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ICI Reduction Methods in OFDM Systems

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1. Introduction

The principles of multicarrier modulation have been in existence for several decades. However, in recent years these techniques have quickly moved out of textbooks and into practice in modern communications systems in the form of orthogonal frequency division multiplexing (OFDM). OFDM is a special form of multicarrier modulation technique which is used to generate waveforms that are mutually orthogonal and then distributes the data over a large number of carriers that are spaced apart at precise frequencies. This spacing provides the "orthogonality" in this technique which prevents the demodulators from seeing frequencies other than their own. In an OFDM scheme, a large number of orthogonal, overlapping, narrow band subcarriers are transmitted in parallel. These carriers divide the available transmission bandwidth. The separation of the subcarriers is such that there is a very compact spectral utilization. With OFDM, it is possible to have overlapping sub channels in the frequency domain (Figure 1), thus increasing the transmission rate.

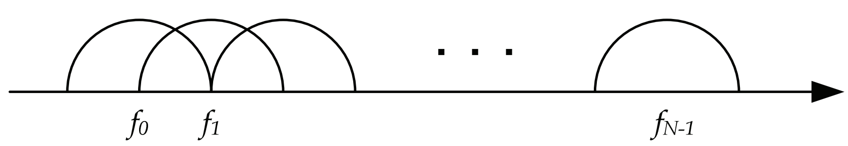

Fig. 1. Power spectrum of the transmitted signal

In order to avoid a large number of modulators and filters at the transmitter and complementary filters and demodulators at the receiver, it is desirable to be able to use modern digital signal processing techniques, such as fast Fourier transform (FFT).
OFDM is a promising candidate for achieving high data rates in mobile environment because of its multicarrier modulation technique and ability to convert a frequency selective fading channel into several nearly flat fading channels.
This technology has been chosen as the transmission method of many standards, such as Digital Subscribe Line (DSL), European Digital Audio and Video Broadcasting terrestrial (DAB/DVB-T), European HIPERLAN/2 and IEEE 802.11 a/g for wireless local area networks (WLAN), Worldwide Interoperability for Microwave Access (WiMAX), etc. However, OFDM systems exhibit a sensitivity to phase noise higher than single carrier modulations due to its long symbol period. Because carriers are kept very close to each other, OFDM is very sensitive to distortion that may remove the orthogonality between carriers. The crystal oscillator used in a mixer generates phase noise. It can also be caused by

AWGN present at the input of a Phase Locked Loop (PLL) in a coherent receiver. Phase noise can cause several types of signal degradation that are usually very difficult to quantify analytically. When the modulation experiences phase noise, it encounters two problems: 1) a common phase rotation over all the carrier frequencies which rotate the entire signal space for a given OFDM symbol and 2) inter-carrier interference due to the loss of orthogonality between subcarriers. Especially, the ICI seriously degrades system predominance because it may break down the orthogonality between subcarriers.
There have been many previous works on the phase noise, frequency offset and reduction of ICI. Among them the following methods are discussed and compared in this chapter. In the next section the OFDM system is introduced and its benefits along with its drawbacks are analyzed. ICI reduction methods such as pulse shaping and self-cancellation are given in section 3 and the last section concludes the chapter.

2. OFDM system

Figure 2 shows the block diagram of a typical OFDM system. The transmitter section converts digital data to be transmitted, into a mapping of subcarrier amplitude and phase. It then transforms this spectral representation of the data into the time domain using an Inverse Discrete Fourier Transform (IDFT). The Inverse Fast Fourier Transform (IFFT) performs the 20 same operations as an IDFT, except that it is much more computationally efficient, and so is used in all practical systems. In order to transmit the OFDM signal the calculated time domain signal is then mixed up to the required frequency. The receiver performs the reverse operation of the transmitter, mixing the RF signal to base band for processing, then using a Fast Fourier Transform (FFT) to analyze the signal in the frequency domain. The amplitude and phase of the subcarriers is then picked out and converted back to digital data. The IFFT and the FFT are complementary function and the most appropriate term depends on whether the signal is being received or generated. In cases where the signal is independent of this distinction then the term FFT and IFFT is used interchangeably.
The high data rate serial input bit stream is fed into serial to parallel converter to get low data rate output parallel bit stream. Input bit stream is taken as binary data. The low data rate parallel bit stream is modulated in Signal Mapper. Modulation can be BPSK, QPSK, QAM, etc. The modulated data are served as input to inverse fast Fourier transform so that each subcarrier is assigned with a specific frequency. The frequencies selected are orthogonal frequencies. In this block, orthogonality in subcarriers is introduced. In IFFT, the frequency domain OFDM symbols are converted into time domain OFDM symbols. Guard interval is introduced in each OFDM symbol to eliminate inter symbol interference (ISI). All the OFDM symbols are taken as input to parallel to serial data. These OFDM symbols constitute a frame. A number of frames can be regarded as one OFDM signal. This OFDM signal is allowed to pass through digital to analog converter (DAC). In DAC the OFDM signal is fed to RF power amplifier for transmission. Then the signal is allowed to pass through additive white Gaussian noise channel (AWGN channel). At the receiver part, the received OFDM signal is fed to analog to digital converter (ADC) and is taken as input to serial to parallel converter. In these parallel OFDM symbols, Guard interval is removed and it is allowed to pass through Fast Fourier transform. Here the time domain OFDM symbols are converted into frequency domain. After this, it is fed into Signal Demapper for demodulation purpose. And finally the low data rate parallel bit stream is converted into high data rate serial bit stream which is in form of binary.

