Rectifier Low Order Harmonics Reduction Technique — Educational Approach

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1. Introduction

The rectifier is an electrical device that converts alternating current (AC) to direct current (DC), a process known as rectification. Rectifiers are usually built to supply polarized voltage power to the DC circuits. Rectifier applications rang from the DC biasing in mVolts to electronic components on the PC motherboards, to the kVolts of DC power supplies in the steel industry.
The wide utilization of DC rectifiers justifies studying the performance of theses electrical devices. Voltage and current harmonics are commonly associated with the function of power electronics devices including full wave rectifiers.
Harmonics in power systems have received increased attention in recent years with the widespread application of advanced solid-state power switching devices in a multitude of power electronic applications. Intensive study and research has concentrated on the input side to prevent the harmonics generated by switching devices to travel back to the network and disturb other connected loads. The voltage harmonics at the load side have not been treated widely in the published literature, although they share many effects with the input side harmonics. In both cases, these harmonic voltages can, in worst cases, result in the following:

- Wiring overheating.
- Capacitor bank damage.
- Electronic equipment malfunctioning.
- Communication interference.
- Resonance.

Furthermore, certain effects are associated with the output harmonics like vibrations and noises in electromagnetic loads such as DC motors. A chopping technique is the control method that will be applied to shape the output waveform; also, it will be used to devolve the mathematical model, perform Pspice simulation and build the experimental prototype. In this method, the output voltage of the bridge rectifier will be "chopped" in a sequential manner with a certain number of pulses and duty cycles to control the flow of current through the load. This technique offers a linear control for the DC component and redistributes the AC components (ripples).

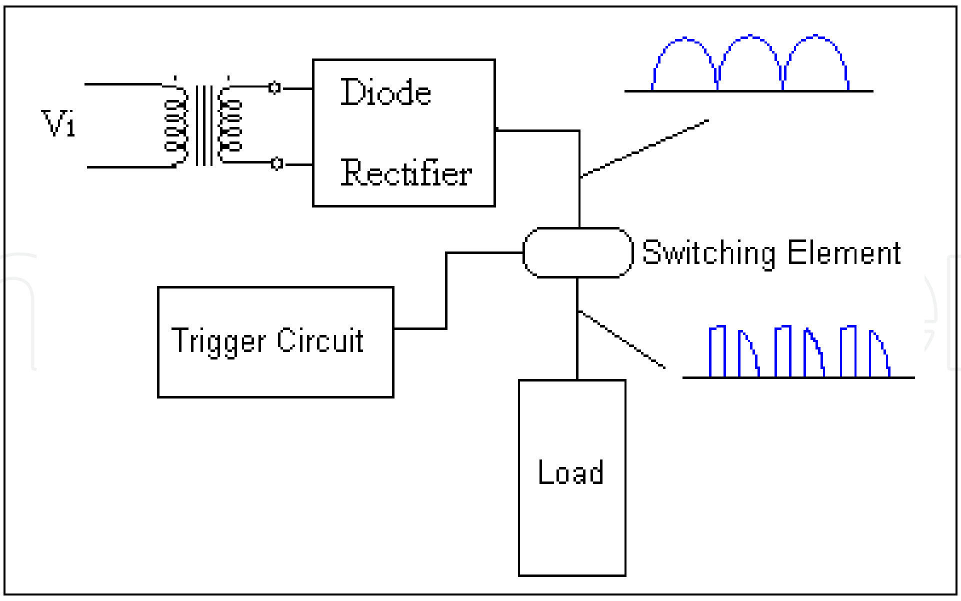


Fig. 1. The basic configuration of the controlled output bridge rectifier.

2. Mathematical Modeling

Based on the basic configuration of the controlled rectifier shown in Figure 1, we can start with the simplest case to calculate the coefficients of Fourier series as follows:

Period:	$T = 2\pi$
Number of Pulses:	$N = 2$
Duty Cycle:	$k = 50\%$

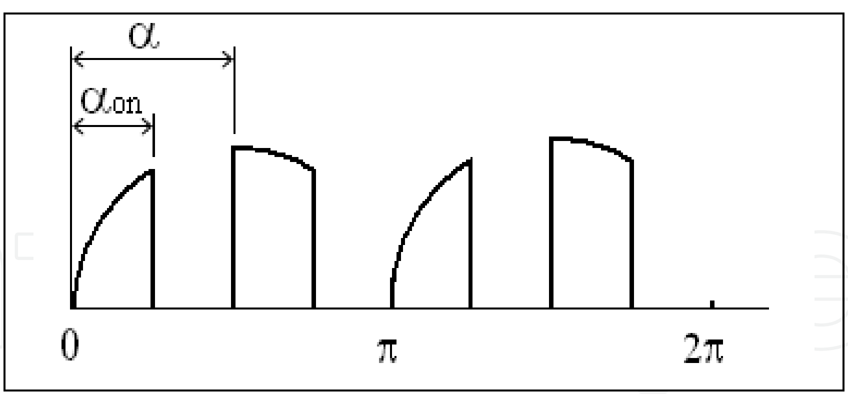


Fig. 2. 2-pulse shaped output of controlled rectifier with 50% dutycycle used to develop the mathematical model

