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# Sliding Mode Control for a Class of Multiple Time-Delay Systems

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## 1. Introduction

Time-delay frequently occurs in many practical systems, such as chemical processes, manufacturing systems, long transmission lines, telecommunication and economic systems, etc. Since time-delay is a main source of instability and poor performance, the control problem of time-delay systems has received considerable attentions in literature, such as [1]-[9]. The design approaches adopt in these literatures can be divided into the delay-dependent method [1]-[5] and the delay-independent method [6]-[9]. The delay-dependent method needs an exactly known delay, but the delay-independent method does not. In other words, the delay-independent method is more suitable for practical applications. Nevertheless, most literatures focus on linear time-delay systems due to the fact that the stability analysis developed in the two methods is usually based on linear matrix inequality techniques [10]. To deal with nonlinear time-delay systems, the Takagi-Sugeno (TS) fuzzy model-based approaches [11]-[12] extend the results of controlling linear time-delay systems to more general cases. In addition, some sliding-mode control (SMC) schemes have been applied to uncertain nonlinear time-delay systems in [13]-[15]. However, these SMC schemes still exist some limits as follows: i) specific form of the dynamical model and uncertainties [13]-[14]; ii) an exactly known delay time [15]; and iii) a complex gain design [13]-[15]. From the above, we are motivated to further improve SMC for nonlinear time-delay systems in the presence of matched and unmatched uncertainties.

The fuzzy control and the neural network control have attractive features to keep the systems insensitive to the uncertainties, such that these two methods are usually used as a tool in control engineering. In the fuzzy control, the TS fuzzy model [16]-[18] provides an efficient and effective way to represent uncertain nonlinear systems and renders to some straightforward research based on linear control theory [11]-[12], [16]. On the other hand, the neural network has good capabilities in function approximation which is an indirect compensation of uncertainties. Recently, many fuzzy neural network (FNN) articles are proposed by combining the fuzzy concept and the configuration of neural network, e.g., [19]-[23]. There, the fuzzy logic system is constructed from a collection of fuzzy If-Then rules while the training algorithm adjusts adaptable parameters. Nevertheless, few results using FNN are proposed for time-delay nonlinear systems due to a large computational load and a vast amount of feedback data, for example, see [22]-[23]. Moreover, the training algorithm is difficultly found for time-delay systems.

In this paper, an adaptive TS-FNN sliding mode control is proposed for a class of nonlinear time-delay systems with uncertainties. In the presence of mismatched uncertainties, we introduce a novel sliding surface design to keep the sliding motion insensitive to uncertainties and time-delay. Although the form of the sliding surface is as similar as conventional schemes [13]-[15], a delay-independent sufficient condition for the existence of the asymptotic sliding surface is obtained by appropriately using the Lyapunov-Krasovskii stability method and LMI techniques. Furthermore, the gain condition is transformed in terms of a simple and legible LMI. Here less limitation on the uncertainty is required. When the asymptotic sliding surface is constructed, the ideal and TS-FNN-based reaching laws are derived. The TS-FNN combining TS fuzzy rules and neural network provides a near ideal reaching law. Meanwhile, the error between the ideal and TS-FNN reaching laws is compensated by adaptively gained switching control law. The advantages of the proposed TS-FNN are: i) allowing fewer fuzzy rules for complex systems (since the Then-part of fuzzy rules can be properly chosen); and ii) a small switching gain is used (since the uncertainty is indirectly cancelled by the TS-FNN). As a result, the adaptive TS-FNN sliding mode controller achieves asymptotic stabilization for a class of uncertain nonlinear time-delay systems.

This paper is organized as follows. The problem formulation is given in Section 2. The sliding surface design and ideal sliding mode controller are given in Section 3. In Section 4, the adaptive TS-FNN control scheme is developed to solve the robust control problem of time-delay systems. Section 5 shows simulation results to verify the validity of the proposed method. Some concluding remarks are finally made in Section 6.

## 2. Problem description

Consider a class of nonlinear time-delay systems described by the following differential equation:

$$\begin{aligned}\dot{x}(t) &= (A + \Delta A)x(t) + \sum_{k=1}^h (A_{dk} + \Delta A_{dk})x(t - d_k) \\ &\quad + Bg^{-1}(x)(u(t) + h(\bar{x})) \\ x(t) &= \psi(t), t \in [-d_{\max}, 0]\end{aligned}\quad (1)$$

where  $x(t) \in R^n$  and  $u(t) \in R$  are the state vector and control input, respectively;  $d_k \in R$  ( $k = 1, 2, \dots, h$ ) is an unknown constant delay time with upper bounded  $d_{\max}$ ;  $A$  and  $A_{dk}$  are nominal system matrices with appropriate dimensions;  $\Delta A$  and  $\Delta A_{dk}$  are time-varying uncertainties;  $x(t)$  is defined as  $\bar{x}(t) = [x(t) \ x(t - d_1) \ \dots \ x(t - d_h)]^T$ ;  $h(\cdot)$  is an unknown nonlinear function containing uncertainties;  $B$  is a known input matrix;  $g(\cdot)$  is an unknown function presenting the input uncertainties; and  $\psi(t)$  is the initial of state. In the system (1), for simplicity, we assume the input matrix  $B = [0 \ \dots \ 0 \ 1]^T$  and partition the state vector  $x(t)$  into  $[x_1(t) \ x_2(t)]^T$  with  $x_1(t) \in R^{n-1}$  and  $x_2(t) \in R$ . Accompanying the state partition, the system (1) can be decomposed into the following:

$$\begin{aligned}\dot{x}_1(t) &= (A_{11} + \Delta A_{11})x_1(t) + \sum_{k=1}^h (A_{dk11} + \Delta A_{dk11})x_1(t - d_k) \\ &\quad + (A_{12} + \Delta A_{12})x_2(t) + \sum_{k=1}^h (A_{dk12} + \Delta A_{dk12})x_2(t - d_k)\end{aligned}\quad (2)$$

$$\begin{aligned}\dot{x}_2(t) = & (A_{21} + \Delta A_{21})x_1(t) + \sum_{k=1}^h (A_{dk11} + \Delta A_{dk11})x_1(t - d_k) \\ & + (A_{22} + \Delta A_{22})x_2(t) + \sum_{k=1}^h (A_{dk22} + \Delta A_{dk22})x_2(t - d_k) \\ & + g^{-1}(x)(u(t) + h(\bar{x}(t)))\end{aligned}\quad (3)$$

where  $A_{ij}$ ,  $A_{dkij}$ ,  $\Delta A_{ij}$ , and  $\Delta A_{dkij}$  (for  $i, j = 1, 2$  and  $k = 1, \dots, h$ ) with appropriate dimension are decomposed components of  $A$ ,  $A_{dk}$ ,  $\Delta A$ , and  $\Delta A_{dk}$ , respectively.

Throughout this study we need the following assumptions:

**Assumption 1:** For controllability,  $g(x) > 0$  for  $x(t) \in U_c$ , where  $U_c \subset \mathbb{R}^n$ . Moreover,  $g(x) \in L_\infty$  if  $x(t) \in L_\infty$ .

**Assumption 2:** The uncertainty  $h(\bar{x})$  is bounded for all  $\bar{x}(t)$ .

**Assumption 3:** The uncertain matrices satisfy

$$[\Delta A_{11} \quad \Delta A_{12}] = D_1 C_1 [E_{11} \quad E_{12}] \quad (4)$$

$$[\Delta A_{dk11} \quad \Delta A_{dk12}] = D_2 C_2 [E_{dk11} \quad E_{dk12}] \quad (5)$$

for some known matrices  $D_i$ ,  $C_i$ ,  $E_{1i}$ , and  $E_{dk1i}$  (for  $i = 1, 2$ ) with proper dimensions and unknown matrices  $C_i$  satisfying  $\|C_i\| \leq 1$  (for  $i = 1, 2$ ).

Note that most nonlinear systems satisfy the above assumptions, for example, chemical processes or stirred tank reactor systems, etc. If  $g(x)$  is negative, the matrix  $B$  can be modified such that Assumption 1 is obtained. Assumption 3 often exists in robust control of uncertainties. Since uncertainties  $\Delta A$  and  $\Delta A_d$  are presented, the dynamical model is closer to practical situations which are more complex than the cases considered in [13]-[15].

Indeed, the control objective is to determine a robust adaptive fuzzy controller such that the state  $x(t)$  converges to zero. Since high uncertainty is considered here, we want to derive a sliding-mode control (SMC) based design for the control goal. Note that the system (1) is not the Isidori-Bynes canonical form [21], [24] such that a new design approaches of sliding surface and reaching control law is proposed in the following.

