# Feedback Control of Marangoni Convection with Magnetic Field

Norihan Md. Arifin, Haliza Rosali and Norfifah Bachok
*Department of Mathematics and Institute For Mathematical Research*
*Universiti Putra Malaysia, 43400 UPM Serdang, Selangor*
*Malaysia*

## 1. Introduction

Convection in a plane horizontal fluid layer heated from below, initially at rest and subject to an adverse temperature gradient, may be produced either by buoyancy forces or surface tension forces. These convective instability problems are known as the Rayleigh-Benard convection and Marangoni convection, respectively. The determination of the criterion for the onset of convection and the mechanism to control has been a subject of interest because of its applications in the heat and momentum transfer research. Rayleigh (1916) was the first to solve the problem of the onset of thermal convection in a horizontal layer of fluid heated from below. His linear analysis showed that Benard convection occurs when the Rayleigh number exceeds a critical value. Theoretical analysis of Marangoni convection was started with the linear analysis by Pearson (1958) who assumed an infinite fluid layer, a nondeformable case and zero gravity in the case of no-slip boundary conditions at the bottom. He showed that thermocapillary forces can cause convection when the Marangoni number exceeds a critical value in the absence of buoyancy forces.
The determination of the criterion for the onset of convection and the mechanism to control convective flow patterns is important in both technology and fundamental Science. The problem of suppressing cellular convection in the Marangoni convection problem has attracted some interest in the literature. The effects of a body force due to an externally-imposed magnetic field on the onset of convection has been studied theoretically and numerically. The effect of magnetic field on the onset of steady buoyancy-driven convection was treated by Chandrasekhar (1961) who showed that the effect of magnetic field is to increase the critical value of Rayleigh number and hence to have a stabilising effect on the layer. The effect of a magnetic field on the onset of steady buoyancy and thermocapillary-driven (Benard-Marangoni) convection in a fluid layer with a nondeformable free surface was first analyzed by Nield (1966). He found that the critical Marangoni number monotonically increased as the strength of vertical magnetic field increased. This indicates that Lorentz force suppressed Marangoni convection. Later, the effect of a magnetic field on the onset of steady Marangoni convection in a horizontal layer of fluid has been discuss in a series of paper by Wilson (1993, 1994). The influence of a uniform vertical magnetic field on

the onset of oscillatory Marangoni convection was treated by Hashim & Wilson (1999) and Hashim & Arifin (2003).
The present work attempts to delay the onset of convection by applying the control. The objective of the control is to delay the onset of convection while maintaining a state of no motion in the fluid layer. Tang and Bau (1993,1994) and Howle (1997) have shown that the critical Rayleigh number for the onset of Rayleigh-Bénard convection can be delayed. Or et al. (1999) studied theoretically the use of control strategies to stabilize long wavelength instabilities in the Marangoni-Bénard convection. Bau (1999) has shown independently how such a feedback control can delay the onset of Marangoni-Bénard convection on a linear basis with no-slip boundary conditions at the bottom. Recently, Arifin et. al. (2007) have shown that a control also can delay the onset of Marangoni-Bénard convection with free-slip boundary conditions at the bottom.
Therefore, in this paper, we use a linear controller to delay the onset of Marangoni convection in a fluid layer with magnetic field. The linear stability theory is applied and the resulting eigenvalue problem is solved numerically. The combined effect of the magnetic field and the feedback control on the onset of steady Marangoni convection are studied.

## 2. Problem Formulation

Consider a horizontal fluid layer of depth $d$ with a free upper surface heated from below and subject to a uniform vertical temperature gradient. The fluid layer is bounded below by a horizontal solid boundary at a constant temperature $T_1$ and above by a free surface at constant temperature $T_2$ which is in contact with a passive gas at constant pressure $P_0$ and constant temperature $T_\infty$ (see Figure 1)

