Application of consistent aggregate functions, inequality constraints and ranking methods in fuzzy query languages

Marcel Shirvanian and Wolfram Lippe
University of Muenster, Institute of Computer Science
Germany

1. Introduction

The data of almost every application are stored in databases controlled by a database management system (DBMS). Since crisp data are unavailable for several real world applications, vague data, which are usually expressed by means of the fuzzy theory, have to be stored in databases. Conventional DBMS cannot effectively handle vague data so that a fuzzy DBMS has to be utilized for this kind of data.
Generating a fuzzy DBMS, however, is a difficult task because many important issues have to be considered, one of which is the creation of a suitable fuzzy query language. By using a query language designed for a DBMS that bases on the relational model, it is possible, amongst others, to determine the minimum and the maximum of all values of an attribute, to restrict a relation by means of inequality constraints and to sort the tuples of a relation according to the values of an attribute. Assuming that a fuzzy DBMS is required whose relations potentially contain fuzzy numbers, the belonging fuzzy query language must also provide these procedures.
There are some works dealing with one of these issues as well as a few papers in which more than one of them is covered. Unfortunately, even in the last-mentioned papers, these components of a fuzzy query language have been examined independently from each other. Independent proposals of such a procedure are extremely problematic because they are in relationship to each other.
Since a ranking of fuzzy numbers implicitly determines the smallest and the greatest item, it is reasonable to utilize one ranking method for the calculation of minimum and maximum. Moreover, the reduction of a relation by discarding all tuples whose fuzzy numbers are greater than another one is equivalent to the ranking of two fuzzy numbers. In contrast to real numbers, it is not clear whether a fuzzy number is greater or less than another one so that different fuzzy ranking methods are available. Hence, in order to achieve consistent results, a single fuzzy ranking method has to be utilized by which each of the three named procedures of the fuzzy query language can be evaluated effectively and efficiently.
The remaining chapter is organized as follows. Section 2 presents some fundamentals of the fuzzy database systems that are considered in the following. Concerning the three

operations of a fuzzy query language mentioned above, several approaches are presented in section 3. In doing so, the conditions to be satisfied by such a procedure are indicated. The identification of fuzzy ranking methods which theoretically can be used for each of these three operations is subject of section 4. In section 5, these measures are applied on various examples in order to evaluate their quality on the basis of their results. Section 6 deals with the performance of the selected method by both introducing an algorithm which computes the results efficiently and comparing that algorithm with alternative calculations. Finally, a short conclusion is drawn in section 7.

2. Fuzzy relational databases

The majority of the analyses of fuzzy databases refer to the relational variant. Thus, it is not remarkable that also most fuzzy query languages are designed for fuzzy relational databases. Equivalent to the traditional relational query languages, fuzzy relational query languages are predominantly characterized by fuzzy relational algebras. Some of these algebras are defined in (Bosc & Pivert, 2005; Buckles & Petry, 1985; Caluwe De et al., 1995; Chen, 1991; Ma & Yan, 2007; Medina et al., 1994; Prade & Testemale, 1984; Shenoi & Melton, 1990; Tang & Chen, 2004; Umano & Fukami, 1994). Moreover, there are implementations of specific languages, for example the variants presented in (Galindo et al., 2006; Umano, 1982; Zemankova-Leech & Kandel, 1984).
Most of these languages or algebras respectively operate on relations of a fuzzy database whose relation schemata potentially contain fuzzy attributes. The values of these fuzzy attributes are possibility distributions defined by (1). The possibility distribution a is characterized by the membership function μ_a over the domain Ω. Specific sets of a possibility distribution a, that is the support, the core and the λ-cut of a for a value $\lambda \in [0, 1]$, are used in general as well as in the following; they are defined by (2), (3) and (4) respectively.

$$a = \{(\mu_a(x)/x) \mid x \in \Omega\} \tag{1}$$

$$supp(a) = \{x \mid x \in \Omega \wedge \mu_a(x) > 0\} \tag{2}$$

$$core(a) = \{x \mid x \in \Omega \wedge \mu_a(x) = 1\} \tag{3}$$

$$cut_\lambda(a) = \{x \mid x \in \Omega \wedge \mu_a(x) \geq \lambda\} \tag{4}$$

Since ranking algorithms can only be applied on fuzzy numbers, the membership functions of all possibility distributions regarded in this chapter are convex, semi-continuous and have a linearly ordered domain. Furthermore, solely normalized trapezoidal possibility distributions $[\alpha, \beta, \gamma, \delta]$ characterized by the membership function (5) are considered. Trapezoids are generally sufficient for practical applications (Prade & Testemale, 1984) so that this restriction is not severe.

$$\mu_{[\alpha,\beta,\gamma,\delta]}(x) = \begin{cases} 0 & \text{if } x < \alpha \text{ or } x > \delta \\ \dfrac{x-\alpha}{\beta-\alpha} & \text{if } \alpha \leq x < \beta \\ 1 & \text{if } \beta \leq x \leq \gamma \\ \dfrac{\delta-x}{\delta-\gamma} & \text{if } \gamma < x \leq \delta \end{cases} \qquad \text{with } \begin{matrix} \alpha,\beta,\gamma,\delta,x \in \Omega \\ \alpha \leq \beta \leq \gamma \leq \delta \end{matrix} \tag{5}$$

A fuzzy query language has to provide several methods to process fuzzy numbers, three of which have been indicated in the introduction. But all languages mentioned above pay only marginal attention to these procedures. Thus, the existing approaches as well as the conditions to be satisfied in each case are presented in the next section.

3. Operations of a fuzzy query language

3.1 Minimum and maximum of fuzzy numbers

One of the most important tools of the fuzzy theory is the extension principle with which conventional arithmetic operations, for example the addition or the division, can be applied on fuzzy numbers. In (Dubois & Prade, 1978), the extension principle was also used to calculate the minimum and the maximum of fuzzy numbers. The fuzzy minimum of the possibility distributions *a* and *b* is defined by (6) whereas the fuzzy maximum is specified by (7).

$$\widetilde{\min}(a,b) = \{(\mu_{\widetilde{\min}}(z)/z) \mid z \in \Omega\} \qquad \text{with } \mu_{\widetilde{\min}}(z) = \sup_{\substack{x,y \in \Omega \\ z=\min(x,y)}} \min(\mu_a(x), \mu_b(y)) \tag{6}$$

$$\widetilde{\max}(a,b) = \{(\mu_{\widetilde{\max}}(z)/z) \mid z \in \Omega\} \qquad \text{with } \mu_{\widetilde{\max}}(z) = \sup_{\substack{x,y \in \Omega \\ z=\max(x,y)}} \min(\mu_a(x), \mu_b(y)) \tag{7}$$

The few papers dealing with the determination of minimum and maximum of fuzzy numbers in fuzzy query languages (Dubois & Prade, 1990; Rundensteiner & Bic, 1992) propose the use of (6) and (7). But instead of selecting the smaller or greater fuzzy number, the application of these two functions potentially creates new possibility distributions. Since query languages usually extract data from relations without modifying them, the generation of nonexistent data is incomprehensible. Hence, two methods are necessary that do not alter the fuzzy numbers.

