x_I(n)$  has an odd real part and an even imaginary part. However, as can be seen in Fig. 6, the actual frequency spectrum is the sum of the two individually-calculated spectra. In reality, these two time domain signals are processed simultaneously, which is the whole point of the Fast Fourier Transform (FFT) algorithm.

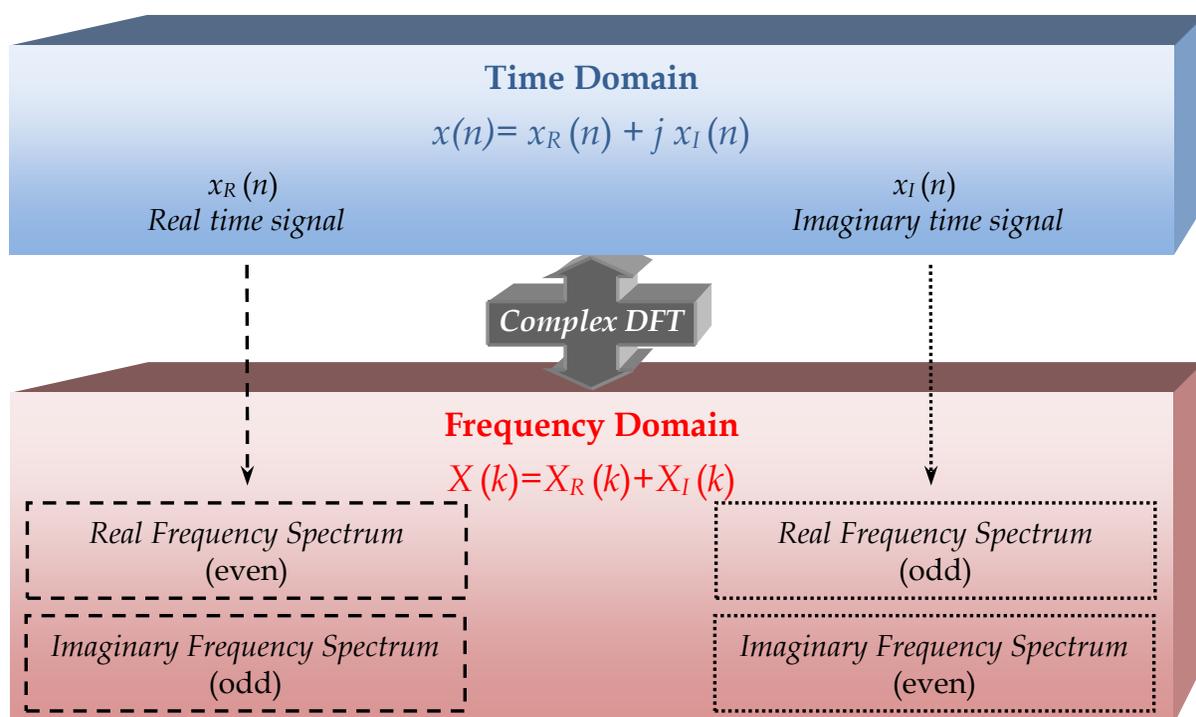


Fig. 6. Forward *complex* DFT algorithm

The imaginary part of the time-domain complex signal can be omitted and the time domain then becomes totally real, as is assumed in the numerical example shown in Fig. 7. A real sinusoidal signal with amplitude  $M$ , represented in a complex form, contains a positive  $\omega_0$  and a negative frequency  $-\omega_0$ . The complex spectrum  $X(k)$  describes the signal in the

frequency domain. The frequency range of its real,  $\text{Re } X(k)$ , and imaginary part,  $\text{Im } X(k)$ , comprises both positive and negative frequencies simultaneously. Since the considered time domain signal is real,  $\text{Re } X(k)$  is even (the spectral values  $A$  and  $B$  have the same sign), while the imaginary part  $\text{Im } X(k)$  is odd ( $C$  is negative,  $D$  is positive).

The amplitude of each of the four spectral peaks is  $M/2$ , which is half the amplitude of the time domain signal. The single frequency interval under consideration  $[-1/4\omega_s \div 1/4\omega_s]$  ( $[-0.5 \div 0.5]$  when normalized frequency is used) is symmetric with respect to a frequency of zero. The real frequency spectrum  $\text{Re } X(k)$  is used to reconstruct a cosine time domain signal, whilst the imaginary spectrum  $\text{Im } X(k)$  results in a negative sine wave, both with amplitude  $M$  in accordance with the complex analysis equation (Table 1). In a way analogous to the example shown in Fig. 7, a complex frequency spectrum can also be derived.