### 3. Sliding surface design

Due to the high uncertainty and nonlinearity in the system (1), an asymptotically stable sliding surface is difficultly obtained in current sliding mode control. This section presents an alternative approach to design an asymptotic stable sliding surface below.

Without loss of generality, let the sliding surface denote

$$S(t) = [-\Lambda \quad 1]x(t) = \tilde{\Lambda}x(t) = 0 \quad (6)$$

where  $\Lambda \in \mathbb{R}^{(n-1)}$  and  $\tilde{\Lambda} = [-\Lambda \quad 1]$  determined later. In the surface, we have  $x_2(t) = \Lambda x_1(t)$ . Thus, the result of sliding surface design is stated in the following theorem.

**Theorem 1:** Consider the system (1) lie in the sliding surface (6). The sliding motion is asymptotically stable independent of delay, i.e.,  $\lim_{t \rightarrow \infty} x_1(t), x_2(t) = 0$ , if there exist positive

symmetric matrices  $X$ ,  $\bar{Q}_k$  and a parameter  $\Lambda$  satisfying the following LMI:

Given  $\varepsilon > 0$ ,  
 Subject to  $X > 0$ ,  $\bar{Q}_k > 0$

$$\begin{bmatrix} N_{11} & (*) \\ N_{21} & -I_\varepsilon \end{bmatrix} < 0 \quad (8)$$

where

$$N_{11} = \begin{bmatrix} N_0 & (*) & (*) & (*) \\ XA_{d11}^T + K^T A_{d12}^T & -\bar{Q}_1 & (*) & (*) \\ \vdots & \vdots & \ddots & (*) \\ XA_{dh11}^T + K^T A_{dh12}^T & 0 & \cdots & -\bar{Q}_h \end{bmatrix}$$

$$N_{21} = \begin{bmatrix} E_{11}X + A_{12}K & 0 & 0 & 0 \\ 0 & E_{111}X + E_{112}K & \cdots & E_{h11}X + E_{h12}K \\ D_1^T & 0 & \ddots & 0 \\ D_2^T & 0 & \cdots & 0_h \end{bmatrix}$$

$$N_0 = A_{11}X + XA_{11}^T + A_{12}K + K^T A_{12}^T + \sum_{k=1}^h \bar{Q}_k ;$$

$K = \Lambda X$ ;  $I_\varepsilon = \text{diag}\{\varepsilon I_a, \varepsilon I_a, \varepsilon^{-1} I_b, \varepsilon^{-1} I_b\}$  in which  $I_a, I_b$  are identity matrices with proper dimensions; and  $(*)$  denotes the transposed elements in the symmetric positions. ■

**Proof:** When the system (1) lie in the sliding surface (6), the sliding motion is described by the dynamics (7). To analysis the stability of the sliding motion, let us define the following Lyapunov-Krasoviskii function

$$V(t) = x_1^T(t) P x_1(t) + \sum_{k=1}^h \int_{t-d_k}^t x_1^T(v) Q_k x_1(v) dv$$

where  $P > 0$  and  $Q_k > 0$  are symmetric matrices. The time derivative of  $V(t)$  along the dynamics (7) is

$$\dot{V}(t) = \bar{x}^T(t) (\Omega_1 + \Omega_2) \bar{x}(t)$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_{10} & (*) & (*) & (*) \\ (A_{d111} + A_{d112}\Lambda)^T P & -Q_1 & (*) & (*) \\ \vdots & \vdots & \ddots & (*) \\ (A_{dh11} + A_{dh12}\Lambda)^T P & 0 & \cdots & -Q_h \end{bmatrix}$$

$$\Omega_{10} = (A_{11} + \Lambda A_{12})^T P + P(A_{11} + \Lambda A_{12}) + \sum_{k=1}^h Q_k$$

$$\Omega_2 = \begin{bmatrix} \Omega_{20} & (*) & (*) & (*) \\ [D_2 C_2 (E_{111} + E_{112} \Lambda)]^T P & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ [D_2 C_2 (E_{h11} + E_{h12} \Lambda)]^T P & 0 & \cdots & 0 \end{bmatrix}$$

$$\Omega_{20} = PD_1 C_1 (E_{11} + E_{12} \Lambda) + [D_1 C_1 (E_{11} + E_{12} \Lambda)]^T P$$

Note that the second term  $\Omega_2$  can be further rewritten in the form:

$$\Omega_2 = \bar{D} \bar{C} \bar{E} + \bar{E}^T \bar{C}^T \bar{D}^T$$

where

$$\bar{D} = \begin{bmatrix} PD_1 & PD_2 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \quad \bar{C}^T = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} E_{11} + E_{12} \Lambda & 0 & 0 & 0 \\ 0 & E_{111} + E_{112} \Lambda & \cdots & E_{h11} + E_{h12} \Lambda \end{bmatrix}$$

with  $\bar{C}$  satisfies  $\bar{C}^T \bar{C} \leq I_d$  for identity matrix  $I_d$  from Assumption 3. According to the matrix inequality lemma [25] (see Appendix I) and the decomposition (9), the stability condition  $\Omega < 0$  is equivalent to

$$\Omega_1 + \begin{bmatrix} \bar{E}^T & \bar{D} \end{bmatrix} I_\varepsilon^{-1} \begin{bmatrix} \bar{E} \\ \bar{D}^T \end{bmatrix} < 0$$

After applying the Schur complement to the above inequality, we further have

$$\begin{bmatrix} \Omega_1 & (*) \\ M_{21} & -I_\varepsilon \end{bmatrix} < 0$$

where

$$M_{21} = \begin{bmatrix} E_{11} + E_{12} \Lambda & 0 & 0 & 0 \\ 0 & E_{121} + E_{122} \Lambda & \cdots & E_{h21} + E_{h22} \Lambda \\ D_1^T P & 0 & \ddots & 0 \\ D_2^T P & 0 & \cdots & 0_h \end{bmatrix}$$

By premultiplying and postmultiplying above inequality by a symmetric positive-definite matrix  $\text{diag}\{X\bar{I}_a, \bar{I}_b\}$  with  $\bar{I}_a, \bar{I}_b$  are identity matrices with proper dimensions, the LMI addressed in (8) is obtained with  $X = P^{-1}$  and  $\bar{Q}_k = XQ_kX$ . Therefore, if the LMI problem

has a feasible solution, then the sliding dynamical system (7) is asymptotically stable, i.e.,  $\lim_{t \rightarrow \infty} x_1(t) = 0$ . In turn, from the fact  $x_2(t) = \Lambda x_1(t)$  in the sliding surface, the state  $x_2(t)$  will asymptotically converge to zero as  $t \rightarrow \infty$ . Moreover, since the gain condition (8) does not contain the information of the delay time, the stability is independent of the delay.

After solving the LMI problem (8), the sliding surface is constructed by  $\Lambda = KP$ . Therefore, the LMI-based sliding surface design is completed for uncertain time-delay systems.

Note that the main contribution of Theorem 1 is solving the following problems: i) the sliding surface gain  $\Lambda$  appears in the delayed term  $x_1(t - d_k)$  such that the gain design is highly coupled; and ii) the mismatched uncertainties (e.g.,  $\Delta A_{11}$ ,  $\Delta A_{dk11}$ ,  $\Delta A_{12}$ ,  $\Delta A_{dk12}$ ) is considered in the design. Compared to current literature, this study proposes a valid and straightforward LMI-based sliding mode control for highly uncertain time-delay systems.

The design of exponentially stable sliding surface, a coordinate transformation is used  $\sigma(t) = e^{\gamma t} x_1(t)$  with an attenuation rate  $\gamma > 0$ . When  $\sigma(t)$  is asymptotically stable, the state  $x_1(t)$  exponentially stable is guaranteed (see **Appendix II or [26],[28] in detail**).

Based on Theorem 1, the control goal becomes to drive the system (1) to the sliding surface defined in (6). To this end, let us choose a Lyapunov function candidate  $V_s = g(x)S^2 / 2$ . Taking the derivative the Lyapunov  $V_s$  along with (1), it renders to

$$\begin{aligned} \dot{V}_s(t) &= g(x)S(t)\dot{S}(t) + \dot{g}(x)S^2(t) / 2 \\ &= S(t) \left[ g(x)\Lambda \left\{ (A + \Delta A)x(t) + \sum_{k=1}^h (A_{dk} + \Delta A_{dk})x(t - d_k) \right\} \right] \\ &\quad + \dot{g}(x)S^2(t) / 2 + u(t) + h(\bar{x}) \end{aligned}$$

If the plant dynamics and delay-time are exactly known, then the control problem can be solved by the so-called feedback linearization method [24]. In this case, the ideal control law  $u^*$  is set to

$$\begin{aligned} u^*(t) &= -\{g(x)\Lambda \left[ \sum_{k=1}^h (A_{dk} + \Delta A_{dk})x(t - d_k) \right. \\ &\quad \left. + (A + \Delta A)x(t) + g(x)S^2(t) / 2 + k_f S(t) + h(\bar{x}) \right\} \end{aligned} \quad (10)$$

where  $k_f$  is a positive control gain. Then the ideal control law (10) yields  $\dot{V}_s(t)$  satisfying  $\dot{V}_s(t) < 0$ .