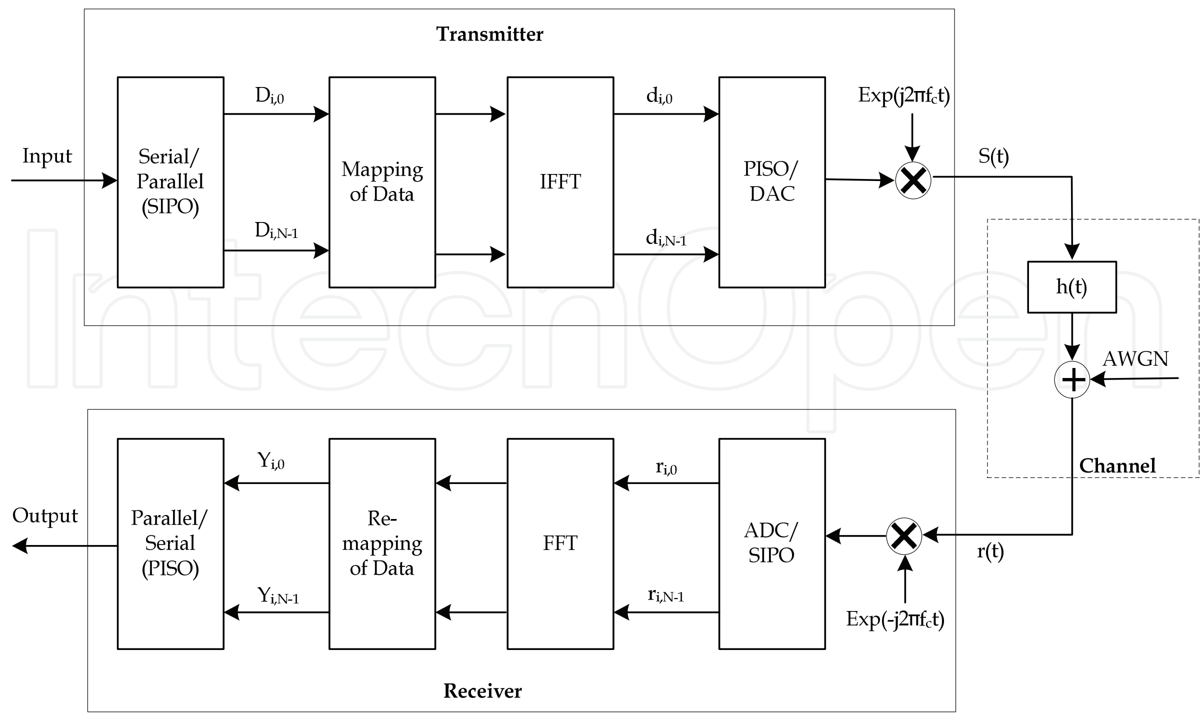

Fig. 2. OFDM system implementation

By the insertion of an extra guard interval between successive OFDM symbols the Inter Symbol Interference (ISI) can be avoided. The guard interval could be a section of all zero samples transmitted in front of each OFDM symbol and its duration should be more than the channel delay spread (L_c). It should be considered that in practical systems the guard interval is not used. Instead, Cyclic Prefix (CP) is inserted to combat the multipath-channel by making the channel estimation simple. The cyclic prefix is a replica of the last L_p samples of the OFDM symbol where $L_p > L_c$. Because of the way in which the cyclic prefix was formed, the cyclically-extended OFDM symbol now appears periodic when convolved with the channel. An important result is that the effect of the channel becomes multiplicative.
For the better understanding of this issue assume that the impulse response of the channel is $h_0, h_1, \dots, h_{L_c}$ and the i-th transmitted signal block in the output of IFFT block is $d_{i,0}, d_{i,1}, \dots, d_{i,N-1}$. In this condition the cyclic prefix would be $d_{i,N-L_c}, d_{i,N-L_c+1}, \dots, d_{i,N-1}$. The symbols of the received baseband signal after the transmission through the channel are equal to:

$$\begin{aligned} r_{i,-L_c} &= h_0 d_{i,N-L_c} + h_1 d_{i-1,N-1} + \cdots + h_{L_c} d_{i-1,N-L_c} \\ r_{i,-L_c+1} &= h_0 d_{i,N-L_c+1} + h_1 d_{i-1,N-L_c} + \cdots + h_{L_c} d_{i-1,N-L_c+1} \\ &\vdots \\ r_{i,0} &= h_0 d_{i,0} + h_1 d_{i,N-1} + \cdots + h_{L_c} d_{i,N-L_c} \\ r_{i,1} &= h_0 d_{i,1} + h_1 d_{i,0} + \cdots + h_{L_c} d_{i,N-L_c+1} \\ &\vdots \\ r_{i,N-1} &= h_0 d_{i,N-1} + h_1 d_{i,N-2} + \cdots + h_{L_c} d_{i,N-L_c-1} \end{aligned} \tag{1}$$

At the receiver the first L_c+*1* symbols are discarded and the N remained symbols are demodulated using an N-point FFT. So the data on the k-th subcarrier is as follows:

$$Y_{i,k} = \sum_{n=0}^{N-1} r_{i,n} e^{-j\frac{2\pi nk}{N}} = D_{i,k} \sum_{n=0}^{Lc} h_n e^{-j\frac{2\pi nk}{N}} = D_{i,k} H_k \tag{2}$$

where H_k is the channel impulse response in the frequency domain. It can also be considered as the channel gain on the k-th subcarrier. $D_{i,k} = \sum_{n=0}^{N-1} d_{i,n} e^{-j\frac{2\pi nk}{N}}$ is the input symbols of the IFFT block at the transmitter as:

$$d_{i,n} = N\_IFFT\{D_{i,k}\} \tag{3}$$

It can be seen that the main symbol stream $\{D_{i,k}\}$ could be detected with the estimation of the channel coefficient at the receiver, while there is no Inter-Symbol Interference (ISI) or Inter-Carrier Interference (ICI).

Thus, a multipath channel is converted into scalar parallel sub-channels in frequency domain, thereby simplifying the receiver design considerably. The task of channel estimation is simplified, as we just need to estimate the scalar coefficients H_k for each sub-channel and once the values of $\{H_k\}$ are estimated, for the duration in which the channel does not vary significantly, merely multiplying the received demodulated symbols by the inverse of H_k yields the estimates of $\{D_{i,k}\}$.

The benefits of OFDM are high spectral efficiency, resiliency to RF interference, and lower multi-path distortion. This is useful because in a typical terrestrial broadcasting scenario there are multipath-channels (i.e. the transmitted signal arrives at the receiver using various paths of different length). Since multiple versions of the signal interfere with each other (inter symbol interference (ISI)) it becomes very hard to extract the original information. With the rapid growth of digital communication in recent years, the need for high speed data transmission is increased. Moreover, future wireless systems are expected to support a wide range of services which includes video, data and voice. OFDM is a promising candidate for achieving high data rates in mobile environment because of its multicarrier modulation technique and ability to convert a frequency selective fading channel into several nearly flat fading channels.

However, there are some non-idealities which can affect the performance of an OFDM system. These non-idealities are as follows:

Noise: Like other communication systems, the performance of an OFDM system is affected by different kind of noise such as uniform noise (AWGN), non-uniform noise (colored noise), and impulse noise.

LO phase offset: This condition occurs when there is a difference between the phase of the output LO and the phase of the received signal.

FFT window location offset: In practice, a correlation is often used with a known preamble sequence located at the beginning of the transmission. This correlation operation aids the receiver in synchronizing itself with the received OFDM symbol boundaries. However, inaccuracies still remain, and they manifest themselves as an offset in the FFT window location. The result is that the N symbols sent to the FFT will not line up exactly with the corresponding OFDM symbol.

Sampling frequency offset: A sampling frequency offset occurs when the A/D converter output is sampled either too fast or too slow.

Non-linearity in the transmitter and receiver circuits: All transmitters and receivers in communication systems contain devices such as amplifiers which are often designed to be non-linear in order to minimize power consumption. On the other hand, an OFDM signal is made up of multiple simultaneous signals that, for a given average power, have a higher peak signal level. Thus, OFDM signals result in an increase in the peak-to-average ratio (PAR) of the signal. Because of the non-linear transfer functions of amplifiers, these higher peak amplitude levels will create more severe distortion than a single carrier case even if the average power levels of each are the same.

Phase noise: OFDM systems are very sensitive to phase noise caused by oscillator instabilities in both the transmitter and the receiver. Without loss of generality, in this study the local oscillator in the receiver will be considered as the phase noise source. As mentioned before, the modulated subcarriers overlap spectrally, but since they are orthogonal over symbol duration, they can be easily recovered as long as the channel and other non-idealities do not destroy the orthogonality. An unwindowed OFDM system has rectangular symbol shapes. Therefore, in the frequency domain the individual sub-channels will have the form of sinc functions where the first sidelobe is only some 13 dB below the main lobe of the subcarrier (Figure 3). A practical oscillator has spectral components around the centre frequency. These components cause the loss of orthogonality of the OFDM carriers. In the frequency domain it can be viewed as interference caused by the high sidelobes of the adjoining carriers on a particular subcarrier.