The Fourier series of the output voltage is given by:

$$V_o = V_{dc} + \sum_{n=1}^{\infty}(a_n \sin n\theta + b_n \cos n\theta) \tag{1}$$

$$V_{dc} = \frac{1}{2\pi}\int_0^{2\pi} V_P \sin\theta \, d\theta \tag{2}$$

$$V_{dc} = \frac{V_P}{2\pi}\left[\int_0^{\pi/4} \sin\theta \, d\theta + \int_{\pi/2}^{3\pi/4} \sin\theta \, d\theta\right] \tag{3}$$

$$= \frac{V_P}{\pi} \tag{4}$$

Equation (3) could be generalized to accommodate the generic case when:

- N = 2, 3, 4, 5, 6… (integer numbers)
- $0 < k \leq 100\%$

Thus,

$$V_{dc} = \frac{V_P}{2\pi}\sum_{I=0}^{N-1}\left(\cos\frac{I}{N}\pi - \cos\frac{k+I}{N}\pi\right) + \frac{V_P}{2\pi}\sum_{I=N}^{2N-1}\left(\cos\frac{k+I}{N}\pi - \cos\frac{I}{N}\pi\right) \tag{5}$$

The resulted formula obtained in equation (5) could be verified by plugging in the conditions of an initial case of 2 pulses with 50% duty cycle; the result is matches with the value obtained by equation (4) above:

$$V_{dc} = k\frac{2V_P}{\pi} = \frac{V_P}{\pi}$$

Coefficients of the Fourier series (a_n and b_n) could be calculated using the same generalized approach.

In this special case, the waveform is considered as an even function, thus the odd components do not exist.

$$a_1 = b_1 = 0 \tag{6}$$

For the cases when harmonic order: $n \geq 2$

$$a_n = \frac{2}{2\pi}\int_0^{2\pi} V_P \sin\theta \cos n\theta \, d\theta \tag{7}$$

Evaluating the integration over the regions where the output voltage ≠0 yields:

$$a_n = \frac{V_P}{\pi}\sum_{I=0}^{N-1}\frac{1}{1-n^2}\left(\cos\frac{(1+n)(k+I)}{N}\pi + \cos\frac{(1+n)I}{N}\pi\right) + \frac{V_P}{\pi}\sum_{I=N}^{2N-1}\frac{1}{1-n^2}\left(\cos\frac{(1+n)(k+I)}{N}\pi + \cos\frac{(1+n)I}{N}\pi\right) \tag{8}$$

Similarly, b_n is calculated as follows:

$$b_n = \frac{2}{2\pi}\int_0^{2\pi} V_P \sin\theta \, \sin n\theta \, d\theta \tag{9}$$

$$b_n = \begin{array}{l} \frac{V_P}{2\pi}\sum_{I=0}^{N-1}\frac{1}{1-n}\left(\sin\frac{(1-n)(k+I)}{N}\pi - \sin\frac{(1-n)I}{N}\pi\right) - \\ \frac{V_P}{2\pi}\sum_{I=0}^{N-1}\frac{1}{1+n}\left(\sin\frac{(1+n)(k+I)}{N}\pi - \sin\frac{(1+n)I}{N}\pi\right) - \\ \frac{V_P}{2\pi}\sum_{I=0}^{2N-1}\frac{1}{1-n}\left(\sin\frac{(1-n)(k+I)}{N}\pi - \sin\frac{(1-n)I}{N}\pi\right) + \\ \frac{V_P}{2\pi}\sum_{I=0}^{N-1}\frac{1}{1+n}\left(\sin\frac{(1+n)(k+I)}{N}\pi - \sin\frac{(1+n)I}{N}\pi\right) \end{array} \tag{10}$$

The values of V_{dc}, a_n and b_n are used to reconstruct the output voltage by developing a calculation tool to obtain the values of V_o as in equation (1). Plotting the voltage versus time resulted in the following waveforms:

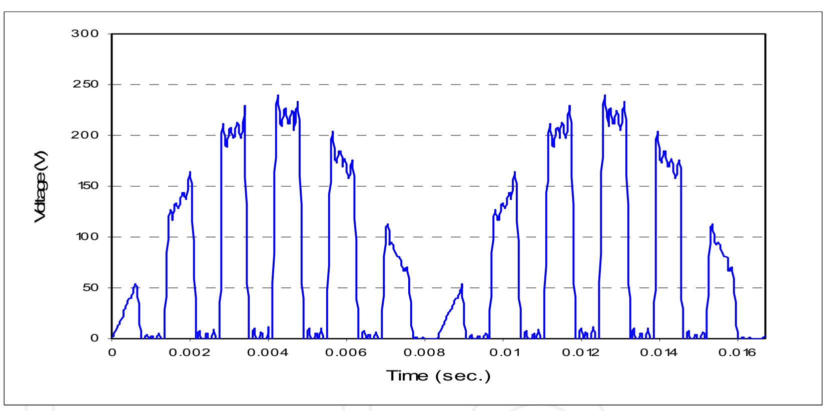


Fig. 3. The reconstructed output voltage as a summation of DC value and 100 harmonics.

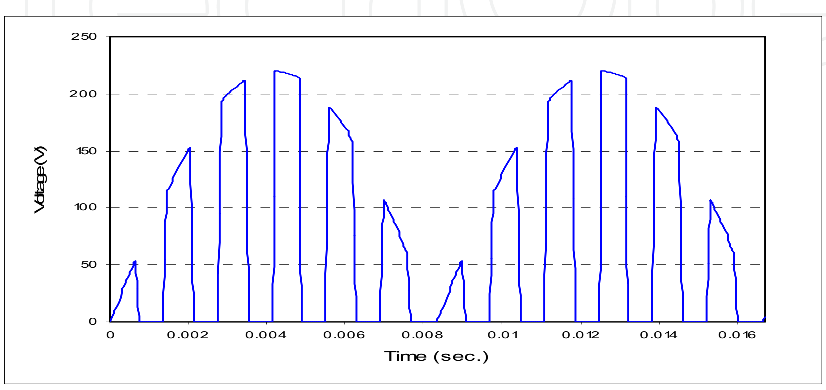


Fig. 4. The reconstructed output voltage as a summation of DC value and 1000 harmonics.

The program generates the values of the AC components (harmonics) of the output voltage for both controlled and uncontrolled rectifiers.

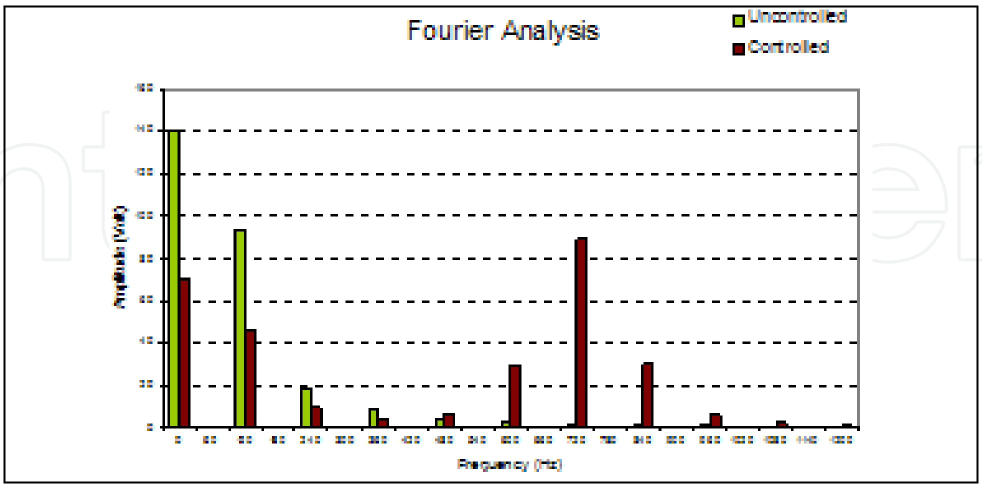


Fig. 5. Spectrum distribution for the calculated AC components of controlled and uncontrolled rectifiers.

Figure 5 shows a comparison of the spectrum distribution of an unchopped output and the output of a chopped rectifier with k=50% and N=6. The spectrum of the chopped rectifier show fewer values of low-order harmonics but increased values at high frequencies. These high order harmonics are easy to filter.

3. Pspice Simulation

The basic circuit of the controlled rectifier is used for simulation using Pspice. Figure 6 shows the circuit representation adopted for Pspice simulation. The resistive load is selected in the circuit above to test the principle without the effect of inductive or capacitive loads that have lagging or leading effects on the voltage as well as current waveforms. Actually, the calculation tool is prepared to reflect such effects.

