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Robust $\mathcal{H}_\infty$ Fuzzy Control Design for Nonlinear Two-Time Scale System with Markovian Jumps based on LMI Approach

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1. Introduction

Markovian jump systems consists of two components; i.e., the state (differential equation) and the mode (Markov process). The Markovian jump system changes abruptly from one mode to another mode caused by some phenomenon such as environmental disturbances, changing subsystem interconnections and fast variations in the operating point of the system plant, etc. The switching between modes is governed by a Markov process with the discrete and finite state space. Over the past few decades, the Markovian jump systems have been extensively studied by many researchers; see (Kushner, 1967; Dynkin, 1965; Wonham, 1968; Feng et al., 1992; Souza & Fragoso, 1993; Boukas & Liu, 2001; Boukas & Yang, 1999; Rami & Ghaoui, 1995; Shi & Boukas, 1997; Benjelloun et al., 1997; Boukas et al., 2001; Dragan et al., 1999). This is due to the fact that jumping systems have been a subject of the great practical importance. For the past three decades, singularly perturbed systems have been intensively studied by many researchers; see (Dragan et al., 1999; Pan & Basar, 1993; Pan & Basar, 1994; Fridman, 2001; Shi & Dragan, 1999; Kokotovic et al., 1986).
Multiple time-scale dynamic systems also known as singularly perturbed systems normally occur due to the presence of small "parasitic" parameters, typically small time constants, masses, etc. In state space, such systems are commonly modelled using the mathematical framework of singular perturbations, with a small parameter, say ε, determining the degree of separation between the "slow" and "fast" modes of the system. However, it is necessary to note that it is possible to solve the singularly perturbed systems without separating between slow and fast mode subsystems. But the requirement is that the "parasitic" parameters must be large enough. In the case of having very small "parasitic" parameters which normally occur in the description of various physical phenomena, a popular approach adopted to handle these systems is based on the so-called reduction technique. According to this technique the fast variables are replaced by their steady states obtained

with "frozen" slow variables and controls, and the slow dynamics is approximated by the corresponding reduced order system. This time-scale is asymptotic, that is, exact in the limit, as the ratio of the speeds of the slow versus the fast dynamics tends to zero.

In the last few years, the research on singularly perturbed systems in the $\mathcal{H}_\infty$ sense has been highly recognized in control area due to the great practical importance. $\mathcal{H}_\infty$ optimal control of singularly perturbed linear systems under either perfect state measurements or imperfect state measurements has been investigated via differential game theoretic approach. Although many researchers have studied the $\mathcal{H}_\infty$ control design of linear singularly perturbed systems for many years, the $\mathcal{H}_\infty$ control design of nonlinear singularly perturbed systems remains as an open research area. This is due to, in general, nonlinear singularly perturbed systems can not be decomposed into slow and fast subsystems.

Recently, a great amount of effort has been made on the design of fuzzy $\mathcal{H}_\infty$ for a class of nonlinear systems which can be represented by a Takagi-Sugeno (TS) fuzzy model; see (Nguang & Shi, 2001; Han & Feng, 1998; Chen et al., 2000; Tanaka et al., 1996). Recent studies (Nguang & Shi, 2001; Han & Feng, 1998; Chen et al., 2000; Tanaka et al., 1996; Wang et al., 1996) show that a fuzzy model can be used to approximate global behaviors of a highly complex nonlinear system. In this fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" of these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling which uses a single model to describe the global behavior of a system, fuzzy modelling is essentially a multi-model approach in which simple submodels (linear models) are combined to describe the global behavior of the system. Employing the existing fuzzy results (Nguang & Shi, 2001; Han & Feng, 1998; Chen et al., 2000; Tanaka et al., 1996; Wang et al., 1996) on the singularly perturbed system, one ends up with a family of ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes. In general, ill-conditioned linear matrix inequalities are very difficult to solve.

What we intend to do in this chapter is to design a robust $\mathcal{H}_\infty$ fuzzy controller for a class of uncertain nonlinear two time-scale dynamic systems with Markovian jumps. First, we approximate this class of uncertain nonlinear two time-scale dynamic systems with Markovian jumps by a Takagi-Sugeno fuzzy model with Markovian jumps. Then based on an LMI approach, we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value. To alleviate the ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes, these ill-conditioned LMIs are decomposed into ε-independent LMIs and ε-dependent LMIs. The ε-independent LMIs are not ill-conditioned and the ε-dependent LMIs tend to zero when ε approaches to zero. If ε is sufficiently small, the original ill-conditioned LMIs are solvable if and only if the ε-independent LMIs are solvable. The proposed approach does not involve the separation of states into slow and fast ones, and it can be applied not only to standard, but also to nonstandard singularly perturbed systems.

This chapter is organized as follows. In Section 2, system descriptions and definition are presented. In Section 3, based on an LMI approach, we develop a technique for designing a robust $\mathcal{H}_\infty$ fuzzy controller such that the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value for the system described in

Section 2. The validity of this approach is demonstrated by an example from a literature in Section 4. Finally, conclusions are given in Section 5.

2. System Descriptions and Definitions

The class of uncertain nonlinear two time-scale dynamic systems with Markovian jumps under consideration is described by the following TS fuzzy model with Markovian jumps:

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\upsilon(t))\Big[\big[A_i(\eta(t)) + \Delta A_i(\eta(t))\big]x(t) \\ &+ \big[B_{1_i}(\eta(t)) + \Delta B_{1_i}(\eta(t))\big]\omega(t) \\ &+ \big[B_{2_i}(\eta(t)) + \Delta B_{2_i}(\eta(t))\big]u(t)\Big], x(0) = 0, \\ z(t) &= \sum_{i=1}^{r} \mu_i(\upsilon(t))\Big[\big[C_{1_i}(\eta(t)) + \Delta C_{1_i}(\eta(t))\big]x(t) \\ &+ \big[D_{12_i}(\eta(t)) + \Delta D_{12_i}(\eta(t))\big]u(t)\Big] \end{aligned} \tag{1}$$

