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The benefits of Schooling: A human capital alocation into a continuous optimal control framework

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1. Introduction

During the last few decades, the multi-scale expansion of scientific and technical knowledge has unavoidably raised the productivity of human capital and other significant parameters in the complex chain of production. Thus, the deeper knowledge, the value of education, the schooling and on-the-job training (i.e. the further growth of scientific and technical knowledge) have become embodied in people-scientists, scholars, technicians, managers and other contributors. Consequently, the lifetime path of education is become one of the most important investments in human capital's growth. In that direction, several empirical studies have showed that good college training, strong influence of family, and lifetime schooling have raised sharply a persons' income and prosperity, see for more details Becker (1994), Murphy and Welch (1989) etc. Thus, the earnings of more educated people are almost always well above average, see Lynn (ed.) (2006), although the gains are generally larger in less-developed countries, see also Becker (1994).
Into our micro-macro economical environment, the investment of an individual in educational matters is a very important, meanwhile a very expensive, decision. For instance, the foregone income alone for the years of primary and secondary (high school) education is likely to be hundreds of thousands euros (or dollar) per student. However, the internal rate of return of this investment has been estimated to be in the rage of ten to twenty, or more in some developing countries, see for instance Bishop (1961), Hansen (1963), and Psacharopoulos and Patrinos (2004). Furthermore, following Southwick, Jr and Zionts (1974), McMahon (2002), the generous investment in education restricts effectively the poverty, as the income earnings are strongly related to the educational level of the individuals. It should be also pointed out that the poor generally undertake less or (in the majority of cases) none education than the non-poor.
The problem of an optimal lifestyle investment in the limited stock of human capital, i.e. the optimal stock of education (knowledge), has been a subject in the literature of financial mathematics for many decades. Analytically, the problem is to determine formalistically (and not merely empirically) the optimal lifetime path of education policy of an average

individual, who can split up its time into learning and working, and it is a subject to negative income by taxation, and by the cost of learning education, as well.
The above interesting problem can be easily reformulated into an optimal control problem. Although, optimal control theory was developed by engineers in order to investigate the properties of dynamic systems of difference or/and differential equations, it has also been applied to financial problems. Tustin (1953) was the first to spot a possible analogy between the industrial and the engineering processes and the post-war macroeconomic policy-making; see Holly and Hallett (1989), for further historical details. Furthermore, the research work of Ben Porath (1967) is one of the earliest applications of optimal control theory that was devoted to this topic.
Meanwhile, in the last five decades, different types of human capital models for the education have been developed to analyse the problem and to optimize several parameters involved, which are penalized by a huge variety of functional criteria. For instance, the interesting readers can advise the pioneer works by Hansen (1963), Southwick, Jr and Zionts (1970, 1974), Blinder and Weiss (1974), Hartl (1983), and other various extensions of these models.
In this book chapter, we present some further extensions of the discussed problem by following the recently proposed research works by Kalogeropoulos and Pantelous (2007), Pantelous and Kalogeropoulos (2008) and Pantelous et al. (2008), which theoretically and practically provide an optimal path of investment in education (i.e. in knowledge) or in other words, it can be approached as a schooling-resources allocation over time for an average individual. The concept of average can be translated that the individual has not so much financial health that he/she can derive some extra benefits of it.
Since the economical and social environment is continually changing, we should also consider the education-investment decision as a dynamic process over the course of a lifetime. In this framework some assumptions must also be made. Thus, as in many developing models, see Ben Porath (1967), Southwick, Jr and Zionts (1974), and Hartl (1983), the individual purchases knowledge solely for its investment value, i.e. the consumption aspects of education is definitely ignored. Moreover, the individual is assumed to act in an optimal way, i.e. it is interesting in purchasing education as long as its incremental value is greater then its respective cost. At last but not at least, two individuals are having the same formal schooling, celeris paribus, they may have quite different levels of acquired knowledge. Since no readily data are available, social indicators such as sex, colour, labour nationality or immigration are not taking into consideration.
Furthermore, the income function used in this chapter is enlarged from earlier studies to include simultaneously profits from risk-free investments (i.e. T-bills, cash accounts etc), from the level of education-including the possibility of the individual to participate in different financed projects or simply to obtain a scholarship, and the experience of individual (i.e. its age), as well. Additionally, it is stretched out that we consider the income function as a dynamic process over the course of a lifetime.
The 2nd section develops the basic continuous-time model in implicit form and some theoretical results are obtained. In 3rd Section, a very interesting case is provided, which is mainly constrained by a special case of the famous Cobb-Douglas production function. Afterwards, a linear case is fully investigated. By using empirical results, an optimal investment-education decision strategy is eventually derived in the 4th Section. In the sequence section, the analytic presentation of the proposed educational optimal-investment

dynamic model is considered transferring the entire discussion and motivation of the previous sections into a stochastic framework. Conclusions and further research proposals are provided in the 6th Section.

2. Development of the optimal dynamic model in deterministic continuous-time framework

In this section, the discussed model is described into a deterministic, continuous-time framework, and it has been recently proposed by Kalogeropoulos and Pantelous (2007). With the next lines, the first step is to describe the necessary notation, and the relative assumptions. It is really worthy to say that our model can be considered as more general comparing with the existence models in the relative literature, see Southwick, Jr and Zionts (1974), Ritzen and Winkler (1979), and Hartl (1983), since the mathematical equations are in a more general form. In addition, as it can be easily verified, the following equations are simpler than those proposed in the 5th section (see stochastic case).
Since the schooling increases the education level, the lack of it may allow the education to decline. The stock of human capital (i.e. knowledge) embodied in an individual may change over the period of schooling, see Checchi (2006).
Analytically, let $x(t) \in C^1(\mathbb{R})$ denote the level of education (i.e. the stock of human capital-knowledge) and $u(t):[0,T] \to [0,1]$, which is a sufficiently differentiable function, denote the investment into that stock. Equivalently, the fraction of time devoted to work is $1-u(t)$.
The change in human capital over time is then given by a *weakly nonlinear ordinary differential equation*

