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Adaptive Control of Chaos

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1. Introduction

Since Ott et al. (1990) presented a feedback control technique for stabilizing unstable fixed points of chaotic maps, many linear and nonlinear control techniques based on feedback were introduced for chaos control of continuous and discrete dynamical systems (Fradkov and Evans 2005) . Consider the following dynamical systems:

$$\dot{x} = f(x, u) \tag{1}$$

$$x[n+1] = f(x[n], u[n]) \tag{2}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the control vector and $f : \Re^n \times \Re^m \to \Re^n$ is a sufficiently smooth function. The first equation presents a continuous time dynamical system and the second one a discrete dynamical system. It is assumed that both equations show chaotic behavior. The techniques of chaos control can be categorized based on different view points.

a) open-loop and closed-loop control

If the control action *u* has the following form:

$$u = u(t) \ \ for \ continuous \ time \ system$$
$$u = u[n] \ \ for \ discrete \ time \ system \tag{3}$$

then the control strategy is called open loop. There are some works on chaos control in an open loop format (Fradkov and Evans 2005, Kiss et al. 2000, Parmanada and Eiswirth 1999). If the control action u has the following form:

$$u = u(x(t)) \ \ for \ continuous \ time \ system$$
$$u = u(x[n]) \ \ for \ discrete \ time \ system \tag{4}$$

then it is called closed loop or feedback control. Feedback control has been frequently used in chaos control. The OGY control method and the delayed feedback control presented by Pyragas (1992, 2006) are the early works on controlling chaos via feedback. Recently, linear feedback control methods based on linearization and dynamic programming (Vincent and Yu 1991, Merat et al. 2009), nonlinear feedback control methods based on Lyapunov stability theory (Alasty and Salarieh 2007, Salarieh et al. 2009), sliding mode control (Konishi et al. 1998, Arjmand et al. 2008), backstepping method (Wu and Lu 2003, Yongguang and

Suochun 2003, Yassen 2006) and some geometrical techniques (Tian and Yu 2000) have been widely used for chaos control in closed loop format.

b) Model based and mode free control:

If the control action u is calculated based on the information obtained from dynamical model of the system, the control technique is called model based method. There are many control methods applied to chaotic systems which are designed based on the system model. When the control action is calculated without using the system model, the control technique is called model free control method. The minimum entropy control of chaos (Salarieh and Alasty 2009, 2009) and controlling chaos on a Poincare section presented by Bonakdar et al. (2008) are two model free control techniques.

c) Adaptive and Non-adaptive control:

When a dynamical system has some unknown parameters in its describing equations, usually an identification algorithm is coupled with the control algorithm to provide an adaptive control system. In adaptive control, parameter estimation and control are performed simultaneously. Usually the following dynamical equations are used for adaptive control applications:

$$\dot{x} = F(x,u)\Theta + g(x,u) \tag{5}$$

$$x[n+1] = F(x[n],u[n])\Theta + g(x[n],u[n]) \tag{6}$$

where $F(x,u)$ and $g(x,u)$ are two known functions and Θ is a vector of unknown parameters which must be estimated for control application. Adaptive control is divided into two categories; indirect and direct methods (Astrom and Wittenmark 1994). When the system parameters are estimated and control action is calculated based on the estimated parameters, the adaptive control scheme is called indirect adaptive control. In direct adaptive control, controller parameters are directly updated using an adaptive law. Adaptive control has been used widely for controlling chaos in discrete and continuous time systems (Hua and Guan 2004). Salarieh and Shahrokhi (2007) have proposed an indirect adaptive control scheme for chaotic maps to stabilize their unstable fixed points when there are some unknown parameters in the model of system. Direct adaptive control of a modified Chua's circuit has been considered by Yassen (2003). Adaptive control of delayed continuous time chaotic systems has been considered by Tian and Gao (1998). Zeng and Singh (1997), Liao and Lin (1999) and Pishkenari et al. (2007) have presented some direct and indirect adaptive controls for the Lorenz system. Adaptive control of the Chua circuit and the Lorenz system has been presented by Ge and Wang (2000) and Pyragas et al. (2004), respectively.

Sometimes in addition to the system parameters, the functions $F(x,u)$ and $g(x,u)$ in Eqs. (5) and (6) are also unknown. In these cases the unknown functions are substituted by a series or a finite summation of other known functions which are called the base functions:

$$F(x,u) = \sum_k \Phi_k^F(x,u)\Gamma_k^F$$
$$g(x,u) = \sum_k \Phi_k^g(x,u)\Gamma_k^g \tag{7}$$

where Φ_k^F and Φ_k^g are base functions which can be selected from neuro, fuzzy or neuro-fuzzy base functions or form polynomial functions. Γ_k^F and Γ_k^g are the unknown parameters which are estimated adaptively. Fuzzy adaptive control of chaos has been used by many authors to

stabilize unstable fixed points and periodic orbits (Chen et al. 1999). An indirect adaptive fuzzy control of chaos based on the sliding mode control has been presented by Layeghi et al. (2008). A fuzzy adaptive control of discrete time chaotic systems based on the Takagi Sugeno Kang fuzzy model has been proposed by Feng and Chen (2005). Adaptive fuzzy model based control of chaos with application to the Lorenz system has been investigated by Park et al. (2002). Guan and Chen (2003) have studied adaptive fuzzy control of chaos in presence of disturbance.

Sometimes an adaptive algorithm is used to update the parameters of the fuzzy IF-THEN rules during control procedures. Wang (1993) proposed a direct fuzzy adaptive control and applied to chaotic systems based on updating the parameters of IF-THEN fuzzy rules of the inference engine.

2. Controlling a class of discrete-time chaotic systems

In this section an adaptive control technique applicable to a class of discrete chaotic systems for stabilizing their unstable fixed points is presented. The method is an indirect adaptive control scheme which is proposed originally by Salarieh and Shahrokhi (2007).

2.1 Problem statement

Consider a discrete chaotic system given below:

$$x_j[k+1] = f_j^T(\underline{x}[k])\theta_j^f + (g_j^T(\underline{x}[k])\theta_j^g)u_j[k], \quad j = 1,...,N \tag{8}$$

where $f_j(.)$ and $g_j(.)$ $j = 1,...,N$ are known smooth functions, and θ_j^f and θ_j^g, $j = 1,...,N$ are unknown constant coefficients. By successive substitution (d times) the following delayed discrete time system is obtained:

$$\begin{aligned} x_j[k+d] = {} & F_j^T(\underline{x}[k],\underline{u}[k],...,\underline{u}[k+d-2])\Theta_j^F \\ & + (G_{j,1}^T(\underline{x}[k],\underline{u}[k],...,\underline{u}[k+d-2])\Theta_{j,1}^G)u_j[k+d-1] + \cdots + (G_{j,d}^T(\underline{x}[k])\Theta_{j,d}^G)u_j[k] \\ & j = 1,...,N \end{aligned} \tag{9}$$

where $F_j(.)$ and $G_{j,i}(.)$, $i = 1,...,d$, $j = 1,...,N$ are known smooth functions, $\Theta_{j,i}^F$ and $\Theta_{j,i}^G$, $i = 1,...,d$, $j = 1,...,N$ are unknown constant coefficients and $\underline{u} = [u_1\ u_2\ ...\ u_N]^T$. Equation (9) is a general delayed form of the discrete system (8) and can be used for stabilizing a d-cycle unstable fixed point of a discrete time chaotic system. It is assumed that the nonlinear coefficient of $u_j[k+d-1]$ is invertible, all of the nonlinear functions are sufficiently smooth and the state variables are available. The main goal is to stabilize the given fixed points of system (9). Note that by using some numerical techniques, the fixed points of a chaotic system whose states are accessible can be calculated without using its dynamic equation (Ramesh and Narayanan 2001, Schmelcher and Diakonos, 1997) hence it is assumed that the fixed points of the system are obtained by a numerical algorithm without using the system parameters.

2.2 Identification method

To identify the unknown parameters of system (9), it is written in the following form:

$$x_j[k+d] = \begin{bmatrix} F_j^T(.) & G_{j,1}^T(.)u_j[k+d-1] & \ldots & G_{j,d}^T(.)u_j[k] \end{bmatrix} \begin{bmatrix} \Theta_j^F \\ \Theta_{j,1}^G \\ \vdots \\ \Theta_{j,d}^G \end{bmatrix} \tag{10}$$

Now define,

$$\eta_j[k] = x_j[k+d] \tag{11}$$

$$\Phi_j[k] = \begin{bmatrix} F_j^T(.) & G_{j,1}^T(.)u_j[k+d-1] & \ldots & G_{j,d}^T(.)u_j[k] \end{bmatrix}^T \tag{12}$$

$$\Theta_j = \begin{bmatrix} \left(\Theta_j^F\right)^T & \left(\Theta_{j,1}^G\right)^T & \ldots & \left(\Theta_{j,d}^G\right)^T \end{bmatrix}^T \tag{13}$$

Using the above definitions Eq. (11) can be written as:

$$\eta_j[k] = \Phi_j^T[k]\Theta_j \tag{14}$$

$\hat{\Theta}_j[k]$ denotes the estimate of Θ_j and it is defined as:

$$\hat{\Theta}_j[k] = \begin{bmatrix} \left(\hat{\Theta}_j^F[k]\right)^T & \left(\hat{\Theta}_{j,1}^G[k]\right)^T & \ldots & \left(\hat{\Theta}_{j,d}^G[k]\right)^T \end{bmatrix}^T \tag{15}$$

