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Improving the Pose Accuracy of Planar Parallel Robots using Mechanisms of Variable Geometry

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1. Introduction

In comparison to classical serial mechanisms parallel kinematic machines (PKM) provide a higher accuracy, a higher stiffness, and higher dynamic properties. However, parallel robots suffer from the presence of singularities within their workspace (Gosselin & Angeles, 1990). In such configurations the moving platform gains at least one degree of freedom (DOF) and the actuation forces become (in theory) infinite. As a result, the kinematic structure can be damaged or even destroyed. Additionally, several performance indices, e.g. the achievable accuracy, are directly related to the singularity loci (Kotlarski, Abdellatif & Heimann, 2008). The closer the endeffector (EE) is 'located' to a singularity the higher is the pose error resulting from the influence of active joint errors, e.g. from limited encoder resolution.

In order to minimize the singularity loci of parallel mechanisms and to increase their performance, e.g. their achievable accuracy, redundancy can be used (Merlet, 1996). Two redundancy approaches are established for PKM, actuation redundancy and kinematic redundancy (Kock & Schumacher, 1998; Wang & Gosselin, 2004). Actuation redundancy can be realized whether by adding a kinematic chain to the mechanism or by actuating a passive joint. Amongst others, it reduces singular configurations and leads to internal preload that can be controlled in order to prevent backlash (Kock, 2001). However, the control of such mechanisms is a challenging task (Müller, 2005). Furthermore, an additional kinematic chain mostly reduces the total workspace. Therefore, kinematic redundancy is proposed realized by adding at least one actuated joint to one kinematic chain (Cha et al., 2007; Mohamed & Gosselin, 2005).

It is well known that the singularity loci as well as the achievable accuracy are greatly affected by the geometrical parameters of a mechanism (Kotlarski, de Nijs, Abdellatif & Heimann, 2009; Merlet & Daney, 2005), and are therefore highly dependent on the mechanism's actual configuration. In this chapter, as examples, kinematically redundant versions of the well known planar 3RRR and 3RPR mechanisms (Gosselin & Angeles, 1988; Zein et al., 2006) are considered. In each case, an additional prismatic actuator is added to an arbitrary base joint. The introduced mechanisms are denoted as 3(P)RRR and 3(P)RPR. Thanks to the additional prismatic actuator, the inverse displacement problem has an infinite number of solutions (Ebrahimi et al., 2007). Hence, reconfigurations of the mechanisms can be performed selectively in order to avoid singularities and to affect their performance directly (Kotlarski, Do Thanh, Abdellatif & Heimann, 2008). It is important to note that with respect to the work of Arakelian et al. (Arakelian et al., 2008), kinematic redundancy can be used to rather change the geometrical parameters of a mechanism than its basic structure. This can be done at the

task planning stage or while operating the manipulator using several different strategies (see Sec. 3.2).

Here, the key idea is to change the position of the redundant actuator in a discrete manner while operating the mechanism, in particular just before shifts in direction of the moving platform. This allows for optimizing the accuracy of the manipulator for a given trajectory segment. After each switching operation, i.e. after each reconfiguration, the additional prismatic actuator is supposed to remain locked. Therefore, the joint clearance as well as the control error corresponding to the redundant actuator can be minimized. The resulting set of discrete actuator positions is called the switching pattern. The optimization of the switching patterns is achieved according to a performance index denoted as the gain of the maximal homogenized pose error (see Sec. 3.2).

The rest of this chapter is organized as follows. In Sec. 2 the geometric and the inverse kinematic models of the proposed mechanisms are given as well as fundamental definitions related to their Jacobian analysis. Sec. 3 clarifies the idea of kinematic redundancy in order to avoid singularities and to increase the performance of a PKM. Additionally, it gives a brief theoretical overview on the determination of the achievable moving platform pose accuracy and introduces the optimization strategy developed for the redundant actuator position. In Sec. 4 several analysis examples are presented in order to validate the proposed redundant schemes with the optimized switching patterns. It is demonstrated that the kinematically redundant mechanisms in combination with the developed optimization procedure lead to a great improvement in terms of singularity avoidance and, therefore, in terms of accuracy and precision. Furthermore, it is shown that the proposed optimization criterion is able to outperform the classical accuracy related performance index, i.e. the condition number of the Jacobian matrix. Sec. 5 concludes this chapter.

2. Kinematically redundant mechanisms

In the following, both the geometrical and the inverse kinematic models of the exemplarily analyzed planar, kinematically redundant mechanisms are given along with fundamental definitions related to the Jacobian analysis. An additional prismatic actuator is proposed allowing one base joint to move linearly. Hence, reconfiguration of the mechanisms can be performed selectively while operating the manipulators.

2.1 Redundant 3(P)RRR mechanism

In (Kotlarski et al., 2007) the kinematically redundant 3(P)RRR planar mechanism (see Fig. 1) was introduced. It is basically similar to the non-redundant 3RRR mechanism studied amongst others in (Gosselin & Angeles, 1988). Three kinematic chains $G_iM_iP_i$ ($i = 1, 2, 3$) connect the moving platform $P_1P_2P_3$ to the base $G_1G_2G_3$. Each kinematic chain consists of two links $l_{i,1}$ and $l_{i,2}$. The position of the joints G_i , M_i , and P_i with respect to the inertial coordinate frame $(CF)_0$ is given by $\mathbf{g}_i = (x_{G_i}, y_{G_i})^T$, $\mathbf{m}_i = (x_{M_i}, y_{M_i})^T$, and $\mathbf{p}_i = (x_{P_i}, y_{P_i})^T$. The base-fixed revolute joints are active while the remaining ones are passive. The orientation of the redundant actuator with respect to the x -axis of $(CF)_0$ is denoted by α . Positions referenced with respect to the platform fixed coordinate frame $(CF)_E$ are marked with $(')$.

In the following the configuration of the moving platform is given by

$$\mathbf{x} = (x_E, y_E, \phi)^T, \quad (1)$$

where x_E and y_E represent the point of origin of the platform fixed coordinate frame $(CF)_E$ with respect to $(CF)_0$ and ϕ is its orientation. The mechanism is driven by the four actuators.