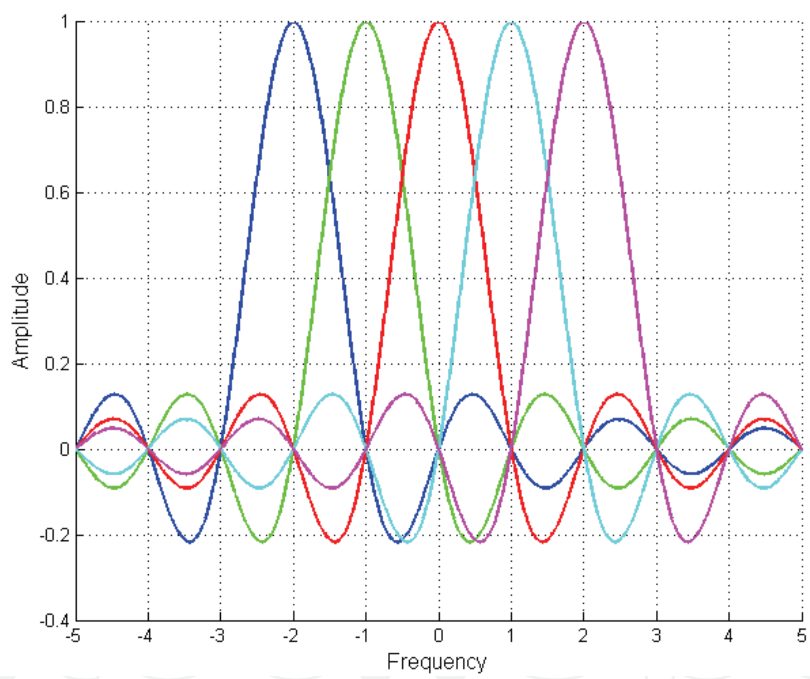

Fig. 3. OFDM spectrum with 5 subcarriers

In this study the phase noise is resolved into two components, namely the Common Phase Error (CPE), which affects all the sub-channels equally and the Inter-Carrier Interference (ICI), which is caused by the loss of orthogonality of the subcarriers.

As described above, the IFFT of $\mathrm{D_i}$ is given by $\mathrm{d_i}$. The k-th sample of d_i can be represented by:

$$d_{i,k} = \frac{1}{N}\sum_{n=0}^{N-1} D_{i,n} e^{\left(j2\pi kn/_N\right)} \qquad k = 0,1,\dots,N-1 \tag{4}$$

The phase noise is modeled as a phasor $e^{j\theta(n)}$, where the phase noise process $\theta(n)$ is zero-mean and wide-sense stationary with a finite variance σ_θ^2. An approximation for the PSD of a free-running oscillator can be found in [Robertson & Kaiser, 1995] is as follows:

$$S_\theta(f) = 10^{-c} + \begin{cases} 10^{-a} & |f| \le f_l \\ 10^{(f_l - |f|)\left(\frac{b}{f_h - f_l}\right) - a} & |f| > f_l \end{cases} \tag{5}$$

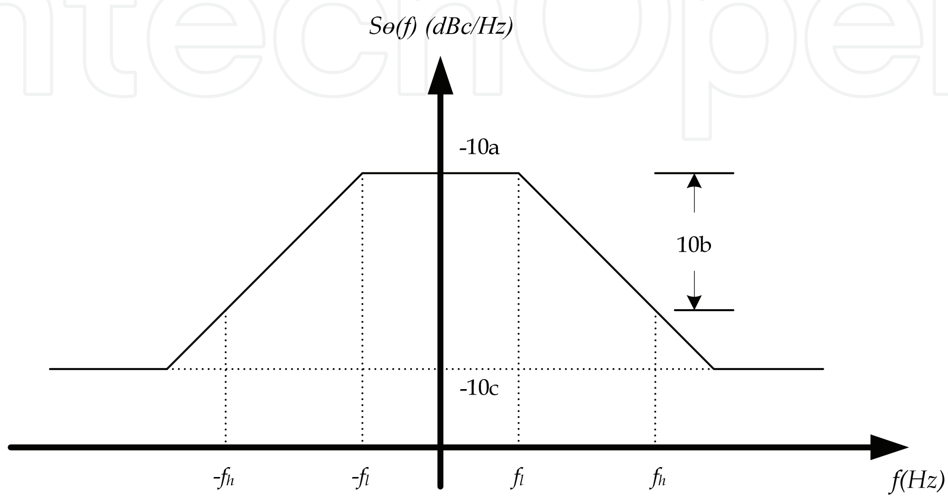

Fig. 4. Phase noise PSD of a typical oscillator

Parameter c determines the noise floor of the oscillator and a determines the noise level in the frequency ranges from the center frequency to f_l. Parameter b gives the noise fall off rate from the noise floor at f_l to the noise level at f_h. (See Figure 4).

The demodulated data symbol of the k-th subcarrier of the i-th OFDM symbol $Y_{i,k}$ with the consideration of phase noise is given by:

$$Y_{i,k} = \sum_{n=0}^{N-1} r_{i,n} e^{-j2\pi kn/N} e^{j\theta(n)} \qquad k = 0,1,\dots,N-1 \tag{6}$$

Using (2), (6) and considering AWGN,

$$Y_{i,k} = \frac{1}{N} \sum_{n=0}^{N-1} \left\{ \left\{ \sum_{l=0}^{N-1} D_{i,l} H_l e^{j2\pi ln/N} \right\} e^{j\theta(n)} \right\} e^{-j2\pi kn/N} + W_k \tag{7}$$

where W_k is the contribution due to AWGN on the k-th subcarrier. We can further simplify (7) as:

$$Y_{i,k} = \frac{1}{N} H_k D_{i,k} \sum_{n=0}^{N-1} e^{j\theta(n)} + \frac{1}{N} \sum_{\substack{l=0 \\ l\neq k}}^{N-1} H_l D_{i,l} \sum_{n=0}^{N-1} e^{j\theta(n)} e^{-j2\pi(k-l)n/N} + W_k \tag{8}$$

The first term on the right-hand side of (8) rotates the useful component $H_k D_{i,k}$ of each subcarrier by an equal amount and is independent of the particular subchannel concerned, k.

This is commonly known as the Common Phase Error (CPE). The second term is the Inter-Carrier Interference (ICI) caused by contributions from all subcarriers $l \neq k$ on k due to the loss of orthogonality. Unlike the CPE, ICI is not easy to estimate.

Local oscillator frequency offset: It drives from the difference between LO frequency at the transmitter and the receiver. In addition to phase noise, the frequency offset which is caused by the high Doppler spread and the mismatch in the oscillator frequency, produces inter-carrier interference (ICI).

For a block of data, the modulated signal at the n-th instant of time can be written as:

$$d_n = \frac{1}{N}\sum_{k=0}^{N-1} D_k\, e^{j2\pi kn/N} \quad , n = 0,1,\dots,N-1 \tag{9}$$

The signal at the receiver, after passing through a frequency selective fading channel is expressed as:

$$r_n = \sum_{l=0}^{Lc} d_{n-l}\, h_l + w_n = \sum_{l=0}^{Lc}\frac{1}{N}\sum_{k=0}^{N-1} D_k\, e^{j2\pi kn/N}\, h_l + w_n \tag{10}$$

At the receiver, the signal is mixed with a local oscillator signal which is Δf above the correct carrier frequency. The signal Y_k received at the k-th subcarrier after performing the FFT is expressed as:

$$\begin{aligned} Y_k = \sum_{n=0}^{N-1} r_n e^{-\frac{j2\pi kn}{N}} e^{\frac{j2\pi n\varepsilon}{N}} &= D_k H_k \frac{1}{N}\sum_{n=0}^{N-1} e^{\frac{j2\pi n\varepsilon}{N}} + \sum_{\substack{l=0\\ l\neq k}}^{N-1} D_l H_l \frac{1}{N}\sum_{n=0}^{N-1} e^{\frac{j2\pi n(l+\varepsilon-k)}{N}} + w'_k \\ &= D_k H_k I(0) + \sum_{\substack{l=0\\ l\neq k}}^{N-1} D_l H_l I(l-k) + w'_k\,, \quad k = 0,1,\dots,N-1 \end{aligned} \tag{11}$$

Where $\varepsilon = \Delta f \times T_s$ is the frequency offset normalized to the OFDM symbol rate $1/T_s$. The T_s denotes the OFDM symbol duration excluding the guard interval. The sequence $I\,(l-k)$ is defined as the ICI coefficient between l-th and k-th subcarriers, which can be expressed as:

$$I(l-k) = \frac{sin\,(\pi(l+\varepsilon-k))}{Nsin(\frac{\pi}{N}(l+\varepsilon-k))} exp\,(j\pi(1-\frac{1}{N})(l+\varepsilon-k)) \tag{12}$$

The first term in the right-hand side of (11) represents the desired signal. Without frequency error ($\varepsilon = 0$), $I\,(0)$ takes its maximum value. The second term is the ICI components, which as ε becomes larger, the desired part $|I\,(0)|$ decreases and the undesired part $|I(l-k)|$ increases.