The error vector can be written as:

$$\varepsilon_j[k] = \eta_j[k] - \Phi_j^T[k]\hat{\Theta}_j[k-1] \tag{16}$$

To obtain the estimated parameters, $\hat{\Theta}_j[k]$, the least squares technique is used. Consider the following objective functions:

$$J_k = \sum_{n=1}^{k} (\eta_j[n] - \Phi_j^T[n]\hat{\Theta}_j[k])^2 \tag{17}$$

By differentiating Eq. (17) with respect to $\hat{\Theta}$ and set it to zero we have:

$$\hat{\Theta}_j[k] = \left(\begin{bmatrix} \Phi_j[1] & \cdots & \Phi_j[k] \end{bmatrix} \begin{bmatrix} \Phi_j^T[1] \\ \vdots \\ \Phi_j^T[k] \end{bmatrix} \right)^{-1} \begin{bmatrix} \Phi_j[1] & \cdots & \Phi_j[k] \end{bmatrix} \begin{bmatrix} \eta_j[1] \\ \vdots \\ \eta_j[k] \end{bmatrix} \tag{18}$$

Let:

$$P_j[k] = \left(\begin{bmatrix} \Phi_j[1] & \cdots & \Phi_j[k] \end{bmatrix} \begin{bmatrix} \Phi_j^T[1] \\ \vdots \\ \Phi_j^T[k] \end{bmatrix} \right)^{-1} \tag{19}$$

Substituting Eq. (19) into Eq. (18) yields:

$$
\begin{aligned}
\hat{\Theta}_j[k] &= P_j[k]\sum_{n=1}^{k}\Phi_j[n]\eta_j[n] \\
&= P_j[k]\left(\sum_{n=1}^{k-1}\Phi_j[n]\eta_j[n]+\Phi_j[k]\eta_j[k]\right) \\
&= P_j[k]P_j^{-1}[k-1]\hat{\Theta}_j[k-1]+P_j[k]\Phi_j[k]\eta_j[k]
\end{aligned}
\tag{20}
$$

Using Eq. (19) one can write:

$$
P_j^{-1}[k-1]=P_j^{-1}[k]-\Phi_j[k]\Phi_j^T[k]
\tag{21}
$$

Substituting Eq. (21) into Eq. (20) results in:

$$
\hat{\Theta}_j[k]=\hat{\Theta}_j[k-1]+P_j[k]\Phi_j[k]\varepsilon_j[k]
\tag{22}
$$

After some matrix manipulations, Eqs. (21) and (22) can be rewritten as:

$$
P_j[k]=P_j[k-1]-\frac{P_j[k-1]\Phi_j[k]\Phi_j^T[k]P_j[k-1]}{1+\Phi_j^T[k]P_j[k-1]\Phi_j[k]}
\tag{23}
$$

$$
\hat{\Theta}_j[k]=\hat{\Theta}_j[k-1]+\frac{P_j[k-1]\Phi_j[k]\varepsilon_j[k]}{1+\Phi_j^T[k]P_j[k-1]\Phi_j[k]}
\tag{24}
$$

The above recursive equations can be solved using a positive definite initial matrix for $P_j[0]$ and an arbitrary initial vector for $\hat{\Theta}_j[0]$.
The identification method given by Eqs. (23) and (24) implies that:

$$
\Phi_j^T[k]\hat{\Theta}_j[k-1]\rightarrow\Phi_j^T[k]\Theta_j \ \ as \ k\rightarrow\infty
\tag{25}
$$

To show the property (25), note Eq. (19) implies that $P[k]$ is a positive definite matrix. Define:

$$
\delta_j[k]=\hat{\Theta}_j[k]-\Theta_j
\tag{26}
$$

Consider the following Lyapunov function:

$$
V_j[k]=\delta_j^T[k]P_j^{-1}[k]\delta_j[k]
\tag{27}
$$

Using Eq. (24), we have:

$$
\delta_j[k]=P_j[k]P_j^{-1}[k-1]\delta_j[k-1]
\tag{28}
$$

$\Delta V_j[k]$ can be obtained as follows:

$$V_j[k] - V_j[k-1] = \delta_j^T[k] P_j^{-1}[k] \delta_j[k] - \delta_j^T[k-1] P_j^{-1}[k-1] \delta_j[k-1]$$

$$= \delta_j^T[k] P_j^{-1}[k-1] \delta_j[k-1] - \delta_j^T[k-1] P_j^{-1}[k-1] \delta_j[k-1]$$

$$= (\hat{\Theta}_j^T[k] - \hat{\Theta}_j^T[k-1]) P_j^{-1}[k-1] (\hat{\Theta}_j[k-1] - \Theta_j) \tag{29}$$

$$= -\frac{(\eta_j[k] - \Phi_j^T[k] \hat{\Theta}_j[k-1])^2}{1 + \Phi_j^T[k] P_j[k-1] \Phi_j[k]} = -\frac{\varepsilon_j^2[k]}{1 + \Phi_j^T[k] P_j[k-1] \Phi_j[k]}$$

Equation (29) shows that $V_j[k]$ is a decreasing sequence, and consequently:

$$\sum_{k=1}^{n} \frac{\varepsilon_j^2[k]}{1 + \Phi_j^T[k] P_j[k-1] \Phi_j[k]} = V_j[0] - V_j[n] < \infty \tag{30}$$

Hence,

$$\lim_{k \to \infty} \frac{\varepsilon_j^2[k]}{1 + \Phi_j^T[k] P_j[k-1] \Phi_j[k]} = 0 \quad \text{or} \quad \lim_{k \to \infty} \varepsilon_j[k] = 0 \tag{31}$$

and consequently:

$$\Phi_j^T[k] \hat{\Theta}_j[k-1] \to \eta_j[k] = \Phi_j^T[k] \Theta_j, \, as \, k \to \infty \tag{32}$$

2.3 Indirect adaptive control of chaos

Assume that $\underline{x}_j^F = \left(x_j^F[0], x_j^F[1], ..., x_j^F[d-1] \right)$ is the fixed point of Eq. (9) when $u[k] \equiv 0$, i.e.

$$x_j^F[0] = x_j^F[d] = F_j^T \left(\underline{x}^F[0], 0, ..., 0 \right) \Theta_j^F$$

$$\text{or} \tag{33}$$

$$x_j^F[k+d] = x_j^F[k]$$

The main goal is stabilizing the fixed point x_j^F. Let,

$$\rho_j[k] = \Big[-F_j^T \left(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2] \right) \Theta_j^F$$

$$- \left(G_{j,2}^T \left(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-3] \right) \Theta_{j,2}^F \right) u[k+d-2] - \cdots \tag{34}$$

$$- \left(G_{j,d}^T \left(\underline{x}[k] \right) \Theta_{j,d}^G + x_j^F[k+d] - \sum_{j=1}^{d} \lambda_j \left(x_j[k+d-j] - x_j^F[k+d-j] \right) \right) \Big]$$

$$\hat{\rho}_j[k] = \Big[-F_j^T \left(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2] \right) \hat{\Theta}_j^F[k-1]$$

$$- \left(G_{j,2}^T \left(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-3] \right) \Theta_{j,2}^F[k-1] \right) u[k+d-2] - \cdots \tag{35}$$

$$- \left(G_{j,d}^T \left(\underline{x}[k] \right) \Theta_{j,d}^G[k-1] + x_j^F[k+d] - \sum_{j=1}^{d} \lambda_j \left(x_j[k+d-j] - x_j^F[k+d-j] \right) \right) \Big]$$

where λ_i's are chosen such that all roots of the following polynomial lie inside the unit circle.

$$z^d + \lambda_1 z^{d-1} + \cdots + \lambda_d = 0 \qquad (36)$$

Assume that $G_{j,1}^T\big(x[k], u[k], ..., u[k+d-2]\big)\Theta_{j,1}^G[k-1] \neq 0$ and consider the conrol law as given below:

$$u_j[k+d-1] = \frac{\hat{\rho}_j[k]}{G_{j,1}^T\big(x[k], u[k], ..., u[k+d-2]\big)\Theta_{j,1}^G[k-1]} \qquad (37)$$

To show that the above controller can stabilize the fixed point of Eq. (9), note that from Eq. (25) we have:

$$\lim_{k \to \infty} F_j^T(.)\hat{\Theta}_j^F[k-1] + \sum_{j=1}^{d} G_{j,i}^T(.)\hat{\Theta}_{j,i}^G[k-1]u[k+d-i]$$
$$= F_j^T(.)\Theta_j^F + \sum_{j=1}^{d} G_{j,i}^T(.)\Theta_{j,i}^G u[k+d-i] \qquad (38)$$

One can write the above equation in the following form:

$$\varepsilon_j[k] = F_j^T(.)\big(\hat{\Theta}_j^F[k-1] - \Theta_j^F\big) + \sum_{i=1}^{d} G_{j,i}^T(.)\big(\hat{\Theta}_{j,i}^G[k-1] - \Theta_{j,i}^G\big)u[k+d-i]$$
$$\lim_{k \to \infty} \varepsilon_j[k] = 0 \qquad (39)$$

From Eqs. (34), (35) and (39) one can obtain:

$$\hat{\rho}_j[k] - \rho_j[k] = \varepsilon_j[k] + G_{j,1}^T(.)\big(\hat{\Theta}_{j,1}^G[k-1] - \Theta_{j,1}^G\big) + u[k+d-1] \qquad (40)$$

Using Eqs. (9), (34), (35) and (37) the controlled system can be written as:

$$x_j[k+d] = -\rho[k] + x_j^F[k+d] - \sum_{i=1}^{d} \lambda_i\big(x_j[k+d-i] - x_j^F[k+d-i]\big)$$
$$+ \frac{G_{j,1}^T\big(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2]\big)\Theta_{j,1}^G}{G_{j,1}^T\big(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2]\big)\hat{\Theta}_{j,1}^G[k-1]}\hat{\rho}[k] \qquad (41)$$