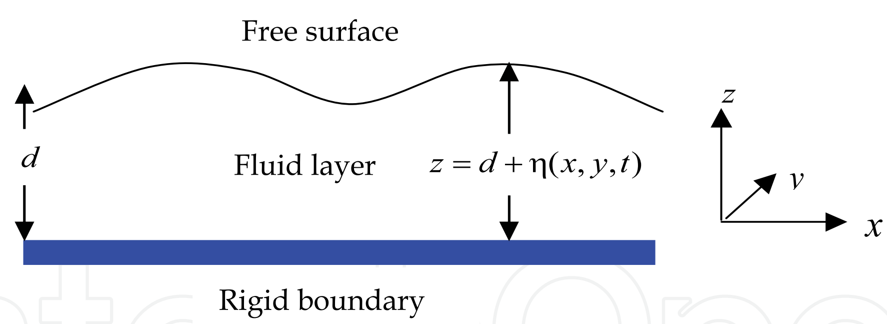


Fig. 1. Problem set up

We use Cartesian coordinates with two horizontal $x$- and $y$- axis located at the lower solid boundary and a positive $z$- axis is directed towards the free surface. The surface tension, $\tau$ is assumed to be a linear function of the temperature

$$\tau = \tau_0 - \gamma\left(T - T_0\right), \tag{1}$$

where $\tau_0$ is the value of $\tau$ at temperature $T_0$ and the constant $\gamma$ is positive for most fluids. The density of the fluid is given by

$$\rho = \rho_0[1 - \alpha(T - T_0)], \tag{2}$$

where $\alpha$ is the positive coefficient of the thermal liquid expansion and $\rho_0$ is the value of $\rho$ at the reference temperature $T_0$. Subject to the Boussinesq approximation, the governing equations for an incompressible, electrically conducting fluid in the presence of a magnetic field are expressed as follows:

Continuity equation:

$$\nabla \cdot \mathbf{U} = 0, \tag{3}$$

Momentum equation:

$$\left(\frac{\partial}{\partial \mathrm{t}} + \mathbf{U} \cdot \nabla\right)\mathbf{U} = -\frac{1}{\rho}\nabla\Pi + \nu\nabla^2\mathbf{U} + \frac{\mu}{4\pi\rho}\left(\mathbf{H} \cdot \nabla\right)\mathbf{H}, \tag{4}$$

Energy equation :

$$\left(\frac{\partial}{\partial \mathrm{t}} + \mathbf{U} \cdot \nabla\right)T = \kappa\nabla^2 T \tag{5}$$

Magnetic field equations:

$$\nabla \cdot \mathbf{H} = 0 \tag{6}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{U}.\nabla\right)\mathbf{H} = \left(\mathbf{H}.\nabla\right)\mathbf{U} + \eta\nabla^2\mathbf{H} \tag{7}$$

where $\mathbf{U}, T, \mathbf{H}, \rho, \nu, \kappa$ and $\eta$ denote the velocity , temperature, magnetic field, pressure, density, kinematic viscosity , thermal diffusivity and electrical resistivity, respectively. $\Pi = p + \mu|\mathbf{H}|^2/8\pi$ is the magnetic pressure where $p$ is the fluid pressure and $\mu$ is the magnetic permeability. When motion occurs, the upper free surface of the layer will be deformable with its position at $z = d + f(x, y, t)$. At the free surface, we have the usual kinematic condition together with the conditions of continuity for the normal and tangential stresses. The temperature obeys the Newton's law of cooling, $k\partial T/\partial \mathrm{n} = h(T - T_\infty)$, where $k$ and $h$ are the thermal conductivity of the fluid and the heat transfer coefficient between the free surface and the air, respectively, and n is the outward unit normal to the free surface. The boundary conditions at the bottom wall, $z = 0$, are no-slip and conducting to the temperature perturbations.

To simplify the analysis, it is convenient to write the governing equations and the boundary conditions in a dimensionless form. In the dimensionless formulation, scales for length, velocity, time and temperature gradient are taken to be $d, \kappa/d, d^2/\kappa$ and $\Delta T$ respectively. Furthermore, six dimensionless groups appearing in the problem are the Marangoni number $M = \gamma\Delta T d/\rho_0\kappa\nu$, the Biot number, $B_\mathrm{i} = hd/k$, the Bond number, $B_\mathrm{o} = \rho_0 g d^2/\tau_0$, the Prandtl number, $P_r = \nu/\kappa$, the Crispation number, $C_r = \rho_0\nu\kappa/\tau_0 d$ and the internal heating, $Q = qd^2/2\kappa\Delta T$.