The undesired ICI degrades the performance of the system. It is not possible to make reliable data decisions unless the ICI powers of OFDM system are minimized. Thus, an accurate and efficient Inter-Carrier Interference (ICI) reduction procedure is essential to demodulate the received data. Several methods have been presented to reduce ICI, including windowing at the receiver [Muschallik, 1996; Müller-Weinfurtner, 2001; Song & Leung, 2005], the use of pulse shaping [Tan & Beaulieu, 2004; Mourad, 2006; Maham & Hjørungnes, 2007], self-cancellation schemes [Zhao & Haggman, 2001], and frequency domain equalization. The next part of this chapter introduces these techniques.

3. ICI reduction techniques

In the OFDM systems, N subcarriers are used for data transmission of N symbols $\{D_{i,0}, D_{i,1}, \dots D_{i,N-1}\}$. By using the IFFT operation for the data modulation, rectangular pulse shaping filter is implicitly applied. Thus, the spectrum of each individual subcarrier equals a sinc-function defined as $sinc(x) = sin(\pi x) / \pi x$ and is given by:

$$S_k(z) = D_{i,k} \times sinc(z - z_{k)}, \qquad k = 0,1, \dots, N-1 \tag{13}$$

where $z \in \mathbb{R}$ represents the frequency f shifted to the carrier frequency of the OFDM system f_c and normalized to the sampling frequency $1/T_s$. The normalized frequency is given by:

$$z = (f - f_c) \times T_s \tag{14}$$

Accordingly, $z_k = (f_k - f_c) \times T_s$ is defined as the normalized center frequency of the k-th subcarrier with f_k representing the center frequency of the k-th subcarrier. The spectrum of the transmitted OFDM symbol is the superposition of the spectra of all individual subcarriers:

$$S(z) = \sum_{k=0}^{N-1} S_k(z) \tag{15}$$

The sidelobe power of this sum signal and also the sidelobe power of each subcarrier spectrum only decays with $1/z^2$ resulting in a high interference caused by the high sidelobes of the adjoining carriers on a particular subcarrier.
Here, some techniques introduced to reduce the power of the interfering components.

3.1 Pulse shaping

As we have seen in the OFDM spectrum each carrier consists of a main lobe followed by a number of sidelobes with reducing amplitudes. As long as orthogonality is maintained, there is no interference among the carriers because at the peak of the every carrier, there exists a spectral null. That is at that point that the component of all other carriers is zero. Hence the individual carrier is easily separated.
In the presence of the frequency offset the orthogonality is lost because the spectral null does not coincide to the peak of the individual carriers. So some power of the sidelobes exists at the centre of the individual carriers which is called ICI power. The ICI power will go on increasing as the frequency offset increases. The purpose of pulse shaping is to reduce the sidelobes which leads to the significant decrease in the ICI power.
In a simple OFDM system, symbols are performed using an N-FFT function. This implies that the received signal *r(k)* is shaped in the time domain by a rectangular pulse function. One possible countermeasure to overcome the interference is making the PDS of the OFDM modulated subcarriers ($S_n(z)$) go down more rapidly by shaping the transmit signal of the OFDM subcarriers. This makes the amplitude go smoothly to zero at the symbol boundaries. The N-subcarrier OFDM block with pulse-shaping is expressed as:

$$s(t) = e^{j2\pi f_c t} \sum_{k=0}^{N-1} D_k p(t) e^{j2\pi f_k t} \tag{16}$$

where *p(t)* is the pulse shaping function. The transmitted symbol D_k is assumed to have zero mean and normalized average symbol energy. Also we assume that all data symbols are uncorrelated, i.e.:

$$E[D_k D_m^*] = \begin{cases} 1, & k = m, \ \ k, m = 0,1, \dots, N-1 \\ 0, & k \neq m, \ \ k, m = 0,1, \dots, N-1 \end{cases} \tag{17}$$

where D_k^* is the complex conjugate of D_k. To ensure the subcarrier orthogonality, which is very important for OFDM systems the equation below has to be satisfied:

$$f_k - f_m = \frac{k-m}{T_s}, \qquad k, m = 0,1, \dots, N-1 \tag{18}$$

In the receiver block, the received signal can be expressed as:

$$r(t) = s(t) \otimes h(t) + w(t) \tag{19}$$

where $\otimes$ denotes convolution and h(t) is the channel impulse response. In (19), w(t) is the additive white Gaussian noise process with zero mean and variance $N_0/2$ per dimension. For this work we assume that the channel is ideal, i.e., h(t) = δ(t) in order to investigate the effect of the frequency offset only on the ICI performance. At the receiver, the received signal $r'(t)$ becomes:

$$r'(t) = e^{j2\pi\Delta f t + \theta} \sum_{k=0}^{N-1} D_k p(t) e^{j2\pi f_k t} + w(t) e^{j(2\pi(-f_c + \Delta f)t + \theta)} \tag{20}$$

Where θ is the phase error and Δf is the carrier frequency offset between transmitter and receiver oscillators. For the transmitted symbol D_m , the decision variable is given as

$$\widehat{D}_m = \int_{-\infty}^{\infty} r'(t) e^{-j2\pi f_m t} dt \tag{21}$$

By using (18) and (21), the decision variable $\widehat{D}_m$ can be expressed as

$$\widehat{D}_m = \left(D_m P(-\Delta f) + \sum_{\substack{k=0 \\ k \neq m}}^{N-1} D_k P(\tfrac{m-k}{T_s} - \Delta f) \right) e^{j\theta} + w_m \quad , m = 0, \dots, N-1 \tag{22}$$

where *P(f)* is the Fourier transform of *p(t)* and w_m is the independent white Gaussian noise component. In (22), the first term contains the desired signal component and the second term represents the ICI component. With respect to (18), *P(f)* should have spectral nulls at the frequencies $\pm(1/T_s), \pm(2/T_s), \dots$ to ensure subcarrier orthogonality. Then, there exists no ICI term if Δf and θ are zero.

The power of the desired signal can be calculated as [Tan & Beaulieu, 2004; Mourad, 2006; Kumbasar & Kucur, 2007]:

$$\sigma_m^2 = E[D_m P(-\Delta f) D_m^* P(-\Delta f)^*] = E[D_m D_m^*] |P(\Delta f)|^2 = |P(\Delta f)|^2 \tag{23}$$

The power of the ICI can be stated as:

$$\sigma_{ICI_m}^2 = \sum_{\substack{k=0 \\ k \neq m}}^{N-1} \sum_{\substack{n=0 \\ n \neq m}}^{N-1} D_n D_k^* P(\frac{k-m}{T_s} + \Delta f) P(\frac{n-m}{T_s} + \Delta f)^* \tag{24}$$

The average ICI power across different sequences can be calculated as:

$$\overline{\sigma_{ICI}^2} = E\left[\sigma_{ICI_m}^2\right] = \sum_{\substack{k=0 \\ k\neq m}}^{N-1} \left|P(\frac{k-m}{T_0} + \Delta f)\right|^2 \tag{25}$$

As seen in (25) the average ICI power depends on the number of the subcarriers and *P(f)* at frequencies: $\left(\frac{k-m}{T_s} + \Delta f\right), \quad k \neq m, \; k = 0,1,\dots,N-1$

The system ICI power level can be evaluated by using the CIR (Carrier-to-Interference power Ratio). While deriving the theoretical CIR expression, the additive noise is omitted.