Fig. 1. Kinematically redundant 3(P)RRR mechanism

Therefore, the system input is given by the according actuator coordinates

$$\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3, \delta)^T.$$

2.1.1 Inverse kinematics

For each kinematic chain i the geometric constraints can be written as

$$\begin{pmatrix} x_{P_i} \\ y_{P_i} \end{pmatrix} = \begin{pmatrix} x_{G_i} \\ y_{G_i} \end{pmatrix} + \begin{pmatrix} \cos(\theta_i) & \cos(\theta_i + \psi_i) \\ \sin(\theta_i) & \sin(\theta_i + \psi_i) \end{pmatrix} \begin{pmatrix} l_{i,1} \\ l_{i,2} \end{pmatrix},$$

where the position of the moving platform’s passive joints with respect to $(CF)_0$ is defined as

$$\begin{pmatrix} x_{P_i} \\ y_{P_i} \end{pmatrix} = \begin{pmatrix} x_E \\ y_E \end{pmatrix} + \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix} \begin{pmatrix} x'_{P_i} \\ y'_{P_i} \end{pmatrix}.$$

In the presented redundant case the position of G_1 depends on the actuator position δ :

$$\begin{pmatrix} x_{G_1} \\ y_{G_1} \end{pmatrix} = \begin{pmatrix} x_{G_1} \\ y_{G_1} \end{pmatrix} \Big|_{\delta=0\text{m}} + \begin{pmatrix} \delta \cos(\alpha) \\ \delta \sin(\alpha) \end{pmatrix}.$$

From (3) the passive joint angles ψ_i can be determined:

$$\psi_i = \pm \arccos \left(\frac{x_{GP_i}^2 + y_{GP_i}^2 - l_{i,1}^2 - l_{i,2}^2}{2 l_{i,1} l_{i,2}} \right),$$

where $x_{GP_i} = x_{P_i} - x_{G_i}$ and $y_{GP_i} = y_{P_i} - y_{G_i}$. The active joint angles θ_i are finally obtained using (3) and (6):

$$\theta_i = \arctan \left(\frac{(l_{i,1} + l_{i,2} \cos(\psi_i)) y_{GP_i} - (l_{i,2} \sin(\psi_i)) x_{GP_i}}{(l_{i,1} + l_{i,2} \cos(\psi_i)) x_{GP_i} + (l_{i,2} \sin(\psi_i)) y_{GP_i}} \right),$$

Additionally, for simplification reasons, the angles ζ_i are defined as the counterclockwise angles that the passive links form with the x -axis:

$$\zeta_i = \arctan \left(\frac{y_{GP_i} (l_{i,2} + l_{i,1} \cos(\psi_i)) + (l_{i,1} \sin(\psi_i)) x_{GP_i}}{x_{GP_i} (l_{i,2} + l_{i,1} \cos(\psi_i)) - (l_{i,1} \sin(\psi_i)) y_{GP_i}} \right). \quad (8)$$

2.1.2 Jacobian formulation

Several performance criteria and indices, e.g. the achievable accuracy, can be calculated based on the Jacobian matrices of a PKM (see Sec. 4). After summing the squares of (3) the Jacobians can be obtained by a derivation of the resulting inverse kinematic equations f_i

$$f_i \equiv 0 = (x_{P_i} - x_{G_i} - l_{i,1} \cos(\theta_i))^2 + (y_{P_i} - y_{G_i} - l_{i,1} \sin(\theta_i))^2 - l_{i,2}^2, \quad (9)$$

with respect to time (Gosselin & Angeles, 1990):

$$\frac{\partial f}{\partial x} \dot{x} + \frac{\partial f}{\partial \theta} \dot{\theta} = 0 \Leftrightarrow A \dot{x} + B \dot{\theta} = 0. \quad (10)$$

For the 3(P)RRR mechanism the direct and the inverse Jacobian matrices A and B result to

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & 0 & 0 & b_{14} \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \end{pmatrix}, \quad (11)$$

with (for $i = 1, 2, 3$)

$$\begin{aligned} a_{i1} &= x_{P_i} - x_{G_i} - l_{i,1} \cos \theta_i, \\ a_{i2} &= y_{P_i} - y_{G_i} - l_{i,1} \sin \theta_i, \\ a_{i3} &= -a_{i1} (x'_{P_i} \sin(\phi) + y'_{P_i} \cos(\phi)) + a_{i2} (x'_{P_i} \cos(\phi) - y'_{P_i} \sin(\phi)), \end{aligned} \quad (12)$$

and

$$\begin{aligned} b_{ii} &= l_{i,1} (a_{i1} \sin(\theta_i) - a_{i2} \cos(\theta_i)), \\ b_{14} &= -(a_{11} \cos(\alpha) + a_{12} \sin(\alpha)). \end{aligned} \quad (13)$$

As long as the Jacobian A is nonsingular, its inverse A^{-1} can be determined analytically:

$$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{33}a_{22} - a_{32}a_{23} & -(a_{33}a_{12} - a_{32}a_{13}) & a_{23}a_{12} - a_{22}a_{13} \\ -(a_{33}a_{21} - a_{31}a_{23}) & a_{33}a_{11} - a_{31}a_{13} & -(a_{23}a_{11} - a_{21}a_{13}) \\ a_{32}a_{21} - a_{31}a_{22} & -(a_{32}a_{11} - a_{31}a_{12}) & a_{22}a_{11} - a_{21}a_{12} \end{pmatrix}. \quad (14)$$

It will be useful for calculating the achievable accuracy as demonstrated in Sec. 3.1.

2.2 Redundant 3(P)RPR mechanism

Additionally, the kinematically redundant 3(P)RPR planar mechanism presented in Fig. 2 is considered. It was firstly introduced in (Kotlarski, Abdellatif, Ortmaier & Heimann, 2009). It is based on the well known 3RPR mechanism (Zein et al., 2006). Three kinematic chains G_iP_i ($i = 1, 2, 3$) consisting of active prismatic joints connect the moving platform $P_1P_2P_3$ to the base $G_1G_2G_3$. In the following, notations and definitions similar to the 3(P)RRR mechanism

Fig. 2. Kinematically redundant 3(P)RPR mechanism

and already introduced are not mentioned again.
The system input is given by the four actuator coordinates

$$\boldsymbol{\theta} = (\rho_1, \rho_2, \rho_3, \delta)^T, \tag{15}$$

where ρ_i defines the lengths of the kinematic chain i .