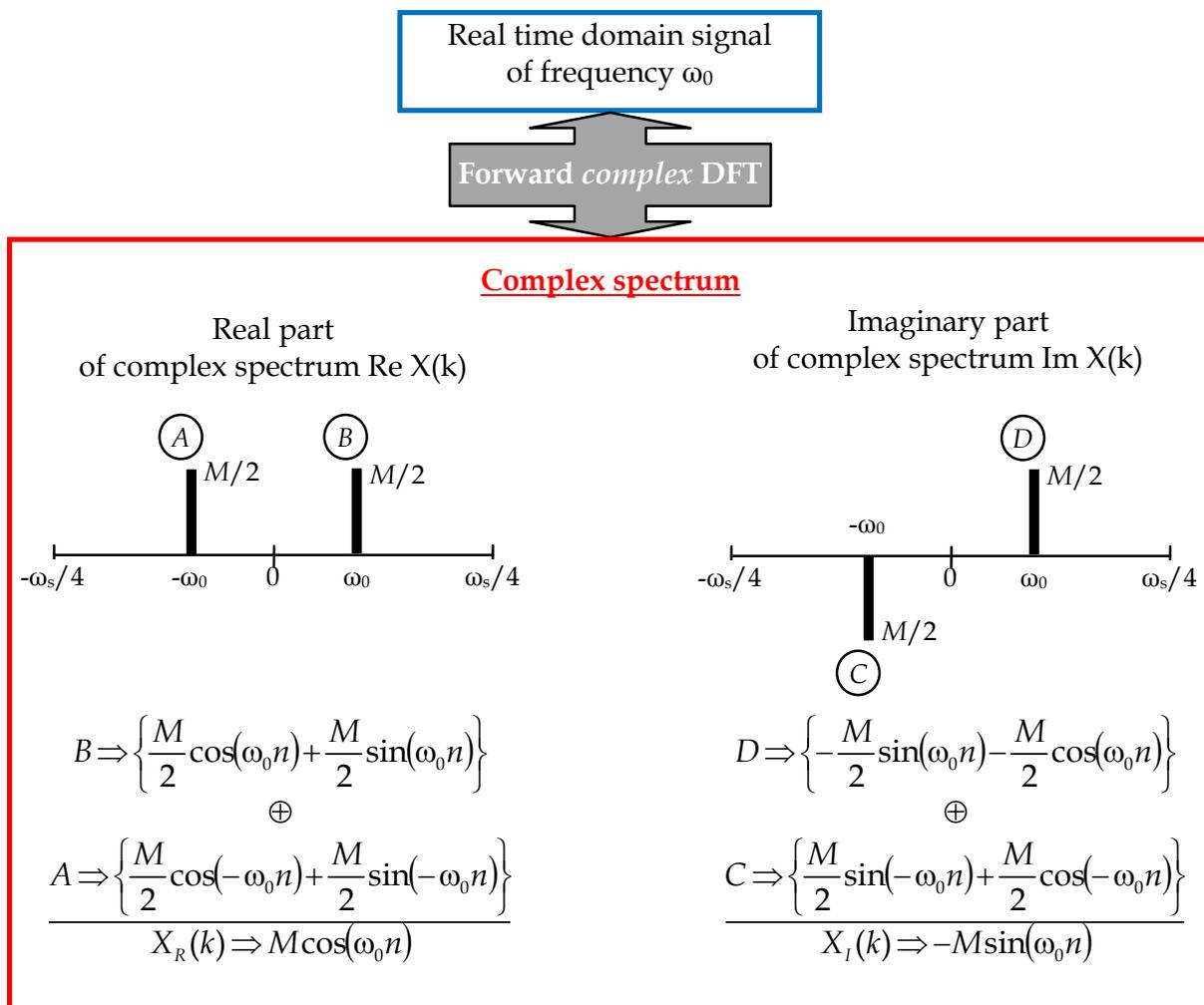


Fig. 7. Inverse complex DFT - reconstruction of a real time domain signal

Why is complex DFT used since it involves intricate complex number math?

Complex DFT has persuasive advantages over real DFT and is considered to be the more comprehensive version. Real DFT is mathematically simpler and offers practical solutions to real world problems; by extension, negative frequencies are disregarded. Negative frequencies are always encountered in conjunction with complex numbers.

A *real* DFT spectrum can be represented in a complex form. Forward *real* DFT results in cosine and sine wave terms, which then form respectively the real and imaginary parts of a complex number sequence. This substitution has the advantage of using powerful complex number math, but this is not true *complex* DFT. Despite the spectrum being in a complex form, the DFT remains *real* and  $j$  is not an integral part of the *complex representation of real* DFT.

Another mathematical inconvenience of *real* DFT is the absence of symmetry between analysis and synthesis equations, which is due to the exclusion of negative frequencies. In order to achieve a perfect reconstruction of the time domain signal, the first and last samples of the *real* DFT frequency spectrum, relating to zero frequency and Nyquist's frequency respectively, must have a scaling factor of  $1/N$  applied to them rather than the  $2/N$  used for the rest of the samples.

In contrast, *complex* DFT doesn't require a scaling factor of 2 as each value in the time domain corresponds to two spectral values located in a positive and a negative frequency; each one contributing half the time domain waveform amplitude, as shown in Fig. 7. The factor of  $1/N$  is applied equally to all samples in the frequency domain. Taking the negative frequencies into account, *complex* DFT achieves a mathematically-favoured symmetry between *forward* and *inverse* equations, i.e. between time and frequency domains.

*Complex* DFT overcomes the theoretical imperfections of *real* DFT in a manner helpful to other basic DSP transforms, such as forward and inverse z-transforms. A bright future is confidently predicted for *complex* DSP in general and the *complex* versions of Fourier transforms in particular.

## 2. Complex DSP – some applications in telecommunications

DSP is making a significant contribution to progress in many diverse areas of human endeavour – science, industry, communications, health care, security and safety, commercial business, space technologies etc.

Based on powerful scientific mathematical principles, *complex* DSP has overlapping boundaries with the theory of, and is needed for many applications in, telecommunications. This chapter presents a short exploration of precisely this common area.

Modern telecommunications very often uses narrowband signals, such as NBI (Narrowband Interference), RFI (Radio Frequency Interference), etc. These signals are complex by nature and hence it is natural for *complex* DSP techniques to be used to process them (Ovtcharov et al, 2009), (Nikolova et al, 2010).

Telecommunication systems very commonly require processing to occur in real time, adaptive complex filtering being amongst the most frequently-used *complex* DSP techniques. When multiple communication channels are to be manipulated simultaneously, parallel processing systems are indicated (Nikolova et al, 2006), (Iliev et al, 2009).

An efficient Adaptive Complex Filter Bank (ACFB) scheme is presented here, together with a short exploration of its application for the mitigation of narrowband interference signals in MIMO (Multiple-Input Multiple-Output) communication systems.

### 2.1 Adaptive complex filtering

As pointed out previously, adaptive complex filtering is a basic and very commonly-applied DSP technique. An adaptive complex system consists of two basic building blocks:

the variable complex filter and the adaptive algorithm. Fig. 8 shows such a system based on a variable complex filter section designated LS1 (Low Sensitivity). The variable complex LS1 filter changes the central frequency and bandwidth independently (Iliev et al, 2002), (Iliev et al, 2006). The central frequency can be tuned by trimming the coefficient  $\theta$ , whereas the single coefficient  $\beta$  adjusts the bandwidth. The LS1 variable complex filter has two very important advantages: firstly, an extremely low passband sensitivity, which offers resistance to quantization effects and secondly, independent control of both central frequency and bandwidth over a wide frequency range.