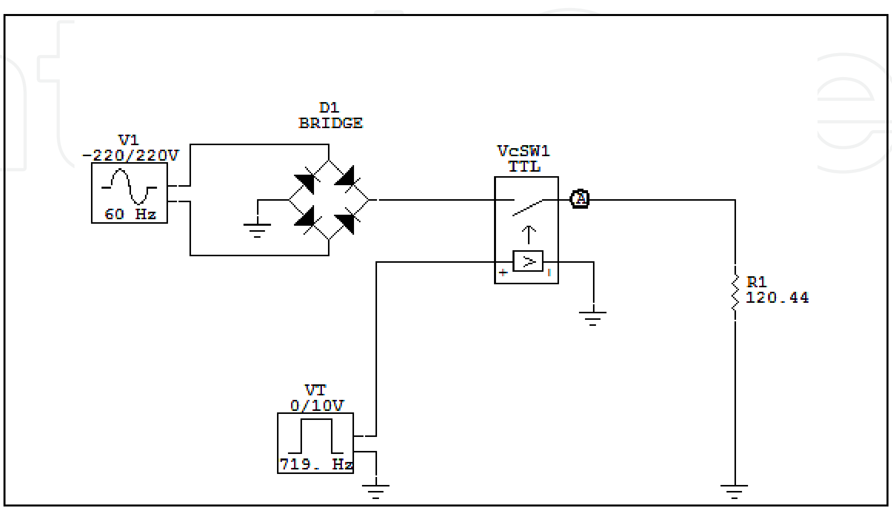


Fig. 6. Pspice simulation model

Figure 7 shows the waveform of output voltage as simulated by Pspice. The package of CircuitMaker V6.2c circuit simulation program