2.2.1 Inverse kinematics

The geometric constraints of the 3(P)RPR mechanism can be written as

$$\begin{pmatrix} x_{P_i} \\ y_{P_i} \end{pmatrix} = \begin{pmatrix} x_{G_i} \\ y_{G_i} \end{pmatrix} + \rho_i \begin{pmatrix} \cos(\xi_i) \\ \sin(\xi_i) \end{pmatrix}, \tag{16}$$

where the passive joint angles ξ_i are obtained by

$$\xi_i = \arctan \left(\frac{y_{GP_i}}{x_{GP_i}} \right). \tag{17}$$

From the Euclidean norm of the vector connecting point G_i to point P_i the lengths of each kinematic chain i are obtained

$$\rho_i^2 = x_{GP_i}^2 + y_{GP_i}^2. \tag{18}$$

2.2.2 Jacobian formulation

Similar to (11) and using f_i (cp. (18))

$$f_i \equiv 0 = x_{GP_i}^2 + y_{GP_i}^2 - \rho_i^2, \tag{19}$$

for the 3(P)RPR mechanism the elements of the direct and inverse Jacobian matrices $\mathbf{A}$ and $\mathbf{B}$ result to

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_{11} & 0 & 0 & b_{14} \\ 0 & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & 0 \end{pmatrix}, \tag{20}$$

with (for $i = 1, 2, 3$)

$$\begin{aligned} a_{i1} &= x_{GP_i}, \\ a_{i2} &= y_{GP_i}, \\ a_{i3} &= -a_{i1} \left(x'_{P_i} \sin(\phi) + y'_{P_i} \cos(\phi) \right) + a_{i2} \left(x'_{P_i} \cos(\phi) - y'_{P_i} \sin(\phi) \right), \end{aligned} \quad (21)$$

and

$$\begin{aligned} b_{ii} &= -\rho_i, \\ b_{14} &= -(a_{11} \cos(\alpha) + a_{12} \sin(\alpha)). \end{aligned} \quad (22)$$

3. Singularity avoidance and accuracy improvement using kinematic redundancy

The condition for type-two singularities of planar mechanisms can be formulated geometrically (Hunt, 1978; Yang et al., 2002). For the here treated and common case of revolute passive joints at the moving platform, a pose of the robot is singular if the three lines passing through the passive links of the kinematic chains (passive lines) intersect at a common point or are all parallel. Thanks to the kinematic redundancy, the direction of the passive lines and, therefore, the Jacobians' elements can be directly affected. As a result, the singularity loci change as shown in Fig. 3, exemplarily for the introduced kinematically redundant versions of the planar 3RRR (top) and 3RPR (bottom) mechanisms with a base joint mounted on an additional prismatic actuator.

Fig. 3. Variation of the singularity loci (solid red) due to the redundant actuator configuration, i.e. the base joint position; top: redundant 3(P)RRR, bottom: redundant 3(P)RPR

The use of kinematic redundancy to avoid singularities is demonstrated in Fig. 4, left. The given path would cross a singularity for the symmetric, i.e. the 'classical', configuration. By

moving the redundant actuator towards the right, the singularity loci could be completely removed. In case of the mechanism performance, e.g. in case of the achievable accuracy, similar effects are given. Regions suffering from high pose errors, i.e. workspace regions that do not provide a certain desired accuracy, can be 'moved' and, therefore, avoided when following a desired trajectory as shown in Fig. 4, right. Hence, the achievable accuracy increases

Fig. 4. Trajectory (solid black) going through a singular configuration (left, solid red) / a region where a certain accuracy in one or more directions is not given (right, yellow); a reconfiguration (from the left to the right) allows to follow the desired path

significantly as demonstrated in Sec. 4.

3.1 The moving platform's pose error

Due to several factors, like manufacturing errors, joint clearance, and active joint errors, the pose of the moving platform can be provided only within a given accuracy. An approach to determine the achievable accuracy of a PKM while considering any kind of uncertainties can be found in (Kotlarski, Abdellatif, Ortmaier & Heimann, 2009). Referring to (Merlet, 2006a), the active joint errors, e.g. the limited resolution of the encoders, are the major sources of error in a calibrated and precisely manufactured PKM. Therefore, the analysis is focussed on the achievable accuracy of a moving platform in the presence of active joint errors only.

By rewriting the velocity equation (10) in incremental form, an approximation that relates the active joint errors (the input error) $\Delta\theta$ to the pose error (the output error) Δx is obtained:

$$A \Delta x + B \Delta \theta \Leftrightarrow A \Delta x = -B \Delta \theta \Rightarrow \Delta x = A^{-1} (-B) \Delta \theta = -J \Delta \theta. \quad (23)$$

Using (23) and incorporating the fact that $|ab| = |a||b|$ and $|a + b| \leq |a| + |b| \forall (a, b) \in R$ the maximal pose error vector $\overline{\Delta x}$ can be calculated by

$$\overline{\Delta x} = \begin{pmatrix} \overline{\Delta x} \\ \overline{\Delta y} \\ \overline{\Delta \phi} \end{pmatrix} = \begin{pmatrix} |J_{11}| & |J_{12}| & |J_{13}| & |J_{14}| \\ |J_{21}| & |J_{22}| & |J_{23}| & |J_{24}| \\ |J_{31}| & |J_{32}| & |J_{33}| & |J_{34}| \end{pmatrix} \begin{pmatrix} |\Delta \theta_1| \\ |\Delta \theta_2| \\ |\Delta \theta_3| \\ |\Delta \delta| \end{pmatrix} \geq |\Delta x|. \quad (24)$$

The Jacobian element of row i and column j is denoted as J_{ij} . Since the Jacobian matrix J highly depends on the actuator position δ , i.e. on the robot geometry, the additional DOF of the proposed kinematically redundant mechanisms can be used to affect the Jacobian elements and, therefore, the robot accuracy directly. Fig. 5 shows the elements of $\overline{\Delta x}$ with respect to the actuator position δ for the chosen kinematically redundant mechanisms (left: 3(P)RRR, right 3(P)RPR) and an arbitrary constant EE pose x . The elements of the active joint error

vector $\Delta\theta$ are taken from data sheets of available actuators. The authors forbear from giving the concrete parameters, i.e. the robot geometry, the EE pose, and the active joint errors, because the characteristics of $\overline{\Delta x}$ with respect to δ is similar in all cases. The dependency of

Fig. 5. Positioning error $\overline{\Delta x}$ and $\overline{\Delta y}$ (solid gray), the overall translational error $\overline{\Delta xy}$ (solid black) and the orientational error $\overline{\Delta\phi}$ (solid red) with respect to the redundant actuator position δ and for a constant EE pose x

the achievable accuracy on the redundant actuator position is well noticeable. Additionally, it can be seen that the elements of the maximal pose error $\overline{\Delta x}$, $\overline{\Delta y}$, and $\overline{\Delta\phi}$ as well as the overall translational error $\overline{\Delta xy} = \|(\overline{\Delta x}, \overline{\Delta y})\|_2$ have similar minima. Hence, in most cases, the maximal accuracy of each DOF can be increased for almost identical actuator positions δ by an appropriate reconfiguration of the mechanism. Therefore, an optimization procedure is required in order to find the best solution for δ .