$$\dot{x}(t) = -a(t)x(t) + f\big(t, x(t), u(t)\big)$$
$$x(0) = x_o \tag{1}$$

where $a(t):[0,T] \to U \subset \mathbb{R}$ the depreciation rate of education and $f(\cdot):[0,T]\times\mathbb{R}\times[0,1] \to \mathbb{R}$ which represents the production of human capital Becker (1994), Glomm, and Ravikumar (1992) are also sufficiently differentiable equations. Note that the deduction factor $a(t)$ varies with the different type of education (i.e. medical, mathematical vs. technical etc), and the different economical-political-social circumstances (i.e. in the western vs. sub-African countries). Moreover, the source of this depreciation may be either from forgetting or from technological obsolescence, or both, see Southwick, Jr and Zionts (1970).
Furthermore, it is assumed that the potential money income that can be earned by an individual is mainly a function of his level of education x, the age t and the risk-free interest rate $r > 0$ is fixed. Thus, we obtain

$$\dot{y}(t) = ry(t) + \big(1-u(t)\big)h_1\big(t, x(t)\big) + u(t)h_2\big(t, x(t)\big)$$
$$y(0) = y_o\,. \tag{2}$$

This differential function implies that money income is evaluated by the risk-free investment (i.e. T-bills, cash accounts etc) by the function

$$h_1(\cdot):[0,T]\times\mathbb{R} \to \mathbb{R}\,,$$

for the time which is being spent at work, $1-u(t)$ and by the function

$$h_2(\cdot):[0,T]\times\mathbb{R} \to \mathbb{R}\,,$$

for the time which is being invested in schooling through a scholarship, or his/her participation into a research programme. Note that it is assumed that part-time is equally paid as the full-time work.

The direct cost of education is assumed to be linearly related to the proportion invested to knowledge, i.e. $g_1(\cdot):[0,1]\to\mathbb{R}$ and to the level of education x, $g_2(\cdot):\mathbb{R}\to\mathbb{R}$.

This yields that

$$c(t)=g_1(u)\cdot g_2(x)\,. \tag{3}$$

A further realistic consideration is the income tax policy, which is surely correlated to the earning and to the different expenditures, as the following equation is devoted

$$T(t)=\tau_o+\tau_1 y(t)-\tau_2 c(t)\,. \tag{4}$$

Analytically, the constant number τ_o can be a negative or positive number analogously to the present or to the proposed welfare tax system, see Southwick, Jr and Zionts (1974). Moreover, eq. (4) takes into consideration a percentage τ_1 of the actual income diminished by a percentage τ_2 of the direct cost of education. However, in practice, the actual tax system is somewhat much more complex because it takes into consideration several others inputs such as capital gains, medical deductions, number of infants etc. Although, a number of empirical studies, see Bishop (1961) etc, have found that the tax system is approximately proportional, which gives support to expression (4).

Now, in the same point of view as in research work proposed by Southwick, Jr and Zionts (1974), Ritzen and Winkler (1979), and Hartl (1983), it can be assumed that the objective function is to optimize (maximize, in this case) the discounted present value of future income streams. The expression under parentheses in the objective function (5) is the net cash flow at time $t\in[0,T]$. Additionally, we can also stress that the controlled interval period is $[0,T]$ (e.g. 0: the starting of working and T: the year of retirement) and the discount rate r is constant and equal to the premium of a T-period government (risk-free) bond. Thus

$$\max_u\left\{\int_0^T e^{-rt}\big(y(t)-T(t)-c(t)\big)dt\right\}. \tag{5}$$

Actually, the individual follows a time-path of education (through seminars, attainting MSc courses or doing MBA etc) into that period in order to maximize the value of (5). Of course, the investment into the knowledge stock via the rate $u(t)$ has a limited range, between 0 and 1, since he/she can not obtain schooling at a negative rate or more than full time.

Historically, the maximum principle, which has been formulated and derived by Potryagin and his group in the 1950s, is truly a milestone to the optimal control theory; see Yong and Zhou (1999) for more details. It states that any optimal control along with the optimal state trajectory must solve the so-called (extended) Hamiltonian system, which is a two-point boundary value problem (and can also called a forward-backward differential equation), plus a maximum condition of a function called the Hamiltonian.

Thus, the Hamiltonian function is given by expression (6)

$$H(t,x,y,u,p,q)=e^{-rt}\big(y(t)-T(t)-c(t)\big)+p(t)\dot{x}(t)+q(t)\dot{y}(t)\,, \tag{6}$$
$$(t,x,y,u,p,q):[0,T]\times\mathbb{R}\times\mathbb{R}\times[0,1]\times\mathbb{R}\times\mathbb{R}$$

where the shadow prices p and q of the level of education and the potential money income, respectively are the solution of

$$\dot{p}(t) = -H_x\left(t, x(t), y(t), u(t), p(t), q(t)\right), \quad (7)$$

$$\dot{q}(t) = -H_y\left(t, x(t), y(t), u(t), p(t), q(t)\right), \quad (8)$$

at a. e. $t \in [0, T]$.

Furthermore, since the objective function is the maximization of cognitive knowledge, x at the end of the period, the following transversely condition applies

$$p(T) = 0, \tag{9}$$

and additionally,

$$q(0) = q_o. \tag{10}$$

The condition for optimality is

$$H\left(t, x(t), y(t), u(t), p(t), q(t)\right) = \max_{u \in [0,1]} H\left(t, x(t), y(t), u, p(t), q(t)\right) \tag{11}$$

In practice, the maximization of the criterion is achieved if the control is chosen to maximize the Hamiltonian at each point in time. Thus, the necessary first-order condition is derived

$$H_u = 0. \tag{12}$$

Note that time dependency (t) of the variables is omitted for notational convenience. By substituting expressions (3) and (4) into (5) it is derived

$$\max_u \left\{ \int_0^T e^{-rt}\left((1-\tau_1)y - \tau_o - (1-\tau_2)c\right)dt \right\}. \tag{13}$$