The adaptive complex system (Fig.8) has a complex input  $x(n)=x_R(n)+jx_I(n)$  and provides both band-pass (BP) and band-stop (BS) complex filtering. The real and imaginary parts of the BP filter are respectively  $y_R(n)$  and  $y_I(n)$ , whilst those of the BS filter are  $e_R(n)$  and  $e_I(n)$ . The cost-function is the power of the BP/BS filter's output signal.

The filter coefficient  $\theta$ , responsible for the central frequency, is updated by applying an adaptive algorithm, for example LMS (Least Mean Square):

$$\theta(n+1) = \theta(n) + \mu \text{Re}[e(n)y'^*(n)]. \quad (11)$$

The step size  $\mu$  controls the speed of convergence, (\*) denotes complex-conjugate,  $y'(n)$  is the derivative of complex BP filter output  $y(n)$  with respect to the coefficient, which is subject to adaptation.

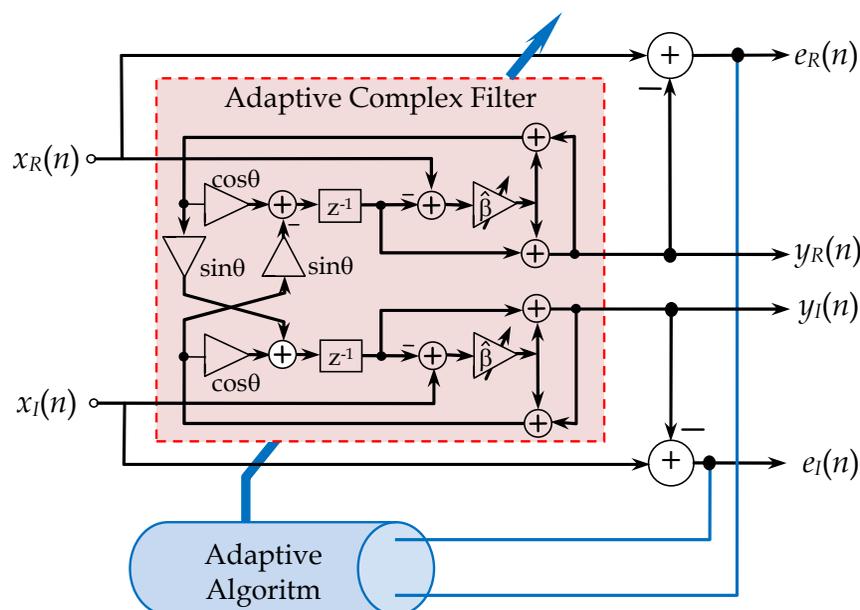


Fig. 8. Block-diagram of an LS1-based adaptive complex system

In order to ensure the stability of the adaptive algorithm, the range of the step size  $\mu$  should be set according to (Douglas, 1999):

$$0 < \mu < \frac{P}{N\sigma^2}. \quad (11)$$

where  $N$  is the filter order,  $\sigma^2$  is the power of the signal  $y'(n)$  and  $P$  is a constant which depends on the statistical characteristics of the input signal. In most practical situations,  $P$  is approximately equal to 0.1.

The very low sensitivity of the variable complex LS1 filter section ensures the general efficiency of the adaptation and a high tuning accuracy, even with severely quantized multiplier coefficients.

This approach can easily be extended to the adaptive complex filter bank synthesis in parallel complex signal processing.

In (Nikolova et al, 2002) a narrowband ACFB is designed for the detection of multiple complex sinusoids. The filter bank, composed of three variable complex filter sections, is aimed at detecting multiple complex signals (Fig. 9).