Since  $V_s(t) > 0$  and  $\dot{V}_s(t) < 0$ , the error signal  $S(t)$  converges to zero in an asymptotic manner, i.e.,  $\lim_{t \rightarrow \infty} S(t) = 0$ . This implies that the system (1) reaches the sliding surface  $S(t) = 0$  for any start initial conditions. Therefore, the ideal control law provides the following result. Unfortunately, the ideal control law (10) is unrealizable in practice applications due to the poor modeled dynamics. To overcome this difficulty, we will present a robust reaching control law by using an adaptive TS-FNN control in next section.

#### 4. TS-FNN-based sliding mode control

In control engineering, neural network is usually used as a tool for modeling nonlinear system functions because of their good capabilities in function approximation. In this



section, the TS-FNN [26] is proposed to approximate the ideal sliding mode control law  $u^*(t)$ . Indeed, the FNN is composed of a collection of T-S fuzzy IF-THEN rules as follows:

Rule  $i$ :

IF  $\bar{z}_1$  is  $\bar{G}_{i1}$  and  $\dots$  and  $\bar{z}_{ni}$  is  $\bar{G}_{ni}$  THEN

$$u_n(t) = z_0 v_{i0} + z_1 v_{i1} + \dots + z_{nv} v_{inv} = z^T v_i$$

for  $i = 1, 2, \dots, n_R$ , where  $n_R$  is the number of fuzzy rules;  $\bar{z}_1 \sim \bar{z}_{ni}$  are the premise variables composed of available signals;  $u_n$  is the fuzzy output with tunable  $v_i = [v_{i0} \ v_{i1} \ \dots \ v_{inv}]^T$  and properly chosen signal  $z = [z_0 \ z_1 \ \dots \ z_{nv}]^T$ ;  $\bar{G}_{ij}(\bar{z}_j)$  ( $j = 1, 2, \dots, n_i$ ) are the fuzzy sets with Gaussian membership functions which have the form  $\bar{G}_{ij}(\bar{z}_j) = \exp(-(\bar{z}_j - m_{ij})^2 / (\sigma_{ij}^2))$  where  $m_{ij}$  is the center of the Gaussian function; and  $\sigma_{ij}$  is the variance of the Gaussian function.

Using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier, the inferred output of the fuzzy neural network is  $u_n = \sum_{i=1}^{n_R} \mu_i(\bar{z}) z^T v_i$  where  $\mu_i(\bar{z}) = \bar{\omega}_i(\bar{z}) / \sum_{i=1}^{n_R} \bar{\omega}_i(\bar{z})$ ,  $\bar{z} = [\bar{z}_1 \ \bar{z}_2 \ \dots \ \bar{z}_{ni}]^T$  and  $\bar{\omega}_i(\bar{z}) = \prod_{j=1}^{n_i} G_{ij}(\bar{z}_j)$ . For simplification, define two auxiliary signals

$$\xi = [z^T \mu_1 \quad z^T \mu_2 \quad \dots \quad z^T \mu_{n_R}]^T$$

$$\theta = [v_1^T \quad v_2^T \quad \dots \quad v_{n_R}^T]^T.$$

In turn, the output of the TS-FNN is rewritten in the form:

$$u_n(t) = \xi^T \theta \quad (13)$$

Thus, the above TS-FNN has a simple structure, which is easily implemented in comparison of traditional FNN. Moreover, the signal  $z$  can be appropriately selected for more complex function approximation. In other words, we can use less fuzzy rules to achieve a better approximation.

According to the uniform approximation theorem [19], there exists an optimal parametric vector  $\theta^*$  of the TS-FNN which arbitrarily accurately approximates the ideal control law  $u^*(t)$ . This implies that the ideal control law can be expressed in terms of an optimal TS-FNN as  $u^*(t) = \xi^T \theta + \bar{\varepsilon}(x)$  where  $\bar{\varepsilon}(x)$  is a minimum approximation error which is assumed to be upper bounded in a compact discussion region. Meanwhile, the output of the TS-FNN is further rewritten in the following form:

$$u_n = u^* \xi^T \tilde{\theta} - \bar{\varepsilon}(x) \quad (14)$$

where  $\tilde{\theta} = \theta - \theta^*$  is the estimation error of the optimal parameter. Then, the tuning law of the FNN is derived below.

Based on the proposed TS-FNN, the overall control law is set to

$$u(t) = u_n(t) + u_c(t) \quad (15)$$



where  $u_n(t)$  is the TS-FNN controller part defined in (13); and  $u_c(t)$  is an auxiliary compensation controller part determined later. The TS-FNN control  $u_n(t)$  is the main tracking controller part that is used to imitate the idea control law  $u^*(t)$  due to high uncertainties, while the auxiliary controller part  $u_c(t)$  is designed to cope with the difference between the idea control law and the TS-FNN control. Then, applying the control law (15) and the expression form of  $u_n(t)$  in (10), the error dynamics of  $S$  is obtained as follows:

$$\begin{aligned} g(x)\dot{S}(t) &= g(x)\Lambda \left[ (A + \Delta A)x(t) + \sum_{k=1}^h (A_{dk} + \Delta A_{dk})x_1(t - d_k) \right] \\ &\quad + h(\bar{x}) + u^*(t) + \xi^T \tilde{\theta} - \bar{\varepsilon}(x) + u_c(t) \\ &= -k_f S(t) - \frac{1}{2} \dot{g}(x) S(t) + \xi^T \tilde{\theta} - \bar{\varepsilon}(x) + u_c(t) \end{aligned}$$

where the definition of  $u^*(t)$  in (10) has been used. Now, the auxiliary controller part and tuning law of FNN are stated in the following.

**Theorem 2:** Consider the uncertain time-delay system (1) using the sliding surface designed by Theorem 1 and the control law (15) with the TS-FNN controller part (14) and the auxiliary controller part

$$u_n(t) = -\hat{\delta} \operatorname{sgn}(S(t))$$

The controller is adaptively tuned by

$$\dot{\theta}(t) = -\eta_\theta S(t) \xi \tag{17}$$

$$\dot{\hat{\delta}}(t) = -\eta_\delta |S(t)| \tag{18}$$

where  $\eta_\theta$  and  $\eta_\delta$  are positive constants. The closed-loop error system is guaranteed with asymptotic convergence of  $S(t)$ ,  $x_1(t)$ , and  $x_2(t)$ , while all adaptation parameters are bounded. ■

**Proof:** Consider a Lyapunov function candidate as

$$V_n(t) = \frac{1}{2} (g(x) S^2(t) + \frac{1}{\eta_\theta} \tilde{\theta}(t)^T \tilde{\theta}(t) + \frac{1}{\eta_\delta} \tilde{\delta}^2(t))$$

where  $\tilde{\delta}(t) = \hat{\delta}(t) - \delta$  is the estimation error of the bound of  $\bar{\varepsilon}(x)$  (i.e.,  $\sup_t |\bar{\varepsilon}(x)| \leq \delta$ ). By taking the derivative the Lyapunov  $V_n(t)$  along with (16), we have

$$\begin{aligned} \dot{V}_n(t) &= \frac{1}{2} g(x) S(t) \dot{S}(t) + \frac{1}{2} \dot{g}(x) S^2(t) + \frac{1}{\eta_\theta} \tilde{\theta}(t)^T \dot{\tilde{\theta}}(t) + \frac{1}{\eta_\delta} \tilde{\delta}(t) \dot{\tilde{\delta}}(t) \\ &= -k_f S^2(t) - S(t) \bar{\varepsilon}(x) - \delta |S(t)| + S(t) \xi^T \tilde{\theta}(t) + \frac{1}{\eta_\theta} \tilde{\theta}(t)^T \dot{\tilde{\theta}}(t) \\ &\quad - (\hat{\delta}(t) - \delta) |S(t)| + \frac{1}{\eta_\delta} \tilde{\delta}(t) \dot{\tilde{\delta}}(t) \end{aligned}$$

When substituting the update laws (17), (18) into the above,  $V_n(t)$  further satisfies

$$\begin{aligned}\dot{V}_n(t) &= -k_f S^2(t) - S(t)\bar{\varepsilon}(x) - \delta|S(t)| \\ &\leq -k_f S^2(t) - (\delta - |\bar{\varepsilon}(x)|)|S(t)| \\ &\leq -k_f S(t)\end{aligned}$$

Since  $V_n(t) > 0$  and  $\dot{V}_n(t) < 0$ , we obtain the fact that  $V_n(t) \leq V_n(0)$ , which implies all  $S(t)$ ,  $\tilde{\theta}(t)$  and  $\tilde{\delta}(t)$  are bounded. In turn,  $\dot{S}(t) \in L_\infty$  due to all bounded terms in the right-hand side of (16). Moreover, integrating both sides of the above inequality, the error signal  $S(t)$  is  $L_2$ -gain stable as

$$k_f \int_0^t S^2(\tau) d\tau \leq V_n(0) - V_n(t) \leq V_n(0)$$

where  $V_n(0)$  is bounded and  $V_n(t)$  is non-increasing and bounded. As a result, combining the facts that  $S(t)$ ,  $\dot{S}(t) \in L_\infty$  and  $S(t) \in L_2$  the error signal  $S(t)$  asymptotically converges to zero as  $t \rightarrow \infty$  by Barbalat's lemma. Therefore, according to Theorem 1, the state  $x(t)$  will be asymptotically sliding to the origin. The results will be similar when we replace another FNN[26] or NN[27] with the TS-FNN, but the slight different to transient.