3.2 Optimization of the redundant actuator position

The optimization of the redundant actuator position δ can be performed based on two main strategies: a classical continuous optimization and a selective discrete optimization. The latter is the key idea of this chapter and is discussed in the following.

Undoubtedly, a continuous optimization leads to an instantly influenceable, i.e. maximal achievable, accuracy. In contrast to the mentioned advantage, it results in a more challenging task concerning the robot control and usually in a higher energy demand. The proposed approach is based on the optimization of δ in a discrete manner while operating the system. Therefore, the trajectory is divided into segments. The starting and final points of the segments are certain poses, e.g. shifts in direction. Appropriate constant values of the actuator position δ corresponding to the different segments of the desired trajectory are determined. The resulting set of discrete actuator positions is called the optimized switching pattern. While moving along the desired trajectory, the position of the redundant actuator is changed according to the switching pattern. This allows for the reconfiguration of the mechanism to influence its accuracy for a given path segment. While performing a reconfiguration the pose of the moving platform is kept constant. After each switching operation, e.g. while moving along a trajectory segment, the additional prismatic actuator is supposed to remain locked. Therefore, compliance, e.g. resulting from joint clearance, as well as the control error corresponding to the redundant actuator are minimized.

In order to further minimize the switching operations the mentioned discrete optimization, i.e. the 'main idea', can be additionally modified in several ways. One possibility is to only

change the redundant actuator position once before starting the desired movement. As a result, the number of reconfigurations is minimized. But, regarding complex trajectories, i.e. trajectories going through a large area of the robot workspace, this may lead to an unacceptable performance, e.g. with respect to the accuracy. Thus, another possible modification is to perform a reconfiguration only if the mechanism is unable to perform the desired operation, e.g. following a singularity-free trajectory and providing a certain accuracy, in its current configuration (Kotlarski, Do Thanh, Abdellatif & Heimann, 2008). Therefore, before moving the EE, for the upcoming trajectory segment the required performance criteria have to be calculated. If any criteria is less than its corresponding threshold a reconfiguration of the mechanism has to be performed. The mentioned modification of the proposed selective discrete optimization strategy further reduces the inconvenient switching operations and guarantees a desired performance. Nevertheless, in this chapter it is focussed on the main idea of the discrete reconfiguration strategy without any of the modifications mentioned in this paragraph. It mostly clarifies the influence of the additional prismatic actuator which is the authors' main purpose.

The optimization can be realized with respect to several criteria and performance indices: a well accepted criterion is the condition number (in general the two-norm condition number) of the Jacobian matrix $\kappa(\mathbf{J})$ and its inverse $\eta = \kappa^{-1}$ called dexterity. In (Gosselin, 1992) it is defined as:

$$\kappa(\mathbf{J}) = \kappa(\mathbf{J}^{-1}) = \|\mathbf{J}^{-1}\|_2 \|\mathbf{J}\|_2, \quad 1 \leq \kappa \leq \infty, \quad (25)$$

where $\kappa = 1$ represents an isotropic configuration without an amplification of the active joint error $\Delta\theta$ and $\kappa = \infty$ represents a singular configuration with an infinite amplification of $\Delta\theta$ leading to (in theory) an infinite $\Delta\mathbf{x}$. However, the moving platforms of the considered mechanisms have two translational as well as one rotational DOF. As a result, the Jacobian matrix $\mathbf{J}$ is not homogeneous in terms of physical units. Therefore, the value of the condition number depends on the unit choice. Hence, a modification of the Jacobian matrix is required in order to obtain appropriate values for κ . Amongst others, the homogeneity can be achieved by transforming the moving platform velocity $\dot{\mathbf{x}}$ into the linear velocity $\dot{\mathbf{x}}_h = (\dot{x}_{P_1}, \dot{y}_{P_1}, \dot{x}_{P_2}, \dot{y}_{P_2})^T$ of two arbitrary points P_1 and P_2 (Pond & Carretero, 2006). Therefore, a transformation matrix $\mathbf{Q}$ has to be found that satisfies the following equation:

$$\dot{\mathbf{x}}_h = \mathbf{Q}\dot{\mathbf{x}}, \quad (26)$$

where the subscript 'h' indicates homogeneous. But, instead of describing a manipulator with three DOF by the four parameters $\dot{\mathbf{x}}_h$, a reduction of the terms describing the velocities of the moving platform to three can be performed (Gosselin, 1992). As a result, the dimension of the Jacobian matrix $\mathbf{J}$ remains constant. Therefore, a coordinate frame $(\text{CF})_{E_h}$, located at P_1 , is attached to the moving platform such that its x -axis passes through P_1 and P_2 . For the proposed mechanisms, by choosing $\dot{\mathbf{x}}_h = (\dot{x}_E, \dot{y}_E, \dot{y}_{P_3})^T$ (cp. Fig. 1 and Fig. 2), the modified transformation matrix $\mathbf{Q}$ results to:

$$\mathbf{Q} = \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ -\sin \beta & \cos \beta & \|\mathbf{p}_3\|_2 \end{bmatrix}, \quad (27)$$

where the angle β gives the orientation of $(\text{CF})_0$ to $(\text{CF})_{E_h}$. It is important to note that the points P_1 and P_2 can be chosen arbitrary as long as they fulfill the mentioned characteris-

tics (Gosselin, 1992). The homogenized Jacobian matrix $\mathbf{J}_h$ can finally be determined using (23) and (26):

$$\mathbf{J}_h = \mathbf{Q}\mathbf{J}. \quad (28)$$

Hence, an optimization of the actuator position δ can be performed by a minimization of the condition number $\kappa(\mathbf{J}_h)$ and by a maximization of the dexterity $\eta(\mathbf{J}_h)$, respectively:

$$\delta_{\text{opt}} = \arg \left(\min_{\delta} \kappa(\mathbf{J}_h) \right) \triangleq \delta_{\text{opt}} = \arg \left(\max_{\delta} \eta(\mathbf{J}_h) \right). \quad (29)$$