By using (23) and (25), the CIR can be derived as [Tan & Beaulieu, 2004; Mourad, 2006; Kumbasar & Kucur, 2007]:

$$CIR = \frac{|P(\Delta f)|^2}{\sum_{\substack{k=0 \\ k\neq m}}^{N-1} \left|P(\frac{k-m}{T_s} + \Delta f)\right|^2} \tag{26}$$

Therefore, the CIR of the OFDM systems only depends approximately on the normalized frequency offset. A commonly used pulse shaping function is the raised cosine function that is defined by:

$$g(t) = \begin{cases} \frac{1}{2} + \frac{1}{2}cos\left(\pi + \frac{\pi t}{\alpha T_s}\right), & for\ 0 \leq t < \alpha T_s \\ 1, & for\ \alpha T_s \leq t < T_s \\ \frac{1}{2} + \frac{1}{2}cos\left(\frac{\pi(t - T_s)}{\alpha T_s}\right), & for\ T_s \leq t < (1+\alpha\beta)T_s \end{cases} \tag{27}$$

where α denotes the rolloff factor and the symbol interval T_s is shorter than the total symbol duration *(1 + α)* T_s because adjacent symbols are allowed to partially overlap in the rolloff region. Simulation shows that the benefit of the raised cosine function with respect to the ICI reduction is fairly low.

A number of pulse shaping functions such as Rectangular pulse (REC), Raised Cosine pulse (RC), Better Than Raised Cosine pulse (BTRC), Sinc Power pulse (SP) and Improved Sinc Power pulse (ISP) have been introduced for ICI power reduction. Their Fourier transforms are given, respectively as [Kumbasar & Kucur, 2007]:

$$P_{REC}(f) = sinc(fT_s), \tag{28}$$

$$P_{RC}(f) = sinc(fT_s)\frac{cos\,(\pi\alpha f T_s)}{1-(2\alpha f T_s)^2}, \tag{29}$$

$$P_{BTRC}(f) = sinc(fT_s)\frac{[2\beta f T_s\, sin(\pi\alpha f T_s) + 2\,cos(\pi\alpha f T_s) - 1]}{1+(\beta f T_s)^2}, \tag{30}$$

$$P_{SP}(f) = sinc^n(fT_s), \tag{31}$$

$$P_{ISP}(f) = exp\,\{-a(fT_s)^2\}sinc^n(fT_s), \tag{32}$$

where α $(0 \leq \alpha \leq 1)$ is the rolloff factor, $\beta = \pi\alpha/ln\,2$, a is a design parameter to adjust the amplitude and n is the degree of the sinc function.

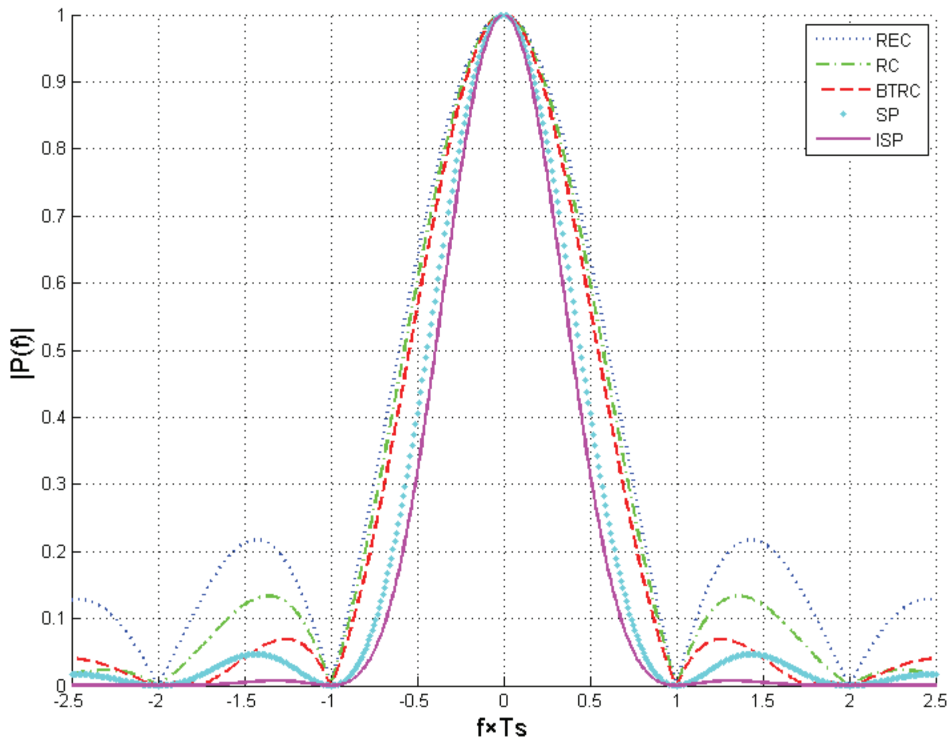

Fig. 5. Comparison of REC, RC, BTRC, SP, and ISP spectrums

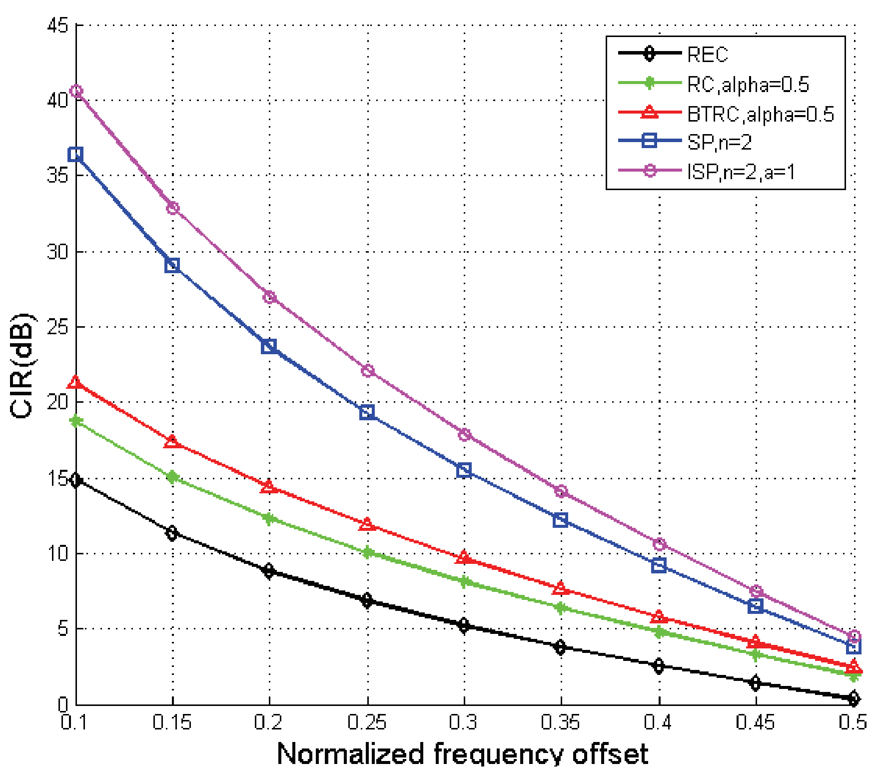

Fig. 6. CIR performance for different pulse shapes

The purpose of pulse shaping is to increase the width of the main lobe and/or reduce the amplitude of sidelobes, as the sidelobe contains the ICI power.