So, from eqs. (7) and (8) by using the reformed Hamiltonian equation (14)

$$\begin{aligned} H(t, x, y, u, p, q) = {} & e^{-rt}\left((1-\tau_1)y - \tau_o - (1-\tau_2)c\right) \\ & + p\{-ax + f\} + q\{ry + (1-u)h_1 + uh_2\} \end{aligned} \tag{14}$$

it is obtained

$$\dot{p} = (a - f_x)p + (1-\tau_2)e^{-rt}g_1 g_{2x} - q\left(h_{1x} + u\left(h_{2x} - h_{1x}\right)\right) \tag{15}$$

and

$$\dot{q} = -rq - (1-\tau_2)e^{-rt}. \tag{16}$$

Moreover, through the eq. (12), it is taken that

$$-(1-\tau_2)e^{-rt}g_{1u}g_2 + pf_u = 0,$$

or equivalently

$$f_u / g_{1u} = (1-\tau_2)e^{-rt}g_2 / p. \tag{17}$$

The Hamiltonian equation (14) and the co-state variables p and q should be analysed by taking into consideration several economical-social interpretations. According to eq. (15), the first co-state variable which reflects to the level of education per individual is very complicated, as many parameters get involved. Thus, the rate $a - f_x$ decreases the ordinary linear differential equation of function p, see eq. (15), whenever the marginal productivity of human capital exceeds the rate of depreciation of knowledge, see also Ritzen and Winkler (1979). Moreover, the second co-state variable which reflects the potential money income per individual is simpler and it depends mainly on the discount rate r. It is intuitively clear that the income is decreasing by the increment of the discount rate.

After these preliminary results and comments, the properties of the optimal investment policy can be determined. Now, differentiate eq. (17) with respect to time t and substitute the necessary equations, we obtain

$$\begin{aligned}\dot{p}f_u + pf_{uu}\dot{u} + pf_{ux}\dot{x} + pf_{ut} + (1-\tau_2)re^{-rt}g_{1u}g_2 - (1-\tau_2)e^{-rt}g_{1uu}\dot{u}g_2 - (1-\tau_2)e^{-rt}g_{1u}g_{2x}\dot{x} \\ = \dot{q}(h_1 - h_2) + q(h_{1t} - h_{2t}) + q(h_{1x} - h_{2x})\dot{x}\end{aligned}$$

or the equivalent non-linear partial differential eq. (18)

$$\begin{aligned}\left[pf_{uu} - (1-\tau_2)e^{-rt}g_{1uu}g_2\right]\dot{u} = \left[q(h_{1x} - h_{2x}) - pf_{ux} + (1-\tau_2)e^{-rt}g_{1u}g_{2x}\right]\dot{x} + \dot{q}(h_1 - h_2) \\ + q(h_{1t} - h_{2t}) - \dot{p}f_u - pf_{ut} - (1-\tau_2)re^{-rt}g_{1u}g_2\end{aligned} \quad (18)$$

Given the functions involved in expression above, the time path of education along the turnpike can be found using (18), and the sufficient boundary conditions.

Finally, the Hamiltonian may be interpreted as the net "profit" at time t from the net investment in human capital. Moreover, by taking also into consideration the above-complicated eq. (18), a much insightful view for the percentage of education that someone should invest into that stock is derived in order to maximize his "profit". Equivalently, the fraction of time devoted to work $1-u$ is also obtained. In the next section, a particular production function f is used, see also eq. (1).

3. A special case: Cobb-Douglas production function

In economics, the Cobb-Douglas functional expression of productivity is widely used to represent the strong relation of an output to inputs. This functional expression has been firstly used in Cobb and Douglas (1928) as a law of production, but as it is mentioned in Fraser (2002), it was already known by Pareto, several decades before.

Therefore, in this section, a special case of function f is considered; i.e. the famous Cobb – Douglas production function, as it has already been used in Southwick and Zionts (1974), Ritzen and Winkler (1979), and Hansen (1963) research works, which are related to our approach.

Thus, we assume that

$$f(t,x,u) = b(t)u^{\beta}(t)x^{\gamma}(t), \quad (19)$$

where $\beta,\ \gamma \in \mathbb{R}$.

Obviously, the eq. (1) can be transposed into

$$\dot{x}(t) = -a(t)x(t) + b(t)u^{\beta}(t)x^{\gamma}(t). \quad (20)$$

Moreover, we assume

$$h_1(t,x(t)) = a_1(t)x(t) \text{ and } h_2(t,x(t)) = a_2(t)x(t).$$

where, the coefficients parameters $a_1(t)$ and $a_2(t)$ are t-continuous functions.

So, the eq. (2) is transposed into the linear equation

$$\dot{y}(t) = r(t)y(t) + a_1(t)x(t) - (a_1(t) - a_2(t))u(t)x(t). \quad (21)$$

Following the Southwick and Zionts (1974), the cost of education is assumed to have the expression

$$c(t) = u(t)\cdot g(x). \quad (22)$$

Then, by using the eq. (17) and noting that time dependency t of the variables is also omitted for notational convenience, a quite complicate expression for the controller is obtained, i.e.

$$u = \left[\frac{pb\beta}{1-\tau_2} e^{rt} \frac{x^{\gamma-1}}{g(x)} \right]^{\frac{1}{1-\beta}} . \tag{23}$$

where $\beta,\ \gamma,\ \tau_2,\ r$ are constant, b is a function of t, and $p,\ x$ are the solution of (15) and (20) respectively.

Now, by substituting the expression above (23) into (20) and (15), the following strong nonlinear system should be solved.

$$\dot{x} = -ax + \left(\frac{\beta}{1-\tau_2} \right)^{\frac{\beta}{1-\beta}} b^{\frac{1}{1-\beta}} e^{\frac{\beta r}{1-\beta} t} p^{\frac{\beta}{1-\beta}} \frac{x^{\frac{\beta}{1-\beta}(\gamma-1)+\gamma}}{g^{\frac{\beta}{1-\beta}}(x)} , \tag{24}$$

$$\dot{p} = ap - a_1 qx - \left\{ \gamma \left[\frac{\beta}{1-\tau_2} e^{rt} \frac{1}{g(x)} \right]^{\beta} + \left[(1-\tau_2) e^{-rt} g_x + (a_1 - a_2) x \right] \right\} \left[\frac{\beta b e^{rt}}{1-\tau_2} \right]^{\frac{1}{1-\beta}} \left[\frac{x^{\gamma-1}}{g(x)} \right]^{\frac{1}{1-\beta}} p^{\frac{1}{1-\beta}} , \tag{25}$$

where $\beta,\ \gamma \in \mathbb{R}$ and q is the solution of ordinary linear differential equation (16). The solution of such nonlinear systems is far beyond the main target of this paper. However, in order to obtain some insightful practical comments, we can denote $\beta = \gamma = 1$.