where $E_\varepsilon = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}, \varepsilon > 0$ is the singular perturbation parameter, $\upsilon(t) = [\upsilon_1(t) \cdots \upsilon_\vartheta(t)]$ is the premise variable that may depend on states in many cases, $\mu_i(\upsilon(t))$denote the normalized time varying fuzzy weighting functions for each rule, ϑ is the number of fuzzy sets, $x(t) \in \Re^n$ is the state vector $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, $z(t) \in \Re^s$ is the controlled output, the matrix functions $A_i(\eta(t))$, $B_{1_i}(\eta(t))$, $B_{2_i}(\eta(t))$, $C_{1_i}(\eta(t))$, $D_{12_i}(\eta(t))$, $\Delta A_i(\eta(t))$, $\Delta B_{1_i}(\eta(t))$, $\Delta B_{2_i}(\eta(t))$, $\Delta C_{1_i}(\eta(t))$, and $\Delta D_{12_i}(\eta(t))$, are of appropriate dimensions. $\{(\eta(t))\}$ is a continuous-time discrete-state Markov process taking values in a finite set S = {1,2,⋯, s} with transition probability matrix $Pr \triangleq \{P_{\iota k}(t)\}$ given by

$$\begin{aligned} P_{\iota k}(t) &= Pr\big(\eta(t+\Delta) = k \,\big|\, \eta(t) = \iota\big) \\ &= \begin{cases} \lambda_{\iota k}\Delta + O(\Delta) & if\ \iota \neq k \\ 1 + \lambda_{\iota\iota}\Delta + O(\Delta) & if\ \iota = k \end{cases} \end{aligned} \tag{2}$$

where Δ > 0, and $\lim_{\Delta \to 0} \frac{O(\Delta)}{\Delta} = 0$. Here $\lambda_{\iota k} \geq 0$ is the transition rate from mode ι (system operating mode) to mode $k(\iota \neq k)$, and

$$\lambda_{\iota\iota} = -\sum_{k=1, k\neq\iota}^{s} \lambda_{\iota k}. \tag{3}$$

For the convenience of notations, we let $\mu_i \triangleq \mu_i(\upsilon(t)), \eta = \eta(t)$, and any matrix $M(\mu,\iota) \triangleq M(\mu, \eta = \iota)$. The matrix functions $\Delta A_i(\eta)$, $\Delta B_{1_i}(\eta)$, $\Delta B_{2_i}(\eta)$, $\Delta C_{1_i}(\eta)$, and $\Delta D_{12_i}(\eta)$,

re-present the time-varying uncertainties in the system and satisfy the following assumption.

Assumption 1:

$$\Delta A_i(\eta) = F(x(t),\eta,t)H_{1_i}(\eta),$$
$$\Delta B_{1_i}(\eta) = F(x(t),\eta,t)H_{2_i}(\eta),$$
$$\Delta B_{2_i}(\eta) = F(x(t),\eta,t)H_{3_i}(\eta),$$
$$\Delta C_{1_i}(\eta) = F(x(t),\eta,t)H_{4_i}(\eta),$$
$$\text{and } \Delta D_{12_i}(\eta) = F(x(t),\eta,t)H_{5_i}(\eta),$$

where $H_{j_i}(\eta), j = 1,2,\cdots,5$ are known matrices which characterize the structure of the uncertainties. Furthermore, there exists a positive function $\rho(\eta)$ such that the following inequality holds:

$$\left\| F(x(t),\eta,t) \right\| \le \rho(\eta). \tag{4}$$

We recall the following definition.

Definition 1: Suppose γ is a given positive number. A system of the form (1) is said to have the $\mathcal{L}_2$-gain less than or equal to γ if

$$\mathrm{E}\left[\int_0^{T_f} \left\{ z^T(t)z(t) - \gamma^2 w^T(t)w(t) \right\} dt \right] \le 0, x(0) = 0 \tag{5}$$

where $\mathrm{E}[\cdot]$ stands for the mathematical expectation, for all T_f and all $w(t) \in \mathcal{L}_2[0, T_f]$.

Note that for the symmetric block matrices, we use (∗) as an ellipsis for terms that are induced by symmetry.

3. Robust $\mathcal{H}_\infty$ Fuzzy Control Design

This section provides the LMI-based solutions to the problem of designing a robust $\mathcal{H}_\infty$ fuzzy controller that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value.

First, we consider the following $H\infty$ fuzzy state feedback which is inferred as the weighted average of the local models of the form:

$$u(t) = \sum_{j=1}^{r} \mu_j K_j(\iota) x(t). \tag{6}$$

Then, we describe the problem under our study as follows.

***Problem Formulation*:** Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, design a robust $\mathcal{H}_\infty$ fuzzy state-feedback controller of the form (6) such that the inequality (5) holds.

Before presenting our first main result, the following lemma is needed.

Lemma 1: Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, for $\iota = 1, 2, \cdots, s$, if there exist matrices $P_\varepsilon(\iota) = P_\varepsilon^T(\iota)$, positive constants $\delta(\iota)$ and matrices $Y_j(\iota)$, $j = 1, 2, \cdots, r$ such that the following ε-dependent linear matrix inequalities hold:

$$P_\varepsilon(\iota) > 0 \tag{7}$$

$$\psi_{ii}(\iota,\varepsilon) < 0, \quad i = 1,2,\cdots,r \tag{8}$$

$$\psi_{ij}(\iota,\varepsilon) + \psi_{ji}(\iota,\varepsilon) < 0, \quad i < j \le r \tag{9}$$

where

$$\Psi_{ij}(\iota,\varepsilon) = \begin{pmatrix} \Phi_{ij}(\iota,\varepsilon) & (*)^T & (*)^T & (*)^T \\ \mathrm{R}(\iota)B_{1_i}^T(\iota) & -\gamma\mathrm{R}(\iota) & (*)^T & (*)^T \\ \varphi_{ij}(\iota,\varepsilon) & 0 & -\gamma\mathrm{R}(\iota) & (*)^T \\ \mathrm{Z}^T(\iota,\varepsilon) & 0 & 0 & -\mathrm{P}(\iota,\varepsilon) \end{pmatrix} \tag{10}$$

$$\Phi_{ij}(\iota,\varepsilon) = A_i(\iota)E_\varepsilon^{-1}P_\varepsilon(\iota) + E_\varepsilon^{-1}P_\varepsilon(\iota)A_i^T(\iota) + B_{2_i}(\iota)Y_j(\iota) + Y_j^T(\iota)B_{2_i}^T(\iota) + \lambda_{\iota\iota}E_\varepsilon^{-1}P_\varepsilon(\iota), \tag{11}$$