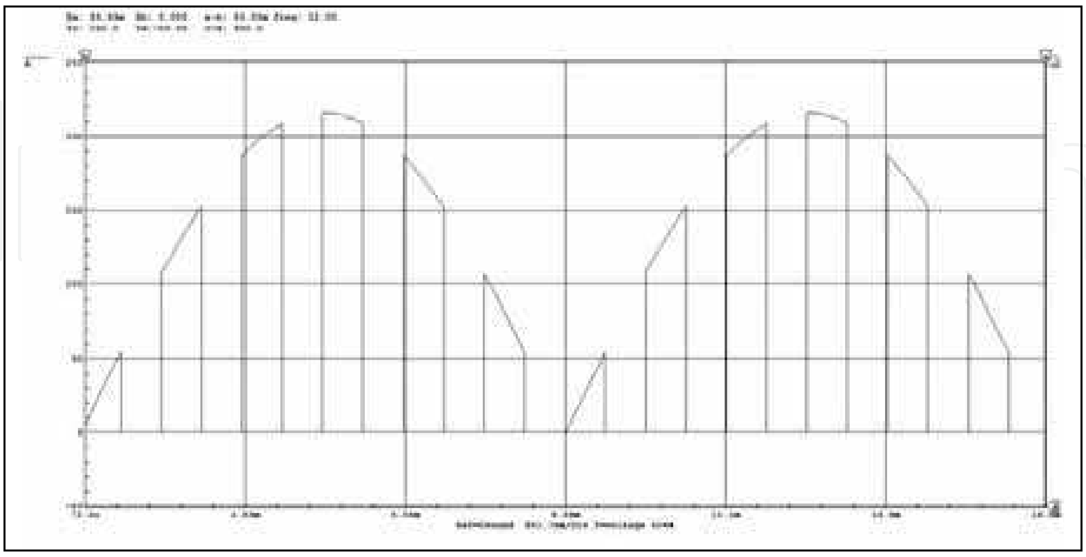


Fig. 7. Pspice output waveform for the controlled rectifier

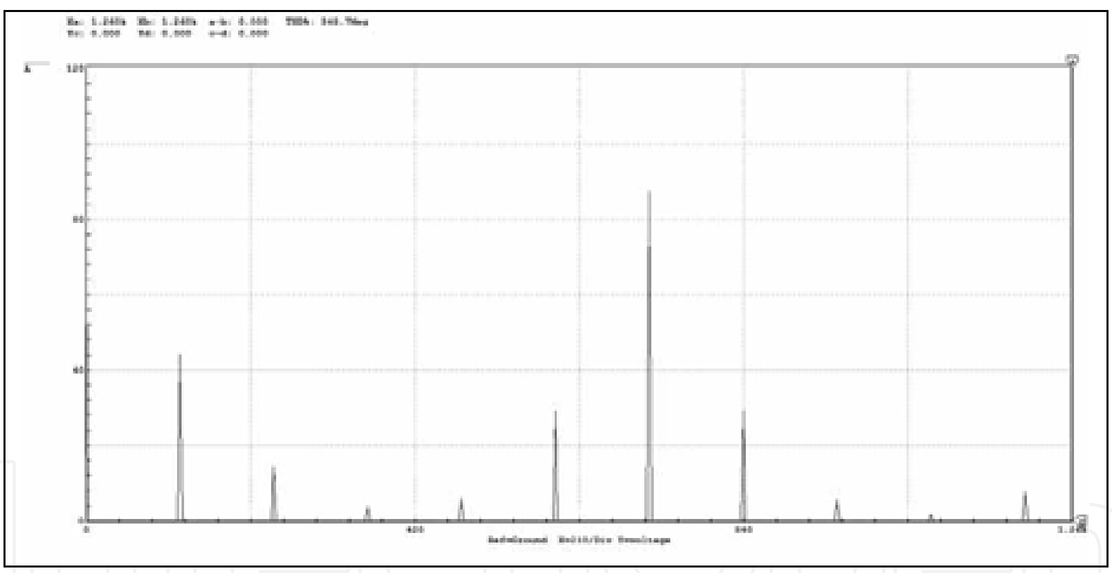


Fig. 8. Spectrum analyzer trace resulted by Fourier analysis tool of Pspice simulation

4. Experimental Modeling

To support the theoretical and simulation results, it is advisable to build an experimental prototype at the lab. to conduct a complete comparison of three different approaches.

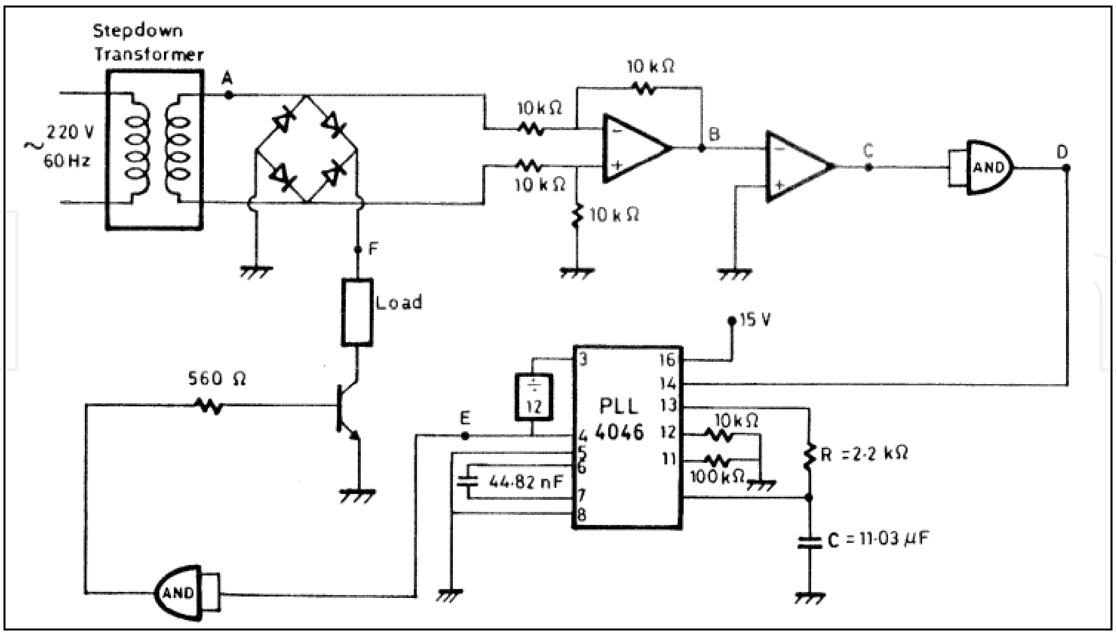


Fig. 9. Experimental prototype

Getting a synchronized trigger signal to switch on/off the power transistor (or GTO) that connects the load to the GND was the most important part in the prototyping. It is essential to fire the switching element at certain sequence with ON, OFF, ON, OFF,…etc as follows:

$$\alpha_{on} = 0, \frac{\pi}{2N}, \frac{2\pi}{2N}, \frac{3\pi}{2N}, \frac{4\pi}{2N}, ...\text{etc.} \tag{11}$$

(for 2-pulse with 50% duty cycle)

The experiential model is built using a zero-crossing detector to insure the synchronization with the input waveform.