Thus, eq. (19) is rewritten as

$$f(t,x,u) = b(t)u(t)x(t) .$$

Moreover, we denote $c(t) = cu(t)x(t)$, where c is constant.

Then, considering the eq. (17), it is derived that

$$p(t) = \frac{c}{b(t)} (1-\tau_2) e^{-rt} , \tag{26}$$

using the differential equation (15), and after some simple calculus, the following closed form expression for the controller $u(t)$ is taken,

$$u(t) = \frac{\dot{p}(t) - a(t)p(t) + a_1(t)q(t)}{(1-\tau_2)ce^{-rt} - b(t)p(t) - (a_1(t) - a_2(t))q(t)} . \tag{27}$$

Now, the solution of eq. (16) is given by eq. (28)

$$q(t) = (q_o - (1-\tau_1)t) e^{-rt} , \tag{28}$$

and finally, the controller, i.e. the fraction of time invested into education, is given

$$u(t) = \frac{a_1(t)(q_o - (1-\tau_1)t) - \frac{c}{b(t)}(1-\tau_2)(r - a(t))}{(1-\tau_2)(1-c) - (a_1 - a_2)(q_o - (1-\tau_1)t)} . \tag{29}$$

Thus, through the eq. (29), we can efficiently control the pattern of human capital (i.e. knowledge) in order to maximize the financial profits of the individuals. Moreover, according to the eq. (20), the change in human capital over time is then obtained by the following linear ordinary linear equation

$$\dot{x}(t) = \left[b(t)u(t) - a(t) \right] x(t) . \tag{30}$$

Obviously, the above results are apparently interesting and very helpful to practitioners, as well. In the following section, an interesting numerical application is analytically presented.

4. Using some empirical numerical results to obtain the optimal education investment decision

In general, the form of the optimal solution is the initial full time schooling, i.e. $u(t)=1$ (scaled down to part time if consumption requirements are sufficient high) followed by alternating periods of part-time schooling and zero schooling, with the last period prior to retirement one of zero schooling. However, in this numerical application, we are mainly being interested about the period of part-time schooling.

Analytically, the observed relations on income of age and education are used to develop the optimal allocation of effort between work (i.e. employment) and education (i.e. lifetime schooling). Moreover, it is underlying that our results are based on the eq. (29) and its parameters relatively involved.

Furthermore, we would like to mention that the key factor for the optimal allocation policy is the measure of the depreciation rate of education. This estimation is remarkably important; since it could be answered several serious questions, which are naturally derived, see Groot (1998). To be honest, fairly little things are known about that rate. However, quite recently a simple methodology to the determination of the depreciation rate is proposed and it has been applied into real data sets from Great Britain; see Groot (1998) and Lynn (ed.) (2006). In this numerical application, we use the results of the empirical application mentioned above for the population of Great Britain.

The findings suggest that the rate of depreciation is 11-17% per year. Those quite high depreciation rates (compared with those that have been used in the early application of Southwick and Zionts (1974)) emphasize more the importance of the lifetime learning.

Before we go further, it is important to determine the values of the variables, which are taken into consideration in eq. (29):

$a(t)$: The depreciation rate of education $t \in [0,T]$. According to the results of Groot (1998), it takes values into the interval $[11\%, 17\%]$, see eq. (30).

$a_1(t)$: The increment of earnings through work, see eqs. (2) and (21). Suppose that it is a stable 3% increment for each year.

$a_2(t)$: The increment of earnings through scholarships or participation into a research programme, see also eqs (2) and (21). Suppose that it is almost unchangeable 0.5%, i.e. almost no serious increment at all.

T: The end of the time-period, i.e. the year of retirement. In our application, it is 35 years of full time work.

r: The risk-free interest rate.

c: The proportion of cost for the relative education, see eqs. (3) and $c(t) = cu(t)x(t)$. This proportion depends of the quality of education, i.e. the cost of attainting seminar, doing MSc courses or MBA, which can be quite different. In our application, it is supposed that the three choices mentioned above of training are the only available. Thus, we consider the proportion of cost to be equal to $1/3$.

τ_1 : The percentage of the actual income, see eq. (4). It is suppose to be stable and it takes the value of 10%.
τ_2 : The percentage of direct cost of education, see also eq. (4). It is also suppose to be stable and it takes the value of 8%. Obviously, it can be either smaller or greater of τ_1, as a mere consequence of the government policy. In the particular case that $\tau_2 > \tau_1$, the government provides extra tax motivation for lifelong learning. In our example the tax policy does not provide any extra motivation.
$b(t)$: Without further details, we suppose that the t -continuous parameter of the production function f is constant and equal to 1, see eq. (19). Obviously, it can be any smooth real function which is really feasible or, in practice, it depends upon the empirical data of each special problem.
Finally, since we have assumed that at the beginning of the part-time period, i.e. at time $t = 0$, the proportion $u(t)$ is equal to 1, we can obtain

$$u(0) = \frac{a_1 q_o - c(1-\tau_2)(r-a(t))}{(1-\tau_2)(1-c)-(a_1-a_2)(q_o)}$$

or equivalently

$$q_o = \frac{(1-\tau_2)\left[1-c(1+a(t)-r)\right]}{2a_1 - a_2}$$

Thus, we conclude the discussion above by presenting the following collective Table.

Table

Application Parameters

a1 = 3%	T = 35	τ1 = 10%	c = 1/3	r = 4%
a2 = 0.5%	t < T	τ2 = 8%	b(t) = 1	a in [11%,17%]

Now, in the figure 1, the different values of the controller $u(t)$ are observed for the different values of the depreciation rates. It is clear that the depreciation rates have a large impact on the fraction of time invested into education. The larger the depreciation rate is the more time should be spent in schooling. This result is apparently obvious.
Moreover, the following comment is easily derived; since someone has a depreciation rate of 11%, it needs to spend almost one third of his time in schooling in order to receive optimal income results.