## 5. Simulation results

In this section, the proposed TS-FNN sliding mode controller is applied to two uncertain time-delay system.

**Example 1:** Consider an uncertain time-delay system described by the dynamical equation (1) with  $x(t) = [x_1(t) \ x_2(t) \ x_3(t)]^T$ ,

$$\begin{aligned}A + \Delta A(t) &= \begin{bmatrix} -10 + \sin(t) & 1 & 1 + \sin(t) \\ 1 & -8 - \cos(t) & 1 - \cos(t) \\ 5 + \cos(t) & 4 + \sin(t) & 2 + \cos(t) \end{bmatrix} \\ A_d + \Delta A_d(t) &= \begin{bmatrix} 1 + \sin(t) & 0 & 1 + \sin(t) \\ 0 & 1 + \cos(t) & 1 + \cos(t) \\ 3 + \sin(t) & 4 + \cos(t) & 2 + \sin(t) \end{bmatrix}\end{aligned}$$

$$B = [0 \ 0 \ 1]^T, \quad g(\bar{x}) = 1 \text{ and } h(\bar{x}) = 0.5\|x\| + \|x(t-d)\| + \sin(t).$$

It is easily checked that Assumptions 1~3 are satisfied for the above system. Moreover, for Assumption 3, the uncertain matrices  $\Delta A_{11}(t)$ ,  $\Delta A_{12}(t)$ ,  $\Delta A_{d11}(t)$ , and  $\Delta A_{d12}(t)$  are decomposed with

$$\begin{aligned}D_1 = D_2 = E_{11} = E_{21} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_{12} = E_{22} = [1 \ 1]^T, \\ C_1(t) &= \begin{bmatrix} \sin(t) & 0 \\ 0 & -\cos(t) \end{bmatrix}, \quad C_2(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}\end{aligned}$$

First, let us design the asymptotic sliding surface according to Theorem 1. By choosing  $\varepsilon = 0.2$  and solving the LMI problem (8), we obtain a feasible solution as follows:

$$\Lambda = [0.4059 \quad 0.4270]$$

$$P = \begin{bmatrix} 9.8315 & 0.2684 \\ 0.2684 & 6.1525 \end{bmatrix},$$

$$Q = \begin{bmatrix} 85.2449 & 2.9772 \\ 2.9772 & 51.2442 \end{bmatrix}$$

The error signal  $S$  is thus created from (6).

Next, the TS-FNN (11) is constructed with  $n_i = 1$ ,  $n_R = 8$ , and  $n_v = 4$ . Since the T-S fuzzy rules are used in the FNN, the number of the input of the TS-FNN can be reduced by an appropriate choice of THEN part of the fuzzy rules. Here the error signal  $S$  is taken as the input of the TS-FNN, while the discussion region is characterized by 8 fuzzy sets with Gaussian membership functions as (12). Each membership function is set to the center  $m_{ij} = -2 + 4(i-1)/(n_R - 1)$  and variance  $\sigma_{ij} = 10$  for  $i = 1, \dots, n_R$  and  $j = 1$ . On the other hand, the basis vector of THEN part of fuzzy rules is chosen as  $z = [1 \quad x_1(t) \quad x_2(t) \quad x_3(t)]^T$ . Then, the fuzzy parameters  $v_j$  are tuned by the update law (17) with all zero initial condition (i.e.,  $v_j(0) = 0$  for all  $j$ ).

In this simulation, the update gains are chosen as  $\eta_\theta = 0.01$  and  $\eta_\delta = 0.01$ . When assuming the initial state  $x(0) = [2 \quad 1 \quad 1]^T$  and delay time  $d(t) = 0.2 + 0.15\cos(0.9t)$ , the TS-FNN sliding controller (17) designed from Theorem 3 leads to the control results shown in Figs. 1 and 2. The trajectory of the system states and error signal  $S(t)$  asymptotically converge to zero. Figure 3 shows the corresponding control effort.

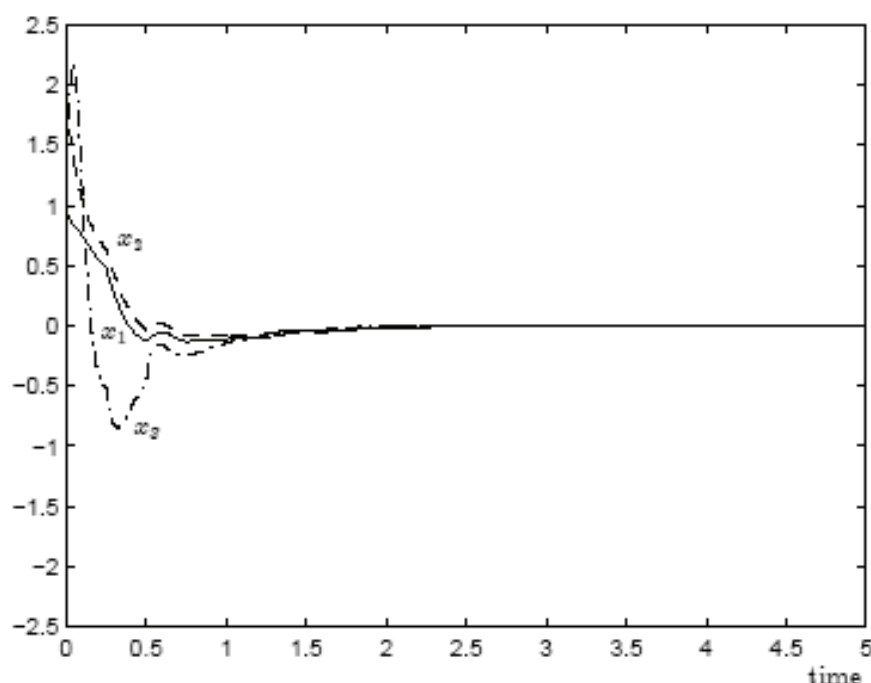


Fig. 1. Trajectory of states  $x_1(t)$  (solid);  $x_2(t)$  (dashed);  $x_3(t)$  (dotted).

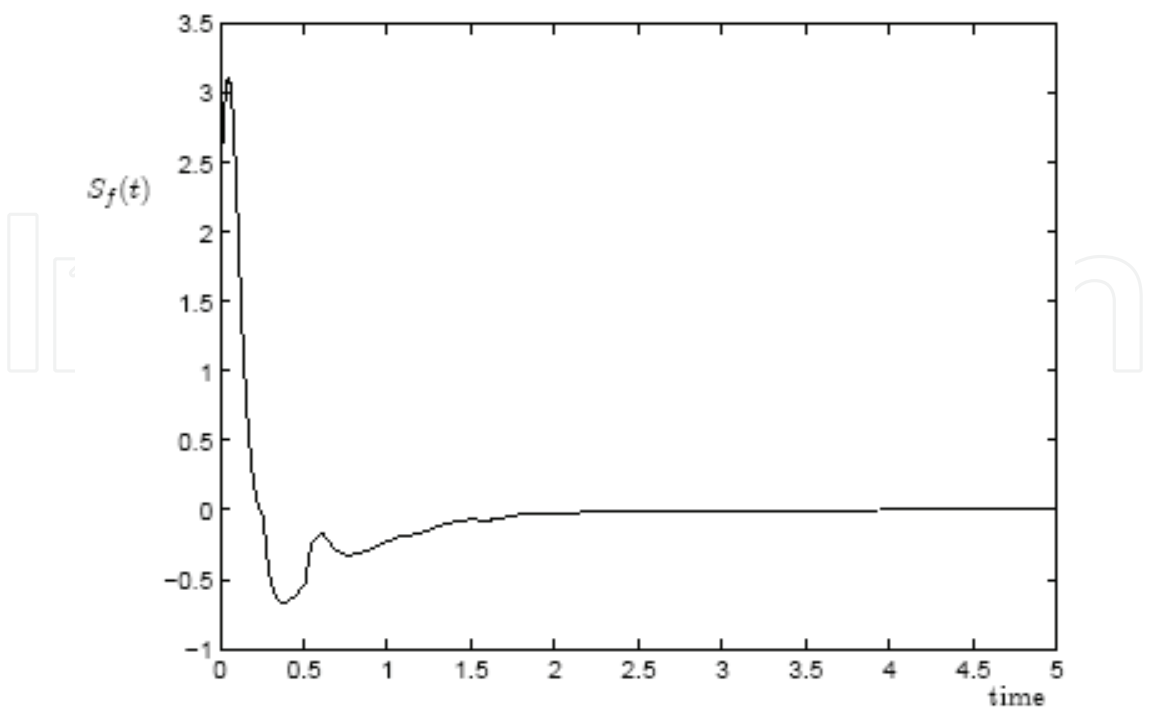


Fig. 2. Dynamic sliding surface  $S(t)$ .