$$\varphi_{ij}(\iota,\varepsilon) = \widetilde{C}_{1_i}(\iota)E_\varepsilon^{-1}P_\varepsilon(\iota) + \widetilde{D}_{12_i}(\iota)Y_j(\iota), \tag{12}$$

$$\mathrm{R}(\iota) = diag\{\delta(\iota)I, I, \delta(\iota)I, I\}, \tag{13}$$

$$\mathrm{Z}(\iota,\varepsilon) = \left(\sqrt{\lambda_{\iota 1}}E_\varepsilon^{-1}P_\varepsilon(\iota)\cdots\sqrt{\lambda_{\iota(\iota-1)}}E_\varepsilon^{-1}P_\varepsilon(\iota)\ \sqrt{\lambda_{\iota(\iota+1)}}E_\varepsilon^{-1}P_\varepsilon(\iota)\cdots\sqrt{\lambda_{\iota s}}E_\varepsilon^{-1}P_\varepsilon(\iota)\right), \tag{14}$$

$$\mathrm{P}(\iota,\varepsilon) = diag\left\{E_\varepsilon^{-1}P_\varepsilon(1),\cdots,E_\varepsilon^{-1}P_\varepsilon(\iota-1),E_\varepsilon^{-1}P_\varepsilon(\iota+1),\cdots,E_\varepsilon^{-1}P_\varepsilon(\mathrm{s}),\right\}, \tag{15}$$

with

$$\widetilde{B}_{1_i}(\iota) = \begin{bmatrix} I & I & I & B_{1_i}(\iota) \end{bmatrix}$$

$$\widetilde{C}_{1_i}(\iota) = \begin{bmatrix} \gamma\rho(\iota)H_{1_i}^T(\iota) & \sqrt{2}\aleph(\iota)\rho(\iota)H_{4_i}^T(\iota) & 0 & \sqrt{2}\aleph(\iota)C_{1_i}^T(\iota) \end{bmatrix}^T$$

$$\widetilde{D}_{12_i}(\iota) = \begin{bmatrix} 0 & \sqrt{2}\aleph(\iota)\rho(\iota)H_{5_i}^T(\iota) & \gamma\rho(\iota)H_{3_i}^T(\iota) & \sqrt{2}\aleph(\iota)D_{12_i}^T(\iota) \end{bmatrix}^T$$

$$\aleph(\iota) = \left(I + \rho^2(\iota)\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\left\|H_{2_i}^T(\iota)H_{2_j}(\iota)\right\|\right]\right)^{\frac{1}{2}}$$

then the inequality (5) holds. Furthermore, a suitable choice of the fuzzy controller is

$$u(t) = \sum_{j=1}^{r}\mu_j K_{\varepsilon_j}(\iota)x(t) \tag{16}$$

where

$$K_{\varepsilon_j}(\iota) = Y_j(\iota)\left(P_\varepsilon(\iota)\right)^{-1} E_\varepsilon \cdot \tag{17}$$

Proof: The closed-loop state space form of the fuzzy system model (1) with the controller (16) is given by

$$\begin{aligned} E_\varepsilon \dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \left(\left[A_i(\iota) + B_{2_i}(\iota) K_{\varepsilon j}(\iota) \right] x(t) \right. \\ &+ \left[\Delta A_i(\iota) + \Delta B_{2_i}(\iota) K_{\varepsilon j}(\iota) \right] x(t) \\ &\left. + \left[B_{1_i}(\iota) + \Delta B_{1_i}(\iota) \right] w(t) \right), x(0) = 0, \end{aligned} \tag{18}$$

or in a more compact form

$$E_\varepsilon \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \left(\left[A_i(\iota) + B_{2_i}(\iota) K_{\varepsilon j}(\iota) \right] x(t) + \tilde{B}_{1_i}(\iota) R(\iota) \tilde{w}(t) \right), x(0) = 0 \tag{19}$$

where

$$\tilde{B}_{1_i}(\iota) = \begin{bmatrix} I & I & I & B_{1_i}(\iota) \end{bmatrix} \tag{20}$$

$$\tilde{w}(t) = R^{-1}(\iota) \begin{bmatrix} F(x(t),\iota,t) H_{1_i}(\iota) x(t) \\ F(x(t),\iota,t) H_{2_i}(\iota) w(t) \\ F(x(t),\iota,t) H_{3_i}(\iota) K_{\varepsilon j}(\iota) x(t) \\ w(t) \end{bmatrix} \tag{21}$$

Consider a Lyapunov functional candidate as follows:

$$V(x(t),\iota,\varepsilon) = \gamma x^T(t) Q_\varepsilon(\iota) x(t), \quad \forall \iota \in \mathrm{S} \tag{22}$$

Note that $Q_\varepsilon(\iota)$ is constant for each ι. For this choice, we have $V(0, \iota_0, \varepsilon) = 0$ and $V(x(t), \iota, \varepsilon) \to \infty$ only when $\|x(t)\| \to \infty$.

Now let consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{(x(t), \iota, \varepsilon), t \geq 0\}$, which is the stochastic analog of the deterministic derivative. $\{(x(t), \iota, \varepsilon), t \geq 0\}$ is a Markov process with infinitesimal operator given by (Souza & Fragoso, 1993),

$$\begin{aligned} \tilde{\Delta} V(x(t),\iota,\varepsilon) &= \gamma\, \dot{x}^T(t) Q_\varepsilon(\iota) x(t) + \gamma x^T(t) Q_\varepsilon(\iota) \dot{x}(t) + \gamma x^T(t) \sum_{k=1}^{s} \lambda_{\iota k} Q_\varepsilon(k) x(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i \mu_j \left(\gamma x^T(t) Q_\varepsilon(\iota) \left[\left(A_i(\iota) + B_{2_i}(\iota) K_{\varepsilon j}(\iota) \right) \right] x(t) \right. \\ &+ \gamma x^T(t) \left[A_i(\iota) + B_{2_i}(\iota) K_{\varepsilon j}(\iota) \right]^T Q_\varepsilon(\iota) x(t) \\ &+ \gamma x^T(t) Q_\varepsilon(\iota) \tilde{B}_{1_i}(\iota) \mathrm{R}(\iota) \tilde{w}(t) \\ &\left. + \gamma \tilde{w}^T(t) \mathrm{R}(\iota) \tilde{B}_{1_i}^T(\iota) Q_\varepsilon(\iota) x(t) \right) \end{aligned}$$

$$+\gamma x^T(t)\sum_{k=1}^{s}\lambda_{tk}Q_\varepsilon(k)x(t)\Bigg)\cdot \tag{23}$$