As demonstrated by Merlet (Merlet, 2006b) and shown later in Sec. 4 the condition number does not necessarily exhibit a complete consistent behavior with respect to the pose error of a robot. Therefore, an optimization of the actuator position δ based on minimizing the two-norm of the maximal homogenized pose error $\Delta \mathbf{x}_h$ is proposed:

$$\gamma(\overline{\Delta \mathbf{x}_h}) = \|\overline{\Delta \mathbf{x}_h}\|_2 = \left\| \begin{pmatrix} |J_{h11}| & |J_{h12}| & |J_{h13}| & |J_{h14}| \\ |J_{h21}| & |J_{h22}| & |J_{h23}| & |J_{h24}| \\ |J_{h31}| & |J_{h32}| & |J_{h33}| & |J_{h34}| \end{pmatrix} \begin{pmatrix} |\Delta \theta_1| \\ |\Delta \theta_2| \\ |\Delta \theta_3| \\ |\Delta \delta| \end{pmatrix} \right\|_2, \quad (30)$$

where the elements of the active joint error vector $\Delta \boldsymbol{\theta}$, i.e. their limited resolutions, are well known from the data sheets of the actuators. This index is called the gain $\gamma(\overline{\Delta \mathbf{x}_h})$ of the maximal homogenized pose error $\Delta \mathbf{x}_h$. Although the influence of the prismatic actuator joint error $\Delta \delta$ on the pose error $\Delta \mathbf{x}$ is small only (see Sec. 4) it should not be neglected. The cost function to be minimized results to:

$$\delta_{\text{opt}} = \arg \left(\min_{\delta} \gamma(\overline{\Delta \mathbf{x}_h}) \right). \quad (31)$$

There might be trajectories, e.g. regarding special applications, for which a high accuracy is required in certain DOF only. In this case, the optimization criterion (31) can be adopted such that the corresponding elements (or a single element) of $\overline{\Delta \mathbf{x}_h}$ are solely minimized.

4. Accuracy analysis - numerical results

Several examples are presented in order to validate the proposed redundant scheme with the developed optimized switching patterns. The advantage of the approach is verified for different trajectories. Additionally, the influence of the redundant prismatic actuator on the moving platform pose accuracy is demonstrated. Moreover, in order to further confirm the results given in Sec. 4.1, the useable workspace, i.e. the singularity-free part of the workspace providing a certain performance, of the considered mechanisms is determined (and compared).

4.1 Simulation of single trajectories

Accuracy analysis along selected simulated trajectories were performed. The geometrical parameters of the analyzed kinematically redundant mechanisms and their non-redundant counterparts are given in Table 1. In the redundant case, one prismatic actuator is attached to G_1 of the basic structure. Keeping the design space in mind the orientation of the redundant actuator was set to $\alpha = 0^\circ$. At this point, it is important to note that the design of the additional prismatic actuator, i.e. its stroke as well as its orientation, was more or less chosen intuitively. Future work will deal with an optimization of the parameters related to the redundant actuator.

	$i = 1$	$i = 2$	$i = 3$	
$x_{G_i}[\text{m}]$	0.6	0	1.2	} 3(P)RRR & 3(P)RPR
$y_{G_i}[\text{m}]$	$\sqrt{27}/5$	0	0	
$x'_{P_i}[\text{m}]$	0	-0.125	0.125	
$y'_{P_i}[\text{m}]$	0	$-\sqrt{3}/8$	$-\sqrt{3}/8$	
$l_{i,1}[\text{m}]$	0.6	0.6	0.6	} 3(P)RRR
$l_{i,2}[\text{m}]$	0.6	0.6	0.6	
$\rho_{i,\text{min}}[\text{m}]$	0.1	0.1	0.1	} 3(P)RPR
$\rho_{i,\text{max}}[\text{m}]$	1.2	1.2	1.2	

Table 1. Design parameters of the analyzed 3(P)RRR and 3(P)RPR mechanisms ($-0.5 \text{ m} \leq \delta \leq 0.5 \text{ m}$)

4.1.1 Redundant 3(P)RRR mechanism

First, the accuracy analysis of the kinematically redundant 3(P)RRR mechanism is performed. Exemplarily, simulation results of the three triangular trajectories (t_I, t_{II}, t_{III}) shown in Fig. 6 are presented. In order to clarify the effectiveness of the proposed concept the trajectories were chosen within the workspace of the mechanisms (solid black) such that the non-redundant mechanism ($\delta = 0 \text{ m}$) does not pass any singular configurations when $\phi = 0^\circ$. Without loss of generality, the regarded 3RRR-based mechanisms are in the following assumed to remain in the same working mode which is shown in Fig. 1.

Fig. 6. Exemplarily chosen trajectories t_I, t_{II}, t_{III} (solid gray) for the 3RRR-based mechanisms, the solid red lines represent the singularity loci within the workspace (solid black)

The EE was moved counterclockwise along the depicted trajectories with a constant orientation. The trajectories were divided such that each side of a triangular represents a segment. Hence, at every corner $c_{i,1}, c_{i,2}$, and $c_{i,3}$ ($i = I, II, III$) the position of the redundant actuator δ is switched according to the optimized switching pattern. During each switching operation the moving platform pose is kept constant. The optimization was performed based on the introduced cost functions (29) and (31). Even though the prismatic joint is locked between two switching phases, its joint error, e.g. the limited resolution of the encoder, has to be taken into account in order to obtain a realistic and practical accuracy analysis. Therefore, the active

joint errors were chosen based on data sheets of commercially available standard actuators to $\Delta\theta(3(\underline{P})\underline{RRR}) = (0.025^\circ, 0.025^\circ, 0.025^\circ, 40\ \mu\text{m})^T$. It is important to note that in the non-redundant case the last element of $\Delta\theta$ vanishes.