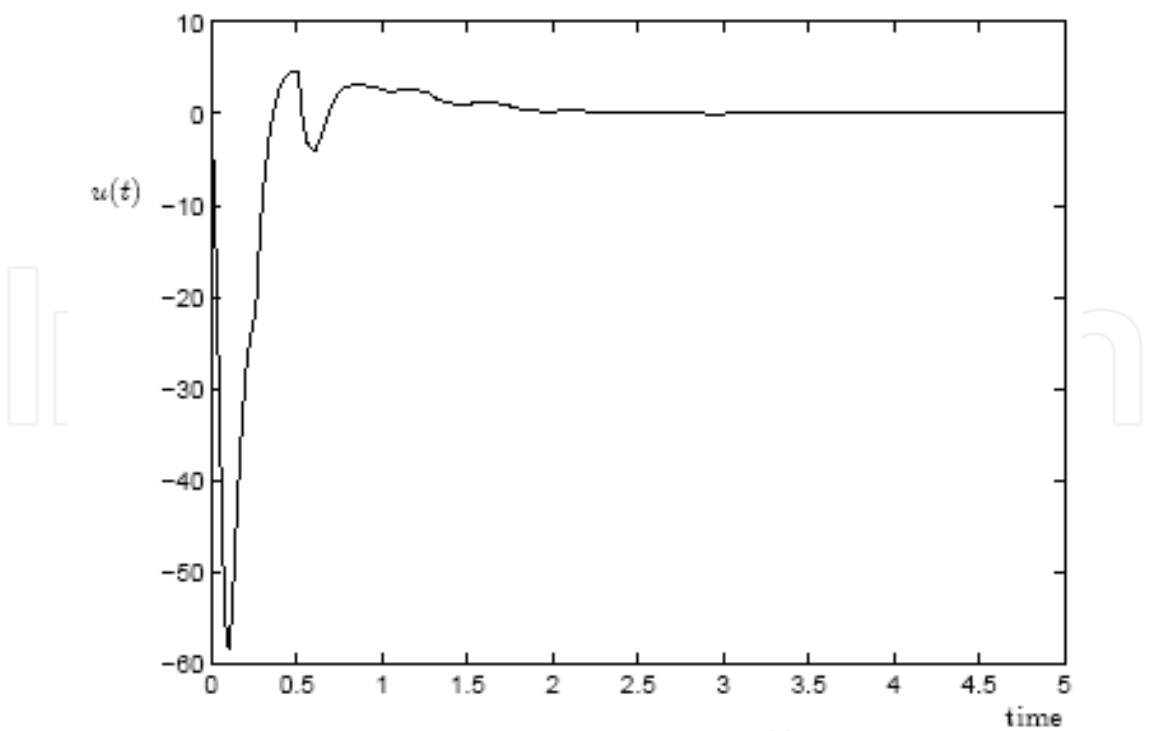


Fig. 3. Control effort  $u(t)$ .

**Example 2:** Consider a chaotic system with multiple time-dely system. The nonlinear system is described by the dynamical equation (1) with  $x(t) = [x_1(t) \ x_2(t)]$ ,

$$\begin{aligned}\dot{x}(t) = & (A + \Delta A)x(t) + Bg^{-1}(x)(u(t) + h(\bar{x})) \\ & + (A_{d1} + \Delta A_{d1})x(t - 0.02) \\ & + (A_{d2} + \Delta A_{d2})x(t - 0.015)\end{aligned}$$

where  $g^{-1}(x) = 4.5$ ,

$$A = \begin{bmatrix} 0 & 2.5 \\ -\frac{1}{2.5} & -0.1 \end{bmatrix}, \Delta A = \begin{bmatrix} \sin(t) & \sin(t) \\ 0 & 0 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.01 \end{bmatrix}, \Delta A_{d1} = \begin{bmatrix} \cos(t) & \cos(t) \\ \cos(t) & \sin(t) \end{bmatrix}$$

$$A_{d2} = \begin{bmatrix} 0 & 0 \\ 0.01 & 0.01 \end{bmatrix}, \Delta A_{d2} = \begin{bmatrix} \cos(t) & \cos(t) \\ \sin(t) & \cos(t) \end{bmatrix},$$

$$\begin{aligned}h(\bar{x}) = & \frac{1}{4.5} \left[ -\left(\frac{1}{2.5}x_1(t)\right)^3 + 0.01x_2^2(t - 0.02) \right. \\ & \left. + 0.01x_2^2(t - 0.015) + 25\cos(t) \right]\end{aligned}$$

If both the uncertainties and control force are zero the nonlinear system is chaotic system (c.f. [23]). It is easily checked that Assumptions 1~3 are satisfied for the above system. Moreover, for Assumption 3, the uncertain matrices  $\Delta A_{11}$ ,  $\Delta A_{12}$ ,  $\Delta A_{d11}$ ,  $\Delta A_{d12}$ ,  $\Delta A_{d21}$ , and  $\Delta A_{d22}$  are decomposed with

$$D_1 = D_2 = E_{11} = E_{21} = E_{111} = E_{112} = E_{211} = E_{212} = 1$$

$$C_1 = \sin(t), C_2 = \cos(t)$$

First, let us design the asymptotic sliding surface according to Theorem 1. By choosing  $\varepsilon = 0.2$  and solving the LMI problem (8), we obtain a feasible solution as follows:  $\Lambda = 1.0014$ ,  $P = 0.4860$ , and  $Q_1 = Q_2 = 0.8129$ . The error signal  $S(t)$  is thus created.

Next, the TS-FNN (11) is constructed with  $n_i = 1$ ,  $n_R = 8$ , and. Since the T-S fuzzy rules are used in the FNN, the number of the input of the TS-FNN can be reduced by an appropriate choice of THEN part of the fuzzy rules. Here the error signal  $S$  is taken as the input of the TS-FNN, while the discussion region is characterized by 8 fuzzy sets with Gaussian membership functions as (12). Each membership function is set to the center  $m_{ij} = -2 + 4(i - 1) / (n_R - 1)$  and variance  $\sigma_{ij} = 5$  for  $i = 1, \dots, n_R$  and  $j = 1$ . On the other hand, the basis vector of THEN part of fuzzy rules is chosen as  $z = [75 \ x_1(t) \ x_2(t)]^T$ . Then, the fuzzy parameters  $v_j$  are tuned by the update law (17) with all zero initial condition (i.e.,  $v_j(0) = 0$  for all  $j$ ).