Adding and subtracting $-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n \left[\tilde{w}^T(t)\mathrm{R}(\iota)\tilde{w}(t)\right]$ to and from (23), we get

$$\begin{aligned}\widetilde{\Delta}V(x(t),\iota,\varepsilon) &= -\aleph^2(\iota)z^T(t)z(t)+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\mathrm{R}(\iota)\tilde{w}(t)\right]\\ &+\aleph^2(\iota)z^T(t)z(t)+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\times\\ &\begin{pmatrix}\begin{pmatrix}\left[A_i(\iota)+B_{2_i}(\iota)K_{\varepsilon j}(\iota)\right]^T Q_\varepsilon(\iota)\\ Q_\varepsilon(\iota)\left[A_i(\iota)+B_{2_i}(\iota)K_{\varepsilon j}(\iota)\right]+\sum_{k=1}^{s}\lambda_{tk}Q_\varepsilon(k)\end{pmatrix} & (*)^T\\ \mathrm{R}(\iota)\widetilde{B}_{1_i}^T(\iota)Q_\varepsilon(\iota) & -\gamma\mathrm{R}(\iota)\end{pmatrix}\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}\cdot\end{aligned} \tag{24}$$

Now let us consider the following terms:

$$\begin{aligned}&\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\mathrm{R}(\iota)\tilde{w}(t)\right]\\ &\le\frac{\rho^2(\iota)\gamma^2}{\delta(\iota)}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n x^T(t)\times\\ &\left\{H_{1_i}^T(\iota)H_{1_m}(\iota)+K_{\varepsilon j}^T(\iota)H_{3_i}^T(\iota)H_{3_m}(\iota)K_{\varepsilon n}(\iota)\right\}x(t)+\aleph^2(\iota)\gamma^2 w^T(t)w(t)\end{aligned} \tag{25}$$

and

$$\begin{aligned}\aleph^2(\iota)z^T(t)z(t)\le 2\aleph^2(\iota)&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n x^T(t)\left\{\left[C_{1_i}(\iota)+D_{12_i}(\iota)K_{\varepsilon j}(\iota)\right]^T\times\right.\\ &\left[C_{1_m}(\iota)+D_{12_m}(\iota)K_{\varepsilon n}(\iota)\right]+\rho^2(\iota)\left[H_{4_i}(\iota)+H_{5_i}(\iota)K_{\varepsilon j}(\iota)\right]^T\times\\ &\left.\left[H_{4_m}(\iota)+H_{5_m}(\iota)K_{\varepsilon n}(\iota)\right]\right\}x(t)\end{aligned} \tag{26}$$

where $\aleph(\iota)=\left(1+\rho^2(\iota)\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\left\|H_{2_i}^T(\iota)H_{2_j}(\iota)\right\|\right]\right)^{\frac{1}{2}}\cdot$ Hence,

$$\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\mathrm{R}(t)\tilde{w}(t)\right]+\aleph^2(t)z^T(t)z(t)$$

$$\le\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\Big(x^T(t)\left[\tilde{C}_{1_i}(t)+\tilde{D}_{12_i}(t)K_{\varepsilon j}(t)\right]^T\mathrm{R}^{-1}(t)\times$$

$$\left[\tilde{C}_{1_m}(t)+\tilde{D}_{12_m}(t)K_{\varepsilon n}(t)\right]x(t)+\aleph^2(t)\gamma^2w^T(t)w(t) \tag{27}$$

where

$$\tilde{C}_{1_i}(t)=\left[\gamma\rho(t)H_{1_i}^T(t)\quad\sqrt{2}\aleph(t)\rho(t)H_{4_i}^T(t)\quad 0\quad\sqrt{2}\aleph(t)C_{1_i}^T(t)\right]^T$$

$$\tilde{D}_{12_i}(t)=\left[0\quad\sqrt{2}\aleph(t)\rho(t)H_{5_i}^T(t)\quad\gamma\rho(t)H_{3_i}^T(t)\quad\sqrt{2}\aleph(t)D_{12_i}^T(t)\right]^T$$

Substituting (27) into (24), we have

$$\tilde{\Delta}V\left(x(t),\iota,\varepsilon\right)\le-\aleph^2(t)z^T(t)z(t)+\gamma^2\aleph^2(t)w^T(t)w(t)$$

$$+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\times\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\Phi_{ijmn}(\iota,\varepsilon)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}. \tag{28}$$

where

$$\Phi_{ijmn}(\iota,\varepsilon)=\begin{pmatrix}\begin{pmatrix}\left(A_i(t)+B_{2_i}(t)K_{\varepsilon j}(t)\right)^TQ_\varepsilon(t)\\ +Q_\varepsilon(t)\left[A_i(t)+B_{2_i}(t)K_{\varepsilon j}(t)\right]+\frac{1}{\gamma}\left[\tilde{C}_{1_i}(t)+\tilde{D}_{12_i}(t)K_{\varepsilon j}(t)\right]^T\times\\ \mathrm{R}^{-1}(t)\left[\tilde{C}_{1_m}(t)+\tilde{D}_{12_m}(t)K_{\varepsilon n}(t)\right]+\sum_{k=1}^{s}\lambda_{tk}Q_\varepsilon(k)\end{pmatrix} & (*)^T\\ \mathrm{R}(t)\tilde{B}_{1_i}^T(t)Q_\varepsilon(t) & -\gamma\mathrm{R}(t)\end{pmatrix}$$