Our control strategy basically applies a principle similar to that used by Bau (1999), which is as follows:
Assumed that the sensors and actuators are continuously distributed and that each sensor directs an actuator installed directly beneath it at the same {*x,y*} location. The sensor detects the deviation of the free surface temperature from its conductive value. The actuator modifies the heated surface temperature according to the following rule Bau (1999) :

$$T(x,y,0,t) = \frac{1+B_i}{B_i} - K\left(T\left(x,y,1,t\right) - \frac{1}{B_i}\right) \tag{8}$$

where $K$ is the scalar controller gain. Equation (8) can be rewritten more conveniently as

$$T'(x,y,0,t) = -K\left(T'(x,y,1,t)\right) \tag{9}$$

where $T'$ is the deviation of the fluid's temperature from its conductive value. The control strategy in equation (9), in which $K$ is a scalar will be used to demonstrate that our system can be controlled.

## 3. Linearized Problem

We study the linear stability of the basic state by seeking perturbed solutions for any quantity $\Phi\left(x,y,z,t\right)$ in terms of normal modes in the form

$$\Phi(x,y,z,t) = \Phi_0(x,y,z) + \phi(z)\,exp[\,i(\alpha_x x + \alpha_y y) + st\,], \tag{10}$$

where $\Phi_0$ is the value of $\Phi$ in the basic state, $a = (\alpha_x^2 + \alpha_y^2)^{1/2}$ is the total horizontal wave number of the disturbance and $s$ is a complex growth rate with a real part representing the growth rate of the instability and the imaginary part representing its frequency. At marginal stability, the growth rate $s$ of perturbation is zero and the real part of $s$, $\Re(s) > 0$ represents unstable modes while $\Re(s) < 0$ represents stable modes. Substituting equation (10) into equations (3) - (7) and neglecting terms of the second and higher orders in the perturbations we obtain the corresponding linearized equations involving only the $z$-dependent parts of the perturbations to the temperature and the $z$-components of the velocity denoted by $T$ and $w$ respectively,

$$\left[(D^2 - a^2)^2 - H^2 D^2\right] w = 0 \tag{11}$$

$$\left(D^2 - a^2\right) T + w = 0, \tag{12}$$

subject to

$$sf - w(1) = 0, \tag{13}$$

$$P_1 C_r \left[\left(D^2 - 3a^2 - H^2\right) Dw\left(1\right)\right] - a^2\left(a^2 + B_o\right) f = 0, \tag{14}$$

$$\left(D^2 + a^2\right) w(1) + a^2 M\left(T(1) - \left(1 + Q\right) f\right) = 0, \tag{15}$$

$$h_z(1) = 0\,, \tag{16}$$

$$DT(1) + B_i\left(T(1) - (1+Q)f\right) = 0, \tag{17}$$

$$w(0) = 0, \tag{18}$$

$$Dw(0) = 0, \tag{19}$$

$$h_z(0) = 0, \tag{20}$$

and

$$T(0) + KT(1) = 0. \tag{21}$$

on the lower rigid boundary $z = 0$. The operator D = $d/dz$ denotes the differentiation with respect to the vertical coordinate z. The variables $w$, $T$ and $f$ denote respectively the vertical variation of the $z$-velocity, temperature and the magnitude of the free surface deflection of the linear perturbation to the basic state with total wave number $a$ in the horizontal $x$-$y$ plane and complex growth rates.

## 4. Results and disussion

The effect of feedback control on the onset of Marangoni convection in a fluid layer with a magnetic field in the case of a deformable free surface $(C_r \neq 0)$ is investigated numerically. The marginal stability curves in the $(a, M)$ plane are obtained numerically where $M$ is a function of the parameters $a$, $B_i$, $B_o$, $C_r$ and $Q$. For a given set of parameters, the critical Marangoni number for the onset of convection defined as the minimum of the global minima of marginal curve. We denote this critical value by $M_c$ and the corresponding critical wave number, $a_c$. The problem has been solved to obtain a detail description of the marginal stability curves for the onset of Marangoni convection when the free surface is perfectly insulated ($B_i = 0$).
Figure 2 shows the numerically calculated Marangoni number, $M$ as a function of the wavenumber, $a$ for different values of $K$ in the case $C_r = 0$. From Figure 4 it is seen that the critical Marangoni number increase with an increase of $K$. Thus, the magnetic always has a stabilizing effect on the flow. In the absence of controller gain, $K$ = 0 and magnetic field, $Q$ = 0, the present calculation reproduce closely the stability curve obtained by Pearson (1958). The present calculation are also reproduced the stability curve obtained by Wilson for $K$ =0 and $Q$ =100. It can been seen that the feedback control and magnetic field suppresses Marangoni convection. The critical Marangoni number, $M_c$ increases monotonically as the controller gain, $K$ increases. In the case of non-deformable free surface $C_r = 0$, the controller can suppress the modes and maintain a no-motion state, but this situation is significantly different if the free surface is deformable, $C_r \neq 0$.