Using Eq. (40) in Eq. (41), we have:

$$x_j[k+d] = -\hat{\rho}[k] + \varepsilon_j[k] + x_j^F[k+d] - \sum_{i=1}^{d} \lambda_i\big(x_j[k+d-i] - x_j^F[k+d-i]\big)$$
$$+ \frac{G_{j,1}^T\big(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2]\big)\Theta_{j,1}^G}{G_{j,1}^T\big(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2]\big)\hat{\Theta}_{j,1}^G[k-1]}\hat{\rho}[k] \qquad (42)$$

After some manipulations we get:

$$x_j[k+d] = x_j^F[k+d] - \sum_{i=1}^{d} \lambda_i(x_j[k+d-i] - x_j^F[k+d-i]) + \varepsilon_j[k] \qquad (43)$$

or,

$$(x_j[k+d] - x_j^F[k+d]) + \sum_{i=1}^{d} \lambda_i(x_j[k+d-i] - x_j^F[k+d-i]) = \varepsilon_j[k] \qquad (44)$$

Define,

$$y_1[k] = x_j[k] - x_j^F[k], \ y_2[k] = x_j[k+1] - x_j^F[k+1], \ \ldots, \ y_d[k] = x_j[k+d-1] - x_j^F[k+d-1], \quad (45)$$

Eq. (44) can be re-written as:

$$\begin{bmatrix} y_1[k+1] \\ y_2[k+1] \\ \vdots \\ y_d[k+1] \end{bmatrix} = \begin{bmatrix} 0 & & & \\ 0 & & I_{(d-1)\times(d-1)} & \\ 0 & & & \\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_d \end{bmatrix} \begin{bmatrix} y_1[k] \\ y_2[k] \\ \vdots \\ y_d[k] \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \varepsilon_j[k] \quad (46)$$

Let:

$$G = \begin{bmatrix} 0 & & & \\ 0 & & I_{(d-1)\times(d-1)} & \\ 0 & & & \\ -\lambda_1 & -\lambda_2 & \cdots & -\lambda_d \end{bmatrix}, \ H = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -1 \end{bmatrix} \quad (47)$$

Equation (45) can be written as:

$$Y[k+1] = GY[k] + H\varepsilon_j[k] \quad (48)$$

Taking z-transform from both sides of Eq. (48) we get:

$$Y(z) = (zI - G)^{-1} zY(0) + (zI - G)^{-1} H\varepsilon_j(z) \quad (49)$$

Taking inverse yields:

$$Y(k) = G^k Y[0] + z^{-1}[(zI - G)^{-1} H\varepsilon_j(z)] \quad (50)$$

Note that $\lim G^k Y[0] = 0$ as $k \to \infty$ (because all eigen-values of G lie inside the unit circle), besides $\lim \varepsilon_j[k] = 0$ as $k \to \infty$ and G is a stable matrix therefore:

$$\lim_{k \to \infty} z^{-1}[(zI - G)^{-1} H\varepsilon_j(z)] = 0 \quad (51)$$

Consequently we have:

$$\lim_{k \to \infty} Y[k] = 0 \Rightarrow x_j[k] \to x_j^F[k], \ldots, x_j[k+d-1] \to x_j^F[k+d-1] \quad (52)$$

From Eq. (52) the stability of the proposed controller is established.

Remark 1

In practice, control law (37) works well but theoretically there is the remote possibility of division by zero in calculating $u[.]$. This can be easily avoided. For example $u[.]$ can be calculated as follows:

$$u_j[k+d-1] = \begin{cases} \dfrac{\hat{\rho}_j[k]}{\mu_j[k]}, & |\mu_j[k]| \neq 0 \\ \dfrac{\hat{\rho}_j[k]}{m_\varepsilon} & |\mu_j[k]| = 0 \end{cases} \quad (53)$$

where

$$\mu_j[k] = G_{j,1}^T \left(\underline{x}[k], \underline{u}[k], ..., \underline{u}[k+d-2] \right) \hat{\Theta}_{j,1}^G[k-1] \quad (54)$$

and $m_\varepsilon > 0$ is a small positive real number.

Remark 2

For the time varying systems, least squares algorithm with variable forgetting factor can be used. The corresponding updating rule is given below (Fortescue et al. 1981):

$$P_j[k] = \frac{1}{v[k]} \left[P_j[k-1] - \frac{P_j[k-1]\Phi_j[k]\Phi_j^T[k]P[k-1]}{v_j[k] + \Phi_j^T[k]P_j[k-1]\Phi_j[k]} \right] \quad (55)$$

$$\hat{\Theta}_j[k] = \hat{\Theta}_j[k-1] + \frac{P_j[k-1]\Phi_j[k]\varepsilon_j[k]}{v_j[k] + \Phi_j^T[k]P_j[k-1]\Phi_j[k]}$$

where,

$$v_j[k] = \max \left\{ 1 - \frac{\varepsilon_j^2[k]}{1 + \varepsilon_j^2[k]}, \ \lambda_{\min} \right\} \quad (56)$$

and $0 < \lambda_{\min} < 1$ usually set to 0.95.

2.4. Simulation results

In this section through simulation, the performance of the proposed adaptive controller is evaluated.

Example 1:

Consider the logistic map given below:

$$x[k+1] = \mu x[k] (1 - x[k]) + u[k] \quad (57)$$

For $\mu \geq 3.567$ and $u[k] = 0$ the behavior of the system is chaotic. In this example stabilization of the 2-cycle fixed point of the following logistic map is considered. In this case the governing equation is:

$$x[k+2] = \mu^2 x[k] - (\mu^2 + \mu^3)x^2[k] + 2\mu^3 x^3[k] - \mu^3 x^4[k]$$
$$+ \mu u[k] - 2\mu^2 x[k]u[k] + 2\mu^2 x^2[k]u[k] - \mu u^2[k] + u[k+1] \quad (58)$$

and the system fixed points for $\mu = 3.6$ are $x_F[1] = 0.8696$, $x_F[2] = 0.4081$ and for $\mu = 3.9$ are $x_F[1] = 0.8974$, $x_F[2] = 0.3590$. Equation (58) can be written in the following form:
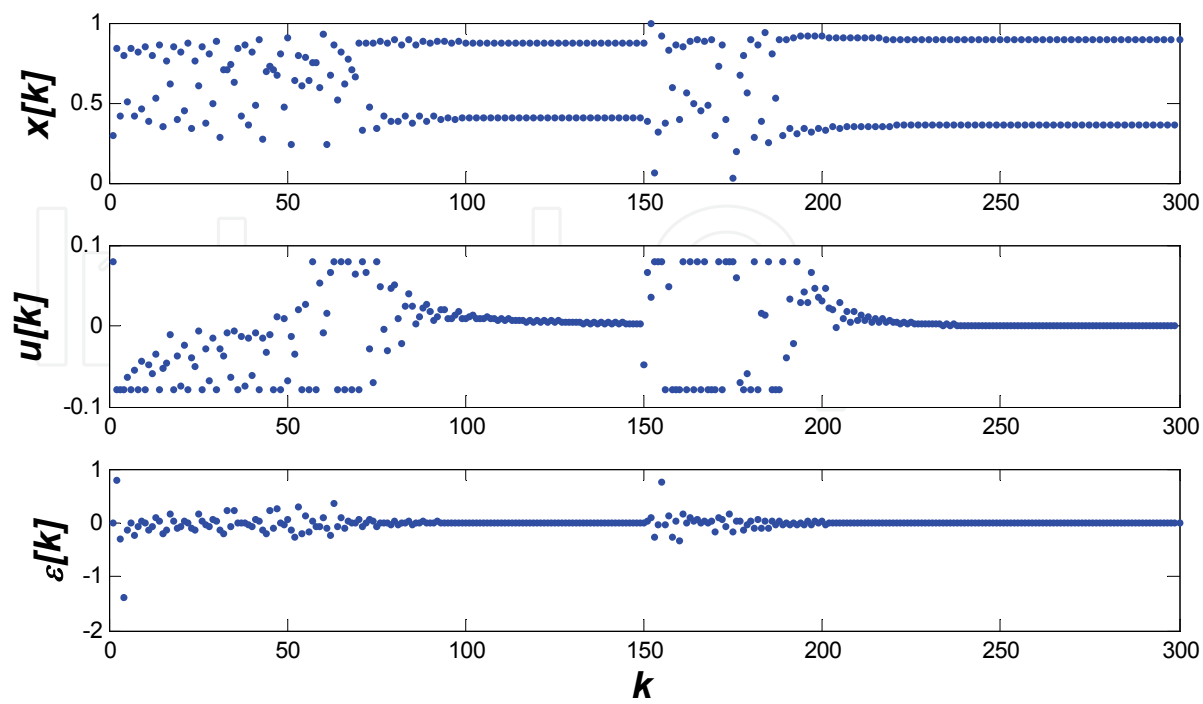
Fig. 1. Closed-loop response of the logistic map (59), for stabilizing the 2-cycle fixed point, when the parameter μ changes from $\mu = 3.6$ to $\mu = 3.9$ at $k = 150$.
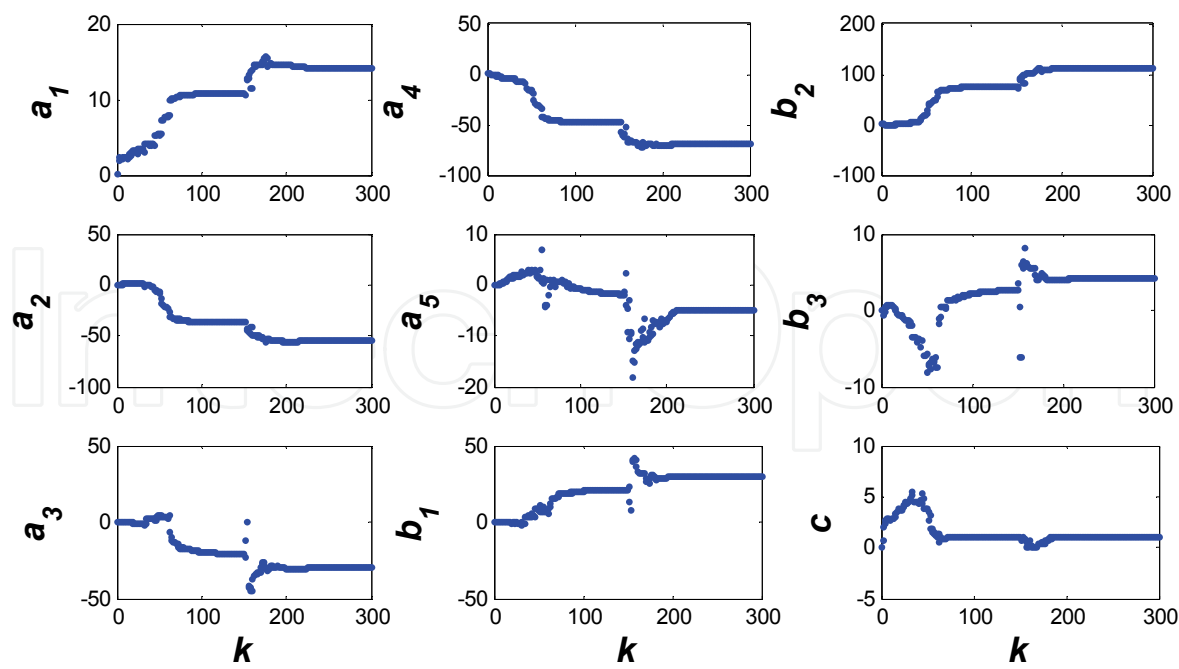