In Fig. 7 the optimized switching patterns δ_{opt} of the actuator position δ as well as the resulting mechanism pose errors $\overline{\Delta xy}$ and $\overline{\Delta\phi}$ are presented. The EE was moved along trajectory t_I with a constant orientation of $\phi = -30^\circ$, $\phi = 0^\circ$, and $\phi = 30^\circ$ denoted as $t_I(-30^\circ)$, $t_I(0^\circ)$, and $t_I(30^\circ)$, respectively. The distance the EE moved along the trajectory is denoted as s . A significant improvement of the accuracy due to the kinematic redundancy

Fig. 7. Simulation results while moving along trajectory $t_I(-30^\circ)$ (left), $t_I(0^\circ)$ (center), and $t_I(30^\circ)$ (right); solid gray: non-redundant mechanism; dashed black: optimized redundant mechanism using $\eta(\mathbf{J}_h)$; solid red: optimized redundant mechanism using $\gamma(\Delta\mathbf{x}_h)$

is well noticeable. E.g. regarding $t_I(-30^\circ)$ and $t_I(0^\circ)$, the maximal pose error occurring close to $c_{I,2}$ is minimized by a reconfiguration of the mechanism according to the optimized switching patterns. Fig. 7 shows that both optimization criteria ($\eta(\mathbf{J}_h)$ and $\gamma(\Delta\mathbf{x}_h)$) lead to similar switching patterns and to similar achievable accuracies. In Table 2 an overview of the maximal errors of the three triangular trajectories shown in Fig. 6 are given. In order to quantify the accuracy improvement the maximal translational $\overline{\Delta xy}_{\text{max}}$ and rotational error $\overline{\Delta\phi}_{\text{max}}$ of the moving platform over a complete trajectory was determined. The values represent the achievable accuracy of the associated mechanism. Additionally, the percentage increase/decrease of the kinematically redundant PKM in comparison to its non-redundant counterpart is given. Significant improvements of the achievable accuracy are well noticeable in most cases. Furthermore, e.g. for $t_{III}(30^\circ)$, it can be seen that an optimization based on the gain $\gamma(\Delta\mathbf{x}_h)$ leads

$t_i(\phi)$	Value	3RRR	3(P)RRR	
			using $\eta(\mathbf{J}_h)$	using $\gamma(\Delta \mathbf{x}_h)$
$t_I(-30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	7.13	1.34 (-88.3%)	1.34 (-88.3%)
	$\overline{\Delta \phi}_{\max}$ [°]	1.93	0.23 (-81.2%)	0.23 (-81.2%)
$t_I(0^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	1.44	1.02 (-28.8%)	0.91 (-36.9%)
	$\overline{\Delta \phi}_{\max}$ [°]	0.36	0.21 (-42.3%)	0.16 (-57.2%)
$t_I(30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.90	0.90 (-0.5%)	0.81 (-9.5%)
	$\overline{\Delta \phi}_{\max}$ [°]	0.32	0.32 (+2.2%)	0.31 (-2.6%)
$t_{II}(-30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	∞	0.69 (-)	0.58 (-)
	$\overline{\Delta \phi}_{\max}$ [°]	∞	0.14 (-)	0.12 (-)
$t_{II}(0^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	3.25	0.75 (-77.1%)	0.69 (-78.9%)
	$\overline{\Delta \phi}_{\max}$ [°]	1.50	0.22 (-85.3%)	0.22 (-85.3%)
$t_{II}(30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.63	0.70 (+10.3%)	0.68 (+6.8%)
	$\overline{\Delta \phi}_{\max}$ [°]	0.37	0.43 (+14.5%)	0.40 (+7.6%)
$t_{III}(-30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.57	0.48 (-15.3%)	0.48 (-15.3%)
	$\overline{\Delta \phi}_{\max}$ [°]	0.30	0.26 (-12.0%)	0.26 (-12.0%)
$t_{III}(0^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.70	0.86 (+22.8%)	0.60 (-14.9%)
	$\overline{\Delta \phi}_{\max}$ [°]	0.41	0.31 (-25.0%)	0.35 (-13.3%)
$t_{III}(30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	1.40	1.43 (+2.2%)	1.10 (-21.6%)
	$\overline{\Delta \phi}_{\max}$ [°]	0.41	0.44 (+7.0%)	0.35 (-14.9%)

Table 2. Redundant 3(P)RRR mechanism: maximal translational $\overline{\Delta xy}_{\max}$ and rotational error $\overline{\Delta \phi}_{\max}$ of the moving platform while moving along trajectory t_I , t_{II} , and t_{III}

to more appropriate switching patterns in comparison to an optimization based on the condition of the Jacobian $\eta(\mathbf{J}_h)$. Regarding $t_{II}(30^\circ)$, due to the additional active joint error $\Delta \delta$ there might be trajectory segments suffering from a decreased performance when using the proposed discrete optimization, i.e. the proposed switching patterns. This could be avoided using a continuous optimization. However, due to the mentioned advantages of the discrete switching patterns and due to the minimal decrease of the achievable accuracy only (Δxy : 0.05 mm and $\Delta \phi$: 0.03°), the authors still propose the selective discrete optimization of the redundant actuator position.

4.1.2 Redundant 3(P)RPR mechanism

Similar to Sec. 4.1.1, an accuracy analysis of the kinematically redundant 3(P)RPR mechanism is performed. Exemplarily, simulation results of the three triangular trajectories (t_I , t_{II} , t_{III}) which are shown in Fig. 8 are presented. In the following, facts and definitions similar to the analysis of the 3(P)RRR mechanism and already introduced are not mentioned again. Based on the data sheets of commercially available standard actuators, the active joint errors were chosen to $\Delta \theta(3(P)RPR) = (0.2 \text{ mm}, 0.2 \text{ mm}, 0.2 \text{ mm}, 40 \text{ }\mu\text{m})^T$. As well, in the non-redundant case the last element of $\Delta \theta$ vanishes.