3.2 Evaluation of inequality constraints

In order to determine the maximum of two fuzzy numbers, it is not necessary to know to what extent a fuzzy number is greater than the other one. By contrast, it may be requested to remove a tuple from a relation if the fuzzy number of that tuple is not greater than another fuzzy number to a certain degree which means that the inequality constraint is not satisfied to that degree. Thus, exclusively those measures computing an adequate value to which a fuzzy number is greater than another one can be applied.

Equivalent to the determination of the fuzzy minimum and the fuzzy maximum, the evaluation of inequality constraints is either ignored or only mentioned marginally in most of the papers concerning fuzzy query languages. In contrast to these papers, the work presented in (Galindo et al., 2006) deals with this issue in details by specifying the fuzzy query language FSQL. This language provides four methods for the computation of the degree of the possibility and the necessity respectively to which a value of *a* is greater than a value of *b*. The four measures that can only process normalized trapezoidal possibility distributions are defined by (8) to (11).

$$FGEQ(a,b)=\begin{cases}1 & \text{if } \gamma_a \geq \beta_b \\ \dfrac{\delta_a-\alpha_b}{\delta_a-\alpha_b+\beta_b-\gamma_a} & \text{if } \gamma_a < \beta_b \text{ and } \delta_a > \alpha_b \\ 0 & \text{else}\end{cases} \tag{8}$$

$$FGT(a,b)=\begin{cases}1 & \text{if } \gamma_a \geq \delta_b \\ \dfrac{\delta_a-\gamma_b}{\delta_a-\gamma_b+\delta_b-\gamma_a} & \text{if } \gamma_a < \delta_b \text{ and } \delta_a > \gamma_b \\ 0 & \text{else}\end{cases} \tag{9}$$

$$NFGEQ(a,b)=\begin{cases}1 & \text{if } a_a \geq \beta_b \\ \dfrac{\beta_a-a_b}{\beta_a-a_b+\beta_b-a_a} & \text{if } a_a < \beta_b \text{ and } \beta_a > a_b \\ 0 & \text{else}\end{cases} \tag{10}$$

$$NFGT(a,b)=\begin{cases}1 & \text{if } a_a \geq \delta_b \\ \dfrac{\beta_a-\gamma_b}{\beta_a-\gamma_b+\delta_b-a_a} & \text{if } a_a < \delta_b \text{ and } \beta_a > \gamma_b \\ 0 & \text{else}\end{cases} \tag{11}$$

It has to be mentioned that these four measures match the methods introduced in (Dubois & Prade, 1983) if exclusively trapezoids are permitted. One of the referenced measures is defined by (12) and produces the same results as (8).

$$PD(a,b)=\sup_{\substack{x,y\in\Omega \\ x\geq y}} \min(\mu_a(x),\mu_b(y)) \tag{12}$$

It is obvious that (12) generates the extremal values if and only if the first or the last case of (8) holds. Concerning the evaluation of the remaining situation, the output of (12) is the membership degree of the intersection between the right edge of *a* and the left edge of *b*. The *x*-coordinate value of the intersection is determined by equating the two respective linear functions displayed by the first part of the following calculation. This value must be used in any of these two functions in order to determine the membership degree; this process is described by the second part of the calculation.

$$-\frac{1}{\delta_a-\gamma_a}x+\frac{\delta_a}{\delta_a-\gamma_a}=\frac{1}{\beta_b-\alpha_b}x-\frac{\alpha_b}{\beta_b-\alpha_b}\Leftrightarrow\frac{\delta_a(\beta_b-\alpha_b)+\alpha_b(\delta_a-\gamma_a)}{(\delta_a-\gamma_a)(\beta_b-\alpha_b)}=\frac{\delta_a-\gamma_a+\beta_b-\alpha_b}{(\delta_a-\gamma_a)(\beta_b-\alpha_b)}x$$
$$\Leftrightarrow x=\frac{\delta_a(\beta_b-\alpha_b)+\alpha_b(\delta_a-\gamma_a)}{\delta_a-\gamma_a+\beta_b-\alpha_b}$$

$$\frac{1}{\beta_b-\alpha_b}x-\frac{\alpha_b}{\beta_b-\alpha_b}=\frac{\delta_a(\beta_b-\alpha_b)+\alpha_b(\delta_a-\gamma_a)-\alpha_b(\delta_a-\gamma_a+\beta_b-\alpha_b)}{(\beta_b-\alpha_b)(\delta_a-\gamma_a+\beta_b-\alpha_b)}=\frac{\delta_a-\alpha_b}{\delta_a-\alpha_b+\beta_b-\gamma_a}$$

The result matches the second case of (8) so that the correlation between (8) and (12) is shown. The equality of the other measures can be proven in a similar way. Furthermore, the formulas proposed in (Galindo et al., 2006) that compute the degree to which a value of *a* is less than a value of *b* can be derived from (8) to (11). Thus, they do not need to be examined. The four measures defined in (Dubois & Prade, 1983), however, have to be used together in order to determine the minimum or maximum fuzzy number. Since a single method is required, the measures (8) to (11) are not feasible.

3.3 Fuzzy ranking

Sorting a relation by means of the values of a fuzzy attribute is equivalent to ranking fuzzy numbers. First of all, it is necessary to find out which kinds of fuzzy ranking methods are compatible with the issues covered in the previous subsections. Different attempts were undertaken to classify various fuzzy ranking methods and to explore - partly by applying them to some examples - their strengths and weaknesses (Bortolan & Degani, 1985; Chen & Chen, 2009; Chen & Hwang, 1992; Deng, 2007; Dubois & Prade, 1999; Rommelfanger, 1986; Wang & Kerre, 1996; Wang & Kerre, 2001; Zhu & Lee, 1992). The most detailed work is (Chen & Hwang, 1992) whose classification, which is illustrated in Figure 1, is examined now. Some of the methods that are indicated in this figure were not part of the original paper because mostly they were published after this classification.