Using the fact

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_nM_{ij}^T(t)N_{mn}(t)\le\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\left[M_{ij}^T(t)M_{ij}(t)+N_{ij}(t)N_{ij}^T(t)\right],$$

we can rewrite (28) as follows:

$$\begin{aligned}\tilde{\Delta}V(x(t),\iota,\varepsilon) &\le -\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\\ &\quad+\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\Phi_{ij}(\iota,\varepsilon)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}\\ &=-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\\ &\quad+\gamma\sum_{i=1}^{r}\mu_i^2\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\Phi_{ii}(\iota,\varepsilon)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}\\ &\quad+\gamma\sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\mu_j\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\left(\Phi_{ij}(\iota,\varepsilon)+\Phi_{ij}(\iota,\varepsilon)\right)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}\end{aligned} \tag{29}$$

where

$$\Phi_{ij}(\iota,\varepsilon)=\begin{pmatrix}\begin{pmatrix}\left(A_i(\iota)+B_{2_i}(\iota)K_{\varepsilon j}(\iota)\right)^T Q_\varepsilon(\iota)\\ +Q_\varepsilon(\iota)\left(A_i(\iota)+B_{2_i}(\iota)K_{\varepsilon j}(\iota)\right)+\frac{1}{\gamma}\left[\tilde{C}_{1_i}(\iota)+\tilde{D}_{12_i}(\iota)K_{\varepsilon j}(\iota)\right]^T\times\\ \mathrm{R}^{-1}(\iota)\left[\tilde{C}_{1_m}(\iota)+\tilde{D}_{12_m}(\iota)K_{\varepsilon n}(\iota)\right]+\sum_{k=1}^{s}\lambda_{\iota k}Q_\varepsilon(k)\end{pmatrix} & (*)^T\\ \mathrm{R}(\iota)\tilde{B}_{1_i}^T(\iota)Q_\varepsilon(\iota) & -\gamma\mathrm{R}(\iota)\end{pmatrix} \tag{30}$$

Pre and post multiplying (30) by

$$\Xi(\iota)=\begin{pmatrix}P_\varepsilon(\iota) & 0\\ 0 & I\end{pmatrix},$$

with $P_\varepsilon(\iota)=Q_\varepsilon^{-1}(\iota)$ we obtain

$$\Xi(\iota)\Phi_{ij}(\iota,\varepsilon)\Xi(\iota)=\begin{pmatrix}\begin{pmatrix}P_\varepsilon(\iota)A_i^T(\iota)+Y_j^T(\iota)B_{2_i}^T(\iota)+A_i(\iota)P_\varepsilon(\iota)+B_{2_i}(\iota)Y_j(\iota)\\ +\frac{1}{\gamma}\left[\tilde{C}_{1_i}(\iota)P_\varepsilon(\iota)+\tilde{D}_{12_i}(\iota)Y_j(\iota)\right]^T\times\\ \mathrm{R}^{-1}(\iota)\left[\tilde{C}_{1_m}(\iota)P_\varepsilon(\iota)+\tilde{D}_{12_m}(\iota)Y_j(\iota)\right]\\ +\sum_{k=1}^{s}\lambda_{\iota k}P_\varepsilon(\iota)P_\varepsilon^{-1}(k)P_\varepsilon(\iota)\end{pmatrix} & (*)^T\\ \mathrm{R}(\iota)\tilde{B}_{1_i}^T(\iota) & -\gamma\mathrm{R}(\iota)\end{pmatrix} \tag{31}$$

Note that (31) is the Schur complement of $\Psi_{ij}(\iota, \varepsilon)$ defined in (10). Using (8)-(9), we learn that

$$\Phi_{ii}(\iota,\varepsilon) < 0, \quad i=1,2,\cdots,r \tag{32}$$

$$\Phi_{ij}(\iota,\varepsilon)+\Phi_{ji}(\iota,\varepsilon) < 0, \quad i<j\le r\cdot \tag{33}$$

Following from (29), (32) and (33) , we know that

$$\tilde{\Delta}V(x(t),\iota,\varepsilon)<-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\cdot \tag{34}$$

Applying the operator $E\left[\int_0^{T_f}(\cdot)dt\right]$ on both sides of (34), we obtain

$$E\left[\int_0^{T_f}\tilde{\Delta}V(x(t),\iota,\varepsilon)dt\right]<E\left[\int_0^{T_f}\left(-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\right)dt\right]\cdot \tag{35}$$

From the Dynkin's formula [2], it follows that

$$E\left[\int_0^{T_f}\tilde{\Delta}V(x(t),\iota,\varepsilon)dt\right]=E\left[V(x(T_f),\iota(T_f),\varepsilon)\right]-E\left[V(x(0),\iota(0),\varepsilon)\right]\cdot \tag{36}$$

Substitute (36) into (35) yields

$$0<E\left[\int_0^{T_f}\left(-\aleph^2(\iota)z^T(t)z(t)+\gamma^2\aleph^2(\iota)w^T(t)w(t)\right)dt\right]$$
$$-E\left[V(x(T_f),\iota(T_f),\varepsilon)\right]+E\left[V(x(0),\iota(0),\varepsilon)\right]\cdot$$

Using (34) and the fact that we have

$$E\left[\int_0^{T_f}\left\{z^T(t)z(t)-\gamma^2w^T(t)w(t)\right\}dt\right]<0\cdot \tag{37}$$

Hence, the inequality (5) holds. This completes the proof of Lemma 1. ■

Remark 1: The linear matrix inequalities given in Lemma 1 becomes ill-conditioned when ε is sufficiently small, which is always the case for the uncertain nonlinear two time-scale dynamic systems. In general, these ill-conditioned linear matrix inequalities are very difficult to solve. Thus, to alleviate these ill-conditioned linear matrix inequalities, we have the following theorem which does not depend on ε. □

Now we are in the position to present our first result.

Theorem 1: Consider the system (1). Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, for $\iota = 1, 2, \cdots, s$, if there exist matrices $P(\iota)$, positive constants $\delta(\iota)$ and matrices $Y_j(\iota)$, $j = 1, 2, \cdots, r$ such that the following ε-independent linear matrix inequalities hold:

$$EP(\iota)+P(\iota)D > 0 \tag{38}$$

$$\Psi_{ii}(\iota) < 0, \quad i=1,2,\cdots,r \tag{39}$$

$$\Psi_{ij}(\iota)+\Psi_{ji}(\iota) < 0, \quad i<j\le r \tag{40}$$

where $EP(\iota)=P^T(\iota)E$, $P(\iota)D=DP^T(\iota)$, $E=\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $D=\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$,

$$\Psi_{ij}(\iota)=\begin{pmatrix} \Phi_{ij}(\iota) & (*)^T & (*)^T & (*)^T \\ R(\iota)B_{1_i}^T(\iota) & -\gamma\mathrm{R}(\iota) & (*)^T & (*)^T \\ \varphi_{ij}(\iota) & 0 & -\gamma\mathrm{R}(\iota) & (*)^T \\ \mathrm{Z}^T(\iota) & 0 & 0 & -\mathrm{P}(\iota) \end{pmatrix} \tag{41}$$