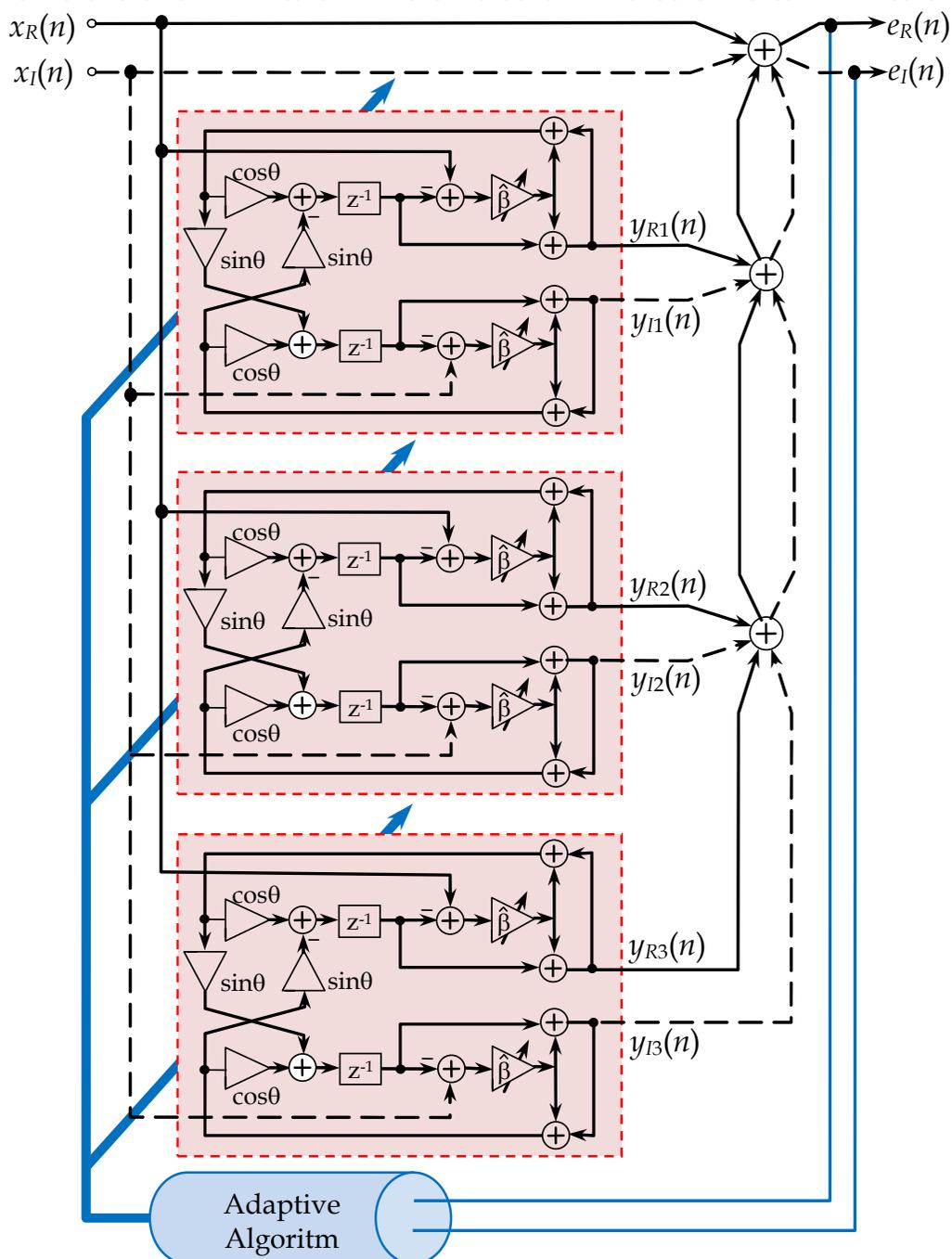


Fig. 9. Block-diagram of an adaptive complex filter bank system

The experiments are carried out with an input signal composed of three complex sine-signals of different frequencies, mixed with white noise. Fig. 10 displays learning curves for the coefficients  $\theta_1$ ,  $\theta_2$  and  $\theta_3$ . The ACFB shows the high efficacy of the parallel filtering process. The main advantages of both the adaptive filter structure and the ACFB lie in their low computational complexity and rapid convergence of adaptation.

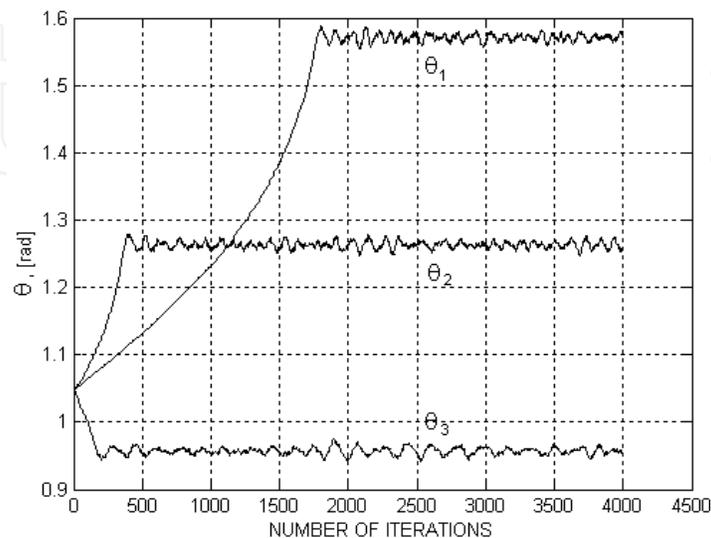


Fig. 10. Learning curves of an ACFB consisting of three complex LS1-sections

## 2.2 Narrowband interference suppression for MIMO systems using adaptive complex filtering

The sub-sections which follow examine the problem of narrowband interference in two particular MIMO telecommunication systems. Different NBI suppression methods are observed and experimentally compared to the *complex* DSP technique using adaptive complex filtering in the frequency domain.

### 2.2.1 NBI Suppression in UWB MIMO systems

Ultrawideband (UWB) systems show excellent potential benefits when used in the design of high-speed digital wireless home networks. Depending on how the available bandwidth of the system is used, UWB can be divided into two groups: *single-band* and *multi-band* (MB). Conventional UWB technology is based on *single-band* systems and employs carrier-free communications. It is implemented by directly modulating information into a sequence of impulse-like waveforms; support for multiple users is by means of time-hopping or direct sequence spreading approaches.

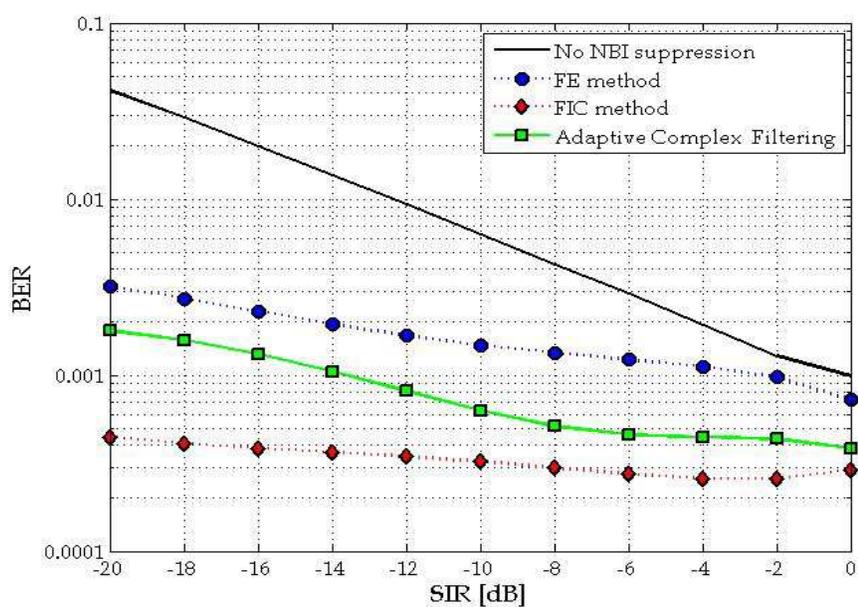
The UWB frequency band of *multi-band* UWB systems is divided into several sub-bands. By interleaving the symbols across sub-bands, multi-band UWB can maintain the power of the transmission as though a wide bandwidth were being utilized. The advantage of the multi-band approach is that it allows information to be processed over a much smaller bandwidth, thereby reducing overall design complexity as well as improving spectral flexibility and worldwide adherence to the relevant standards. The constantly-increasing demand for higher data transmission rates can be satisfied by exploiting both multipath- and spatial-diversity, using MIMO together with the appropriate modulation and coding techniques

(Iliev et al, 2009). The multipath energy can be captured efficiently when the OFDM (Orthogonal Frequency-Division Multiplexing) technique is used to modulate the information in each sub-band. Unlike more traditional OFDM systems, the MB-OFDM symbols are interleaved over different sub-bands across both time and frequency. Multiple access of multi-band UWB is enabled by the use of suitably-designed frequency-hopping sequences over the set of sub-bands.