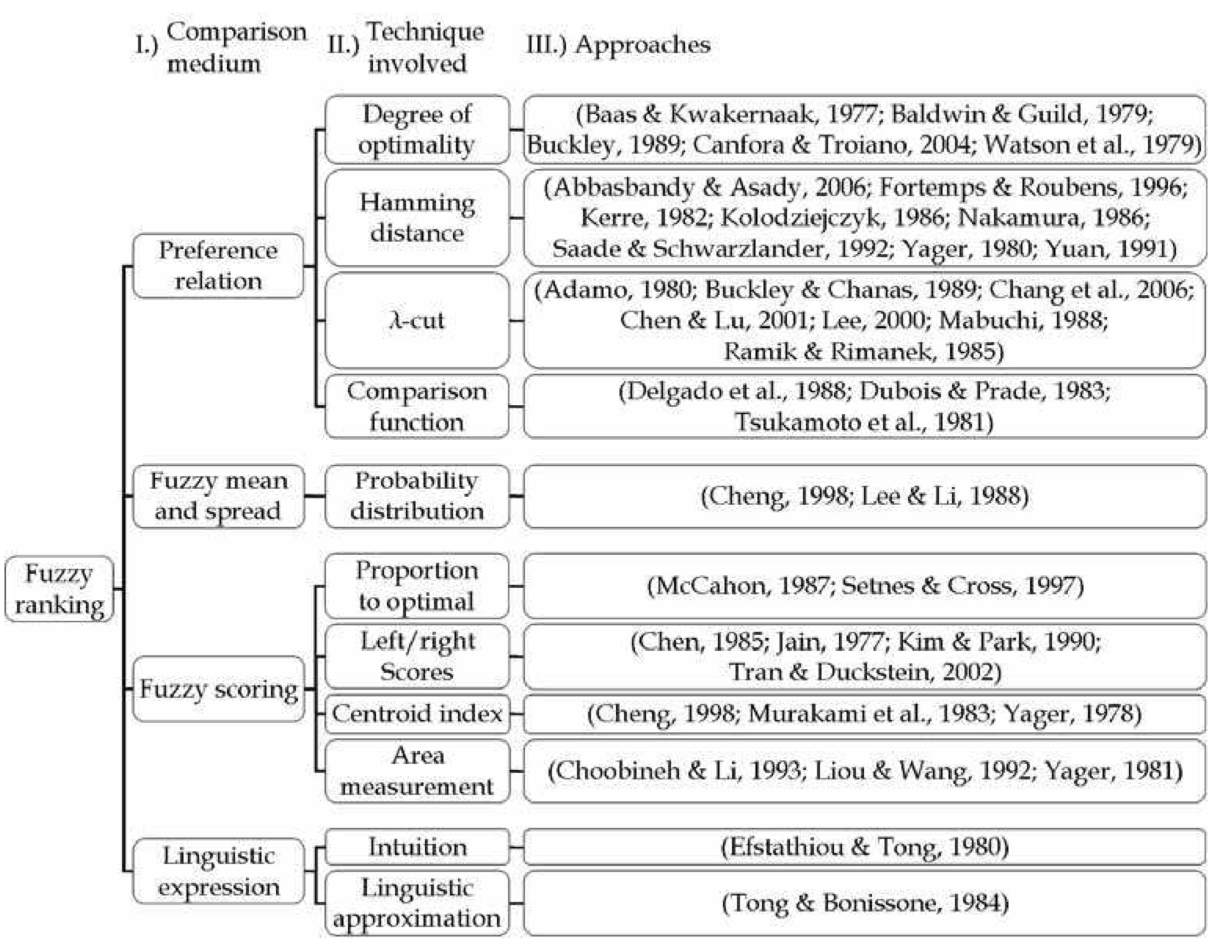

Fig. 1. A taxonomy of fuzzy ranking methods

Apart from these approaches, there are some methods that cannot be assigned to a single technique. On the one hand, a universal method is shown in (Campos-Ibanez De & Gonzales-Munoz, 1989) which can be transformed into particular ranking methods of the class λ-cut as well as the class area measurement. On the other hand, the variant presented in (Chu & Tsao, 2002) contains elements of the centroid index and the area measurement. Moreover, the fuzzy ranking method in (Lian et al., 2006) is based on the probability distribution and the area measurement which belong to different comparison media.

In addition to the generation of a comprehensible order, a fuzzy ranking method must both derive the order of fuzzy numbers from pairwise comparisons and generate a membership degree for each pair. To be more precise, a fuzzy ranking method $R(a, b)$ for two arbitrary trapezoids a and b has to produce a value in [0, 1] declaring to what extent a is greater than b. Since the degree to which a is less than b must not be calculated independently, the fuzzy reciprocal, that is $R(a, b) + R(b, a) = 1$, proposed in (Nakamura, 1986) must hold for each method $R(a, b)$.

The concept of the linguistic expression that produces not a single membership degree therefore can be ignored. The techniques probability distribution, left/right scores, centroid index and area measurement also have to be rejected because they include the distance between the fuzzy numbers in their computations. Concerning the question whether a real number is greater than another one, it is irrelevant if they are close-by or far away from each other. Thus, $R(a, b)$ has to generate the maximum - 1 - or the minimum value - 0 - if and only if the supports of a and b are disjoint. The fuzzy ranking methods of the remaining techniques, that is degree of optimality, λ-cut, hamming distance, comparison function and proportion to optimal must be analyzed individually in order to decide whether they are suitable with respect to the operations of a fuzzy query language described in this section. These analyses also have been presented partly in (Shirvanian & Lippe, 2008).

4. Selection of theoretically appropriate fuzzy ranking methods

4.1 Comparison function

The approaches of the technique comparison function calculate the degree of the possibility and the necessity respectively to which a value of a fuzzy number is greater than a value of another one. Hence, the measures (8) to (11) that have already been characterized as inadequate belong to this class. Alternative methods of the technique comparison function have not to be considered because all of them can be seen as special cases of these four procedures. Although the concept of this technique is compatible with the characteristics of a ranking method required in this work, the specific approaches are not applicable regarding the named operations of a fuzzy query language.

4.2 Degree of optimality

The procedures of the class degree of optimality rank a group of fuzzy numbers by checking them against the maximum. Most of these methods appear to be plain or inadequate. By contrast, the function declared in (Canfora & Troiano, 2004) evaluates two fuzzy numbers by comparing all elements of Ω with each other. In doing so, an element of a which is greater than an element of b receives the degree 1. Conversely, the value 0 and in case of equality, the value 0.5 have to be assigned. The computation, which does not have to include the part with the factor 0, is defined by (13).

$$R_{CT}(a,b)=\frac{\sum_{y\in\Omega}\left(1*\sum_{\substack{x\in\Omega\\x>y}}\min(\mu_a(x),\mu_b(y))+0.5*\sum_{\substack{x\in\Omega\\x=y}}\min(\mu_a(x),\mu_b(y))\right)}{\sum_{y\in\Omega}\sum_{x\in\Omega}\min(\mu_a(x),\mu_b(y))} \tag{13}$$

In theory, the sums have to be replaced by integrals if two continuous fuzzy numbers, for example trapezoidal possibility distributions, are evaluated. But concerning the following analysis, (13) generates suitable results so that no integrals have to be computed. In addition, it is also recommended in (Canfora & Troiano, 2004) to perform an approximate calculation.