Fig. 2. Parameter estimates for the logistic map (59), when the parameter μ changes from $\mu = 3.6$ to $\mu = 3.9$ at $k = 150$.

$$x[k+2] = \begin{bmatrix} x[k] & x^2[k] & x^3[k] & x^4[k] & u^2[k] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} + \begin{bmatrix} 1 & x[k] & x^2[k] \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} u[k] + cu[k+1] \qquad (59)$$

It is assumed that the system parameter μ changes from $\mu = 3.6$ to $\mu = 3.9$ at $k = 150$.

The results are shown in Figs. (1) and (2). As can be seen the 2-cycle fixed point of the system is stabilized and the tracking error tends to zero, although the parameter estimates are not converged to their actual values.

Example 2:

For the second example, the Henon map is considered,

$$\begin{aligned} x_1[k+1] &= 1 - ax_1^2[k] + x_2[k] + u_1[k] \\ x_2[k+1] &= bx_1[k] + u_2[k] \end{aligned} \qquad (60)$$

where for $a = 1.4$, $b = 0.3$ and $u_1[k] = u_2[k] = 0$ the behavior of the system is chaotic. The 1-cycle fixed point is regarded for stabilization. Equation (60) can be written in the following form:

$$\begin{aligned} x_1[k+1] &= f_1^T \left(x_1[k], x_2[k] \right) \Theta_1^f + g_1^T \left(x_1[k], x_2[k] \right) \Theta_1^g u_1[k] \\ x_2[k+1] &= f_2^T \left(x_1[k], x_2[k] \right) \Theta_2^f + g_2^T \left(x_1[k], x_2[k] \right) \Theta_2^g u_2[k] \end{aligned} \qquad (61)$$

where,

$$\begin{aligned} f_1 \left(x_1[k], x_2[k] \right) &= \begin{bmatrix} 1 & x_2[k] & x_1^2[k] \end{bmatrix}^T, & f_2 \left(x_1[k], x_2[k] \right) &= 1 \\ G_1 \left(x_1[k], x_2[k] \right) &= 1, & G_2 \left(x_1[k], x_2[k] \right) &= 1 \end{aligned} \qquad (62)$$

and

$$\Theta_1^f = \begin{bmatrix} \theta_{1,1}^f & \theta_{1,2}^f & \theta_{1,3}^f \end{bmatrix}^T, \qquad \Theta_1^f = \theta_1^f, \qquad \Theta_i^g = \theta_i^g, \quad i = 1, 2 \qquad (63)$$

Again the 1-cycle fixed point is obtained, using numerical methods, $(x_1^F = 0.6314, x_2^F = 0.1894)$. Figures (3) and (4) show the results of applying the proposed adaptive controller to the Henon map. It is observed that the 1-cycle fixed point of the system is stabilized.

It must be noted that if in the system model the exact functionality is not know, a more general form with additional parameters can be considered. For example system (61) can be modeled as follows:

$$\begin{aligned} f_i \left(x_1[k], x_2[k] \right) &= g_i \left(x_1[k], x_2[k] \right) \\ &= \begin{bmatrix} 1 & x_1[k] & x_2[k] & x_1^2[k] & x_1[k]x_2[k] & x_2^2[k] \end{bmatrix}^T \end{aligned} \qquad (64)$$
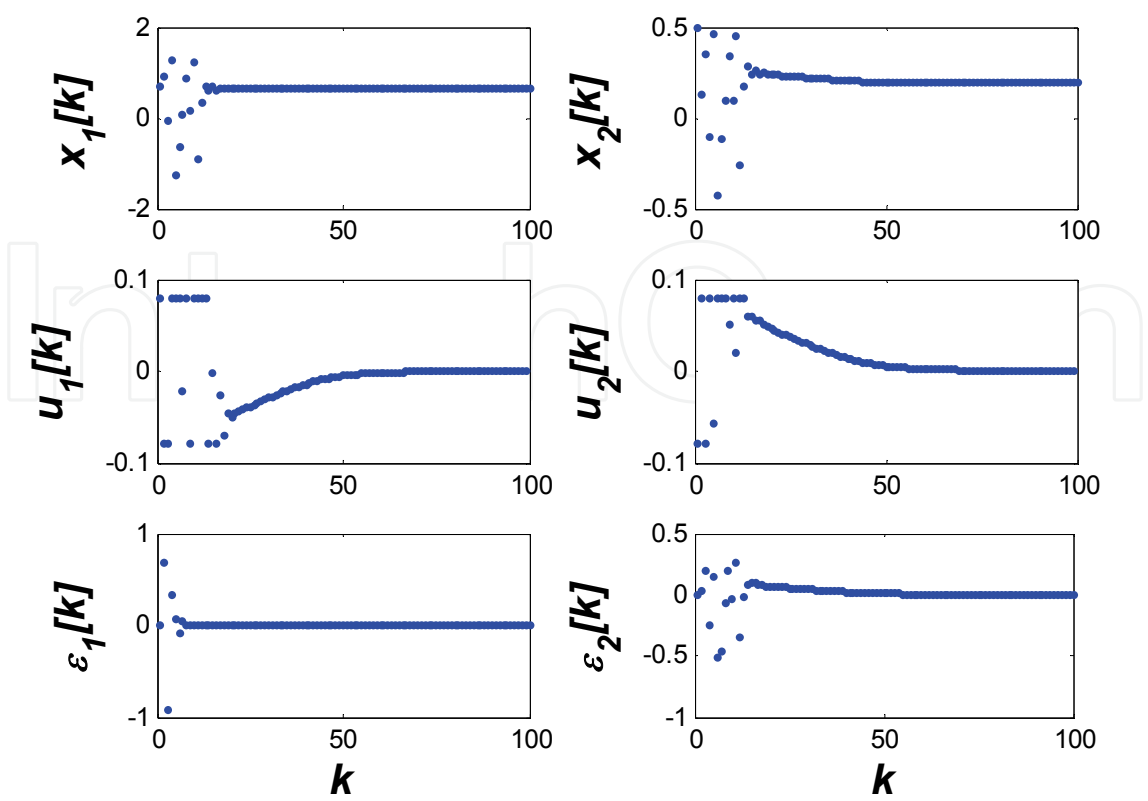
Fig. 3. Closed-loop response of the Henon map (61), for stabilizing the 1-cycle fixed point.
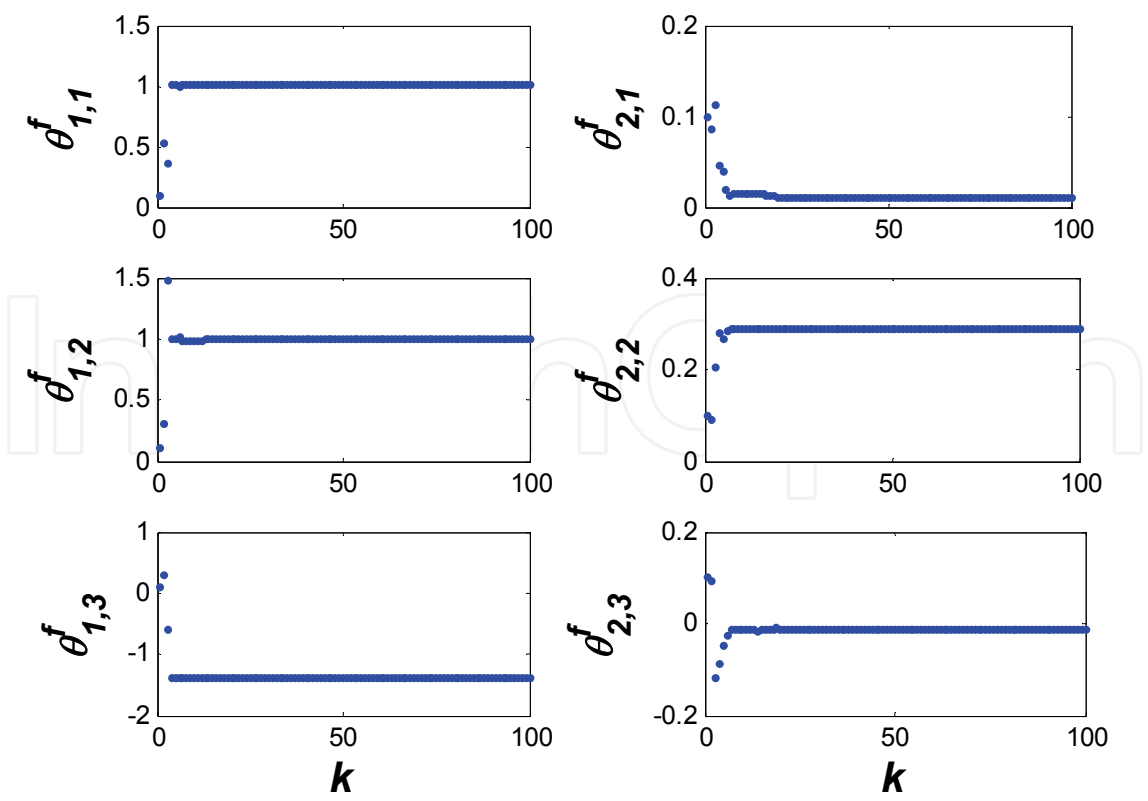