Item	Function
AND Gate	Buffer, level shutter and Q driver
Difference Amp.	Isolation and grounding elimination (G=1)
PLL	Frequency doubler
Q	Darlington transistor for switching
Zero Crossing Detector	Synchronization

Table 1. Experimental circuit component

The waveform of the rectifier output voltage, together with input and trigger signals are shown in Figure 11.

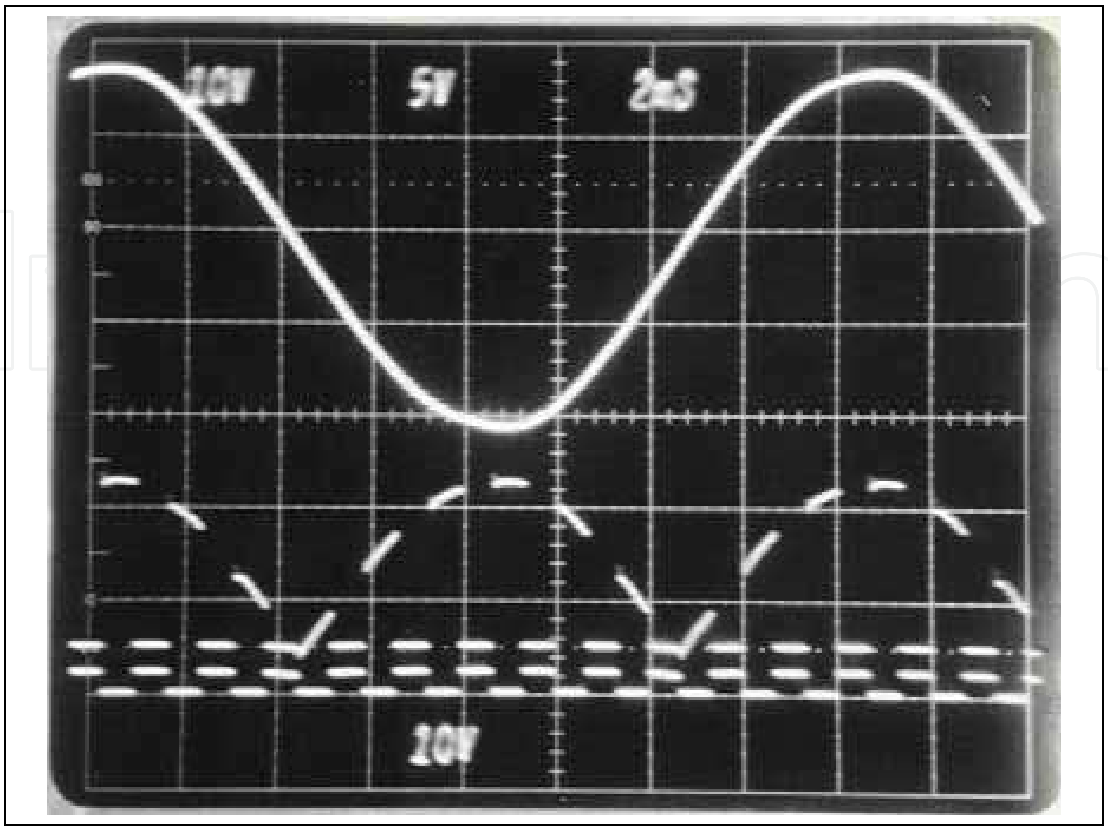


Fig. 10. Oscilloscope traces for input, trigger and output waveforms.