4.3 λ-cut

The technique λ-cut, whose methods are regarded in this subsection, determines the ranking by means of some cuts of the fuzzy numbers to the degree λ. Most of the procedures displayed before use only one particular λ-cut so that important information are ignored. The only fuzzy ranking method that includes a sufficiently great number of λ-cuts is specified in (Mabuchi, 1988) and is presented now. In order to determine the degree to which a is greater than b, the fuzzy number d must be generated by means of (14). The not yet normalized value for a λ-cut arises by using (15).

$$d=\{(\mu_{a-b}(z)/z)\mid z\in\Omega\} \qquad \text{with } \mu_{a-b}(z)=\sup_{\substack{x,y\in\Omega\\z=x-y}}\min(\mu_a(x),\mu_b(y)) \tag{14}$$

$$J_{a,b}(\lambda)=\begin{cases}0 & \text{if } v=w=0 \\ \dfrac{\max(v,0)-\max(-w,0)}{\max(v,0)+\max(-w,0)} & \text{else}\end{cases} \qquad \text{with } v=\sup_{u\in cut_\lambda(d)} u \text{ and } w=\inf_{u\in cut_\lambda(d)} u \tag{15}$$

The values of (15) for the particular degrees of λ must be merged next. But the regular formula is too complex so that an approximation is proposed in (Mabuchi, 1988) which is also utilized here. In the same paper, an accurate computation for trapezoids as fuzzy numbers is indicated which, however, can be ignored. The reasons for it are that the calculation is very complex and sufficiently precise values result from the approximation defined by (16). The accuracy of the output is improved by increasing the value of the natural number N.

$$R_M(a,b)=\frac{\frac{2}{N^2}\left(\sum_{n=0}^{N} n*J_{a,b}\left(\frac{n}{N}\right)-\frac{N}{2}*J_{a,b}(1)\right)+1}{2} \tag{16}$$

4.4 Hamming distance

The next fuzzy ranking methods to be presented work with the hamming distance of two possibility distributions which is described by (17). Furthermore, both the greatest upper set and the greatest lower set defined by (18) and (19) respectively are utilized.

$$H(a,b)=\int_{-\infty}^{\infty}\left|\mu_a(x)-\mu_b(x)\right|dx \tag{17}$$

$$\bar{a}=\left\{\left(\mu_{\bar{a}}(z)/z\right)\mid z\in\Omega\right\} \qquad \text{with } \mu_{\bar{a}}(z)=\sup_{\substack{x\in\Omega\\ x\leq z}}\mu_a(x) \tag{18}$$

$$\underline{a}=\left\{\left(\mu_{\underline{a}}(z)/z\right)\mid z\in\Omega\right\} \qquad \text{with } \mu_{\underline{a}}(z)=\sup_{\substack{x\in\Omega\\ x\geq z}}\mu_a(x) \tag{19}$$

Equivalent to the previous techniques, the class hamming distance also contains only a few fuzzy ranking methods which meet the requirements indicated before. One of them is illustrated in (Nakamura, 1986), is defined by (20) and is dependent on a specific value for λ that must be set in advance.

$$R_N(a,b)=\begin{cases}\dfrac{\lambda * H\left(\underline{a},\widetilde{\min}\left(\underline{a},\underline{b}\right)\right)+(1-\lambda)* H\left(\bar{a},\widetilde{\min}\left(\bar{a},\bar{b}\right)\right)}{\Delta_\lambda} & \text{if } \Delta_\lambda\neq 0\\ 0.5 & \text{if } \Delta_\lambda = 0\end{cases} \tag{20}$$

$$\text{with } \Delta_\lambda=\lambda *\left(H\left(\underline{a},\widetilde{\min}\left(\underline{a},\underline{b}\right)\right)+H\left(\underline{b},\widetilde{\min}\left(\underline{a},\underline{b}\right)\right)\right)+(1-\lambda)*\left(H\left(\bar{a},\widetilde{\min}\left(\bar{a},\bar{b}\right)\right)+H\left(\bar{b},\widetilde{\min}\left(\bar{a},\bar{b}\right)\right)\right)$$

The measure (20), however, can be disregarded because drastic deficits were revealed in (Kolodziejczyk, 1986). To be more precise, R_N can assign the extremal values to both two almost identical and two remote fuzzy numbers. Due to these disadvantages, three alternatives defined by (21) to (23) were developed in the last-mentioned work. The minimum is utilized as the t-norm for the computation of the intersection in R_{K1} and R_{K3}. Interestingly, R_{K2} generates exactly the same results than R_N if λ = 0.5 (Wang & Kerre, 2001) so that only the methods R_{K1} and R_{K3} remain.

$$R_{K1}(a,b)=\frac{H(a\cap b,0)+H\left(b,\widetilde{\max}(a,b)\right)}{H(a,0)+H(b,0)} \tag{21}$$

$$R_{K2}(a,b)=\frac{H\left(\underline{b},\widetilde{\max}\left(\underline{a},\underline{b}\right)\right)+H\left(\bar{b},\widetilde{\max}\left(\bar{a},\bar{b}\right)\right)}{H\left(\underline{a},\underline{b}\right)+H\left(\bar{a},\bar{b}\right)} \tag{22}$$

$$R_{K3}(a,b)=\frac{H(a\cap b,0)+H\left(\underline{b},\widetilde{\max}\left(\underline{a},\underline{b}\right)\right)+H\left(\bar{b},\widetilde{\max}\left(\bar{a},\bar{b}\right)\right)}{2H(a\cap b,0)+H\left(\underline{a},\underline{b}\right)+H\left(\bar{a},\bar{b}\right)} \tag{23}$$

Although R_{K1} and R_{K3} appear relatively diverse at first sight, their results vary only minimally from each other; this effect will be demonstrated later. Nevertheless, both procedures are analyzed because this difference will be pointed out to be significant. But R_{K1} and R_{K3} have a shortcoming, that is, a division by zero can take place which must be avoided. By applying (21), this happens if a and b are crisp numbers whereas concerning (23), the two numbers additionally must be identical. The values 0, 0.5 and 1 have to be produced for these particular scenarios.