In this simulation, the update gains are chosen as  $\eta_\theta = 0.01$  and  $\eta_\delta = 0.01$ . When assuming the initial state  $x(0) = [2 \ -2]$ , the TS-FNN sliding controller (15) designed from Theorem 3

leads to the control results shown in Figs. 4 and 5. The trajectory of the system states and error signal  $S$  asymptotically converge to zero. Figure 6 shows the corresponding control effort. In addition, to show the robustness to time-varying delay, the proposed controller set above is also applied to the uncertain system with delay time  $d_2(t) = 0.02 + 0.015\cos(0.9t)$ . The trajectory of the states and error signal  $S$  are shown in Figs. 7 and 8, respectively. The control input is shown in Fig. 9

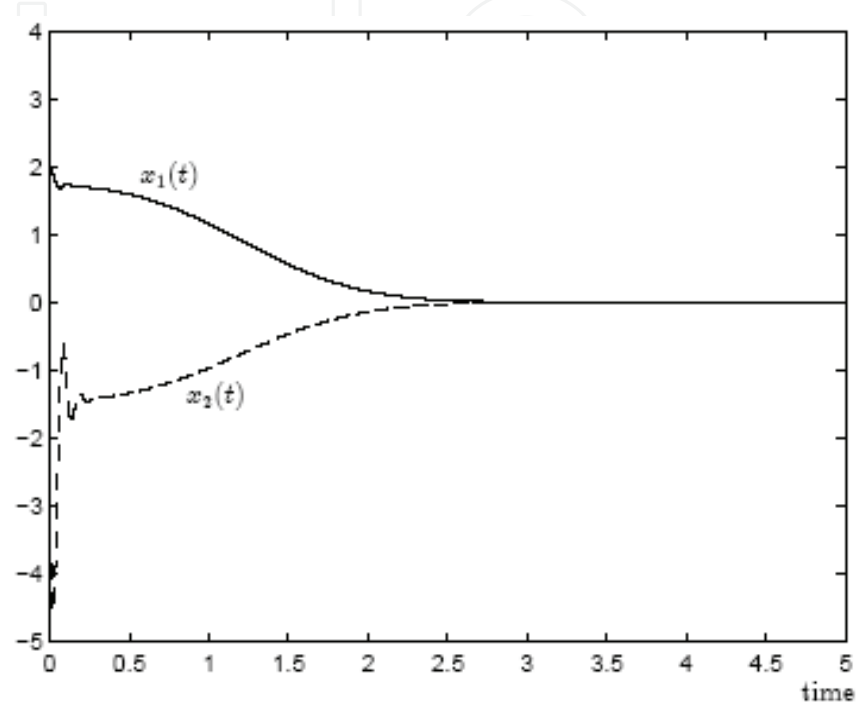


Fig. 4. Trajectory of states  $x_1(t)$  (solid);  $x_2(t)$  (dashed).

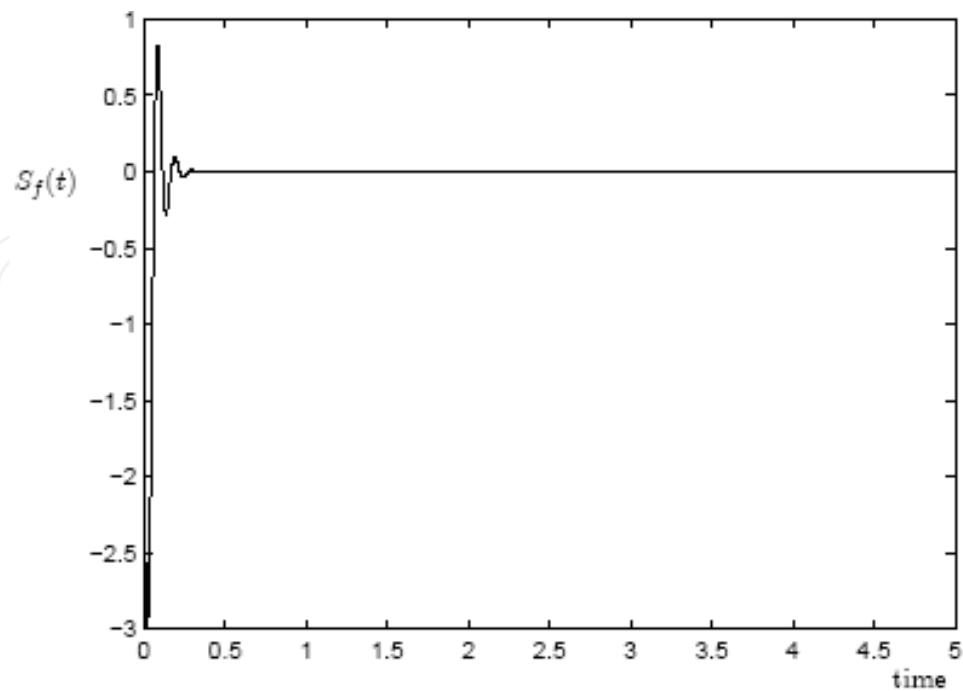


Fig. 5. Dynamic sliding surface  $S(t)$ .

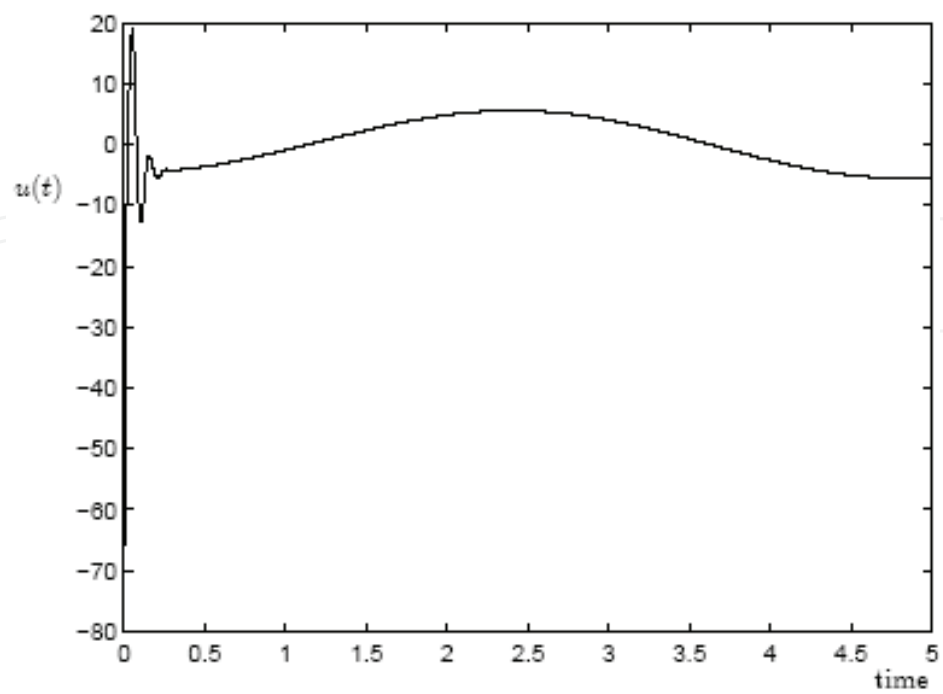


Fig. 6. Control effort  $u(t)$ .

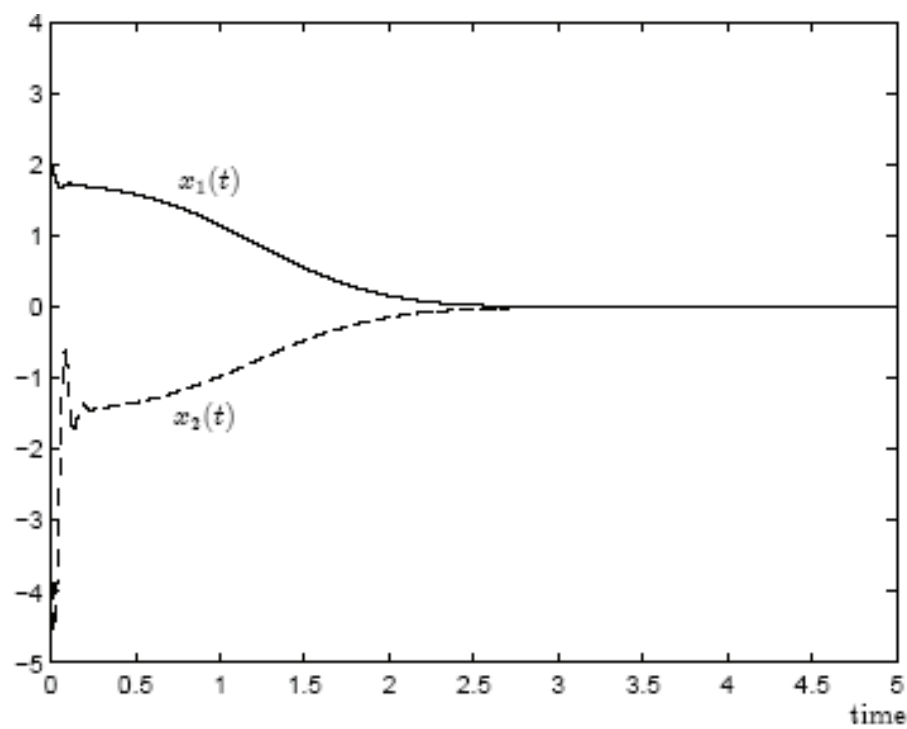


Fig. 7. Trajectory of states  $x_1(t)$  (solid);  $x_2(t)$  (dashed).