$$\Phi_{ij}(\iota)=A_i(\iota)P(\iota)+P^T(\iota)A_i^T(\iota)+B_{2_i}(\iota)Y_j(\iota)+Y_j^T(\iota)B_{2_i}^T(\iota)+\lambda_{\iota\iota}\widetilde{\widetilde{P}}(\iota), \tag{42}$$

$$\varphi_{ij}(\iota)=\widetilde{C}_{1_i}(\iota)P(\iota)+\widetilde{D}_{12_i}(\iota)Y_j(\iota), \tag{43}$$

$$\mathrm{R}(\iota)=diag\{\delta(\iota)I,I,\delta(\iota)I,I\}, \tag{44}$$

$$\mathrm{Z}(\iota)=\left(\sqrt{\lambda_{\iota 1}}\widetilde{\widetilde{P}}(\iota)\cdots\sqrt{\lambda_{\iota(\iota-1)}}\widetilde{\widetilde{P}}(\iota)\ \sqrt{\lambda_{\iota(\iota+1)}}\widetilde{\widetilde{P}}(\iota)\cdots\sqrt{\lambda_{\iota 1}}\widetilde{\widetilde{P}}(\iota)\right), \tag{45}$$

$$\mathrm{P}(\iota)=diag\left\{\widetilde{\widetilde{P}}(1),\cdots,\widetilde{\widetilde{P}}(\iota-1),\widetilde{\widetilde{P}}(\iota+1),\cdots,\widetilde{\widetilde{P}}(s)\right\}, \tag{46}$$

$$\widetilde{\widetilde{P}}(\iota)=\frac{P(\iota)+P^T(\iota)}{2}, \tag{47}$$

with

$$\tilde{B}_{1_i}(\iota)=\begin{bmatrix} I & I & I & B_{1_i}(\iota) \end{bmatrix}$$

$$\tilde{C}_{1_i}(\iota)=\begin{bmatrix} \gamma\rho(\iota)H_{1_i}^T(\iota) & \sqrt{2}\aleph(\iota)\rho(\iota)H_{4_i}^T(\iota) & 0 & \sqrt{2}\aleph(\iota)C_{1_i}^T(\iota) \end{bmatrix}^T$$

$$\tilde{D}_{12_i}(\iota)=\begin{bmatrix} 0 & \sqrt{2}\aleph(\iota)\rho(\iota)H_{5_i}^T(\iota) & \gamma\rho(\iota)H_{3_i}^T(\iota) & \sqrt{2}\aleph(\iota)D_{12_i}^T(\iota) \end{bmatrix}^T$$

$$\aleph(\iota)=\left(I+\rho^2(\iota)\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\left\|H_{2_i}^T(\iota)H_{2_j}(\iota)\right\|\right]\right)^{\frac{1}{2}}$$

then there exists a sufficiently small $\hat{\varepsilon}>0$ such that the inequality (5) holds for $\varepsilon\in(0,\hat{\varepsilon}]$. Furthermore, a suitable choice of the fuzzy controller is

$$u(\iota)=\sum_{j=1}^{r}\mu_j K_j(\iota)x(t)\cdot \tag{48}$$

where

$$K_j(\iota) = Y_j(\iota)\left(P(\iota)\right)^{-1}. \tag{49}$$

Proof: Suppose there exists a matrix $P(\iota)$ such that the inequality (38) holds, then $P(\iota)$ is of the following form:

$$P(\iota) = \begin{pmatrix} P_1(\iota) & 0 \\ P_2^T(\iota) & P_3(\iota) \end{pmatrix} \tag{50}$$

with $P_1(\iota) = P_1^T(\iota) > 0$ and $P_3(\iota) = P_3^T(\iota) > 0$. Let

$$P_\varepsilon(\iota) = E_\varepsilon\left(P(\iota) + \varepsilon\tilde{P}(\iota)\right) \tag{51}$$

with

$$\tilde{P}(\iota) = \begin{pmatrix} 0 & P_2(\iota) \\ 0 & 0 \end{pmatrix}. \tag{52}$$

Substituting (50) and (52) into (51), we have

$$P_\varepsilon(\iota) = \begin{pmatrix} P_1(\iota) & \varepsilon P_2(\iota) \\ \varepsilon P_2^T(\iota) & \varepsilon P_3(\iota) \end{pmatrix}. \tag{53}$$

Clearly, $P_\varepsilon(\iota) = P_\varepsilon^T(\iota)$, and there exists a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}], P_\varepsilon(\iota) > 0$.

Using the matrix inversion lemma, we learn that

$$P_\varepsilon^{-1}(\iota) = \left[P^{-1}(\iota) + \varepsilon M_\varepsilon(\iota)\right]E_\varepsilon^{-1} \tag{54}$$

where $M_\varepsilon(\iota) = -P^{-1}(\iota)\tilde{P}(\iota)\left(I + \varepsilon P^{-1}(\iota)\tilde{P}(\iota)\right)^{-1} P^{-1}(\iota)$. Substituting (51) and (54) into (10), we obtain

$$\Psi_{ij}(\iota) + \psi_{ij}(\iota) \tag{55}$$

where the ε-independent linear matrix $\Psi_{ij}(\iota)$ is defined in (41) and the ε-dependent linear matrix is

$$\Psi_{ij}(\iota)=\varepsilon\begin{pmatrix}\begin{pmatrix}A_i(\iota)\widetilde{P}(\iota)+\widetilde{P}^T(\iota)A_i^T(\iota)\\+B_{2_i}(\iota)Y_{\varepsilon_j}(\iota)+Y_{\varepsilon_j}^T(\iota)B_{2_i}^T(\iota)\\+\lambda_{\iota\iota}\widetilde{\widetilde{P}}(\iota)\end{pmatrix} & (*)^T & (*)^T & (*)^T\\ 0 & 0 & (*)^T & (*)^T\\ \left(\widetilde{C}_{1_i}(\iota)\widetilde{P}(\iota)+\widetilde{D}_{12_i}(\iota)Y_{\varepsilon_j}(\iota)\right)^T & 0 & 0 & (*)^T\\ Z_\varepsilon^T(\iota) & 0 & 0 & -\mathrm{P}_\varepsilon(\iota)\end{pmatrix} \tag{56}$$