4.5 Proportion to optimal

The last class - proportion to optimal - shares properties with the technique degree of optimality. But the optimum now arises out of the given fuzzy numbers. The method introduced in (McCahon, 1987) is not presented here because it is a special case of the fuzzy ranking method declared in (Setnes & Cross, 1997). The approach in (Setnes & Cross, 1997) does not describe a specific fuzzy ranking method, but a group of procedures. In each case, the optima are given by the fuzzy maximum and the fuzzy minimum because the classical maximum and minimum can be used to determine the greatest and the smallest item. The degree to which a is greater than b is computed by (24) whose components φ and S are defined next.

$$\mu_{SC1}(a,b) = \varphi\left(S\left(a, \widetilde{\max}(a,b)\right), S\left(b, \widetilde{\min}(a,b)\right)\right) \tag{24}$$

S is a similarity measure and therefore calculates the similarity of a to the fuzzy maximum and the similarity of b to the fuzzy minimum. Since the issue of similarity measures will not be addressed in this work, only the measure named in (Setnes & Cross, 1997) in the first place is utilized; this function is defined by (25). Moreover, different kinds of comparison measures, for example inclusion measures, can be applied which, however, are neglected here as well.

$$S_1(a,b) = \frac{\int\limits_{x\in\Omega} \min\left(\mu_a(x), \mu_b(x)\right) dx}{\int\limits_{x\in\Omega} \max\left(\mu_a(x), \mu_b(x)\right) dx} \tag{25}$$

According to (Setnes & Cross, 1997), φ is a function in which, amongst others, the arithmetic mean or any t-norm can be used. But if the similarity measure (25) is selected and a t-norm is applied on a comparison between a continuous fuzzy number and a crisp number, the output of (24) is always 0. Thus, the arithmetic mean, whose insertion in (24) leads to (26), is utilized exclusively in the following.

$$\mu_{SC2}(a,b) = \frac{1}{2}\left(S_1\left(a, \widetilde{\max}(a,b)\right) + S_1\left(b, \widetilde{\min}(a,b)\right)\right) \tag{26}$$

Unfortunately, the fuzzy reciprocal does not hold for this formula. Hence, in (Setnes & Cross, 1997), it is recommended to include also the result with swapped arguments into the fuzzy ranking method. In order to obtain a method that satisfies all of the conditions outlined before, the ratio of both terms must be calculated. The approach, which together with the other presented procedures is analyzed in the next section on the basis of examples, is specified by (27). It is necessary again to pay attention that the denominator does not become 0. This event only arises if two crisp numbers are compared with each other.

$$R_{SC}(a,b) = \frac{\mu_{SC2}(a,b)}{\mu_{SC2}(a,b) + \mu_{SC2}(b,a)} \tag{27}$$

5. Comparison of fuzzy ranking methods

Before the quality of an order generated by the fuzzy ranking methods can be examined, it is important to find out whether they produce suitable degrees for different pairs of fuzzy numbers. Concerning the determination of meaningful scenarios, the analyses listed in subsection 3.3 are hardly helpful because mostly comparisons with more than two fuzzy numbers are examined. Consequently, a new collection of scenarios is used now which is observable in Figure 2 and whose items partly derive from some of the examples given in (Deng, 2007; Zhu & Lee, 1992). Naturally, fuzzy numbers with disjoint supports do not need to be taken into account. The fuzzy number marked by the continuous line represents in each case the first argument *a* whereas the other fuzzy number illustrates the second argument *b*. Since the fuzzy reciprocal holds for all fuzzy ranking methods, it is not necessary to apply them with reversed arguments.