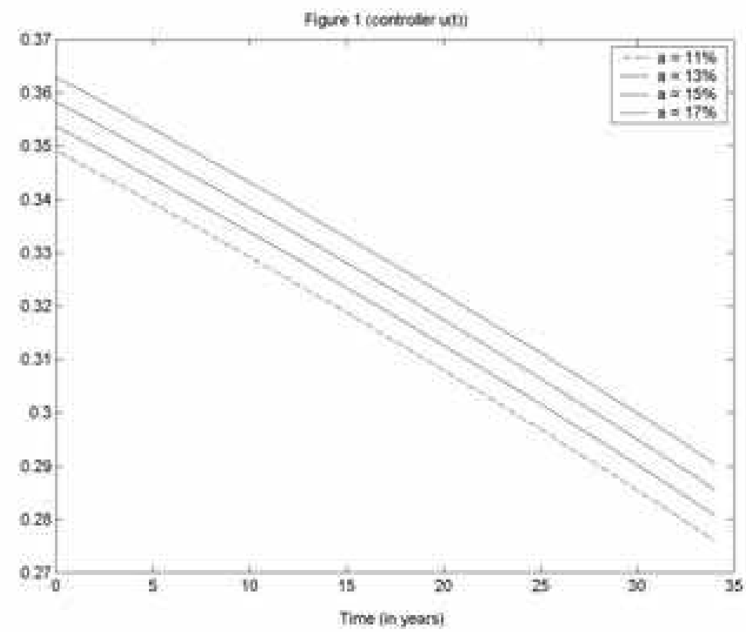

Fig. 1. The fraction of time invested into education for different depreciation rates.

The next figure shows something really astonishing. First, we have computed the interest rate of income return for various values. Then, by using an average deprecation rate of 13%, we observe that our optimal controller is become a strongly decreasing function for the different values of interest rate. Thus, the more someone earns the less time has to spend in schooling.

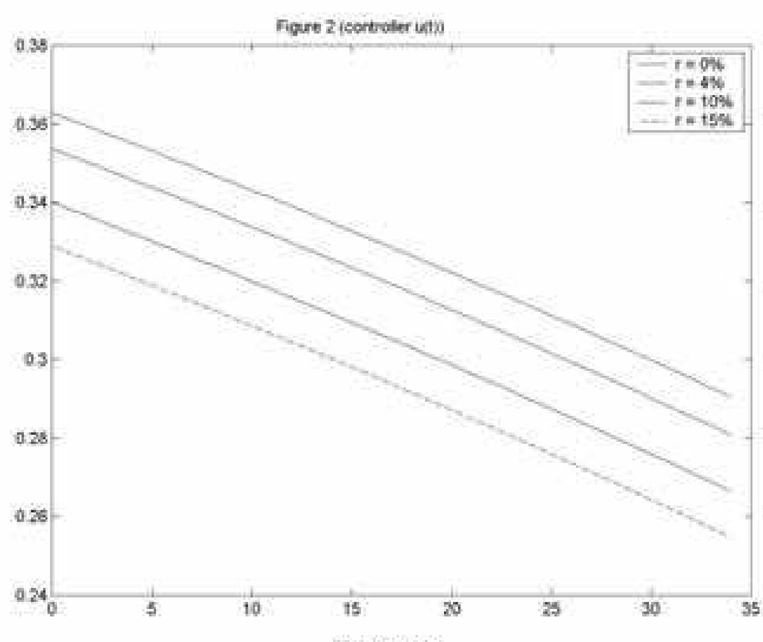

Fig. 2. The fraction of time invested into education for different interest rate values.

Finally, it turns out that the increasing of the depreciation rate also increases the fraction of time invested into education (i.e. knowledge), see eq. (29). The opposite direction is derived when the interest rate is increased.

5. Development of the optimal dynamic model in continuous-time stochastic framework

In this part of the book chapter, we proceed with the analytic presentation of the proposed educational optimal-investment dynamic stochastic model transferring the entire discussion and motivation of the previous section into stochastic framework, see also Kalogeropoulos and Pantelous (2008).

First, the necessary symbols and the relative notations are defined keeping in mind the continuous-time, stochastic framework. For the deeper understanding of the extending model, it is important to construct the main equations (see also 2nd section) in a more general form, following as close as it is possible the known -already used- literature.

Thus, we recall the strong formulation of the stochastic optimal control problem. In that direction, we need to obtain a filtered probability space $\left(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathcal{P}\right)$ satisfying the usual conditions, i.e. if $(\Omega,\mathcal{F},\mathcal{P})$ is complete, $\mathcal{F}_o$ contains all the $\mathcal{P}$-null sets in $\mathcal{F}$, and $\{\mathcal{F}_t\}_{t\geq 0}$ is right continuous. On this probability space, we obtain a standard Brownian motion (sBm) $W(t)$ (with $W(0)=0$).

Moreover, into this classical stochastic framework, let us remind the *stock of human capital* (knowledge) which embodied in an individual may change over the period of schooling, see also 2nd section. Analytically, $x(t)$ denotes *the level of education* (i.e. the stock of human capital-knowledge) and $u(t)\triangleq u\left(t,x(t,\omega)\right):[0,T]\times\Omega\to[0,1]$ denotes the investment into that stock. Equivalently, the fraction of time devoted to work is $1-u(t)$.

Thus, the change in human capital over time is given by a weakly non linear stochastic differential equation

$$dx(t)=\left[-\alpha(t)x(t)+f\left(t,x(t),u(t)\right)\right]dt+g\left(t,x(t)\right)dW(t),$$
$$x(0)=x_o \tag{31}$$

where $\alpha(t):[0,T]\to\mathbb{R}$, is the *depreciation rate of education*, $f(\cdot):[0,T]\times\mathbb{R}\times[0,1]\to\mathbb{R}$ and $g(\cdot):[0,T]\times\mathbb{R}\to\mathbb{R}$ represents the *production of human capital*, see Becker (1994) and Glomm, and Ravikumar (1990), and the *diffusion parameter* (which does not contain the control variable) perturbates the stock of human capital, respectively.