with $Z_\varepsilon(\iota)=\left(\sqrt{\lambda_{\iota 1}}\hat{\widetilde{P}}(\iota)\cdots\sqrt{\lambda_{\iota(\iota-1)}}\hat{\widetilde{P}}(\iota)\ \sqrt{\lambda_{\iota(\iota+1)}}\hat{\widetilde{P}}(\iota)\cdots\sqrt{\lambda_{\iota 1}}\hat{\widetilde{\widetilde{P}}}(\iota)\right)$, $\mathrm{P}_\varepsilon(\iota)=diag\left\{\hat{\widetilde{P}}(1),\ \cdots,\right.$ $\left.\hat{\widetilde{P}}(\iota-1),\hat{\widetilde{P}}(\iota+1),\ \cdots,\hat{\widetilde{P}}(s)\right\}$, $\hat{\widetilde{P}}(\iota)=\dfrac{\tilde{P}(\iota)+\tilde{P}^T(\iota)}{2}$ and $Y_{\varepsilon_j}(\iota)=K_j(\iota)M_\varepsilon^{-1}(\iota)$. Note that the ε-dependent linear matrix tends to zero when ε approaches zero.

Employing (38)-(40) and knowing the fact that for any given negative definite matrix W, there exists an $\varepsilon > 0$ such that $W + \varepsilon I < 0$, one can show that there exists a sufficiently small $\hat{\varepsilon}>0$ such that for $\varepsilon\in(0,\hat{\varepsilon}]$, (8) and (9) hold. Since (7)-(9) hold, using Lemma 1, the inequality (5) holds for $\varepsilon\in(0,\hat{\varepsilon}]$. ■

4. Illustrative Example

Consider a modified series dc motor model based on (Mehta & Chiasson, 1998) as shown in Figure 1 which is governed by the following difference equations:

$$\begin{aligned}J\frac{d\tilde{\omega}(t)}{dt}&=K_mL_f\tilde{i}^2(t)-(D+\Delta D)\tilde{\omega}(t)\\ L\frac{d\tilde{i}(t)}{dt}&=-R\tilde{i}(t)-K_mL_f\tilde{i}(t)\tilde{\omega}(t)+\tilde{V}(t)\end{aligned} \tag{57}$$

where $\tilde{\omega}(t)=\omega(t)-\omega_{ref}(t)$ is the deviation of the actual angular velocity from the desired angular velocity, $\tilde{i}(t)=i(t)-i_{ref}(t)$ is the deviation of the actual current from the desired current, $\tilde{V}(t)=V(t)-V_{ref}(t)$ is the deviation of the actual input voltage from the desired input voltage, J is the moment of inertia, K_m is the torque/back emf constant, D is the viscous friction coefficient, and R_a, R_f, L_a and L_f are the armature resistance, the field winding resistance, the armature inductance and the field winding inductance, respectively, with $R\triangleq R_f+R_a$ and $L\triangleq L_f+L_a$. Note that in a typical series-connected dc motor, the condition $L_f >> L_a$ holds. When one obtains a series connected dc motor, $i(t)=i_a(t)=i_f(t)$ we have $L_f >> L_a$. Now let us assume that $|\Delta J| \le 0.1J$.

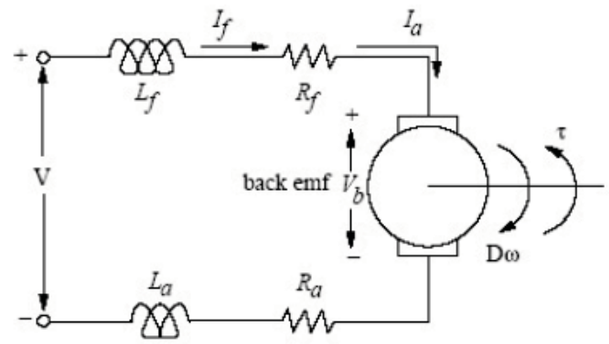


Fig. 1. A modified series dc motor equivalent circuit.

Giving $x_1(t)=\tilde{\omega}(t)$, $x_2(t)=\tilde{i}(t)$ and $u(t)=\tilde{V}(t)$, (57) becomes

$$\begin{bmatrix} \dot{x}_1(t) \\ \varepsilon\,\dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{D}{J+\Delta J} & \frac{K_m L_f}{J+\Delta J}x_2(t) \\ -K_m L_f x_2(t) & -R \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{58}$$

where $\varepsilon = L$ represents a small parasitic parameter. Assume that, the system is aggregated into 3 modes as shown in Table 1:

Mode i	Moment of Inertia	$J(i) \pm \Delta J(i)$ (kg·m^2)
1	Small	0.0005 ±10%
2	Normal	0.005 ±10%
3	Large	0.05 ±10%

Table 1. System Terminology.

The transition probability matrix that relates the three operation modes is given as follows:

$$P_{ik} = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.30 & 0.47 & 0.23 \\ 0.26 & 0.10 & 0.64 \end{bmatrix}.$$

The parameters for the system are given as R = 10 Ω, L_f = 0.005 H, D = 0.05 N·m/rad/s and K_m = 1 N·m/A. Substituting the parameters into (58), we get

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \varepsilon\,\dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} -\frac{0.05}{J(i)} & \frac{0.005}{J(i)}x_2(t) \\ -0.005x_2(t) & -10 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} -\frac{0.05}{\Delta J(i)} & \frac{0.005}{\Delta J(i)}x_2(t) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ z(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \end{aligned} \tag{59}$$

where $x(t)=\begin{bmatrix}x_1^T(t) & x_2^T(t)\end{bmatrix}^T$ is the state variables, $w(t)=\begin{bmatrix}w_1^T(t) & w_2^T(t)\end{bmatrix}^T$ is the disturbance input, $u(t)$ is the controlled input and $z(t)$ is the controlled output.

The control objective is to control the state variable $x_2(t)$ for the range $x_2(t) \in [N1\ N2]$. For the sake of simplicity, we will use as few rules as possible.