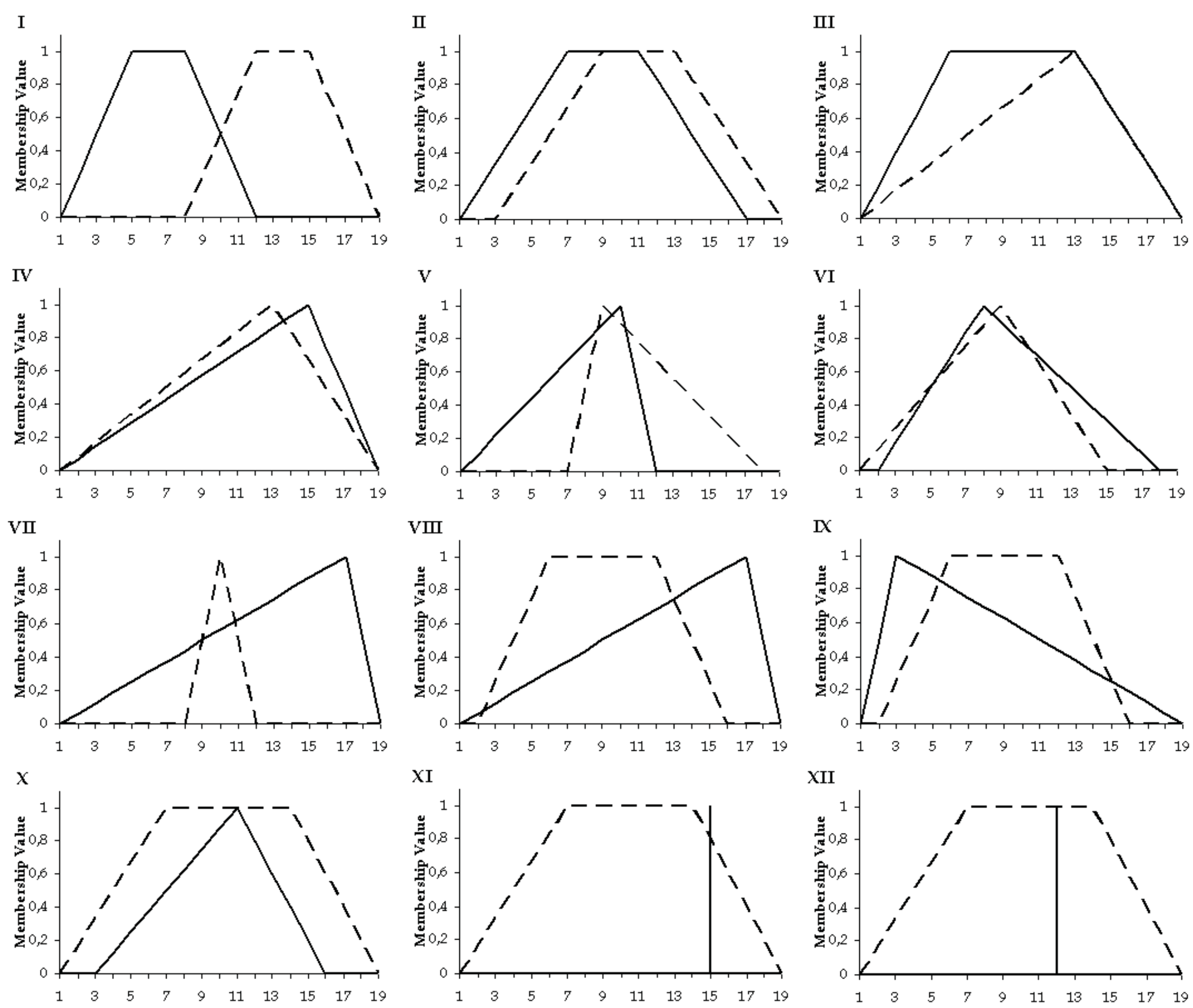

Fig. 2. A set of fuzzy numbers to be evaluated

The fuzzy ranking methods which have been regarded as adequate in the previous section, that is R_{CT}, R_M, R_{K1}, R_{K3} and R_{SC}, now are applied on each of the scenarios presented above. The degrees displayed in Table 1 are rounded after the fifth decimal place. Concerning R_M, the variable N was set to 100 because a sufficiently accurate approximation is achieved with it.

No. \ R	$R_{CT}(a, b)$	$R_M(a, b)$	$R_{K1}(a, b)$	$R_{K3}(a, b)$	$R_{SC}(a, b)$
I	0.02315	0.0187	0.07143	0.0625	0.07143
II	0.33881	0.35859	0.4	0.4	0.4
III	0.41878	0.31114	0.41861	0.41861	0.41861
IV	0.5492	0.70593	0.55	0.55495	0.55
V	0.12412	0.447	0.2702	0.27387	0.26935
VI	0.59305	0.40553	0.53519	0.53321	0.53522
VII	0.66528	0.90659	0.69444	0.75506	0.7018
VIII	0.72494	0.86832	0.70395	0.72222	0.70431
IX	0.39759	0.2078	0.38026	0.37147	0.37966
X	0.48794	0.51917	0.5	0.5	0.5
XI	0.928	0.93589	0.872	0.87402	0.872
XII	0.72	0.66267	0.64	0.64	0.64

Table 1. Results of the proposed fuzzy ranking methods

Studying the results of the first potentially suitable fuzzy ranking method presented in section 4 – R_{CT} –, it is clear that appropriate data exist for many situations. But there are also scenarios in which the output of this method must be interpreted as quite questionable, in particular No. V. Although it appears to be that a is less than b, the disproportionately small degree does not express the situation adequately because, for example, a has the greater maximum value.

By contrast, R_M produces a very high degree for the same example because the zones with high membership values are weighted too heavily. This action especially has an impact on the scenarios VI and VII. In the first case, the fuzzy number a is slightly greater than b but its maximum value is somewhat less than the one of b. All other fuzzy ranking methods accordingly generate a result greater than 0.5. The method R_M, however, clearly declares b as the greater fuzzy number. The result for the second case is close to the maximum value so that this scenario is also characterized inappropriately. Therefore, the utilization of R_M cannot be recommended.

The degrees of the remaining methods differ only minimally from each other or even are partly identical so that R_{K1} can be used as a reference. Since the results of this procedure appear to be consistent with respect to the examined examples, it has to be found out whether the use of R_{K3} or R_{SC} is more preferable. In order to compare R_{K1} with the other methods, its concept has to be described.

The numerator consists of two parts, one of which computes the area of the intersection whereas the other one simply determines the zones in which a dominates b. Dividing the sum of these two parts by the sum of the areas of both fuzzy numbers leads to the final result. The calculation of this method can be explained even more easily by regarding an example in details. Thus, the application of $R_{K1}(a, b)$ on the two fuzzy numbers $a = [1, 10, 12, 15]$ and $b = [3, 5, 6, 17]$ illustrated in Figure 3 is analyzed next.

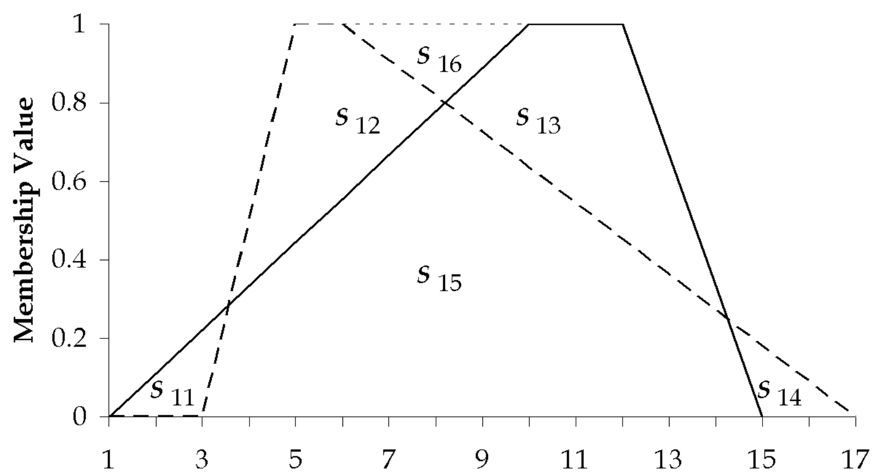

Fig. 3. A pair of fuzzy numbers to be analyzed in details

Six different areas are displayed in Figure 3, one of which, that is s_{16}, does not belong to one of the two trapezoids. The intersection area is denoted as s_{15} and there are two zones in which a dominates b, namely s_{12} and s_{13}. Hence, the numerator consists of the sum of these three areas whereas the denominator arises from the sum of the areas s_{11}, s_{12}, s_{13}, s_{14} and twice s_{15} representing the areas of both trapezoids.

Despite the varying equations (21) and (23), Table 1 indicates that the output of R_{K3} differs from the one of R_{K1} if and only if the cores of both fuzzy numbers do not intersect. The reason for it is that R_{K3} processes the same subareas but additionally includes the area between the two cores, that is s_{16}, in its calculation. Concerning the current example, R_{K3} divides the sum $s_{12} + s_{13} + s_{15} + 2s_{16}$ by the sum $s_{11} + s_{12} + s_{13} + s_{14} + 2s_{15} + 2s_{16}$. Besides, the numerator of R_{K3} contains the area $2s_{16}$ if and only if the core of a is greater than the core of b. Thus, equivalent to R_M, the zones with high membership values are weighted more heavily. But in contrast to R_M, the method R_{K3} does not generate any implausible degrees. Since this kind of weighting is advantageous, R_{K3} is more favorable than R_{K1}.