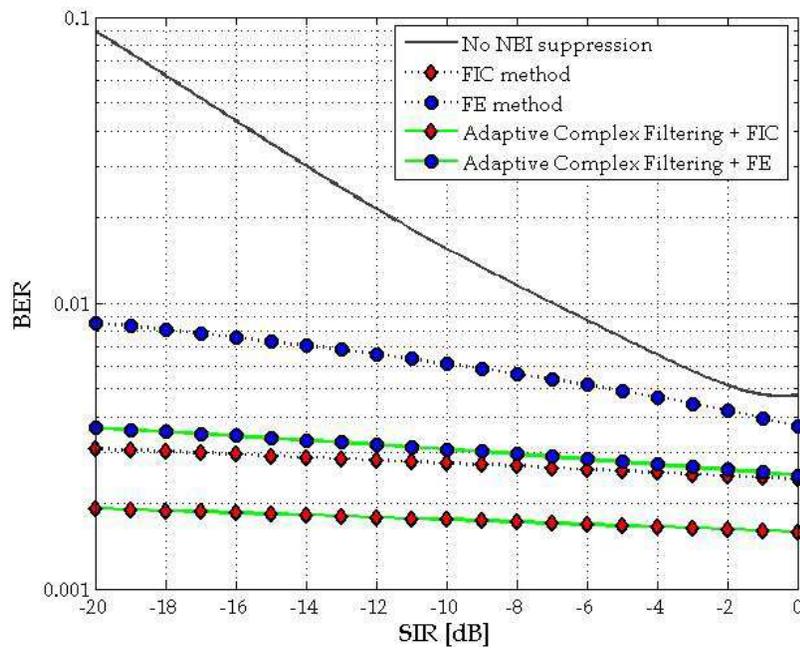
In contrast to conventional MIMO OFDM systems, the performance of MIMO MB-OFDM UWB systems does not depend on the temporal correlation of the propagation channel. However, due to their relatively low transmission power, such systems are very sensitive to NBI. Because of the spectral leakage effect caused by DFT demodulation at the OFDM receiver, many subcarriers near the interference frequency suffer from serious Signal-to-Interference Ratio (SIR) degradation, which can adversely affect or even block communications (Giorgetti et al, 2005).

In comparison with the wideband information signal, the interference occupies a much narrower frequency band but has a higher-power spectral density (Park et al, 2004). On the other hand, the wideband signal usually has autocorrelation properties quite similar to those of AWGN (Adaptive Wide Gaussian Noise), so filtering in the frequency domain is possible. The *complex* DSP technique for suppressing NBI by the use of adaptive complex narrowband filtering, which is an optimal solution offering a good balance between computational complexity and interference suppression efficiency, is put forward in (Iliev et al, 2010). The method is compared experimentally with two other often-used algorithms Frequency Excision (FE) (Juang et al, 2004) and Frequency Identification and Cancellation (FIC) (Baccarelli et al, 2002) for the identification and suppression of complex NBI in different types of IEEE UWB channels.

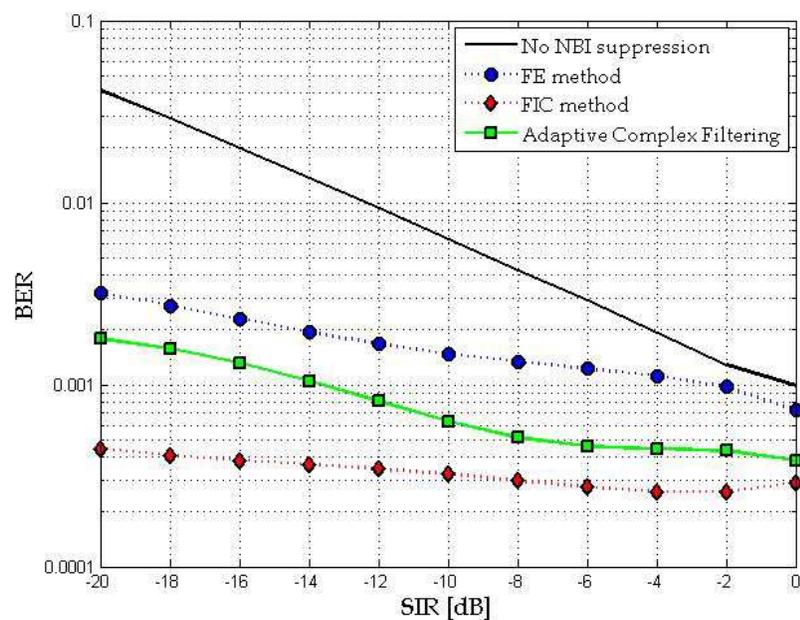
A number of simulations relative to complex baseband presentation are performed, estimating the Bit Error Ratio (BER) as a function of the SIR for the CM3 IEEE UWB channel (Molish & Foerster, 2003) and some experimental results are shown in Fig. 10.



(a)



(b)



(c)

Fig. 10. BER as a function of SIR for the CM3 channel (a) complex NBI; (b) multi-tone NBI; (c) QPSK modulated NBI

The channel is subject to strong fading and, for the purposes of the experiments, background AWGN is additionally applied, so that the Signal-to-AWGN ratio at the input of the OFDM receiver is 20 dB. The SIR is varied from -20 dB to 0 dB. It can be seen (Fig. 10a) that for high NBI, i.e. where the SIR is less than 0 dB, all methods lead to a significant improvement in performance. The adaptive complex filtering scheme gives better performance than the FE method. This could be explained by the NBI spectral leakage effect caused by DFT demodulation at the OFDM receiver, when many sub-carriers near the

interference frequency suffer degradation. Thus, filtering out the NBI before demodulation is better than frequency excision. The FIC algorithm achieves the best result because there is no spectrum leakage, as happens with frequency excision, and there is no amplitude and phase distortion as seen in the case of adaptive complex filtering.