In Fig. 9 the optimized switching patterns δ_{opt} of the actuator position δ as well as the resulting pose errors $\overline{\Delta xy}$ and $\overline{\Delta \phi}$ of the mechanisms are presented. Again, the EE was moved counter-clockwise along trajectory t_I with a constant orientation of $\phi = -30^\circ$, $\phi = 0^\circ$, and $\phi = 30^\circ$. It is important to note that the symmetrical non-redundant mechanism suffers from a completely singular. i.e. useless, workspace for $\phi = 0^\circ$ (indicated by $\overline{\Delta xy} = \overline{\Delta \phi} = \infty$). This is

Fig. 8. Exemplarily chosen trajectories t_I , t_{II} , t_{III} (solid gray) for the 3(P)RPR mechanism, the solid red lines represent the singularity loci within the workspace (solid black); note: the workspace for $\phi = 0^\circ$ is completely singular

Fig. 9. Simulation results while moving along trajectory $t_I(-30^\circ)$ (left), $t_I(0^\circ)$ (center), and $t_I(30^\circ)$ (right); solid gray: non-redundant mechanism; dashed black: optimized redundant mechanism using $\eta(\mathbf{J}_h)$; solid red: optimized redundant mechanism using $\gamma(\Delta \mathbf{x}_h)$

not the case for the kinematically redundant 3(P)RPR mechanism where the symmetry can be affected, i.e. avoided, thanks to the additional prismatic actuator. Regarding Fig. 9 and Table 3 similar to the 3RRR-based structure (see Sec. 4.1.1) a significant improvement of the achiev-

able accuracy due to the kinematic redundancy is well noticeable. Again, in most cases (except

$t_i(\phi)$	Value	3RPR	3(P)RPR	
			using $\eta(\mathbf{J}_h)$	using $\gamma(\Delta\mathbf{x}_h)$
$t_I(-30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	4.87	0.70 (-85.7%)	0.70 (-85.7%)
	$\overline{\Delta\phi}_{\max}$ [°]	1.54	0.16 (-89.9%)	0.16 (-89.9%)
$t_I(0^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	∞	0.90 (-)	0.90 (-)
	$\overline{\Delta\phi}_{\max}$ [°]	∞	0.53 (-)	0.48 (-)
$t_I(30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.97	0.66 (-31.8%)	0.66 (-32.5%)
	$\overline{\Delta\phi}_{\max}$ [°]	0.60	0.35 (-41.1%)	0.32 (-46.6%)
$t_{II}(-30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.97	0.66 (-31.9%)	0.86 (-11.7%)
	$\overline{\Delta\phi}_{\max}$ [°]	0.60	0.32 (-46.6%)	0.35 (-41.9%)
$t_{II}(0^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	∞	0.91 (-)	0.78 (-)
	$\overline{\Delta\phi}_{\max}$ [°]	∞	0.48 (-)	0.44 (-)
$t_{II}(30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	4.87	0.70 (-85.7%)	0.64 (-86.8%)
	$\overline{\Delta\phi}_{\max}$ [°]	1.54	0.16 (-89.9%)	0.15 (-90.2%)
$t_{III}(-30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	0.98	0.93 (-4.6%)	0.93 (-4.9%)
	$\overline{\Delta\phi}_{\max}$ [°]	0.35	0.29 (-17.4%)	0.28 (-21.2%)
$t_{III}(0^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	∞	∞ (-)	∞ (-)
	$\overline{\Delta\phi}_{\max}$ [°]	∞	∞ (-)	∞ (-)
$t_{III}(30^\circ)$	$\overline{\Delta xy}_{\max}$ [mm]	1.20	0.93 (-22.2%)	0.93 (-22.2%)
	$\overline{\Delta\phi}_{\max}$ [°]	0.41	0.27 (-34.1%)	0.27 (-34.1%)

Table 3. Redundant 3(P)RPR mechanism: maximal translational $\overline{\Delta xy}_{\max}$ and rotational error $\overline{\Delta\phi}_{\max}$ of the moving platform while moving along trajectory t_I , t_{II} , and t_{III}

for $t_{II}(-30^\circ)$) the optimization based on the gain $\gamma(\overline{\Delta\mathbf{x}_h})$ leads to more appropriate switching patterns (in terms of accuracy improvement) in comparison to an optimization based on the Jacobian’s condition $\eta(\mathbf{J}_h)$. It is important to note, that even the redundant mechanism suffers from singularities (see $t_{III}(0^\circ)$). This might be overcome by an optimization of the redundant actuator’s design which will be subject to future work.

4.1.3 Influence of the redundant actuator’s joint error

An additional test was performed to clarify the influence of the redundant prismatic actuator joint error $\Delta\delta$ on the moving platform pose error $\Delta\mathbf{x}$. Therefore, for different $\Delta\delta$ the EE was moved along $I(-30^\circ)$. The actuator position δ was changed according to the optimized switching pattern shown in Fig. 7 and Fig. 9 (based on the gain $\gamma(\overline{\Delta\mathbf{x}_h})$). The results are presented in Fig. 10. The plots clearly demonstrate the marginal influence of $\Delta\delta$ on $\Delta\mathbf{x}$ when realistic values for the remaining active joint errors are chosen (cp. Sec. 4.1.1 and 4.1.2). It can be seen that even in the case of an unrealistic high joint error $\Delta\delta$ a significant increase of the mechanism’s achievable accuracy in comparison to the non-redundant case is still obtained (cp. Fig. 7, left column).

4.1.4 Switching operations - accuracy progress

There might be the case that the EE passes a singular configuration while performing a re-configuration of the mechanism, i.e. while changing the singularity loci. As a result, the performance of the PKM decreases dramatically. Hence, the switching operations have to be

Fig. 10. Influence of $\Delta\delta$ on $\Delta\mathbf{x}$ while moving the EE along trajectory $I(-30^\circ)$ (solid black: $\Delta\delta = 0 \mu\text{m}$; solid red: $\Delta\delta = 50 \mu\text{m}$; solid gray: $\Delta\delta = 100 \mu\text{m}$; solid light gray: $\Delta\delta = 250 \mu\text{m}$)

considered within the optimization procedure. While performing a reconfiguration (moving δ while keeping $\mathbf{x}$ constant) the possibility of passing any singularities is taken into account. Additionally, configurations of low performance are avoided. Exemplarily, the behavior of the achievable accuracy obtained while moving the EE along $t_I(-30^\circ)$ (including the switching operations) is given in Fig. 11. It can be clearly seen that the achievable accuracy does not increase during reconfigurations of the mechanism. This is valid for all the trajectories the authors tested so far. A problem however is the additional operation time necessary to follow a desired path. This, i.e. the number of reconfigurations, could be reduced according to the modifications mentioned in Sec. 3.2, e.g. only change δ once before starting the desired movement or if the mechanism is unable to perform a desired operation. Furthermore, the switching time itself could be reduced by a 'semi discrete' optimization strategy, e.g. start moving δ shortly before arriving at the ending point $c_{i,j}$ of the segment j of trajectory i .