The method R_{SC} can also be ignored because the results of this method are very similar to the outputs of R_{K1}. By including an alternative similarity measure in (24), varying fuzzy ranking methods could be designed. Anyhow, the procedure R_{K3} represents an appropriate solution so that this task seems to be superfluous.

Furthermore, due to the analyses in (Wang & Kerre, 2001), an order produced by R_{K3} is reasonable. Consequently, suitable results for all operations of a fuzzy query language indicated in section 3 can be achieved by utilizing this fuzzy ranking method. Although the calculation of (23) appears to be very complex, the application on the previous example has demonstrated the straightforward concept. In the next section, the performance of ranking fuzzy numbers by means of R_{K3} is analyzed.

6. Performance of the selected fuzzy ranking method

6.1 Algorithm for the efficient computation of R_{K3}

It has already been mentioned that R_{K3} compares two fuzzy numbers with each other by calculating the ratio of particular partial areas of them. Since only trapezoids are utilized

here, it is not difficult to determine the relevant partial areas as it has been indicated in Figure 3. Nevertheless, this process can be simplified further by using the fuzzy ranking method introduced in (Yuan, 1991).
This procedure that is defined by (28) derives from the fuzzy ranking method specified by (20). But instead of using the two fuzzy numbers as the arguments, the difference of them calculated by means of (14) represents the first argument. The second argument is the fuzzy number (29) which can also be described by the trapezoid [0, 0, 0, 0]. Furthermore, the variable λ is set to 0.5. In order to explain this procedure as well as its relationship to R_{K3}, Figure 4 is examined.

$$R_Y(a,b) = R_N\left(a-b,\tilde{0}\right) \qquad \text{with } \lambda = 0.5 \tag{28}$$

$$\tilde{0} = \left\{\left(\mu_{\tilde{0}}(x)/x\right) \mid x \in \Omega\right\} \qquad \text{with } \mu_{\tilde{0}}(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases} \tag{29}$$

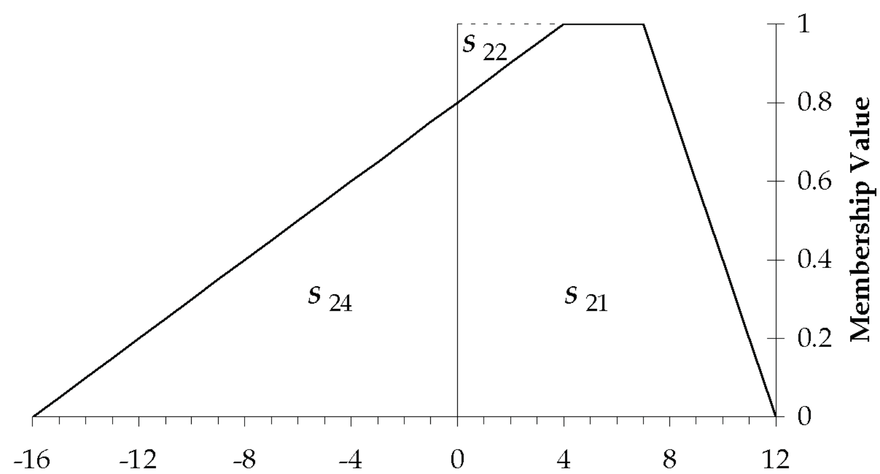

Fig. 4. The fuzzy difference of the previous example

Figure 4 shows the difference of the two fuzzy numbers displayed in Figure 3; the difference d = [–16, 4, 7, 12] is determined by applying (14). If a and b are two normalized trapezoidal possibility distributions, the fuzzy difference $d = a - b$ is also a normalized trapezoid with the values $[\alpha_a - \delta_b, \beta_a - \gamma_b, \gamma_a - \beta_b, \delta_a - \alpha_b]$. The trapezoid arising out of the fuzzy difference is subdivided into the four partial areas s_{21}, s_{22}, s_{23} and s_{24}, three of which are visible in Figure 4. The areas s_{21} and s_{24} represent the positive and the negative part of d whereas s_{22} and s_{23} are the positive and the negative areas between that trapezoid and (29). It is obvious that at most one of the two last-mentioned partial areas can exist.
As it is evident from (20), the denominator of (28) contains a sum of four areas, two of which are also part of the numerator. Regarding the current example, the sum $s_{21} + 2s_{22}$ is divided by the sum $s_{21} + 2s_{22} + 2s_{23} + s_{24}$. In the following, it is shown that R_Y produces the same values as R_{K3}, at least if exclusively normalized trapezoidal possibility distributions are processed. First of all, according to the next equation, the area of d is equal to the sum of the areas of a and b.

$$\frac{\delta_d-\alpha_d+\gamma_d-\beta_d}{2}=\frac{(\delta_a-\alpha_b)-(\alpha_a-\delta_b)+(\gamma_a-\beta_b)-(\beta_a-\gamma_b)}{2}=\frac{\delta_a-\alpha_a+\gamma_a-\beta_a}{2}+\frac{\delta_b-\alpha_b+\gamma_b-\beta_b}{2}$$

Thus, $s_{21} + s_{24} = s_{11} + s_{12} + s_{13} + s_{14} + 2s_{15}$ holds so that the denominators of R_{K3} and R_Y are identical if $s_{22} + s_{23} = s_{16}$ holds. Since $\beta_d = \beta_a - \gamma_b$ and $\gamma_d = \gamma_a - \beta_b$ hold, disjoint cores of a and b, which imply a positive value for s_{16}, are equivalent to a positive value for $s_{22} + s_{23}$. Therefore, intersecting cores lead to equal denominators. Moreover, assuming that the scenario in Figure 3 is given, $s_{22} = s_{16}$ holds if the membership value of the intersection between the left edge of a and the right edge of b is equal to the membership value of d at the point 0; the last value results from $-\alpha_d/(\beta_d - \alpha_d)$. Considering the equivalence of (8) and (12) shown in subsection 3.2, the two membership values are obviously equal. The last case, that is the occurrence of a positive value for s_{23}, can be neglected because the fuzzy ranking methods fulfil the fuzzy reciprocal.

In order to prove the equivalence of R_{K3} and R_Y, the equivalence of the numerators must be shown. Once again, it is not necessary to take the case with a positive value for s_{23} into account so that the area $s_{21} + s_{22}$ is represented by the trapezoid $[0, 0, \gamma_a - \beta_b, \delta_a - \alpha_b]$. The area of this trapezoid is equal to the area of the trapezoid formed by the partial areas s_{12}, s_{13}, s_{15} and s_{16}. Due to the equivalence of s_{16} and s_{22}, the numerators of R_Y and R_{K3} for the current scenario, that is $s_{21} + 2s_{22}$ and $s_{12} + s_{13} + s_{15} + 2s_{16}$, are identical. Intersecting cores of a and b do not influence this result because $s_{21} = s_{12} + s_{13} + s_{15}$ holds in that case.