It should be noted that the adaptive filtering scheme and frequency cancellation scheme lead to a degradation in the overall performance when  $SIR > 0$ . This is due either to the amplitude and phase distortion of the adaptive notch filter or to a wrong estimation of NBI parameters during the identification. The degradation can be reduced by the implementation of a higher-order notch filter or by using more sophisticated identification algorithms. The degradation effect can be avoided by simply switching off the filtering when  $SIR > 0$ . Such a scheme is easily realizable, as the amplitude of the NBI can be monitored at the BP output of the filter (Fig. 8).

In Fig. 10b, the results of applying a combination of methods are presented. A multi-tone NBI (an interfering signal composed of five sine-waves) is added to the OFDM signal. One of the NBI tones is 10 dB stronger than the others. The NBI filter is adapted to track the strongest NBI tone, thus preventing the loss of resolution and *Automatic Gain Control* (AGC) saturation. It can be seen that the combination of FE and Adaptive Complex Filtering improves the performance, and the combination of FIC with Adaptive Complex Filtering is even better.

Fig. 10c shows BER as a function of SIR for the CM3 channel when QPSK modulation is used, the NBI being modelled as a complex sine wave. It is evident that the relative performance of the different NBI suppression methods is similar to the one in Fig. 10a but the BER is higher due to the fact that NBI is QPSK modulated.

The experimental results show that the FIC method achieves the highest performance. On the other hand, the extremely high computational complexity limits its application in terms of hardware resources. In this respect, Adaptive Complex Filtering turns out to be the optimal NBI suppression scheme, as it offers very good performance and reasonable complexity. The FE method shows relatively good results and its main advantage is its computational efficiency. Therefore the *complex* DSP filtering technique offers a good compromise between outstanding NBI suppression efficiency and computational complexity.

### 2.2.2 RFI mitigation in GDSL MIMO systems

The Gigabit Digital Subscriber Line (GDSL) system is a cost-effective solution for existing telecommunication networks as it makes use of the existing copper wires in the last distribution area segment. Crosstalk, which is usually a problem in existing DSL systems, actually becomes an enhancement in GDSL, as it allows the transmission rate to be extended to its true limits (Lee et al, 2007). A symmetric data transmission rate in excess of 1 Gbps using a set of 2 to 4 copper twisted pairs over a 300 m cable length is achievable using vectored MIMO technology, and considerably faster speeds can be achieved over shorter distances.

In order to maximize the amount of information handled by a MIMO cable channel via the cable crosstalk phenomenon, most GDSL systems employ different types of precoding algorithms, such as Orthogonal Space-Time Precoding (OSTP), Orthogonal Space-Frequency Precoding (OSFP), Optimal Linear Precoding (OLP), etc. (Perez-Cruz et al, 2008). GDSL systems use the leading modulation technology, Discrete Multi-Tone (DMT), also known as OFDM, and are very sensitive to RFI. The presence of strong RFI causes nonlinear

distortion in AGC and Analogue-to-Digital Converter (ADC) functional blocks, as well as spectral leakage in the DFT process. Many DMT tones, if they are located close to the interference frequency, will suffer serious SNR degradation. Therefore, RFI suppression is of primary importance for all types of DSL communications, including GDSL.

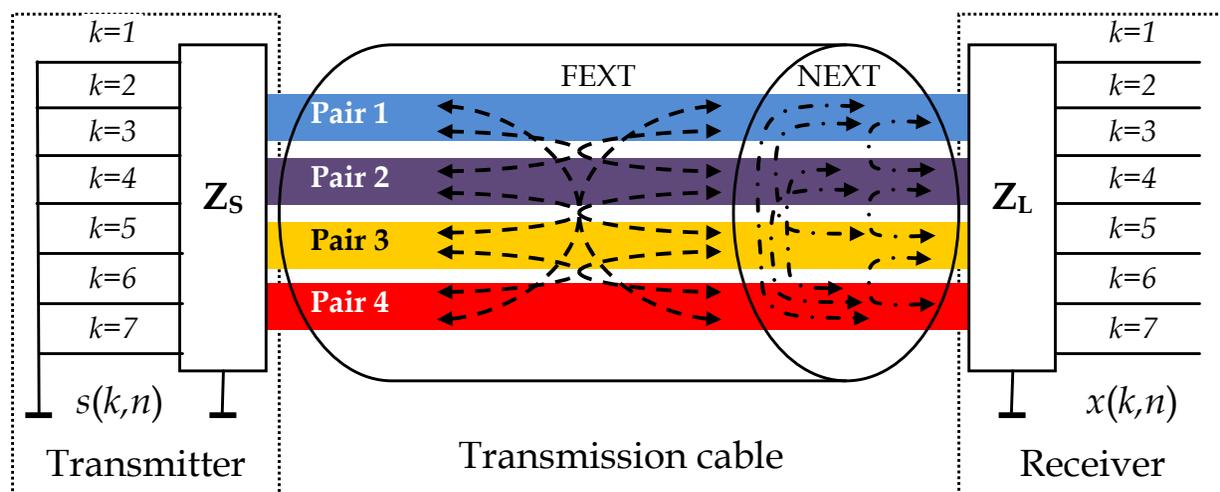


Fig. 11. MIMO GDSL Common Mode system model

The present section considers a MIMO GDSL Common Mode system, with a typical MIMO DMT receiver, using vectored MIMO DSL technology (Fig. 11) (Poulikov et al, 2009).

To achieve the outstanding data-rate of 1 Gbps, the GDSL system requires both source and load to be excited in Common Mode (Starr et al, 2003). The model of a MIMO GDSL channel depicted in Fig. 11 includes 8 wires that create  $k=7$  channels all with the 0 wire as reference.  $Z_S$  and  $Z_L$  denote the source and load impedance matrices respectively;  $s(k,n)$  is the  $n$ -th sample of  $k$ -th transmitted output, whilst  $x(k,n)$  is the  $n$ -th sample of  $k$ -th received input. Wide-scale frequency variations together with standard statistics determined from measured actual Far End Crosstalk (FEXT) and Near End Crosstalk (NEXT) power transfer functions are also considered and OLP, 64-QAM demodulation and Error Correction Decoding are implemented (ITU-T Recommendation G.993.2, 2006), (ITU-T Recommendation G.996.1, 2006). As well as OLP, three major types of general RFI mitigation approaches are proposed.