Fig. 4. Parameter estimates of the Henon map (61).

and

$$\Theta_i^f = \begin{bmatrix} \theta_{i,1}^f & \theta_{i,2}^f & \theta_{i,3}^f & \theta_{i,4}^f & \theta_{i,5}^f & \theta_{i,6}^f \end{bmatrix}^T$$
$$\Theta_i^g = \begin{bmatrix} \theta_{i,1}^g & \theta_{i,2}^g & \theta_{i,3}^g & \theta_{i,4}^g & \theta_{i,5}^g & \theta_{i,6}^g \end{bmatrix}^T$$

(65)

As it is illustrated in Fig. (5), the 1-cycle fixed point is stabilized successfully. It shows that in cases where the system dynamics is not known completely, the proposed method can be applied successfully using the over-parameterized model.
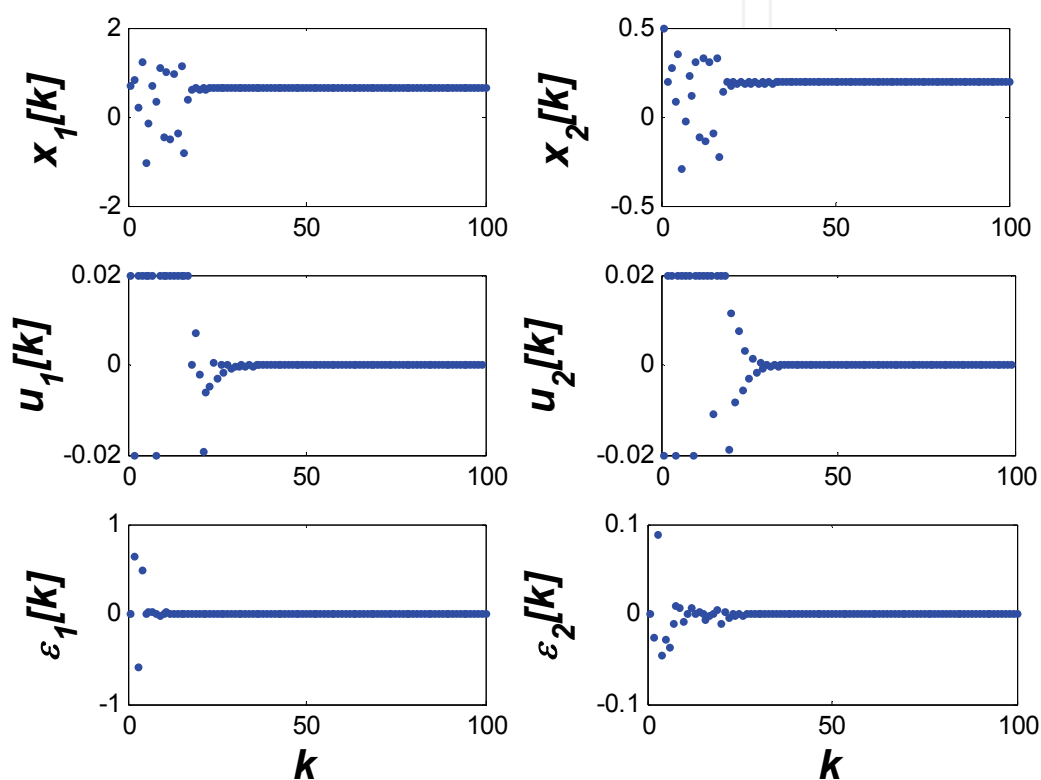
Fig. 5. Closed-loop response of the Henon map (61), for stabilizing the 1-cycle fixed point, using over-parameterized model.

3. Controlling a class of continuous-time chaotic systems

In this section a direct adaptive control scheme for controlling chaos in a class of continuous-time dynamical system is presented. The method is based on the proposed adaptive technique by Salarieh and Alasty (2008) in which the unstable periodic orbits of a stochastic chaotic system with unknown parameters are stabilized via adaptive control. The method is simplified and applied to a non-stochastic chaotic system.

3.1 Problem statement

It is assumed that the dynamics of the under study chaotic system is given by:

$$\dot{x} = f(x) + F(x)\theta + G(x)u$$

(66)

where $x \in \Re^n$ is the state vector of the system, $\theta \in \Re^m$ is the vector of the system parameters, $u \in \Re^n$ is the control vector, $f \in C^1(\Re^n, \Re^n)$, $F \in C^1(\Re^n, \Re^{n \times m})$ and for all x $\|F(x)\|_2 < M$, where M is a positive constant, i.e. F is bounded, and $G \in C^1(\Re^n, \Re^{n \times n})$. The Unstable Periodic Orbit (UPO) of chaotic system (66) with $u = 0$ is denoted by $\bar{x}$, and consequently we have:

$$\dot{\bar{x}} = f(\bar{x}) + F(\bar{x})\theta \qquad (67)$$

It is assumed that all states of the chaotic system are available, G is invertible, functions f, F and G are known, and the system parameters, θ, are unknown. The main objective is designing a feedback direct adaptive controller for stabilizing the unstable periodic orbit, $\bar{x}$ such that:

$$\|x - \bar{x}\| \to 0, \text{ as } t \to \infty \qquad (68)$$

3.2 Direct adaptive control of chaos

By using the following theorem, a direct adaptive controller can be designed which fulfills the above objective:

Theorem

Let $e = x - \bar{x}$ and $k > 0$, than the control and adaptation laws given by:

$$u = -G^{-1}(x)[f(x) - f(\bar{x}) + (F(x) - F(\bar{x}))\alpha + \eta sign(e) + k\ e] \qquad (69)$$

$$\dot{\alpha} = [F(x) - F(\bar{x})]^T e \qquad (70)$$

make the UPO defined by Eq. (67) asymptotically stable.

∎

Note that the *sign* function in Eq. (69), acts on each elements of the $x - \bar{x}$ vector.

Proof:

To design a control and adaptation law, a Lyapunov function is defined as:

$$V = \frac{1}{2}\|x(t) - \bar{x}(t)\|^2 + \frac{1}{2}\|\alpha(t) - \theta\|^2 \qquad (71)$$

where α is the estimate of θ obtained from the adaptive law (71). Differentiating both sides of Eq. (70) yields:

$$\dot{V} = \left(x(t) - \bar{x}(t)\right)^T \left(\dot{x}(t) - \dot{\bar{x}}(t)\right) + \left(\alpha(t) - \theta\right)^T \dot{\alpha} \qquad (72)$$

Using Eqs. (66), (67) and (69), Eq. (72) can be written as:

$$\dot{V} = (x - \bar{x})^T \left(-(F(x) - F(\bar{x}))(\alpha - \theta) - \eta sign(x - \bar{x}) - k(x - \bar{x})\right) + (\alpha(t) - \theta)^T [F(x) - F(\bar{x})]^T (x - \bar{x})$$
$$= -k\|e\|^2 - \eta|e| \qquad (73)$$

Since $\dot{V}$ is negative semi-definite, so the closed-loop system is stable, and $e = x - \bar{x}$ and x are bounded. In addition $\dot{e}$ is bounded too, because it satisfies the following equation:

$$\dot{e} = \dot{x} - \dot{\overline{x}} = F(x)\theta + f(\overline{x}) - \left(F(x) - F(\overline{x})\right)\alpha - \eta sign(e) - k\ e - \dot{\overline{x}} \tag{74}$$

in which $f(.)$ and $F(.)$ are continuous functions. From Eq. (73) we have

$$-\int_{t_0}^{\infty} \dot{V}(t)dt = \int_{t_0}^{\infty} \left(k\|e\|^2 + \eta|e|\right)dt = V(0) - V(\infty) < \infty \tag{75}$$

Since $e \in L^2 \bigcap L^\infty$ and $\dot{e} \in L^\infty$, due to Barbalat's Lemma (Sastry, Bodson 1994) we have

$$\lim_{t \to \infty} e(t) = 0 \implies \lim_{t \to \infty} x(t) = \overline{x}(t) \tag{76}$$

∎

3.3 Simulation results

In this section the presented controller has been used for stabilizing the UPOs of the Lorenz, and the Rossler dynamical systems.