REC, RC, BTRC, SP, and ISP pulse shapes are depicted in Figure 5 for *a=1*, *n=2*, and $\alpha = 0.5$. SP pulse shape has the highest amplitude in the main lobe, but at the sidelobes it has lower amplitude than BTRC. This property provides better CIR performance than that of BTRC as shown in [Mourad, 2006]. As seen in this figure the amplitude of ISP pulse shape is the lowest at all frequencies. This property of ISP pulse shape will provide better CIR performance than those of the other pulse shapes as shown in Figure 6 [Kumbasar & Kucur, 2007].
Figure 5 shows that the sidelobe is maximum for rectangular pulse and minimum for ISP pulse shapes. This property of ISP pulse shape will provide better performance in terms of ICI reduction than those of the other pulse shapes. Figure 7 compares the amount of ICI for different pulse shapes.

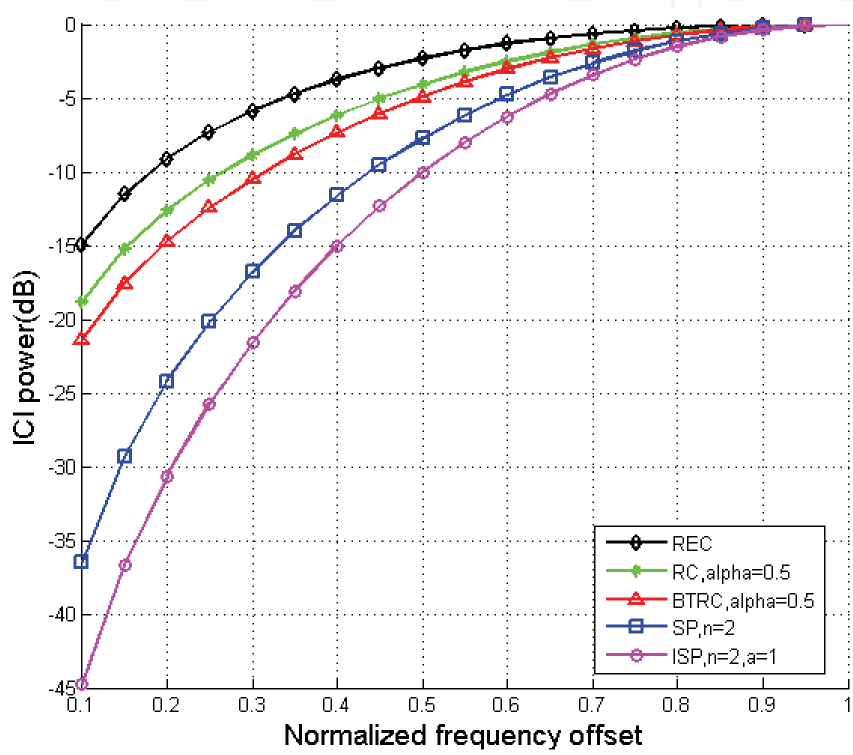

Fig. 7. ICI comparison for different pulse shapes

3.2 ICI self-cancellation methods

In single carrier communication system, phase noise basically produces the rotation of signal constellation. However, in multi-carrier OFDM system, OFDM system is very vulnerable to the phase noise or frequency offset. The serious inter-carrier interference (ICI) component results from the phase noise. The orthogonal characteristics between subcarriers are easily broken down by this ICI so that system performance may be considerably degraded.
There have been many previous works in the field of ICI self-cancellation methods [Ryu et al., 2005; Moghaddam & Falahati, 2007]. Among them convolution coding method, data-conversion method and data-conjugate method stand out.

3.2.1 ICI self-cancelling basis

As it can be seen in eq. 12 the difference between the ICI coefficients of the two consecutive subcarriers are very small. This makes the basis of ICI self cancellation. Here one data symbol is not modulated into one subcarrier, rather at least into two consecutive subcarriers. This is the ICI cancellation idea in this method.

As shown in figure 7 for the majority of *l-k* values, the difference between $I(l-k)$ and $I(l-k+1)$ is very small. Therefore, if a data pair *(a,-a)* is modulated onto two adjacent subcarriers $(l, l+1)$, then the ICI signals generated by the subcarrier will be cancelled out significantly by the ICI generated by subcarrier *l+1* [Zhao & Haggman, 1996, 2001].

Assume that the transmitted symbols are constrained so that $D_1 = -D_0,\ D_3 = -D_2 \ldots D_{N-1} = -D_{N-2}$, then the received signal on subcarrier k considering that the channel coefficients are the same in two adjacent subcarriers becomes:

$$Y'_k = \sum_{\substack{l=0 \\ l=even}}^{N-2} D_l H_l [I(l-k) - I(l-k+1)] + w_k \tag{33}$$

In such a case, the ICI coefficient is denoted as:

$$I'(l-k) = I(l-k) - I(l-k+1) \tag{34}$$

For most of the $l-k$ values, it is found that $|I'(l-k)| << |I(l-k)|$.

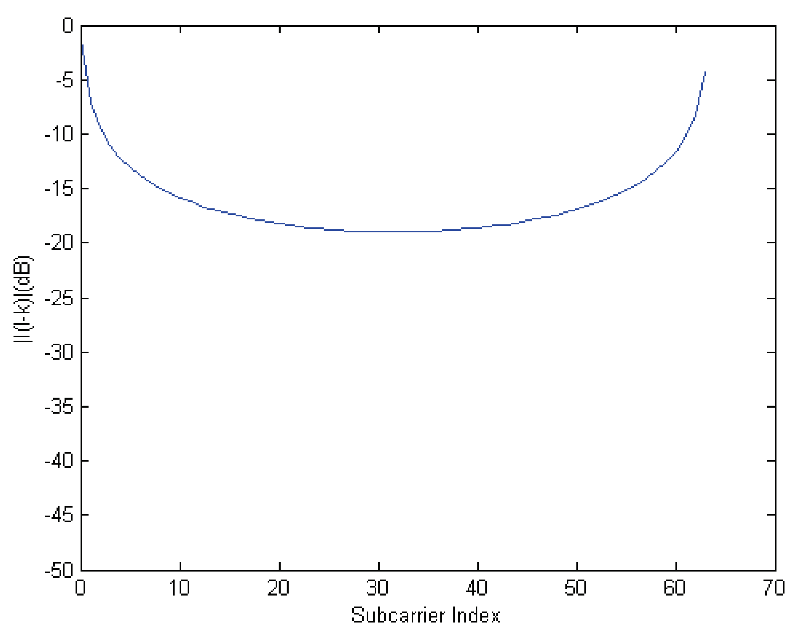

Fig. 7. ICI coefficient versus subcarrier index; N=64

For further reduction of ICI, ICI cancelling demodulation is done. The demodulation is suggested to work in such a way that each signal at the *k+1*-th subcarrier (now *k* denotes even number) is multiplied by *-1* and then summed with the one at the *k*-th subcarrier. Then the resultant data sequence is used for making symbol decision. It can be represented as:

$$Y''_k = Y'_k - Y'_{k+1} = \sum_{\substack{l=0 \\ l=even}}^{N-2} D_l H_l [-I(l-k-1) + 2I(l-k) - I(l-k+1)] + w_k - w_{k+1} \tag{35}$$

The corresponding ICI coefficient then becomes:

$$I''(l-k) = -I(l-k-1) + 2I(l-k) - I(l-k+1) \quad (36)$$

Figure 8 shows the amplitude comparison of $|I(l-k)|$, $|I'(l-k)|$ and $|I''(l-k)|$ for $N=64$ and $\varepsilon = 0.3$. For the majority of l-k values, $|I'(l-k)|$ is much smaller than $|I\ (l-k)|$, and the $|I''(l-k)|$ is even smaller than $|I'(l-k)|$. Thus, the ICI signals become smaller when applying ICI cancelling modulation. On the other hand, the ICI cancelling demodulation can further reduce the residual ICI in the received signals. This combined ICI cancelling modulation and demodulation method is called the ICI self-cancellation scheme.