Altogether, R_Y produces the same values as R_{K3} so that the partial areas of only one trapezoid, namely the fuzzy difference $d = a - b$, have to be computed. The result of $R_Y(a, b)$ is calculated by $R_Y(d)$ defined by (30). The last three cases represent the scenarios, in which the two cores intersect, the core of a is greater than the core of b and vice versa.

$$R_Y(d)=\begin{cases}0.5 & \text{if } \alpha_d=\delta_d=0\\ 0 & \text{if } \alpha_d<0,\delta_d\le 0\\ 1 & \text{if } \alpha_d\ge 0,\delta_d>0\\ \dfrac{\gamma_d+\delta_d}{\delta_d-\alpha_d+\gamma_d-\beta_d} & \text{if } \alpha_d<0,\beta_d\le 0,\gamma_d\ge 0,\delta_d>0\\ \dfrac{\gamma_d+\delta_d+\beta_d^2/(\beta_d-\alpha_d)}{\delta_d-\alpha_d+\gamma_d-\beta_d+2\beta_d^2/(\beta_d-\alpha_d)} & \text{if } \alpha_d<0,\beta_d>0\\ \dfrac{\delta_d^2/(\delta_d-\gamma_d)}{\delta_d-\alpha_d+\gamma_d-\beta_d+2\gamma_d^2/(\delta_d-\gamma_d)} & \text{if } \gamma_d<0,\delta_d>0\end{cases} \tag{30}$$

6.2 Ranking multiple fuzzy numbers

An algorithm efficiently calculating the degree to which a trapezoid is greater than another one has been presented in the previous subsection. But it is not clear how to rank more than two fuzzy numbers. A solution for this problem is the creation of an ordinal scale proposed in (Tseng & Klein, 1989) for the fuzzy ranking method R_{K1}; this concept is also feasible for R_{K3}. In order to obtain a result, $R_{K3}(a, b)$ must be applied on each pair of trapezoids a and b. In doing so, it is noted down which trapezoid is greater than the other one. The order of the fuzzy numbers arises from these data so that a is ranked before b if a has been regarded as the greater fuzzy number more often than b. Assuming that n fuzzy numbers have to be ranked, R_{K3} must be applied $n(n-1)/2$ times.

This procedure is used, amongst others, in psychological researches, more precisely for the ranking of items according to the preferences of a person. Since these preferences usually do not fulfil the transitivity rule, each pair of items has to be evaluated in order to identify and eliminate inconsistencies (Gilbert et al., 1998). The fuzzy ranking method R_{K3}, however, satisfies the transitivity rule specified in (31).

$$\forall a,b,c : \text{if } R(a,b) \geq 0.5 \text{ and } R(b,c) \geq 0.5 \text{ then } R(a,c) \geq 0.5 \tag{31}$$

Moreover, both the fuzzy reciprocal holds for R_{K3} and this method can be applied on any two fuzzy numbers so that R_{K3} induces a weak fuzzy order (Nakamura, 1986). Thus, a distinct order of any set of fuzzy numbers can be generated by R_{K3}; the property to interpret two different fuzzy numbers as equal can be disregarded. In (Tseng & Klein, 1989) as well as in some other papers, for example in (Jaafar & McKenzie, 2008; Lee, 2005; Li, 1999), this fact was apparently not considered. Consequently, fuzzy numbers can be ranked by means of an ordinary sorting algorithm, for example quicksort, so that the complexity is reduced from $O(n^2)$ to $O(n \log n)$.

Although a huge efficiency increase is achieved with this enhancement, a procedure was introduced in (Li, 1999) by which the performance could be improved even more. The reason for it is that – despite the execution of $O(n \log n)$ comparisons – R_{K3} has to be applied only n times. Unfortunately, it was proven in (Lee, 2005) that this procedure is incorrect. In the same paper, it was indicated that the procedure would be valid if a fuzzy ranking method is used that fulfils (32) and (33).

$$\forall a,b,c : R(a,c) > R(b,c) \Leftrightarrow R(a,b) > 0.5 \tag{32}$$
$$\forall a,b,c : R(a,c) = R(b,c) \Leftrightarrow R(a,b) = 0.5 \tag{33}$$

By means of the simple example $a = [1, 2, 6, 7]$, $b = [4, 5, 5, 6]$ and $c = [6, 7, 7, 8]$, it is obvious that (32) does not hold for R_{K3}. Therefore, it would be necessary to determine an alternative fuzzy ranking method. This task can be neglected because, on the one hand, R_{K3} generates a reasonable order. On the other hand, $O(n \log n)$ comparisons have to be performed in any case and by declaring (30), it has been shown that the result of $R_{K3}(a, b)$ can be computed efficiently if both a and b are normalized trapezoidal possibility distributions.

7. Conclusion

Comprehensible and consistent results produced by fuzzy queries are a necessary precondition for the acceptance of a fuzzy DBMS. Three important operations of a query language designed for relational databases are the determination of the minimum and the maximum of all attribute values, the restriction of a relation by discarding those tuples which do not satisfy an inequality constraint and the generation of an order of some tuples according to the values of an attribute. But almost all examinations concerning fuzzy relational query languages that process fuzzy numbers either ignore these operations or consider them independently from each other.

In this work, different fuzzy ranking methods have been analyzed because each of the three tasks mentioned before can be managed by means of a suitable ranking method. First of all, it has been determined which fuzzy ranking methods meet the requirements that are necessary to obtain meaningful results for the three operations. Afterwards, these ranking methods have been applied on several examples in order to identify the variant producing the most plausible results. The method defined by (23) has turned out to be the best choice.
Despite the reasonable results of (23), its calculation appears to be very complex so that it was not clear whether the order of fuzzy numbers could be computed efficiently. But it has been shown that (23) is equivalent to (30), at least if normalized trapezoidal possibility distributions are utilized exclusively. Since on the one hand, this restriction is not severe, and on the other hand, the performance of ranking fuzzy numbers by means of (30) is acceptable, the use of this method is not to be objected.
Altogether, a single fuzzy ranking method remains by which meaningful results for the three named operations of a fuzzy query language can be determined efficiently. That method accordingly should be provided by a fuzzy query language.

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Phone: +86-21-62489820
Fax: +86-21-62489821