When $C_r$ becomes large the long-wavelength instability sets in as a primary one and the critical Marangoni numbers are at $a$ = 0. Figure 3 shows the critical Marangoni number at the onset of convection as a function of the wave number, $a$, for a range of values of the controller gains, $K$ when in the case of $C_r = 0.001$ and $B_o = 0.1$. At $a$ = 0, the critical Marangoni number is zero and in this case, conductive state does not exist. Figure 3 shows that the controller is not effective at the wave number $a$ = 0. Figure 4 shows the critical Marangoni number at the onset of convection as a function of the wave number, $a$ for a range of values of the controller gains, $K$ in the case $C_r = 0.0001$ and $B_o = 0.1$. In this case, the global minimum occurs at $a \neq 0$ and as the controller gain $K$ increases, the curve shifts upwards and most importantly, the controller increases the magnitude of the global minimum, thus it has a stabilizing effect.

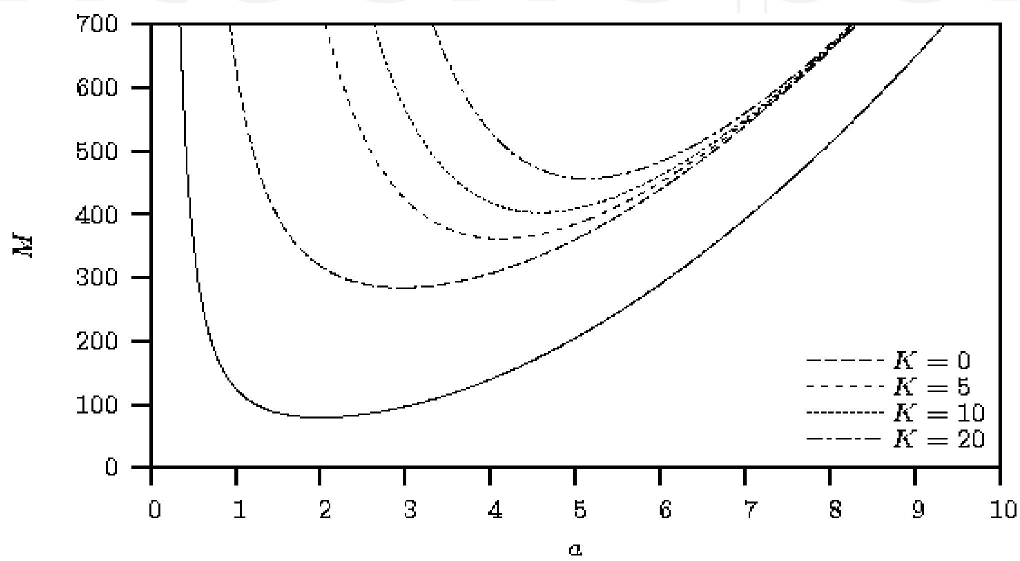


Fig. 2. Numerically-calculated marginal stability curves for $K$ = 0 and $Q$ = 0 (solid line) and for various values of $K$ (dashed line) in the case $Q$ =100 and $C_r = 0$.