Example 1: Stabilizing a UPO of the Lorenz system:

The Lorenz system has the following governing equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_2 - x_1 x_3 \\ x_1 x_2 \end{bmatrix} + \begin{bmatrix} x_2 - x_1 & 0 & 0 \\ 0 & x_1 & 0 \\ 0 & 0 & -x_3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{77}$$

where x_1, x_2 and x_3 are the state variables, θ_1, θ_2 and θ_3 are unknown parameters. For $\theta_1 = 16$, $\theta_2 = 45.92$, $\theta_3 = 4$ and $u_i = 0$, the Lorenz system shows chaotic behavior, and one of its UPOs is initiated from $x_1(0) = 19.926$, $x_2(0) = 30.109$ and $x_3(0) = 40$ with the period of $T = 0.941$, as shown in Fig. (6). The presented adaptive control is applied to the Lorenz dynamical system. Figure (7) shows the states dynamics in the time-domain and in the phase space. The control actions are shown in Fig. (8).
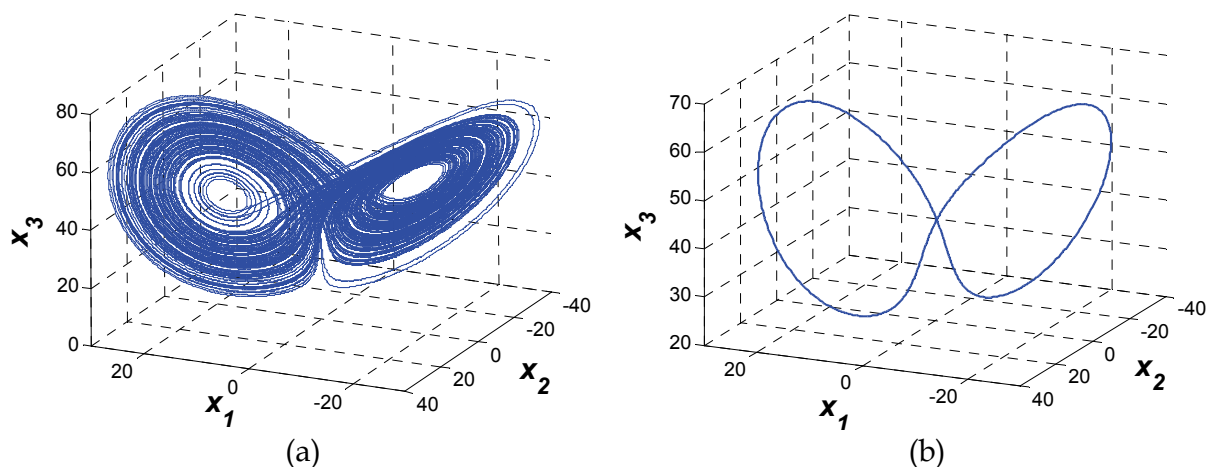

(a)   (b)

Fig. 6. (a) Chaotic attractor of the Lorenz system, (b) the UPO with the period of $T = 0.941$ and the initial conditions of: $x_1(0) = 19.926$, $x_2(0) = 30.109$ and $x_3(0) = 40$ (Salarieh and Alasty, 2008).
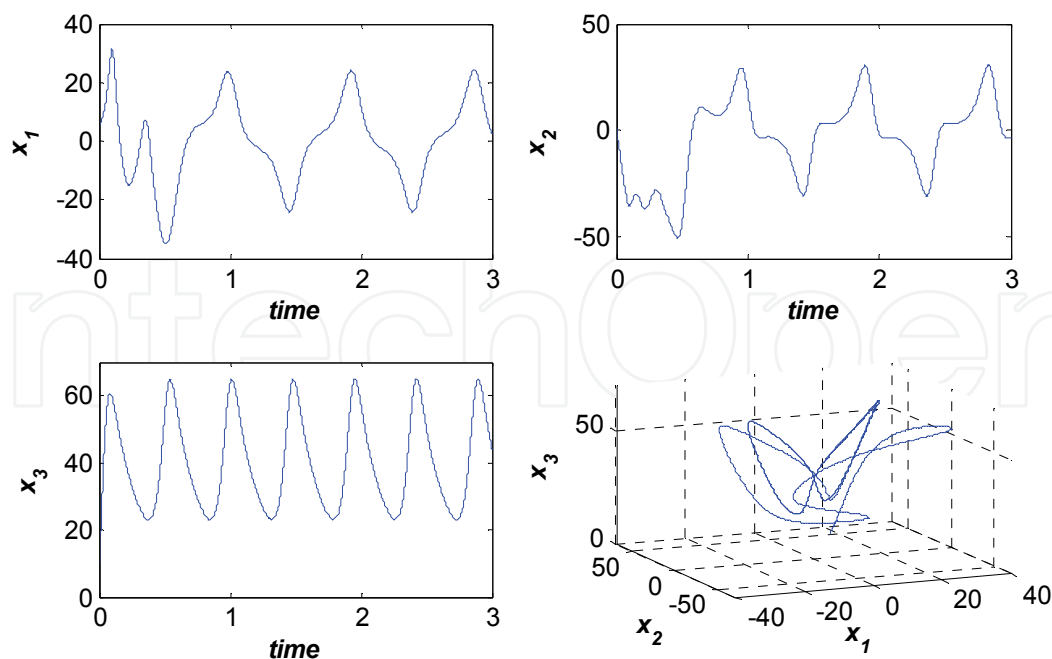
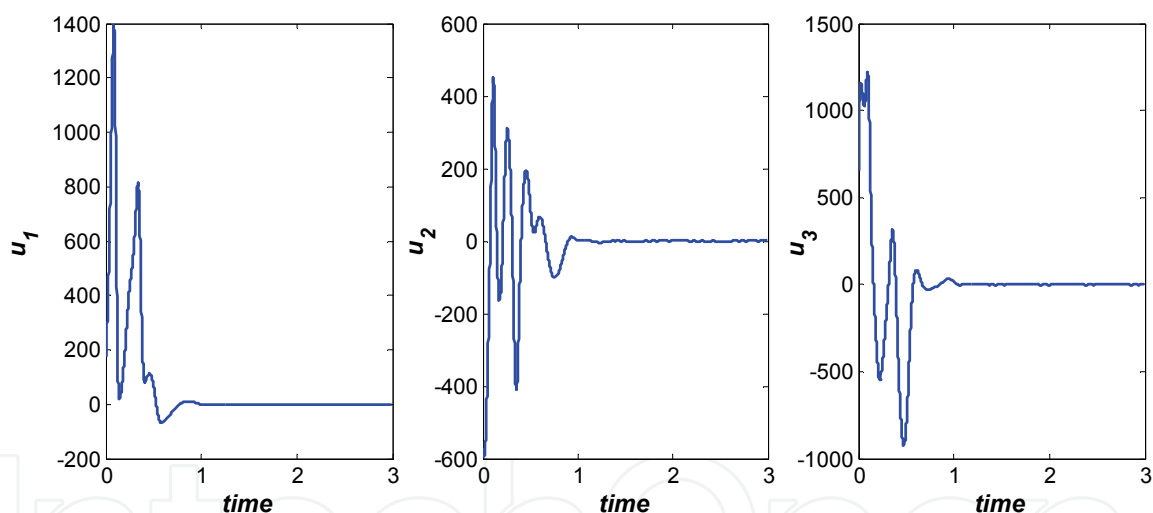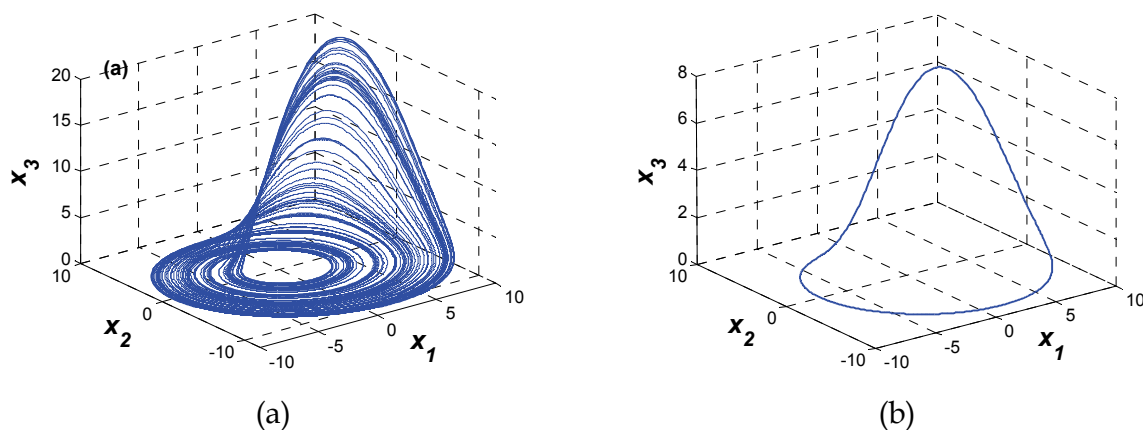Fig. 7. The trajectory of the Lorenz system after applying the adaptive control law (69).

Fig. 8. Corresponding control actions.

Example 2: Stabilizing a UPO of the Rossler system:

The Rossler dynamical system is described by the following equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -(x_2 + x_3) \\ x_1 \\ x_1 x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ x_2 & 0 & 0 \\ 0 & 1 & -x_3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{78}$$

For $\theta_1 = 0.2$, $\theta_2 = 0.2$, $\theta_3 = 5.7$ and $u_i = 0$, the system trajectories are chaotic. The system behavior in the phase space and one UPO of the system are shown in Fig. (9). In this case the UPO has the period of $T = 5.882$ which is obtained from the initial condition $x_1(0) = 0$,

$x_2(0) = 6.089$ and $x_3(0) = 1.301$. The presented adaptive control scheme is applied to the system for stabilizing the UPO shown in Fig. (9). Simulation results are shown in Figs. (10) and (11). Figure (10) shows that the system trajectories converge to the desired UPO.