Due to the repetition coding, the bandwidth efficiency of the ICI self-cancellation scheme is reduced by half. To fulfill the demanded bandwidth efficiency, it is natural to use a larger signal alphabet size. For example, using 4PSK modulation together with the ICI self-cancellation scheme can provide the same bandwidth efficiency as standard OFDM systems (1 bit/Hz/s).

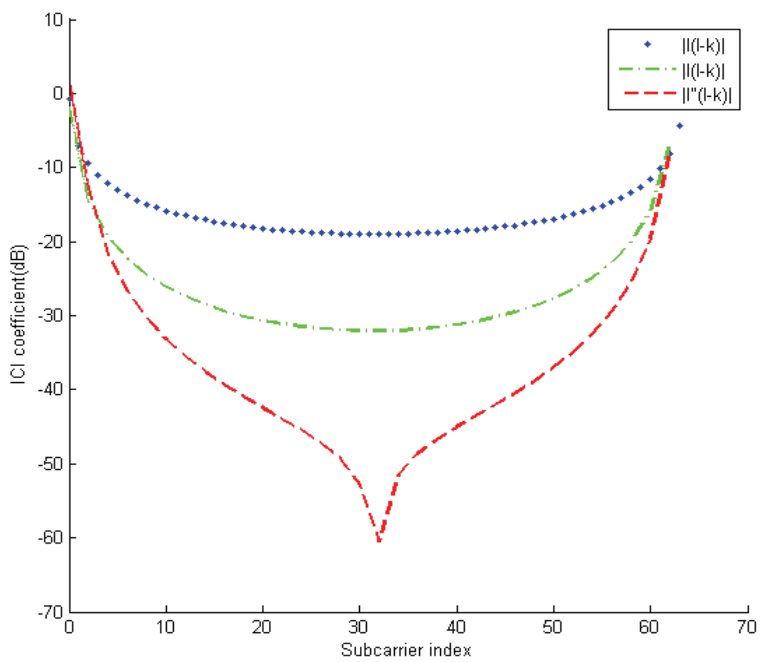

Fig. 8. Amplitude comparison of $|I\ (l-k)|$, $|I'\ (l-k)|$ and $|I''\ (l-k)|$

3.2.1.1 Data-conjugate method

In an OFDM system using data-conjugate method, the information data pass through the serial to parallel converter and become parallel data streams of $N/2$ branch. Then, they are converted into N branch parallel data by the data-conjugate method. The conversion process is as follows. After serial to parallel converter, the parallel data streams are remapped as the form of $D'_{2k} = D_k$, $D'_{2k+1} = -D^*_k$, ($k = 0, \ldots, N/2-1$). Here, D_k is the information data to the k-th branch before data-conjugate conversion, and D'_{2k} is the information data to the $2k$-th branch after ICI cancellation mapping. Likewise, every information data is mapped into a pair of adjacent sub-carriers by data-conjugate method, so the $N/2$ branch data are extended to map onto the N sub-carries.

The original data can be recovered from the simple relation of $Z'_k = (Y_{2k} - Y^*_{2k+1})/2$. Here, Y_{2k} is the $2k$-th sub-carrier data, Z'_k is the k-th branch information data after de-mapping. Finally, the information data can be found through the detection process. The complex base-band OFDM signal after data conjugate mapping is as follows.

$$d(n) = \sum_{i=0}^{N-1} D_i'.e^{j\frac{2\pi}{N}in} = \sum_{k=0}^{\frac{N}{2}-1}\left[D_k.e^{j\frac{2\pi}{N}2kn} - D_k^*.e^{j\frac{2\pi}{N}(2k+1)n}\right], \qquad for\ 0 \le n < N \tag{37}$$

where, N is the total number of sub-carriers, D_k is data symbol for the k-th parallel branch and D_k' is the i–th sub-carrier data symbol after data-conjugate mapping. $d(n)$ is corrupted by the phase noise in the transmitter (TX) local oscillator. Furthermore, the received signal is influenced by the phase noise of receiver (RX) local oscillator. So, it is expressed as:

$$r(t) = \left\{\left(s(t).e^{j\theta_{TX}(t)}\right) \otimes h(t) + w(t)\right\}.e^{j\theta_{RX}(t)} \tag{38}$$

where $s(t)$ is the transmitted signal, $w(t)$ is the white Gaussian noise and $h(t)$ is the channel impulse response. $\theta_{TX}(t)$ and $\theta_{RX}(t)$are the time varying phase noise processes generated in the transceiver oscillators. Here, it is assumed that, $\theta_{TX}(t) = \theta_{RX}(t) = \theta(t)$ and $\theta_{tot}(t) = \theta_{TX}(t) + \theta_{RX}(t)$ for simple analysis. In the original OFDM system without ICI self-cancellation method, the k-th sub-carrier signal after FFT can be written as:

$$Y_k = \frac{1}{N}\sum_{l=0}^{N-1} D_l.H_l \sum_{m=0}^{N-1} e^{j\left(\frac{2\pi}{N}(l-k)m+\theta_{tot}(m)\right)} + w_k \tag{39}$$

In the data-conjugate method, the sub-carrier data is mapped in the form of $D_{2k}^{'} = D_k, D_{2k+1}^{'} = -D_k^*$. Therefore, the $2k$-th sub-carrier data after FFT in the receiver is arranged as:

$$Y_{2k} = \sum_{l=0}^{\frac{N}{2}-1}[D_l H_{2l} Q_{2l-2k} - D_l^* H_{2l+1} Q_{2l+1-2k}] + w_{2k} \tag{40}$$

$$Q_L = \frac{1}{N}\sum_{m=0}^{N-1} e^{j\left(\frac{2\pi}{N}Lm+\theta_{tot}(m)\right)} \tag{41}$$

w_{2k} is a sampled FFT version of the complex AWGN multiplied by the phase noise of RX local oscillator, and random phase noise process $\theta_{tot}(m)$ is equal to $\theta_{TX}(m) + \theta_{RX}(m)$.
Similarly, the $2k+1$-th sub-carrier signal is expressed as:

$$Y_{2k+1} = \sum_{l=0}^{\frac{N}{2}-1}[D_l H_{2l} Q_{2l-2k-1} - D_l^* H_{2l+1} Q_{2l-2k}] + w_{2k+1} \tag{42}$$

In the (40) and (42), $l = k$ corresponds to the original signal with CPE, and $l \neq k$ corresponds to the ICI component. In the receiver, the decision variable Z_k' of the k-th symbol is found from the difference of adjacent sub-carrier signals affected by phase noise. That is,

$$\begin{aligned} Z_k^{'} &= \frac{(Y_{2k} - Y_{2k+1}^*)}{2} = \frac{1}{2}X_k(H_{2k}Q_0 + H_{2k+1}^*Q_0^*) - \frac{1}{2}D_k^*(H_{2k+1}Q_1 + H_{2l}^*Q_{-1}^*) \\ &+ \frac{1}{2}\sum_{\substack{l=0 \\ l\neq k}}^{\frac{N}{2}-1}\{D_l[H_{2l}Q_{2l-2k} + H_{2l+1}^*Q_{2l-2k}^*] - D_l^*[H_{2l+1}Q_{2l+1-2k} + H_{2l}^*Q_{2l-2k-1}^*]\} + w_k \end{aligned} \tag{43}$$

where $w_k = (1/2)(w_{2k} - w_{2k+1}^*)$ is the AWGN of the k–th parallel branch data in the receiver. When channel is flat, frequency response of channel $\{H_k\}$ equals 1. Z'_k is as follows.