The *first* one concerns various FE methods, whereby the affected frequency bins of the DMT symbol are excised or their use avoided. The frequency excision is applied to the MIMO GDSL signal with a complex RFI at each input of the receiver. The signal is converted into the frequency domain by applying an FFT at each input, oversampled by 8, and the noise peaks in the spectra are limited to the pre-determined threshold. After that, the signal is converted back to the time domain and applied to the input of the corresponding DMT demodulator. The higher the order of the FFT, the more precise the frequency excision achieved.

The *second* approach is related to the so-called Cancellation Methods, aimed at the elimination or mitigation of the effect of the RFI on the received DMT signal. In most cases, when the SIR is less than 0 dB, the degradation in a MIMO DSL receiver is beyond the reach of the FE method. Thus, mitigation techniques employing Cancellation Methods, one of which is the RFI FIC method, are recommended as a promising alternative (Juang et

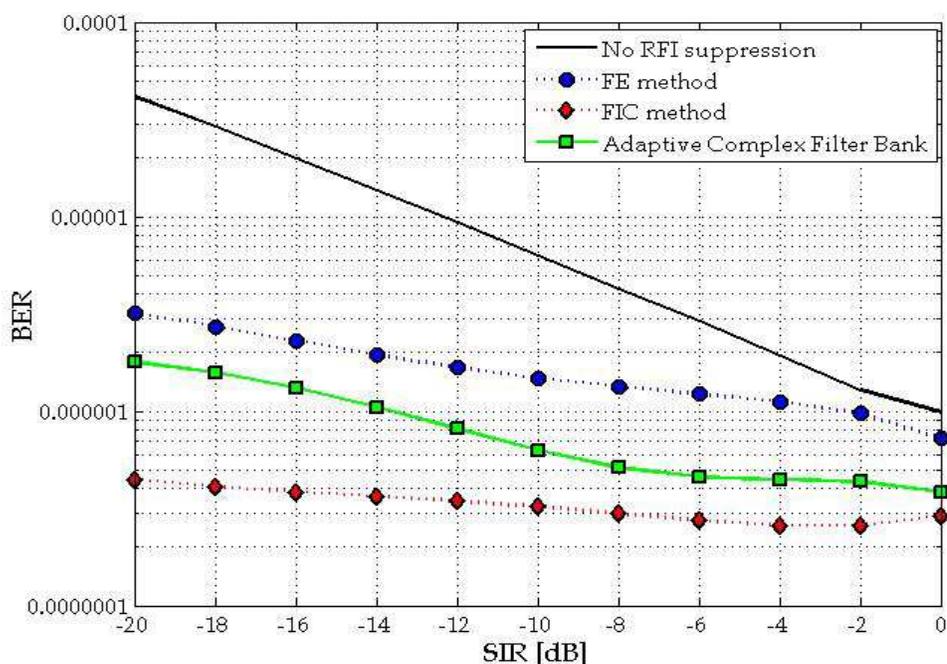
al, 2004). The FIC method is implemented as a two-stage algorithm with the filtering process applied independently at each receiver input. First, the complex RFI frequency is estimated by finding the maximum in the oversampled signal spectrum per each receiver's input. After that, using the Maximum Likelihood (ML) approach, the RFI amplitude and phase are estimated per input. The second stage realizes the Non-Linear Least Square (NLS) Optimization Algorithm, where the RFI complex amplitude, phase and frequency are precisely determined.

The *third* RFI mitigation approach is based on the *complex* DSP parallel adaptive complex filtering technique. A notch ACFB is connected at the receiver's inputs in order to identify and eliminate the RFI signal. The adaptation algorithm tunes the filter at each receiver input in such a way that its central frequency and bandwidth match the RFI signal spectrum (Lee et al, 2007).

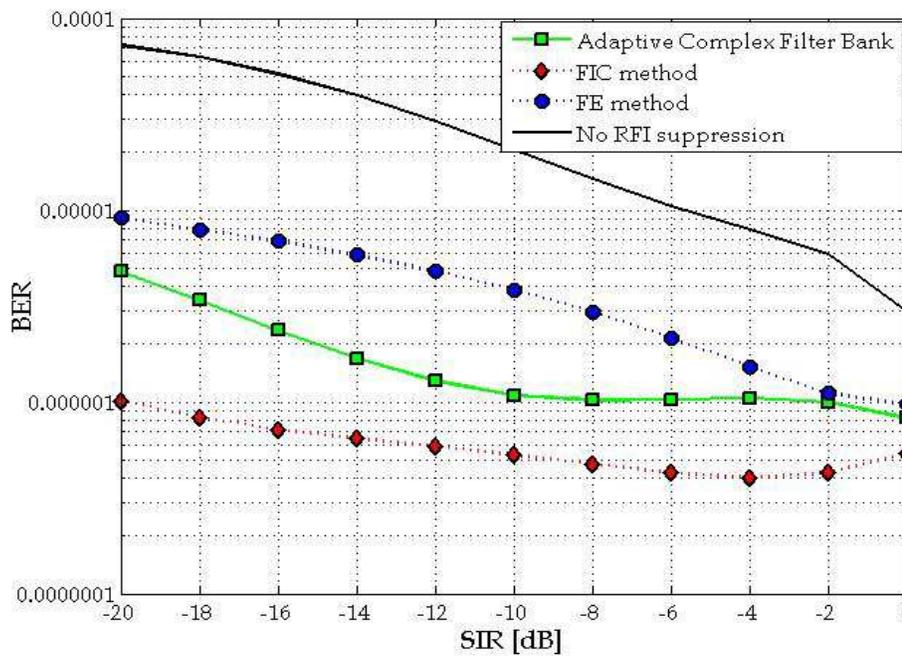
Using the above-described general simulation model of a MIMO GDSL system (Fig. 11), different experiments are performed deriving the BER as a function of the SIR. The RFI is a complex single tone, the frequency of which is centrally located between two adjacent DMT tones. Depending on the number of twisted pairs used 2, 3 or 4-pair MIMO GDSL systems are considered (Fig. 12) (Poulkov et al, 2009).