4. Fuzzy adaptive control of chaos

In this section an adaptive fuzzy control scheme for chaotic systems with unknown dynamics is presented.

4.1 Problem statement

It is assumed that the dynamics of the chaotic system is given by:

$$x^{(n)} = f(X) + g(X)u \qquad (79)$$

Fig. 9. (a) Chaotic attractor of the Rossler system, (b) the UPO with the period of $T = 5.882$ and the initial conditions of: $x_1(0) = 0$, $x_2(0) = 6.089$ and $x_3(0) = 1.301$ (Salarieh and Alasty, 2008).
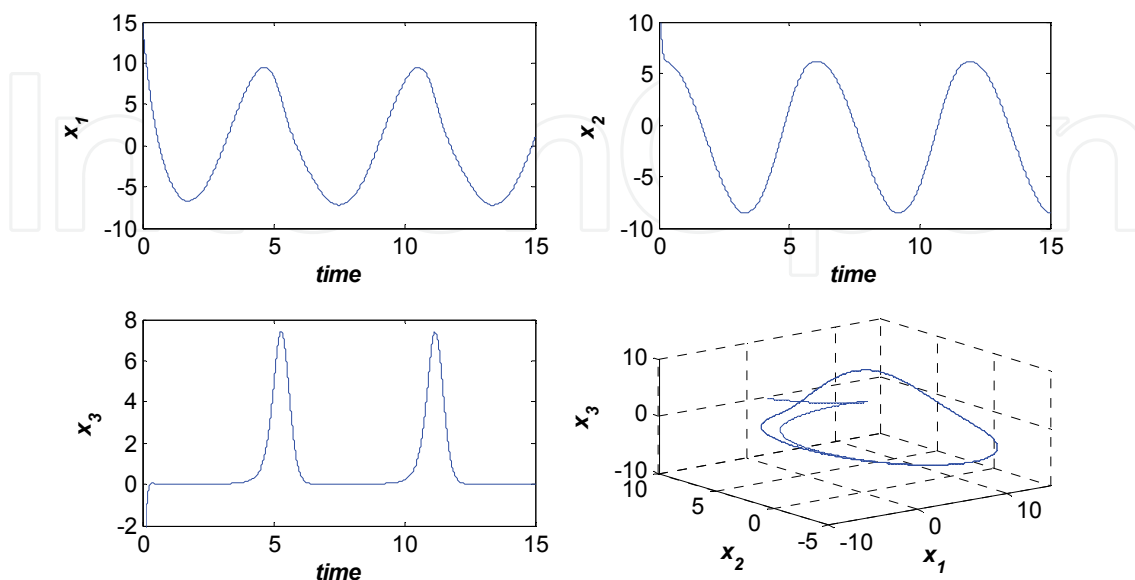

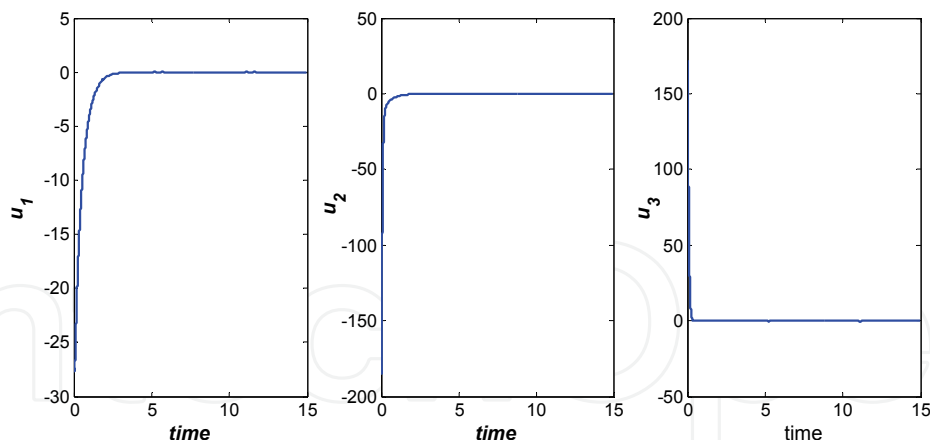Fig. 10. The trajectory of the Rossler system after applying the adaptive control law (69).

Fig. 11. Corresponding control actions.

where $X = [x_1, x_2, ..., x_n]^T = [x, \dot{x}, ..., x^{(n-1)}]^T$ is the system state vector, $u \in \Re$ is the control vector, $f \in C^1(\Re^n, \Re)$ and $g \in C^1(\Re^n, \Re)$. It is also assumed that $f(.)$ is an unknown function but $g(.)$ is a known function.

The Unstable Periodic Orbit (UPO) of chaotic system (79) with $u = 0$ is denoted by $\bar{X}$, therefore:

$$\bar{x}^{(n)} = f(\bar{X}) \tag{80}$$

The main objective is to design a feedback direct adaptive controller for stabilizing the unstable periodic orbit, $\bar{X}$. It is also assumed that all state variables are available for controller design.

4.2 Fuzzy estimation of $f(X)$

Since $f(.)$ is not known, a fuzzy system is used to estimate it. $f(.)$ can be estimated as:

$$\hat{f}(X) = \sum_{k=1}^{N} \theta_k \phi_k(X) = \Theta^T \Phi(X) \tag{81}$$

where $\hat{f}(.)$ is the fuzzy estimation of $f(.)$ and

$$\Phi(X) = [\phi_1(X), ..., \phi_N(X)]^T$$
$$\Theta = [\theta_1, \theta_2, ..., \theta_N]^T \tag{82}$$

$\phi_k(.)$ functions have the following form:

$$\phi_k(X) = \frac{\prod_{i=1}^{n} \mu_k(x_i)}{\sum_{k=1}^{N} \prod_{i=1}^{n} \mu_k(x_i)} \tag{83}$$

where $\mu_k(x_i)$ is the output of fuzzy membership function for ith input argument. Fuzzy systems (81) and (83) are obtained using singleton fuzzifier and product inference engine (PIE) and center average defuzzifier. According to the universal approximation theorem of

Fuzzy systems, for sufficiently large N, i.e. the number of fuzzy rules, one can estimate $f(.)$ such that for every $\varepsilon_f > 0$ the following inequality holds:

$$\left\| f(X) - \hat{f}(X) \right\|_\infty < \varepsilon_f \tag{84}$$

The optimum vector parameter Θ which satisfies Eq. (84) is denoted by Θ_o, and its corresponding estimated $f(.)$ is denoted by $\hat{f}_o(.)$.

4.3 Direct fuzzy adaptive control scheme

The following theorem provides an adaptive control law for system (79).

Theorem 2

Let $E = X - \overline{X}$ and select $\mu_i, i = 0,1,...,n-1$ such that all roots of equation $s^n + \mu_0 s^{n-1} + \cdots + \mu_{n-1} = 0$ have negative real parts. Consider the control and adaptation laws:

$$u = -\frac{1}{g(X)} \left[\hat{f}(X) - \overline{x}^{(n)} + \sum_{i=0}^{n-1} \mu_{n-1-i} \left(x^{(i)} - \overline{x}^{(i)} \right) \right] \tag{85}$$

$$\dot{\Theta} = \Phi(X) B^T P E \tag{86}$$

where $B^T = [0\ 0\ ...\ 0\ 1]$ and P is a positive definite symmetric matrix that satisfies the Lyapunov equation $A^T P + P A = -Q$ with any arbitrary positive definite and symmetric matrix Q and Hurwitz matrix A given below:

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\mu_{n-1} & -\mu_{n-2} & -\mu_{n-3} & \cdots & -\mu_0 \end{bmatrix} \tag{87}$$

Applying the control and adaptation laws given by Eqs. (85) and (86) to the system (79) results in the following tracking error bound:

$$\|E\| \leq \frac{2\varepsilon_f \lambda_{\max}(P)}{\lambda_{\min}(Q)} \tag{88}$$

in which $\lambda(.)$ is the eigen-value and $\|...\|$ is the Euclidian norm.

∎

Proof:

To achieve the control and adaptation laws given by Eq. (85) and (86) consider the following Lyapunov function:

$$V = E^T P E + \frac{1}{2} \left\| \Theta - \Theta_o \right\|^2 \tag{89}$$

Differentiating both sides of Eq. (89) yields:

$$\dot{V} = \dot{E}^T PE + E^T P\dot{E} + \left(\Theta - \Theta_o\right)^T \dot{\Theta} \tag{90}$$

By subtracting Eq. (80) from Eqs. (79) and using the control law (85) we have:

$$
\begin{aligned}
x^{(n)} - \overline{x}^{(n)} &= f(X) - f(\overline{X}) - g(X)\left(\frac{1}{g(X)}\left[\hat{f}(X) - \overline{x}^{(n)} + \sum_{i=0}^{n-1} \lambda_i (x^{(i)} - \overline{x}^{(i)})\right]\right) \\
&= f(X) - f(\overline{X}) - \sum_{i=0}^{n-1} \mu_{n-1-i}(x^{(i)} - \overline{x}^{(i)})
\end{aligned}
\tag{91}
$$

Using Eq. (87), Eq (91) can be rewritten as:

$$\dot{E} = AE + B\left(f(X) - \hat{f}(X)\right) \tag{92}$$

Substituting Eq. (92) and the adaptation law (86) into Eq. (90) get:

$$
\begin{aligned}
\dot{V} &= \left[E^T A^T + \left(f(X) - \hat{f}(X)\right)B^T\right]PE + E^T P\left[AE + B\left(f(X) - \hat{f}(X)\right)\right] + \left(\Theta - \Theta_o\right)^T \Phi(X)B^T PE \\
&= E^T \left(A^T P + PA\right)E + 2\left(f(X) - \hat{f}(X)\right)B^T PE + \left(\Theta - \Theta_o\right)^T \Phi(X)B^T PE
\end{aligned}
\tag{93}
$$