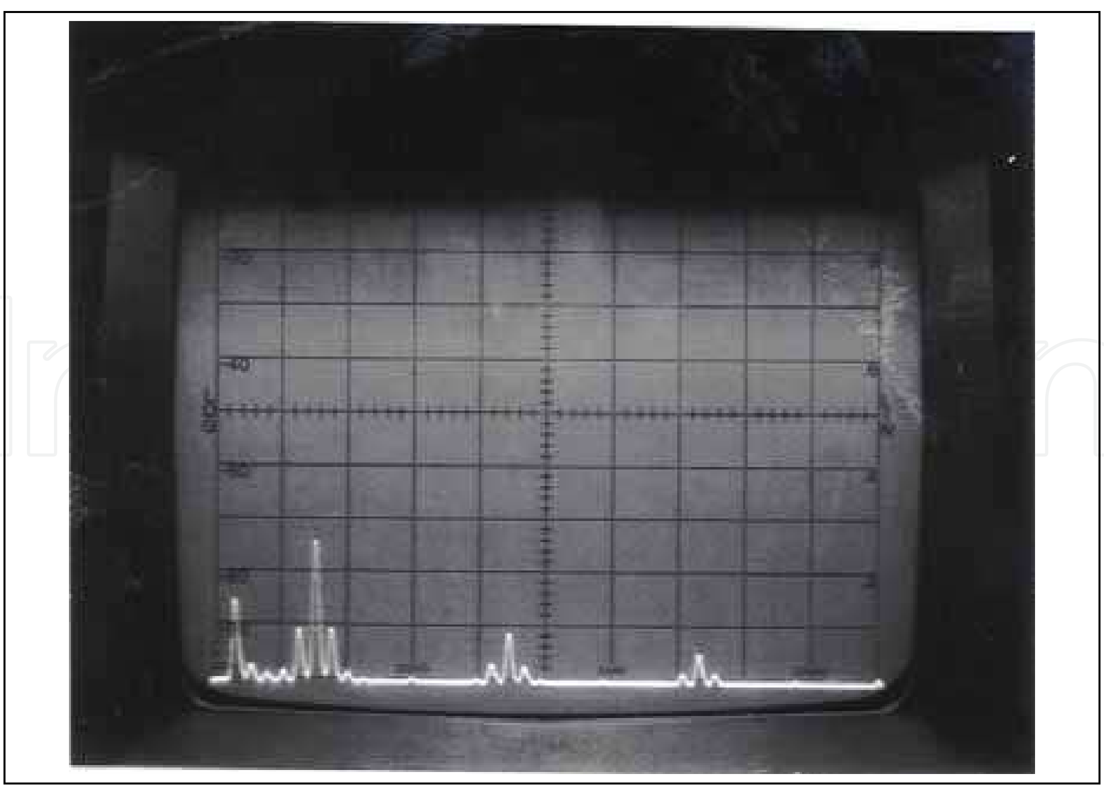
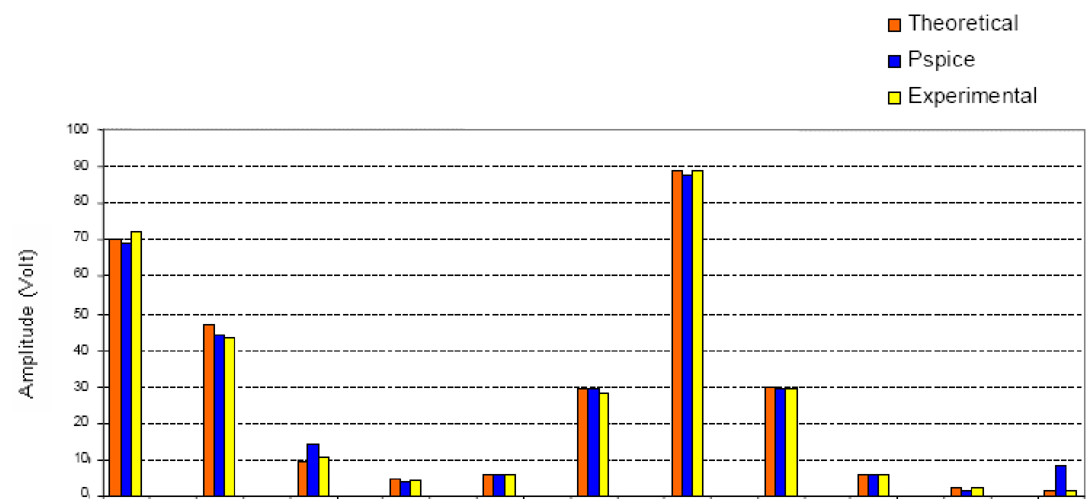


Fig. 11. Spectrum analyzer output for the output waveform.

The comparison of the harmonic spectrum generated through the mathematical distribution, Pspice simulation, and the experimental verification is illustrated in figure 13. The figure depicts close agreement between the three methods. It shows clearly the generation of high order harmonics around the 720 Hz.



	0	60	120	180	240	300	360	420	480	540	600	660	720	780	840	900	960	1020	1080	1140	1200
Theoretical	70.03	0	46.69	0	9.399	0	4.611	0	6.162	0	29.61	0	89.28	0	29.65	0	5.928	0	2.577	0	1.489
Pspice	69.11	0	44	0	14.44	0	3.778	0	6	0	29.33	0	87.78	0	29.33	0	5.541	0	2.122	0	7.84
Experimental	72.5	0	43.25	0	10.7	0	4.25	0	6.25	0	28.3	0	88.5	0	29.25	0	6.1	0	2.25	0	1.25

Frequency (Hz)

Fig. 12. Result comparison for the harmonic redistribution for the theoretical, simulation and experimental approaches.