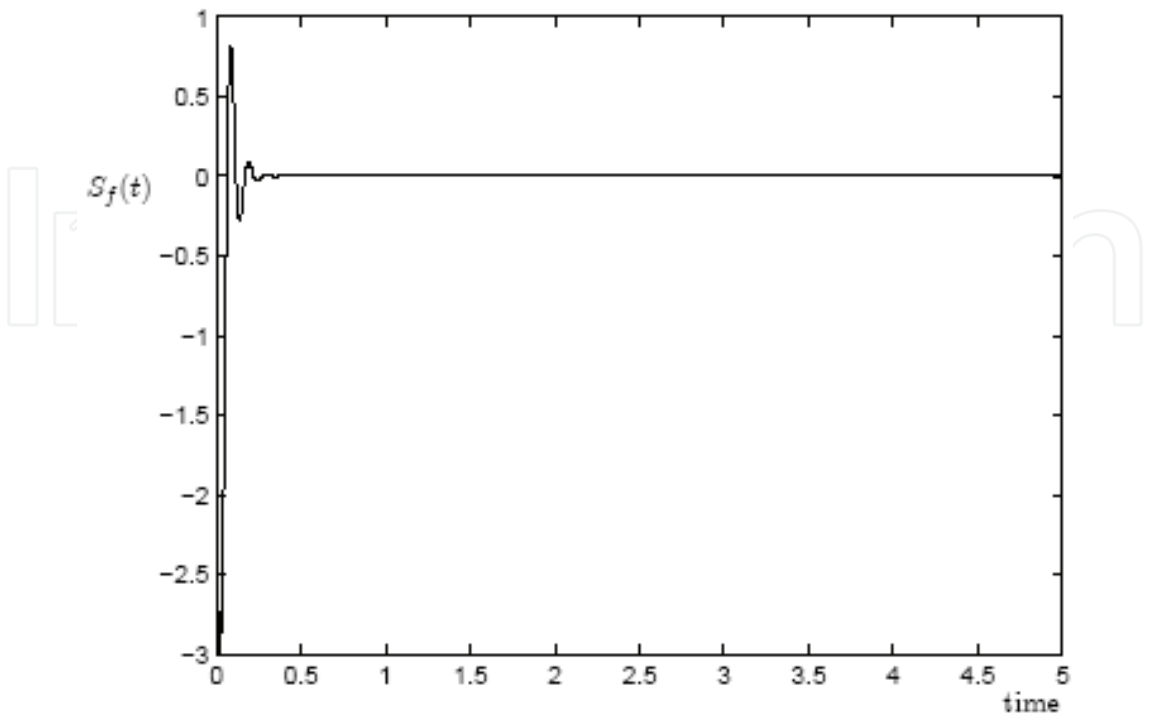


Fig. 8. Dynamic sliding surface  $S(t)$ .

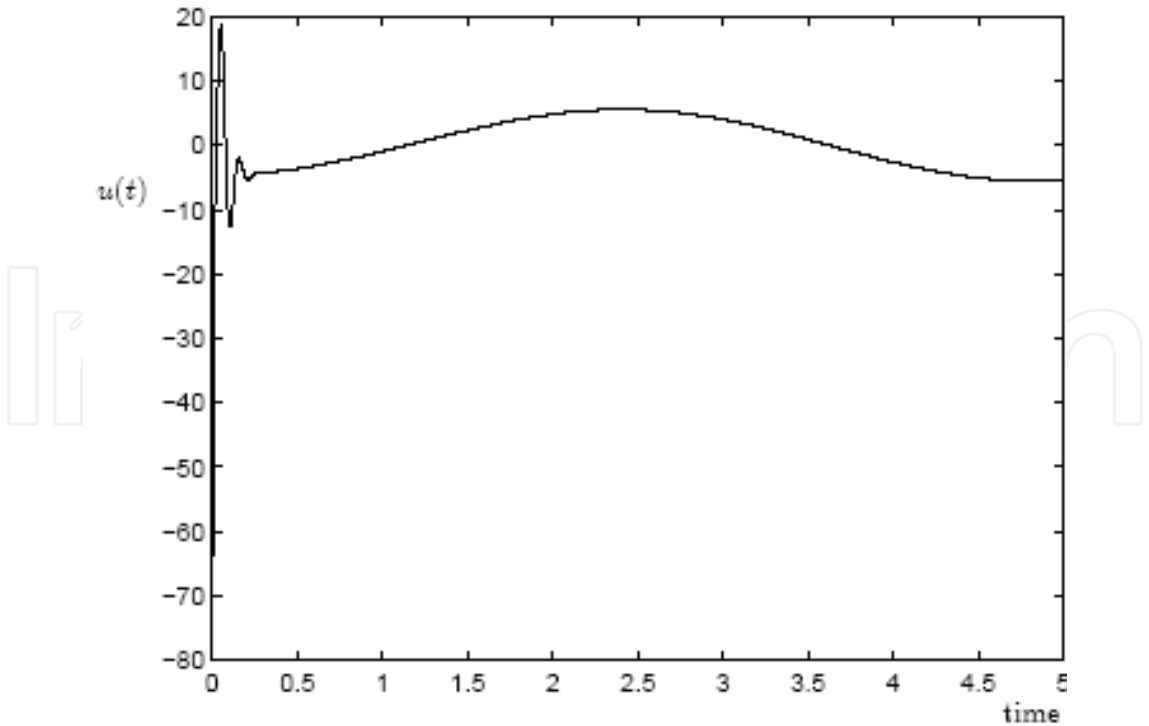


Fig. 9. Control effort  $u(t)$ .

## 5. Conclusion

In this paper, the robust control problem of a class of uncertain nonlinear time-delay systems has been solved by the proposed TS-FNN sliding mode control scheme. Although the system dynamics with mismatched uncertainties is not an Isidori-Bynes canonical form, the sliding surface design using LMI techniques achieves an asymptotic sliding motion. Moreover, the stability condition of the sliding motion is derived to be independent on the delay time. Based on the sliding surface design, and TS-FNN-based sliding mode control laws assure the robust control goal. Although the system has high uncertainties (here both state and input uncertainties are considered), the adaptive TS-FNN realizes the ideal reaching law and guarantees the asymptotic convergence of the states. Simulation results have demonstrated some favorable control performance by using the proposed controller for a three-dimensional uncertain time-delay system.

## 6. References

- [1] Y. Y. Cao, Y. X. Sun, and C. Cheng, "Delay-dependent robust stabilization of uncertain systems with multiple state delays," *IEEE Trans. Automatic Control*, vol. 43, pp.1608-1612, 1998.
- [2] W. H. Chen, Z. H. Guan and X. Lu, "Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay," *IEE Proc.-Control Theory Appl.*, vol. 150, no. 4, pp. 412-416, 2003.
- [3] V. N. Phat, "New stabilization criteria for linear time-varying systems with state delay and norm-bounded," *IEEE Trans. Automatic control*, vol. 47, no. 12, pp. 2095-2098, 2001.
- [4] C. Y. Lu, J. S.-H Tsai, G. J. Jong, and T. J. Su, "An LMI-based approach for robust stabilization of uncertain stochastic systems with time-varying delays," *IEEE Trans. Automatic control*, vol. 48, no. 2, pp. 286-289, 2003.
- [5] C. Hua, X. Guan, and P. Shi, "Robust adaptive control for uncertain time-delay systems," *Int. J. of Adaptive Control and Signal Processing*, vol. 19, no. 7, pp. 531-545, 2005.
- [6] S. Oucheriah, "Adaptive robust control of a class of dynamic delay systems with unknown uncertainty bounds," *Int. J. of Adaptive Control and Signal Processing*, vol. 15, no. 1, pp. 53-63, 2001.
- [7] S. Oucheriah, "Exponential stabilization of linear delayed systems using sliding-mode controllers," *IEEE Trans. Circuit and Systems-I: Fundamental Theory and Applications*, vol. 50, no. 6, pp. 826-830, 2003.
- [8] F. Hong, S. S. Ge, and T. H. Lee, "Delay-independent sliding mode control of nonlinear time-delay systems," *American Control Conference*, pp. 4068-4073, 2005.
- [9] Y. Xia and Y. Jia, "Robust sliding-mode control for uncertain time-delay systems: an LMI approach," *IEEE Trans. Automatic control*, vol. 48, no. 6, pp. 1086-1092, 2003.
- [10] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.
- [11] Y. Y. Cao and P.M. Frank "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Trans. Fuzzy Systems*, vol.8, no.2, pp. 200-210, 2000.