Before, we go further; let us make the following important assumptions.

Assumption 1: $\{\mathcal{F}_t\}_{t\geq 0}$ is the natural filtration generated by $W(t)$, augmented by all the $\mathcal{P}$-null sets in $\mathcal{F}$.

Assumption 2: $\left([0,1],d\right)$ is a separable metric space and $T>0$.

Assumption 3: The maps $f(\cdot)$ and $g(\cdot)$ are measurable, and there exist a constant $L>0$ and a modulus of continuity $\bar{\omega}:[0,\infty)\to[0,\infty)$ such that

$$\left|f\left(t,x(t),u(t)\right)-f\left(t,\hat{x}(t),\hat{u}(t)\right)\right|\le L\left|x(t)-\hat{x}(t)\right|+\bar{\varpi}\left(d\left(u,\hat{u}\right)\right),$$
$$\left|f\left(t,0,u(t)\right)\right|\le L\quad \forall t\in[0,T],\ x,\hat{x}\in\mathbb{R},\ u,\hat{u}\in[0,1]$$

and

$$\left|g\left(t,x(t)\right)-g\left(t,\hat{x}(t)\right)\right|\le L\left|x(t)-\hat{x}(t)\right|,$$
$$\left|g(t,0)\right|\le L\ \ \forall t\in[0,T],\ x,\hat{x}\in\mathbb{R}.$$

Assumption 4: The maps $f(\cdot)$ and $g(\cdot)$ are C^2 in x. Moreover, there exist a constant $L>0$ and a modulus of continuity $\bar{\varpi}:[0,\infty)\to[0,\infty)$ such that

$$\left|f_x\left(t,x(t),u(t)\right)-f_x\left(t,\hat{x}(t),\hat{u}(t)\right)\right|\le L\left|x(t)-\hat{x}(t)\right|+\bar{\varpi}\left(d\left(u,\hat{u}\right)\right),$$
$$\left|f_{xx}\left(t,x(t),u(t)\right)-f_{xx}\left(t,\hat{x}(t),\hat{u}(t)\right)\right|\le\bar{\varpi}\left(\left|x(t)-\hat{x}(t)\right|+\left(d\left(u,\hat{u}\right)\right)\right),$$
$$\forall t\in[0,T],\ x,\hat{x}\in\mathbb{R},\ u,\hat{u}\in[0,1]$$
$$\left|g_x\left(t,x(t)\right)-g_x\left(t,\hat{x}(t)\right)\right|\le L\left|x(t)-\hat{x}(t)\right|,$$
$$\left|g_{xx}\left(t,x(t)\right)-g_{xx}\left(t,\hat{x}(t)\right)\right|\le\bar{\varpi}\left(\left|x(t)-\hat{x}(t)\right|\right),\ \forall t\in[0,T],\ x,\hat{x}\in\mathbb{R}$$

The 1st assumption signifies that the system noise is the only source of uncertainty in the problem, and the past information about the noise is available to the controller. Now, we assume that

$$\mathcal{U}[0,T]\triangleq\left\{u:[0,T]\times\Omega\to[0,1]\,|\,u\text{ is }\{\mathcal{F}_t\}_{t\ge 0}-\textit{adapted}\right\}$$

Given $u(\cdot)\in\mathcal{U}[0,T]$, eq. (31) is a stochastic differential equation with random coefficients. In case that $x(\cdot)$ is the solution of (31) corresponding to $u(\cdot)\in\mathcal{U}[0,T]$, we call $\left(x(\cdot),u(\cdot)\right)$ an *admissible pair*, and $x(\cdot)$ is an admissible *state process (trajectory)*.

Furthermore, we assume that the potential (money) income that can be earned by an individual is mainly a function of his/her level of education x and the age t. Since, the formulation of the stochastic control problem is very difficult (but more natural), we obtain a simpler expression for the income function, comparing (32) with the analogous expression in Kalogeropoulos and Pantelous (2007) (see also 2nd section).

$$y(t)=\left(1-u(t)\right)h_1\left(t,x(t)\right)+u(t)h_2\left(t,x(t)\right)$$
$$y(0)=y_o \tag{32}$$

This function implies that money income is evaluated by the function

$$h_1(\cdot):[0,T]\times\mathbb{R}\to\mathbb{R}$$

for the time which is being spent at work, $1-u(t)$ and by the function

$$h_2(\cdot):[0,T]\times\mathbb{R}\to\mathbb{R}$$

for the time which is being invested in schooling through a scholarship, or his/her participation into a research (co-)funded programme. Note that it is also assumed that part-time work is equally paid as the full-time work.

The direct cost of education is assumed to relate to the time t, to the proportion of knowledge, u and to the level of education x, i.e. $k(\cdot):[0,T]\times\mathbb{R}\times[0,1]\to\mathbb{R}$. This yields

$$c(t)=k(t,u,x) \tag{33}$$

A further realistic consideration is the income tax policy, which is surely correlated to the earning and to the different expenditures, as the eq. (4) is devoted.
Following as close as it is possible the 2nd section, we can assume that the objective function is to maximize the mean value of the discounted present value of the future income streams. The expression under parentheses in the objective function (34) is the net cash flow at time $t \in [0,T]$. Additionally, it can be also strength out that the controlled interval period is $[0,T]$ (e.g. 0: the starting of working and T: the year of retirement) and the discount rate r is constant and equal to the premium of a T-period government (risk-free) bond.

$$\mathbb{E}\left\{\int_0^T e^{-rt}\left(y(t)-T(t)-c(t)\right)dt\right\} \tag{34}$$