Adding and subtracting an $\hat{f}_o(.)$ in the above equation results in

$$
\begin{aligned}
\dot{V} &= E^T \left(A^T P + PA\right)E + 2\left(f(X) - \hat{f}_o(X) + \hat{f}_o(X) - \hat{f}(X)\right)B^T PE + \left(\Theta - \Theta_o\right)^T \Phi(X)B^T PE \\
&= E^T \left(A^T P + PA\right)E + 2\left(\Delta f - \left(\Theta - \Theta_o\right)^T \Phi(X)\right)B^T PE + \left(\Theta - \Theta_o\right)^T \Phi(X)B^T PE
\end{aligned}
\tag{94}
$$

where $\Delta f = f(.) - \hat{f}_o(.)$. Using inequality (84) in Eq. (94) yields:

$$\dot{V} \leq -E^T QE + 2\varepsilon_f B^T PE \tag{95}$$

Since $Q - \lambda_{\min}(Q)I$ and $\lambda_{\max}(P)I - P$ are positive semi-definite (I is the identity matrix), we have:

$$\dot{V} \leq -\lambda_{\min}(Q)\|E\|^2 + 2\varepsilon_f \lambda_{\max}(P)\|E\| \tag{96}$$

From the above inequality it is concluded that:

$$\|E(t)\| \leq \frac{2\varepsilon_f \lambda_{\max}(P)}{\lambda_{\min}(Q)} \quad as \quad t \to \infty \tag{97}$$

because for $\|E\| > \dfrac{2\varepsilon_f \lambda_{\max}(P)}{\lambda_{\min}(Q)}$, we have $\dot{V} < 0$ and consequently the region defined by (97) is an attracting set for the error trajectories.

Remark

From Eq. (97) it is concluded that if $\varepsilon_f \in L^1$, E converges to zero as t approaches to infinity.

Proof:

From inequality (95) we have:

$$\lambda_{\min}(Q)\int_0^\infty \|E\|^2\, dt \le -\int_0^\infty \dot{V}dt + 2\lambda_{\max}(P)\int_0^\infty \varepsilon_f \|E\|dt \tag{98}$$

Or

$$\lambda_{\min}(Q)\int_0^\infty \|E\|^2\, dt \le V(0)-V(\infty) + 2\lambda_{\max}(P)\int_0^\infty \varepsilon_f \|E\|dt \tag{99}$$

If $\varepsilon_f \in L^1$, then from the above equation, it is concluded that $E \in L^2$. Besides, $\dot{E}$ satisfies Eq. (92) which consists of continuously differentiable functions implying that $\dot{E} \in L^\infty$. Since $E \in L^2 \cap L^\infty$ and $\dot{E} \in L^\infty$, due to Barbalat's Lemma (Sastry and Bodson, 1994), the E converges to zero as t approaches to infinity.

4.4 Simulation results

The above described control scheme is used to control states of the modified Duffing system. Consider the following modified Duffing system:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \alpha x_1 + \beta x_1^3 + \delta x_2 + f_0 \cos t + (3 + \cos x_1)u \tag{100}$$

The system shows chaotic behavior for $\alpha = 1$, $\beta = -1$, $\delta = -0.15$, $f_0 = 0.3$ and $u = 0$. For this system $g(X) = 3 + \cos x_1$. The objective is to force the state variables to track the desired trajectory using adaptive control scheme described in the previous section. For $\alpha = 1$, $\beta = -1$, $\delta = -0.15$, $f_0 = 0.3$, the Duffing system (100) has periodic orbits with 2π-period (Fig. (12)) which is selected as desired trajectory in simulation study.
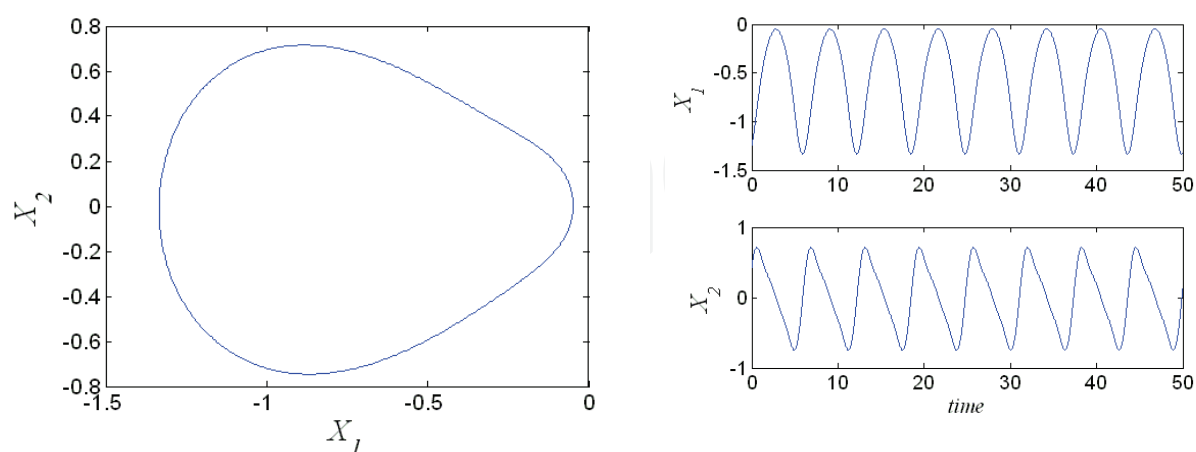

Fig. 12. 2π periodic solution of the Duffing system (Layeghi et al. 2008).

The variation ranges of x_1 and x_2 are partitioned into 3 fuzzy sets with Gaussian membership functions, $\mu(x) = \exp\left[-\left(\frac{x-\bar{x}}{\sigma}\right)^2\right]$, whose centers are at $\{-2,-1,0,1,2\}$. The range

of t is partitioned to 4 fuzzy sets with Gaussian membership functions and centers at $\{0,2,4,6\}$. Note that t in Eq.(100) can be always considered in the interval $(0,2\pi)$. The parameter σ is chosen such that the summation of membership values at the intersection of any two neighboring membership functions be equal to 1. Simulation is performed for $Q = 2I$.

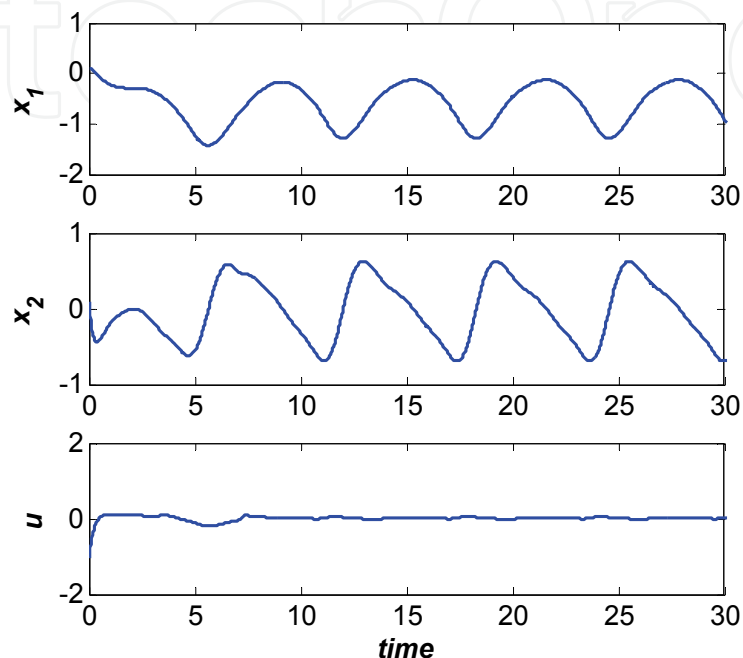Simulation results of the presented method are shown in Fig. (13).

Fig. 13. (a) Trajectory of x_1, x_2 and control action u for fuzzy adaptive control of Duffing system.

As can be seen from Figs. (13), stability of unstable periodic orbits is completely achieved and the tracking errors vanish.

5. Conclusion

In this chapter adaptive control of chaos has been studied. Indirect and direct adaptive control techniques have been presented, and their applications for chaos control of two classes of discrete time and continuous time systems have been considered. The presented methods have been applied to some chaotic systems to investigate the effectiveness and performance of the controllers. In addition a fuzzy adaptive control method has been introduced and it has been utilized for control of a chaotic system.

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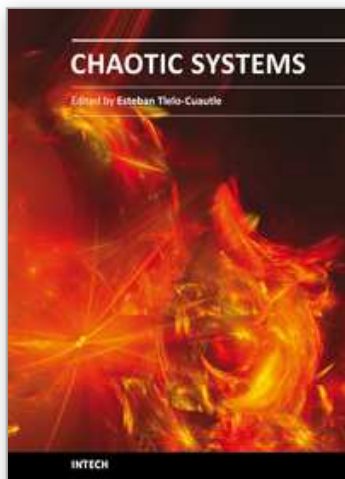
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This book presents a collection of major developments in chaos systems covering aspects on chaotic behavioral modeling and simulation, control and synchronization of chaos systems, and applications like secure communications. It is a good source to acquire recent knowledge and ideas for future research on chaos systems and to develop experiments applied to real life problems. That way, this book is very interesting for students, academia and industry since the collected chapters provide a rich cocktail while balancing theory and applications.

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