- [12] K. R. Lee, J. H. Kim, E. T. Jeung, and H. B. Park, "Output feedback robust  $H_\infty$  control of uncertain fuzzy dynamic systems with time-varying delay," *IEEE Trans. Fuzzy Systems*, vol. 8, no. 6, pp. 657-664, 2003.
- [13] J. Xiang, H. Su, J. Chu, and K. Zhang, "LMI approach to robust delay dependent/independent sliding mode control of uncertain time-delay systems," *IEEE Conf. on Systems, Man, and Cybern.*, pp. 2266-2271, 2003.
- [14] C. C. Chiang, "Adaptive fuzzy sliding mode control for time-delay uncertain large-scale systems," *The 44th IEEE Conf. on Decision and Control*, pp. 4077-4082, 2005.
- [15] C. Hua, X. Guan, and G. Duan, "Variable structure adaptive fuzzy control for a class of nonlinear time-delay systems," *Fuzzy Sets and Systems*, vol. 148, pp. 453-468, 2004.
- [16] K. Y. Lian, T. S. Chiang, C. S. Chiu, and P. Liu, "Synthesis of fuzzy model-based designed to synchronization and secure communication for chaotic systems," *IEEE Trans. Syst., Man, and Cybern. - Part B*, vol. 31, no. 1, pp. 66-83, 2001.
- [17] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. 15, pp. 116-132, 1985.
- [18] H. Ying, "Sufficient conditions on uniform approximation of multivariate functions by general Takagi-Sugeno fuzzy systems with linear rule consequent," *IEEE Trans. Syst., Man, Cybern. - Part A*, vol. 28, no. 4, pp. 515-520, 1998.
- [19] H. Han, C.Y. Su, and Y. Stepanenko, "Adaptive control of a class of nonlinear system with nonlinear parameterized fuzzy approximators," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 2, pp. 315-323, 2001.
- [20] S. Ge Schuzhi and C. Wang, "Direct adaptive NN control of a class of nonlinear systems," *IEEE Trans. Neural Networks*, vol. 13, no. 1, pp. 214-221, 2002.
- [21] C. H. Wang, T. C. Lin, T. T. Lee and H. L. Liu, "Adaptive hybrid Intelligent control for uncertain nonlinear dynamical systems," *IEEE Trans. Syst., Man, and Cybern. - Part B*, vol. 32, no. 5, pp. 583-597, 2002.
- [22] C. H. Wang, H. L. Liu, and T. C. Lin, "Direct adaptive fuzzy-neural control with state observer and supervisory for unknown nonlinear dynamical systems," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 1, pp. 39-49, 2002.
- [23] S. Hu and Y. Liu, "Robust  $H^\infty$  control of multiple time-delay uncertain nonlinear system using fuzzy model and adaptive neural network," *Fuzzy Sets and Systems*, vol. 146, pp. 403-420, 2004.
- [24] A. Isidori, *Nonlinear control system*, 2nd Ed. Berlin, Germany: Springer-Verlag, 1989.
- [25] L. Xie, "Output feedback  $H_\infty$  control of systems with parameter uncertainties," *Int. J. Contr.*, vol. 63, no. 4, pp. 741-750, 1996.
- [26] Tung-Sheng Chiang, Chian-Song Chiu and Peter Liu, "Adaptive TS-FNN control for a class of uncertain multi-time-delay systems: The exponentially stable sliding mode-based approach," *Int. J. Adapt. Control Signal Process*, 2009; 23: 378-399.
- [27] T. S. Chaing, C. S. Chiu and P. Liu, "Sliding model control of a class of uncertain nonlinear time-delay systems using LMI and TS recurrent fuzzy neural network," *IEICE Trans. Fund.* vol. E92-A, no. 1, Jan. 2009.

- [28] T. S. Chiang and P.T Wu, "Exponentially Stable Sliding Mode Based Approach for a Class of Uncertain Multi-Time-Delay System" 22nd IEEE International Symposium on Intelligent Control Part of *IEEE Multi-conference on Systems and Control*, Singapore, 1-3 October 2007, pp. 469-474.

## Appendix I

Refer to the matrix inequality lemma in the literature [25]. Consider constant matrices  $D$ ,  $E$  and a symmetric constant matrix  $G$  with appropriate dimension. The following matrix inequality

$$G + DC(t)E + E^T C^T(t)D^T < 0$$

for  $C(t)$  satisfying  $C^T(t)C(t) \leq R$ , if and only if, is equivalent to

$$G + \begin{bmatrix} E^T & D \end{bmatrix} \begin{bmatrix} \varepsilon^{-1}R & 0 \\ 0 & \varepsilon I \end{bmatrix} \begin{bmatrix} E \\ D^T \end{bmatrix} < 0$$

for some  $\varepsilon > 0$ . ■

## Appendix II

An exponential convergence is more desirable for practice. To design an exponential sliding mode, the following coordinate transformation is used

$$\sigma(t) = e^{\gamma t} x_1(t)$$

with an attenuation rate  $\gamma > 0$ . The equivalent dynamics to (2):

$$\begin{aligned} \dot{\sigma}(t) &= \gamma e^{\gamma t} x_1(t) + e^{\gamma t} \dot{x}_1(t) \\ &= A_{\sigma} \sigma(t) + \sum_{k=1}^h e^{\gamma d_k} \sigma(t - d_k) \end{aligned} \quad (\text{A.1})$$

where

$$A_{\sigma} = \gamma I_{n-1} + A_{11} - A_{12}\Lambda - \Delta A_{11} - \Delta A_{12}\Lambda,$$

$$A_{\sigma d_k} = A_{d_k 11} - A_{d_k 12}\Lambda - \Delta A_{d_k 11} - \Delta A_{d_k 12}\Lambda$$

; the equation (A.1) and the fact  $e^{\gamma d_k} \sigma(t - d_k) = e^{\gamma t} x_1(t - d_k)$  have been applied. If the system (A.1) is asymptotically stable, the original system (1) is exponentially stable with the decay

taking form of  $x_1(t) = e^{-\gamma t} \sigma(t)$  and an attenuation rate  $\gamma$ . Therefore, the sliding surface design problem is transformed into finding an appropriate gain  $\Lambda$  such that the subsystem (A.1) is asymptotically stable.

Consider the following Lyapunov-Krasovskii function

$$V(t) = \sigma^T(t) P \sigma(t) + \sum_{k=1}^h e^{2\gamma d_k} \int_{t-d_k}^t \sigma^T(v) X Q_k \sigma(v) dv$$

where  $P > 0$  and  $Q_k > 0$  are symmetric matrices.

Let the sliding surface  $S(t) = 0$  with the definition (6). The sliding motion of the system (1) is delay-independent exponentially stable, if there exist positive symmetric matrices  $X$ ,  $\bar{Q}_k$  and a parameter  $\Lambda$  satisfying the following LMI:

Given  $\varepsilon > 0$

Subject to  $X > 0$ ,  $\bar{Q}_k > 0$

$$\begin{bmatrix} N_{11} & (*) \\ N_{21} & -I_\varepsilon \end{bmatrix} < 0 \quad (\text{A.2})$$

where

$$N_{11} = \begin{bmatrix} N_0 & (*) & (*) & (*) \\ XA_{d11}^T + K^T A_{d12}^T & -\bar{Q}_1 & (*) & (*) \\ \vdots & \vdots & \ddots & (*) \\ XA_{dh11}^T + K^T A_{dh12}^T & 0 & \dots & -\bar{Q}_h \end{bmatrix}$$

$$N_{21} = \begin{bmatrix} E_{11}X + A_{12}K & 0 & 0 & 0 \\ 0 & E_{111}X + E_{112}K & \dots & E_{h11}X + E_{h12}K \\ D_1^T & 0 & \ddots & 0 \\ D_2^T & 0 & \dots & 0_h \end{bmatrix}$$

$$N_0 = A_{11}X + XA_{11}^T + A_{12}K + K^T A_{12}^T + \sum_{k=1}^h e^{2\gamma d_{\max}} \bar{Q}_k ;$$

$K = \Lambda X$ ;  $I_\varepsilon = \text{diag}\{\varepsilon I_a, \varepsilon I_a, \varepsilon^{-1} I_b, \varepsilon^{-1} I_b\}$  in which  $I_a, I_b$  are identity matrices with proper dimensions; and  $(*)$  denotes the transposed elements in the symmetric positions.

If the LMI problem has a feasible solution, then we obtain  $V(t) > 0$  and  $\dot{V}(t) < 0$ . This implies that the equivalent subsystem (A.1) is asymptotically stable, i.e.,  $\lim_{t \rightarrow \infty} \sigma(t) = 0$ . In turn, the states  $x_1(t)$  and  $x_2(t)$  (here  $x_2(t) = \Lambda x_1(t)$ ) will exponentially converge to zero as  $t \rightarrow \infty$ . As a result, the sliding motion on the manifold  $S(t) = 0$  is exponentially stable.

Moreover, since the gain condition (A.2) does not contain the delay time  $d_k$ , the sliding surface is delay-independent exponentially stable. ■

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## **Time-Delay Systems**

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Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robustness stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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