Thus, our problem is to maximize (34) over $\mathcal{U}[0,T]$.
Any $u^*(\cdot) \in \mathcal{U}[0,T]$ satisfying eq. (35)

$$J\left(u^*(\cdot)\right) = \max_{u \in \mathcal{U}[0,T]} \mathbb{E}\left\{\int_0^T e^{-rt}\left(y(t)-T(t)-c(t)\right)dt\right\} \tag{35}$$

is called an optimal control. The corresponding $x^*(\cdot) \equiv x\left(\cdot;u^*(\cdot)\right)$ and $\left(x^*(\cdot),u^*(\cdot)\right)$ are called an optimal state process/trajectory and optimal pair, respectively.
Actually, the individual follows a time-path of education (through seminars, attainting MSc courses or doing MBA etc) into that time-period in order to maximize the value of (34), see (35). Of course, the investment into the knowledge stock via the rate $u(t)$ has also a limited range, between 0 and 1, since he/she can not obtain schooling at a negative rate or more than full time.
It can be stated that any optimal stochastic control along with the optimal state trajectory must solve the so-called (extended) stochastic Hamiltonian system, which consists a *forward backward stochastic differential equation* and a *maximum condition* with an additional term (which contains the diffusion coefficient).
Thus, the Hamiltonian function is given by expression (36)

$$\begin{aligned} H(t,x,u,p,q) &= e^{-rt}\left(y(t)-T(t)-c(t)\right) \\ &\quad + p(t)\left[-\alpha(t)x(t)+f\left(t,x(t),u(t)\right)\right]+q(t)g\left(t,x(t)\right)' \end{aligned} \tag{36}$$

$(t,x,u,p,q):[0,T]\times\mathbb{R}\times[0,1]\times\mathbb{R}\times\mathbb{R}$,

where the adjoint equation that $p(\cdot)$ satisfies a forward-backward stochastic differential equation.

$$\dot{p}(t) = -H_x\left(t,x(t),u(t),p(t),q(t)\right)+q(t)dW(t), \tag{37}$$

at a.e. $t \in [0,T]$.
Here, the unknown solution is a pair of $\{\mathcal{F}_t\}_{t\geq 0}$-adapted processes $\left(p(\cdot),q(\cdot)\right)$. The key issue is that the equation should be solved backwards (since the terminal value is given, see (38)). Moreover, the solution $\left(p(\cdot),q(\cdot)\right)$ is required to be $\{\mathcal{F}_t\}_{t\geq 0}$-adapted. Any pair of process $\left(p(\cdot),q(\cdot)\right) \in L^2_{\mathcal{F}}(0,T;\mathbb{R})\times L^2_{\mathcal{F}}(0,T;\mathbb{R})$ satisfying (37) is called an adapted solution of (37). For the interested reader, a systematic study of such equations is provided into the Chapter 7 by Yong, and Zhou (1999).

Fortunately, under the above four assumptions, for any $\left(x^*(\cdot),u^*(\cdot)\right)\in L^2_{\mathcal{F}}(0,T;\mathbb{R})\times\mathcal{U}[0,T]$, (37) admits a unique adapted solution $\left(p(\cdot),q(\cdot)\right)$.

Furthermore, since the objective function is the maximization of cognitive knowledge, x, at the end of the period, the following transversely condition applies

$$p(T)=0 \tag{38}$$

The condition for optimality is given by

$$H\left(t,x^*(t),u^*(t),p(t),q(t)\right)=\max_{u\in[0,1]}H\left(t,x^*(t),u,p(t),q(t)\right) \tag{39}$$

a.e. $t\in[0,T]$ and $\mathcal{P}$-a.s., which is parallel to the deterministic case (no risk adjustment is required).

Notice that expression (39) is true, since there is no a control function in the diffusion coefficient. In practice, the maximization of the criterion is achieved if the control is chosen to maximize the Hamiltonian at each point in time. Thus, the necessary first-order condition is

$$H_u=0\,. \tag{40}$$

By substituting expressions (33) and (34) into (35), it is derived that

$$\mathbb{E}\left\{\int_0^T e^{-rt}\left((1-\tau_1)y(t)-\tau_o-(1-\tau_2)c(t)\right)dt\right\}. \tag{41}$$

So, from expression (32)-(34), (37) and (38) by using the reformed Hamiltonian equation (42)

$$\begin{aligned}&H\left(t,x(t),u(t),p(t),q(t)\right)\\&=e^{-rt}\left((1-\tau_1)\left[(1-u(t))h_1(t,x(t))+u(t)h_2(t,x(t))\right]-\tau_o-(1-\tau_2)k(t,u,x)\right)\\&\quad+p(t)\left\{-a(t)x(t)+f(\cdot)\right\}+q(t)g(\cdot)\end{aligned} \tag{42}$$

it is obtained the forward-backward stochastic differential equation

$$\begin{aligned}dp(t)=&-\left\{e^{-rt}\left((1-\tau_1)\left[(1-u(t))h_{1x}(t,x(t))+u(t)h_{2x}(t,x(t))\right]-(1-\tau_2)k_x(\cdot)\right)\right.\\&\left.+p(t)\left\{-a(t)+f_x(\cdot)\right\}+q(t)g_x(\cdot)\right\}dt+q(t)dW(t)\end{aligned}. \tag{43}$$

According to (43) the first co-state variable, which reflects the level of education per individual, is very complicated, as many parameters get involved. Although, the rate $a-f_x(\cdot)$ decreases the stochastic differential equation of function p, in the case that the marginal productivity of human capital exceeds the rate of depreciation of knowledge, see also section 2.

Moreover, through the expression (2.11) it is taken

$$e^{-rt}\left((1-\tau_1)\left[h_2(t,x(t))-h_1(t,x(t))\right]-(1-\tau_2)k_u(t,u,x)\right)+p(t)f_u(\cdot)+q(t)g_u(\cdot)=0$$

or equivalently

$$p(t)f_u(\cdot)+q(t)g_u(\cdot)=-e^{-rt}\left\{(1-\tau_1)\left[h_2(t,x(t))-h_1(t,x(t))\right]-(1-\tau_2)k_u(t,u,x)\right\} \tag{44}$$

Let the optimization problem admits an optimal pair $\left(x^*(\cdot),u^*(\cdot)\right)$.

Then, the optimal 4-tuple $\left(x^*(\cdot),u^*(\cdot),p(\cdot),q(\cdot)\right)$ of the stochastic problem discussed above solves the stochastic Hamiltonian problem.