4.2 Comparing the useable workspace

In order to further clarify the effect of an additional prismatic actuator on the mechanism pose accuracy, in the following, the size of the useable workspaces w_u is determined. The useable workspace is defined as the singularity-free part of the total workspace w_t providing a certain desired performance, in this case a certain desired accuracy. Mathematically, it can be expressed as the largest region where the sign of the determinant of the Jacobian $\det(\mathbf{A})$ does not change and the output error $\Delta\mathbf{x}$ (23) satisfies any thresholds $\Delta\mathbf{x}_{\text{thr}} = (\Delta xy_{\text{thr}}, \Delta\phi_{\text{thr}})^T$, corresponding to $\overline{\Delta xy}$ and $\overline{\Delta\phi}$. Therefore, the Jacobian determinant as well as the moving platform pose error are calculated over the whole workspace. An example clarifying the procedure leading to w_u is given in Fig. 12. The analyzed workspaces for three different EE orientations of the non-redundant 3RRR mechanism ($\delta = 0 \text{ m} = \text{const.}$) is given. The green part is the largest region where the sign of $\det(\mathbf{A})$ does not change whereas the red part is the smallest. The black area is the overlaid region where a required performance, i.e. a required accuracy,

Fig. 11. Simulation results (including switching operations) while moving along trajectory $t_I(-30^\circ)$, reconfigurations are performed based on the gain; left: 3(P)RRR, right: 3(P)RPR, the switching operation is marked by the gray background

Fig. 12. Analyzed workspace of the non-redundant 3RRR mechanism ($\delta = 0 \text{ m} = \text{const.}$); green is largest region where the sign of $\det(\mathbf{A})$ does not change whereas red is the smallest, in the black area the required accuracy can not be provided

can not be provided. Hence, the green color represents the useable workspace with respect to the mentioned requirements. That followed, the connected green area can be determined, i.e. the shape as well as the size of the useable workspace.

Three constant EE orientations $\phi = \{-30^\circ, 0^\circ, 30^\circ\}$ were considered. The design of the exemplarily chosen mechanisms as well as the input error $\Delta \theta$ are equal to the ones chosen in Sec. 4.1. The thresholds are set to $\Delta xy_{\text{thr}} = 0.75 \text{ mm}$ and $\Delta \phi_{\text{thr}} = 0.5^\circ$. The results are given in Fig. 13. In case of the non-redundant mechanisms the total and useable workspace w_t and

Fig. 13. Total (bold lines, filled dots) and useable (light lines, unfilled dots) workspace of the kinematically redundant $3(\underline{\text{P}})\underline{\text{RRR}}$ mechanism (left, solid red), the $3(\underline{\text{P}})\underline{\text{RPR}}$ mechanism (right, solid red), and their non-redundant counterparts (left/right, dotted blue); the dashed red line gives the useable workspace of the redundant mechanisms for $\Delta \mathbf{x}_{\text{thr}} = (0.5 \text{ mm}, 0.35^\circ)^T$

w_u was calculated for different base joint positions G_{1_i} , i.e. for different but constant δ_i . The solid horizontal lines represent w_t and w_u for the redundant case when the base joint G_1 can be moved linearly for $-0.5 \text{ m} \leq \delta \leq 0.5 \text{ m}$. Having a look at Fig. 13 a significant improvement concerning the workspace areas for all the considered EE orientations is well noticeable. Furthermore, for the redundant case the useable workspace for $\Delta \mathbf{x}_{\text{thr}} = (0.5 \text{ mm}, 0.35^\circ)^T$ was determined, i.e. the requested accuracy is increased about one third. It can be clearly seen that in this case similar workspace sizes are obtained in comparison the non-redundant mechanisms with less accuracy requirements. This further demonstrates the use of kinematic redundancy in terms of accuracy improvements.

5. Conclusion

In this paper, the kinematically redundant 3(P)RRR and 3(P)RPR mechanisms were presented. In each case, an additional prismatic actuator was applied to the structure allowing one base joint to move linearly. After a description of some fundamentals of the proposed PKM, the effect of the additional DOF on the moving platform pose accuracy was clarified. An optimization of the redundant actuator position in a discrete manner was introduced. It is based on a minimization of a criterion that the authors denoted the gain $\gamma(\overline{\Delta x_h})$ of the maximal homogenized pose error $\overline{\Delta x_h}$. Using several exemplarily chosen trajectories a significant improvement in terms of accuracy of the proposed redundant mechanisms in combination with the developed optimization procedure was demonstrated. It could be seen that the suggested index $\gamma(\overline{\Delta x_h})$ leads to more appropriate switching patterns than the well known condition number of the Jacobian. Additional simulations demonstrated the marginal influence of the redundant actuator joint error $\Delta\delta$ on the moving platform pose error Δx .

Furthermore, a comparative study on the usable workspaces, i.e. the singularity-free part of the total workspace providing a certain desired performance, of the mentioned mechanisms and their non-redundant counterparts was performed. The results demonstrate a significant increase of the useable workspace of all considered EE orientations thanks to the applied additional prismatic actuator.

To further increase the overall and the operational workspace, future work will deal with the design optimization of the prismatic actuator, e.g. its orientation with respect to the x -axis of the inertial coordinate frame as well as its stroke ('length'). In addition, the simulation will be extended to PKM with higher DOF and an experimental validation of the obtained numerical results will be performed.

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The purpose of this volume is to encourage and inspire the continual invention of robot manipulators for science and the good of humanity. The concepts of artificial intelligence combined with the engineering and technology of feedback control, have great potential for new, useful and exciting machines. The concept of eclecticism for the design, development, simulation and implementation of a real time controller for an intelligent, vision guided robots is now being explored. The dream of an eclectic perceptual, creative controller that can select its own tasks and perform autonomous operations with reliability and dependability is starting to evolve. We have not yet reached this stage but a careful study of the contents will start one on the exciting journey that could lead to many inventions and successful solutions.

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