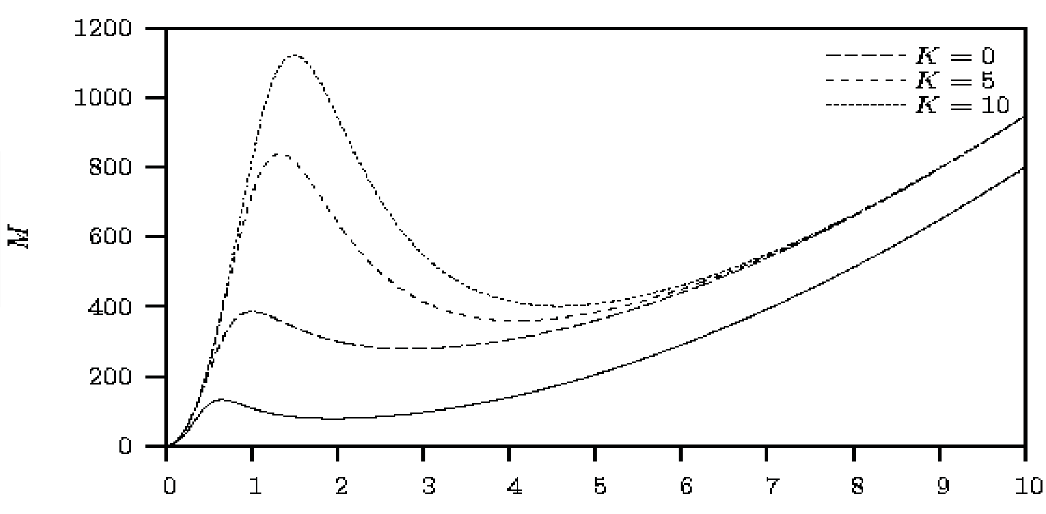


Fig. 3. Numerically-calculated marginal stability curves for $K$ = 0 and $Q$ = 0 (solid line) and for various values of $K$ (dashed line) in the case $Q$ =100, $C_r = 0.001$ and $B_o = 0$.

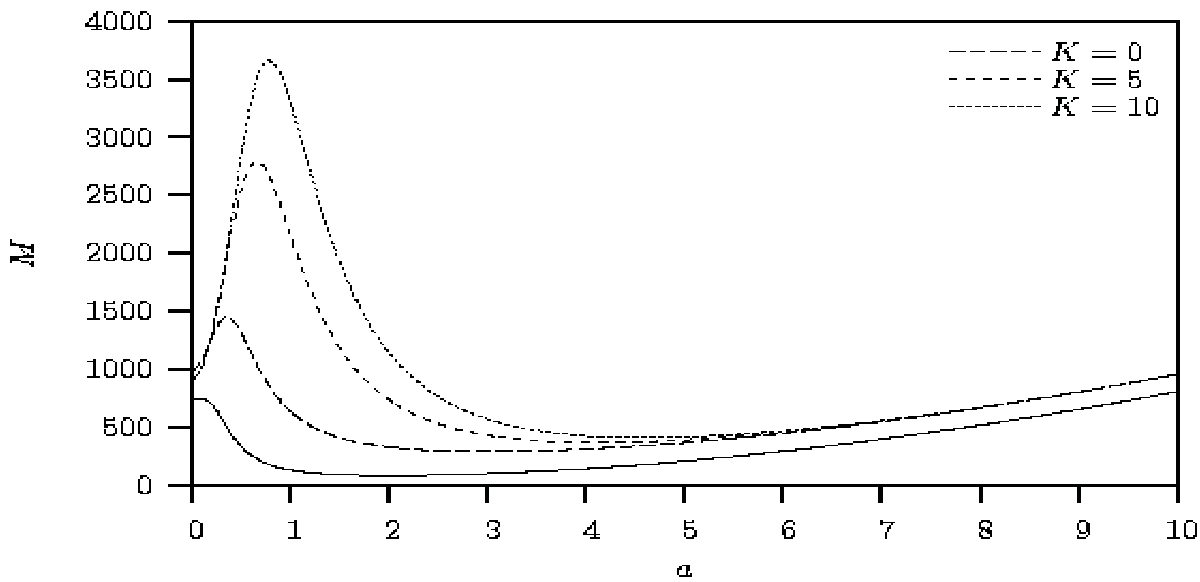


Fig. 4. Numerically-calculated marginal stability curves for $K$ = 0 and $Q$ = 0 (solid line) and for various values of $K$ (dashed line) in the case $Q$ =100, $C_r = 0.0001$ and $B_o = 0.1$ .

## 5. Conclusion

The effect of the feedback control on the onset of steady Marangoni convection instabilities in a fluid layer with magnetic field has been studied. We have shown that the feedback control and magnetic field suppresses Marangoni convection We have also shown numerically that the effect of the controller gain and magnetic field is always to stabilize the layer in the case of a nondeforming surface. However, in the case of a deforming surface, the controller gain is effective depending on the parameter $C_r$ and $B_o = 0.1$ .

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