Finally, the stochastic Hamiltonian system may be interpreted as the net "profit" at time t from the net investment in human capital. Moreover, taking also into consideration the above-complicated expressions, a much insightful view for the percentage of education that someone should invest into that, stock is derived in order to maximize his "profit". Equivalently, the fraction of time devoted to work, $1-u$, is also obtained.

6. Conclusion

The main purpose of this book chapter is to introduce an optimal control theory model to the education-investment decision strategies that maximize a criterion, which is based on the present value of future earnings for an individual. The formulation of this model is quite general including several inputs variables, assuming only the rate of schooling as the control variable.
Using the Potryagin maximum principle and the relative Hamiltonian function, some very general results for the determination of the time path education-pattern, and the optimal lifetime policy are derived into a deterministic (see 2nd section) and stochastic (see 5th section) framework, as well.
Furthermore, some practical and straightforward results are obtained when a special case of the productivity function, the famous Cobb – Douglas, is considered.
The results may be summarized as follows:
a) An analytic control function for the exact determination of the fraction of the time invested into education is derived. The formula, although complicated-since it considers several parameters, is very insightful and presents the efficient way to spend our time between job and schooling.
b) For lower (or higher) depreciation rates, the optimal pattern is a full time education, i.e. $u(t)=1$, for the very first years, followed by a period of education maintenance via higher (lower) part time education (for instance, in our application is almost one third of the time), and finished by zero education for retired persons.
c) Moreover, it appears that the interest rate of return decreases the optimal pattern and obviously, it follows an opposite monotonicity with the depreciation rate.

Furthermore, in this part of the chapter, in order to unify the language that is necessary for further interdisciplinary work, other issues arise, that are focused more on the actual modelling process rather than the concrete mathematical notation of it. Thus, the followings are just few of the areas identified for which thorough understanding and clarification is needed by development of future research work in the modelling process of human capital allocation, see Pantelous et al. (2008):
❖ Question regarding the nature of the building blocks of a model have been raised. These relate to the knowledge we have for the objects, their relationships, their attributes. The knowledge we acquire and the way this knowledge is acquired relevant questions to be investigated. Model definition that fits the existing data and the knowledge of the objects/relations is another issue. From model definition, different kinds of questions come into the surface regarding model "Minimality", as well as model simplification and expansion and how these are achieved. The questions formed provide a framework of problem areas; areas for which little has been done to provide concrete solutions or step-by-step approaches, and thus form open problem areas. These open problem areas have been identified as: *Knowledge extraction, Systems conceptualization, Design of experiments, Model construction, Model minimality, Model expansion, Model reduction* and *Model simplification*. Developing

modelling approaches requires tackling problems of the above classes in a rather substantial way.

❖ The need of data mining and knowledge management has been stressed and the importance of a concrete process of data collection, as well as the extraction of knowledge in a systematic way. The transition from data to information and eventually knowledge is still an open and major challenge. There is a need for a generic framework that should provide the basis for an understanding by answering key questions. Specifically:

1. How is data transformed into information? Issues of data mining and the knowledge brought by the modeller related with the modelling process. A question closely related with the Knowledge extraction problem previously identified, as well as the observer and his previous knowledge.
2. What is the role of data in shaping the structure of the model? Here again, we identify a question that if clearly answered could provide an insight in the model expansion, reduction, as well as simplification problem.
3. Is the role of data purely for quantitative reasons, that is, for providing measures for the variables, constraints, limitations, etc?

An agenda for long-term research is to develop a systemic approach summarizing the above needs that aim at:

(i) Providing a conceptual framework that explains the interrelationships between the different actors of the system notion (objects, interconnection topology, inputs, outputs, environment).

(ii) Select the appropriate modelling tools that describe particular problems and provide qualitative and quantitative means enabling the understanding of hierarchical nesting and system properties emerging at different levels,

(iii) Study control, optimization and state assessment problems in the integrated overall set up; this involves the development of both top-down and bottom-up approaches and related diagnostics-prognostics-control aspects.

(iv) Develop criteria, modelling concepts and methodologies that explain the evolution of physical system structure through the different stages of the cascade design process.

(v) Develop methodologies for redesigning existing systems to meet new operational requirements.

(vi) Explore the system aspects of data merging and transformations, which may provide useful tools that may support the operational and design aspects of integration.

It should be also emphasized that the paper uses a recent method, see Groot (1998), to measure the rate of depreciation. Moreover, the numerical application bases on real data, as well.

Finally, we can stress two other possible directions for further research. The first one considers the same deterministic problem with a generalization as regard the number of individuals, the inputs parameters and consequently the expansion of the number of the control parameters. The second direction considers the stochastic model, which can be benefited more by the introduction of stochastic framework for the Cobb-Douglas production function, the income function, the taxation etc. This approach transforms the simple optimal stochastic allocation problem proposed in section 5 (and Kalogeropoulos and Pantelous (2008)) into a complex optimal multi-stochastic control problem, which has also many mathematical difficulties.

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The widespread deployment and use of Information Technologies (IT) has paved the way for change in many fields of our societies. The Internet, mobile computing, social networks and many other advances in human communications have become essential to promote and boost education, technology and industry. On the education side, the new challenges related with the integration of IT technologies into all aspects of learning require revising the traditional educational paradigms that have prevailed for the last centuries. Additionally, the globalization of education and student mobility requirements are favoring a fluid interchange of tools, methodologies and evaluation strategies, which promote innovation at an accelerated pace. Curricular revisions are also taking place to achieved a more specialized education that is able to responds to the society's requirements in terms of professional training. In this process, guaranteeing quality has also become a critical issue. On the industrial and technological side, the focus on ecological developments is essential to achieve a sustainable degree of prosperity, and all efforts to promote greener societies are welcome. In this book we gather knowledge and experiences of different authors on all these topics, hoping to offer the reader a wider view of the revolution taking place within and without our educational centers. In summary, we believe that this book makes an important contribution to the fields of education and technology in these times of great change, offering a mean for experts in the different areas to share valuable experiences and points of view that we hope are enriching to the reader. Enjoy the book!

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