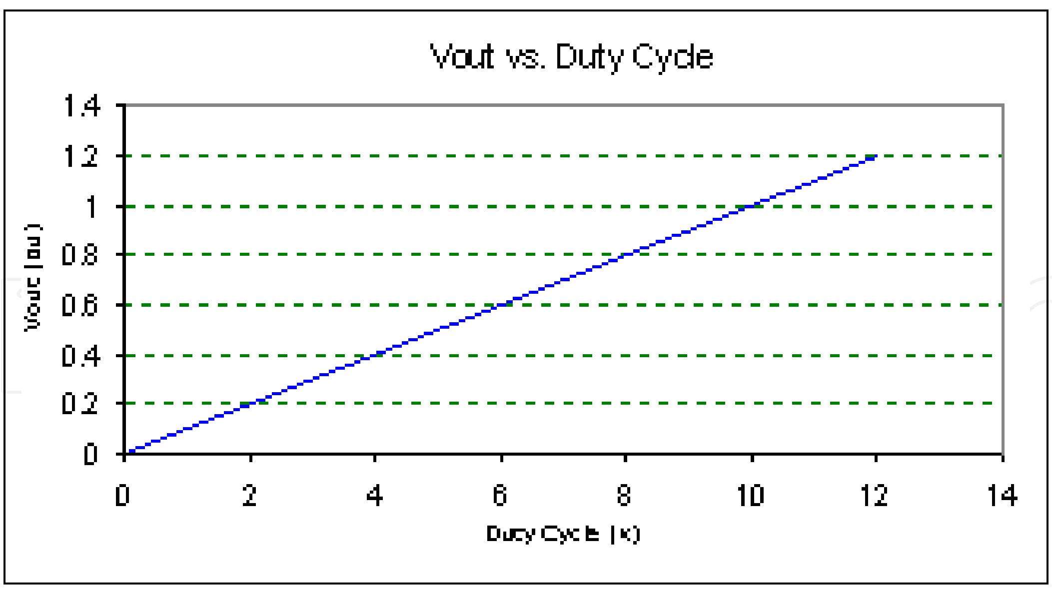


Fig. 13. The relation between the output voltage and the duty cycle

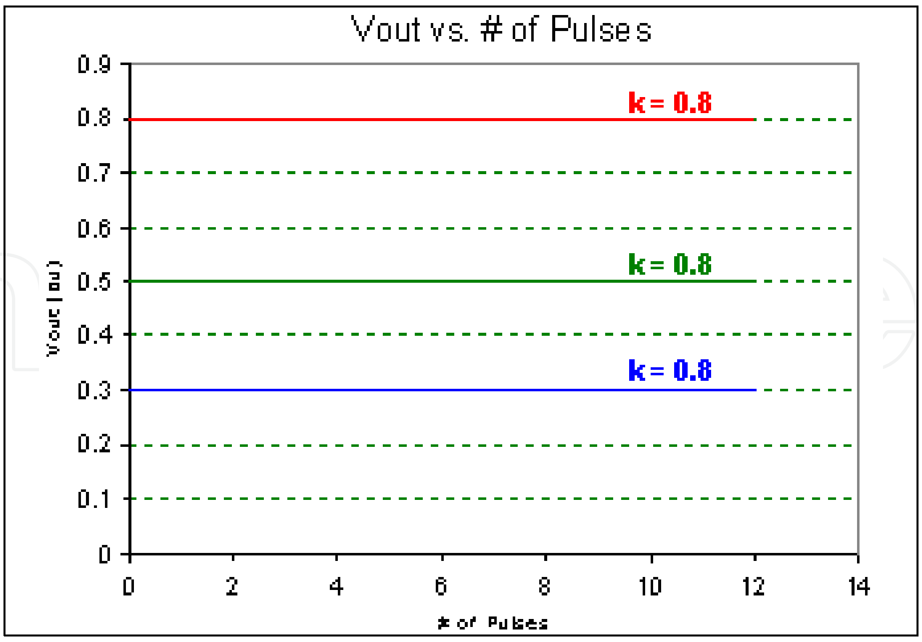


Fig. 14. The relation between the output voltage and the duty cycle

5. Conclusion

The chopping technique was verified as an efficient technique to reduce low-order (high amplitude) harmonics and shift them away from the fundamental at the DC side. Shifting the high amplitude harmonics to higher frequencies resulted in an easier, more effective filter design. The value of the DC average component is directly proportional to the duty cycle k, while the number of PWMr pulses has no effect on the DC value. The highest harmonic amplitude is expected at the frequency equals to the fundamental frequency of the rectified voltage times number of pulses of PWM. The disadvantage of higher THD could be resolved by suppressing the high frequency components using low-pass filter.

6. References

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