$$Z'_k = D_k + \frac{1}{2}\sum_{\substack{l=0 \\ l\neq k}}^{\frac{N}{2}-1}\{D_l[Q_{2l-2k} + Q^*_{2l-2k}] - D^*_l[Q_{2l+1-2k} + Q^*_{2l-2k-1}]\} + w_k \tag{44}$$

3.3 CPE, ICI and CIR analysis

A. Original OFDM

In the original OFDM, the k-th sub-carrier signal after FFT is as follows:

$$Y_k = D_k Q_0 + \sum_{\substack{l=0 \\ l\neq k}}^{N-1} D_l . Q_{l-k} + w_k \tag{45}$$

The received desired signal power on the k-th sub-carrier is:

$$E[|Y_{k1}|^2] = E[|D_k Q_0|^2] \tag{46}$$

ICI power is:

$$E[|Y_{k2}|^2] = E\left[\left|\sum_{\substack{l=0 \\ l\neq k}}^{N-1} D_l Q_{l-k}\right|^2\right] \tag{47}$$

Transmitted signal is supposed to have zero mean and statistically independence. So, the CIR of the original OFDM transmission method is as follows:

$$CIR = \frac{|Q_0|^2}{\sum_{\substack{l=0 \\ l\neq k}}^{N-1}|Q_{l-k}|^2} = \frac{|Q_0|^2}{\sum_{l=1}^{N-1}|Q_l|^2} \tag{48}$$

B. Data-conversion method

In the data-conversion ICI self-cancellation method, the data are remapped in the form of $D'_{2k} = D_k$, $D'_{2k+1} = -D_k$.
So, the desired signal is recovered in the receiver as follows:

$$\begin{aligned} Z'_k = \frac{(Y_{2k} - Y_{2k+1})}{2} &= D_k + \frac{1}{2}D_k[-Q_{-1} + 2(Q_0 - 1) - Q_1] \\ &+ \frac{1}{2}\sum_{\substack{l=0 \\ l\neq k}}^{\frac{N}{2}-1} D_l[-Q_{2l-2k-1} + Q_{2l-2k} - Q_{2l-2k+1}] + w_k \end{aligned} \tag{49}$$

CPE is as follows:

$$CPE = \frac{j2D_k}{N}\sum_{m=0}^{N-1} sin^2\left(\frac{\pi m}{N}\right)\theta_{tot}(m) \tag{50}$$

ICI component of the k-th sub-carrier is as follows:

$$ICI = \frac{2j}{N}\sum_{\substack{l=0\\l\neq k}}^{\frac{N}{2}-1} D_l \sum_{m=0}^{N-1} sin^2\left(\frac{\pi m}{N}\right).exp\left(\frac{j4\pi m(l-k)}{N}\right)\theta_{tot}(m) \tag{51}$$

So

$$CIR = \frac{|-Q_{-1}+2Q_0-Q_1|^2}{\sum_{\substack{l=0\\l\neq k}}^{\frac{N}{2}-1}|-Q_{2l-2k-1}+2Q_{2l-2k}-Q_{2l-2k+1}|^2} = \frac{|-Q_{-1}+2Q_0-Q_1|^2}{\sum_{l=1}^{\frac{N}{2}-1}|-Q_{2l-1}+2Q_{2l}-Q_{2l+1}|^2} \tag{52}$$

C. Data-conjugate method

In the data conjugate method, the decision variable can be written as follows:

$$Z'_k = D_k + \frac{1}{2}\sum_{\substack{l=0\\l\neq k}}^{\frac{N}{2}-1}\{D_l[Q_{2l-2k}+Q^*_{2l-2k}] - D^*_l[Q_{2l+1-2k}+Q^*_{2l-2k-1}]\} + w_k \tag{53}$$

Through the same calculation, CPE, ICI and CIR of the data conjugate method are found.

$$CPE = 0 \tag{54}$$

The fact CPE is zero is completely different from the data conversion method whose CPE is not zero like (14).

Then, ICI of data conjugate method is:

$$ICI = \frac{1}{N}\sum_{\substack{l=0\\l\neq k}}^{\frac{N}{2}-1}\sum_{m=0}^{N-1} sin\left(\frac{4\pi m(l-k)}{N}\right).[D^*_l.e^{j\frac{2\pi}{N}m} - D_l].\theta_{tot}(m) \tag{55}$$

The above term is the summation of the signal of the other sub-carriers multiplied by some complex number resulted from an average of phase noise with spectral shift. This component is added into the k-th branch data Z'_k. It may break down the orthogonalities between sub-carriers. So, CIR is:

$$CIR = \frac{4}{\sum_{l=2}^{\frac{N}{2}-1}\left[\left|Q_{2l}+Q^*_{2l}\right|^2 + \left|Q_{2l+1}+Q^*_{2l-1}\right|^2\right]} \tag{56}$$

4. Conclusion

OFDM has been widely used in communication systems to meet the demand for increasing data rates. It is robust over multipath fading channels and results in significant reduction of the transceiver complexity. However, one of its disadvantages is sensitivity to carrier frequency offset which causes attenuation, rotation of subcarriers, and inter-carrier interference (ICI). The ICI is due to frequency offset or may be caused by phase noise.

The undesired ICI degrades the signal heavily and hence degrades the performance of the system. So, ICI mitigation techniques are essential to improve the performance of an OFDM system in an environment which induces frequency offset error in the transmitted signal. In this chapter, the performance of OFDM system in the presence of frequency offset is

analyzed. This chapter investigates different ICI reduction schemes for combating the impact of ICI on OFDM systems. A number of pulse shaping functions are considered for ICI power reduction and the performance of these functions is evaluated and compared using the parameters such as ICI power and CIR. Simulation results show that ISP pulse shapes provides better performance in terms of CIR and ICI reduction as compared to the conventional pulse shapes.
Another ICI reduction method which is described in this chapter is the ICI self cancellation method which does not require very complex hardware or software for implementation. However, it is not bandwidth efficient as there is a redundancy of 2 for each carrier. Among different ICI self cancellation methods, the data-conjugate method shows the best performances compared with the original OFDM, and the data-conversion method since it makes CPE to be zero along with its role in significant reduction of ICI.

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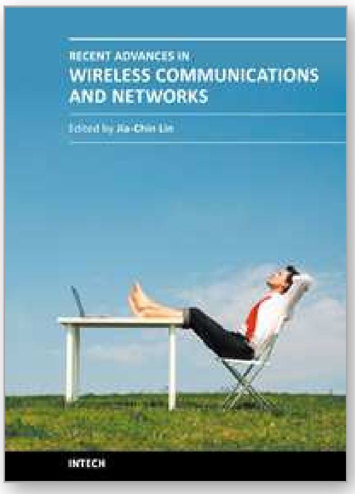

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This book focuses on the current hottest issues from the lowest layers to the upper layers of wireless communication networks and provides â€œreal-timeâ€ research progress on these issues. The authors have made every effort to systematically organize the information on these topics to make it easily accessible to readers of any level. This book also maintains the balance between current research results and their theoretical support. In this book, a variety of novel techniques in wireless communications and networks are investigated. The authors attempt to present these topics in detail. Insightful and reader-friendly descriptions are presented to nourish readers of any level, from practicing and knowledgeable communication engineers to beginning or professional researchers. All interested readers can easily find noteworthy materials in much greater detail than in previous publications and in the references cited in these chapters.