The GDSL channels examined are subjected to FEXT, NEXT and a background AWGN with a flat Power Spectral Density (PSD) of -140 dBm/Hz.

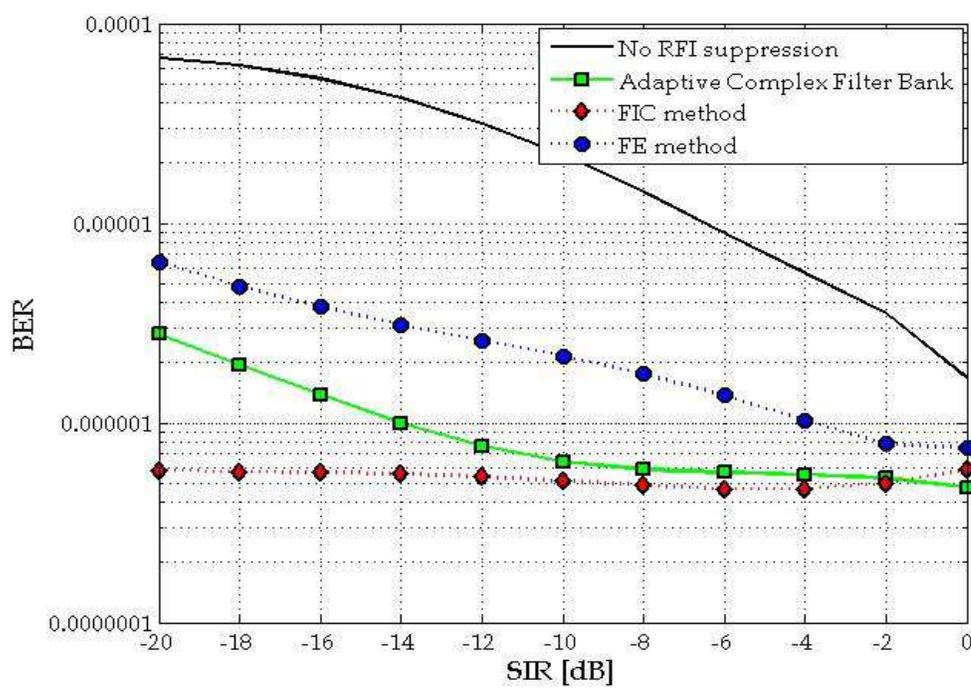
The best RFI mitigation is obtained when the *complex* DSP filtering method is applied to the highest value of channel diversity, i.e. 4-pair GDSL MIMO. The FIC method gives the highest performance but at the cost of additional computational complexity, which could limit its hardware application. The FE method has the highest computational efficiency but delivers the lowest improvement in results when SIR is low: however for high SIR its performance is good.



(a)



(b)



(c)

Fig. 12. BER as a function of SIR for (a) 2-pair; (b) 3-pair; (c) 4-pair GDSL MIMO channels

In this respect, *complex* DSP ACFB filtering turns out to be an optimal narrowband interference-suppression technique, offering a good balance between performance and computational complexity.

### 3. Conclusions

The use of complex number mathematics greatly enhances the power of DSP, offering techniques which cannot be implemented with real number mathematics alone. In comparison with *real* DSP, *complex* DSP is more abstract and theoretical, but also more powerful and comprehensive. Complex transformations and techniques, such as complex modulation, filtering, mixing, z-transform, speech analysis and synthesis, adaptive complex processing, complex Fourier transforms etc., are the essence of theoretical DSP. *Complex* Fourier transforms appear to be difficult when practical problems are to be solved but they overcome the limitations of *real* Fourier transforms in a mathematically elegant way.

*Complex* DSP techniques are required for many wireless high-speed telecommunication standards. In telecommunications, the complex representation of signals is very common, hence complex processing techniques are often necessary.

Adaptive complex filtering is examined in this chapter, since it is one of the most frequently-used real-time processing techniques. Adaptive complex selective structures are investigated, in order to demonstrate the high efficiency of adaptive complex digital signal processing.

The *complex* DSP filtering method, based on the developed ACFB, is applied to suppress narrowband interference signals in MIMO telecommunication systems and is then compared to other suppression methods. The study shows that different narrowband interference mitigation methods perform differently, depending on the parameters of the telecommunication system investigated, but the *complex* DSP adaptive filtering technique offers considerable benefits, including comparatively low computational complexity.

Advances in diverse areas of human endeavour, of which modern telecommunications is only one, will continue to inspire the progress of *complex* DSP.

It is indeed fair to say that complex digital signal processing techniques still contribute more to the expansion of theoretical knowledge rather than to the solution of existing practical problems - but watch this space!

### 4. Acknowledgment

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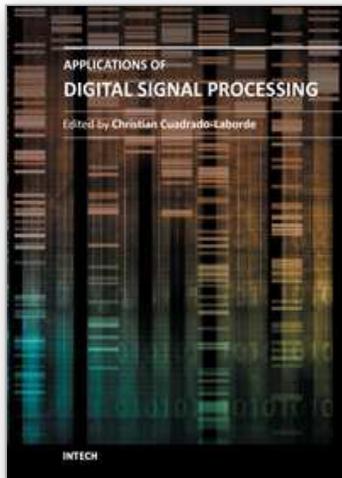
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In this book the reader will find a collection of chapters authored/co-authored by a large number of experts around the world, covering the broad field of digital signal processing. This book intends to provide highlights of the current research in the digital signal processing area, showing the recent advances in this field. This work is mainly destined to researchers in the digital signal processing and related areas but it is also accessible to anyone with a scientific background desiring to have an up-to-date overview of this domain. Each chapter is self-contained and can be read independently of the others. These nineteenth chapters present methodological advances and recent applications of digital signal processing in various domains as communications, filtering, medicine, astronomy, and image processing.

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