Note that Figure 2 shows the plot of the membership function represented by

$$M_1(x_2(t))=\frac{-x_2(t)+N_2}{N_2-N_1} \text{ and } M_2(x_2(t))=\frac{x_2(t)+N_2}{N_2-N_1}$$

Knowing that $x_2(t) \in [N1\ N2]$, the nonlinear system

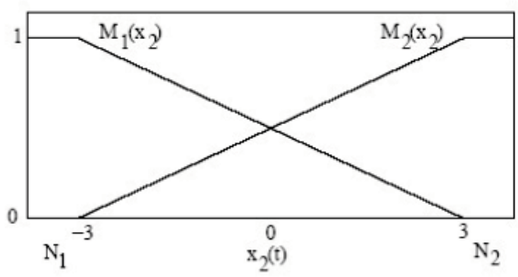


Fig. 2. Membership functions for the two fuzzy set.

(59) can be approximated by the following TS fuzzy model

$$E_\varepsilon \dot{x}(t)=\sum_{i=1}^{r}\mu_i\left[\left[A_i(\iota)+\Delta A_i(\iota)\right]x(t)+B_{1_i}\omega(t)+B_{2_i}u(t)\right], x(0)=0,$$
$$z(t)=\sum_{i=1}^{r}\mu_i\left[C_{1_i}x(t)+D_{12_i}u(t)\right],$$

where μ_i is the normalized time-varying fuzzy weighting functions for each rule, i = 1, 2, $x(t)=\begin{bmatrix}x_1(t)\\x_2(t)\end{bmatrix}$, $E_\varepsilon=\begin{bmatrix}1 & 0\\0 & \varepsilon\end{bmatrix}$, $\Delta A_1(\iota)=F(x(t),\iota,t)H_{1_1}(\iota)$, $\Delta A_2(\iota)=F(x(t),\iota,t)H_{1_2}(\iota)$,

$$A_1(1)=\begin{bmatrix}-100 & 10N_1\\-0.005N_1 & -10\end{bmatrix}, A_2(1)=\begin{bmatrix}-100 & 10N_2\\-0.005N_2 & -10\end{bmatrix},$$
$$A_1(2)=\begin{bmatrix}-10 & N_1\\-0.005N_1 & -10\end{bmatrix}, A_2(2)=\begin{bmatrix}-10 & N_2\\-0.005N_2 & -10\end{bmatrix},$$
$$A_1(3)=\begin{bmatrix}-1 & 0.1N_1\\-0.005N_1 & -10\end{bmatrix}, A_2(3)=\begin{bmatrix}-1 & 0.1N_2\\-0.005N_2 & -10\end{bmatrix},$$
$$B_{1_1}(\iota)=B_{1_2}(\iota)=\begin{bmatrix}0 & 0\\0.1 & 0\end{bmatrix}, B_{2_1}(\iota)=B_{2_2}(\iota)=\begin{bmatrix}0\\1\end{bmatrix}, C_{1_1}(\iota)=C_{1_2}(\iota)=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},$$
$$\text{and } D_{12_1}(\iota)=D_{12_2}(\iota)=\begin{bmatrix}0\\1\end{bmatrix},$$

with $\|F(x(t),\iota,t)\| \le 1$. Then we have

$$H_{1_1}(\iota)=\begin{bmatrix} -\frac{0.05}{J(\iota)} & \frac{0.05}{J(\iota)}N_1 \\ 0 & 0 \end{bmatrix} \text{ and } H_{1_2}(\iota)=\begin{bmatrix} -\frac{0.05}{J(\iota)} & \frac{0.05}{J(\iota)}N_2 \\ 0 & 0 \end{bmatrix}.$$

In this simulation, we select $N_1 = -3$ and $N_2 = 3$. Using the LMI optimization algorithm and Theorem1 with $\varepsilon = 0.005$, $\gamma = 1$ and $\delta(1) = \delta(2) = \delta(3) = 1$, we obtain the results given in Figure 3, Figure 4 and Figure 5

Remark 2: Employing results given in (Nguang & Shi, 2001; Han & Feng, 1998; Chen et al., 2000; Tanaka et al., 1996; Wang et al., 1996) and Matlab LMI solver [28], it is easy to realize that when $\varepsilon < 0.005$ for the state-feedback control design, LMIs become ill-conditioned and Matlab LMI solver yields an error message, "Rank Deficient". However, the state-feedback fuzzy controller proposed in this paper guarantee that the inequality (5) holds for the system (59). Figure 3 shows the result of the changing between modes during the simulation with the initial mode at mode 1 and $\varepsilon = 0.005$. The disturbance input signal, $w(t)$, which was used during simulation is given in Figure 4. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the $\mathcal{H}_\infty$ fuzzy controller is depicted in Figure 5. The ratio of the regulated output energy to the disturbance input noise energy tends to a constant value which is about 0.0094. So $\gamma = \sqrt{0.0094} = 0.0970$ which is less than the pres-cribed value 1. Finally, Table 2 shows the performance index, γ, for different values of ε. □

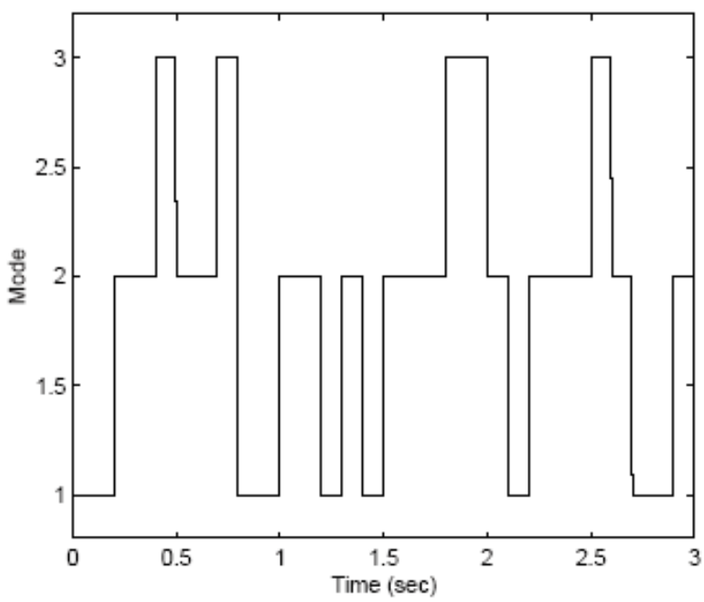


Fig. 3. The result of the changing between modes during the simulation with the initial mode at mode

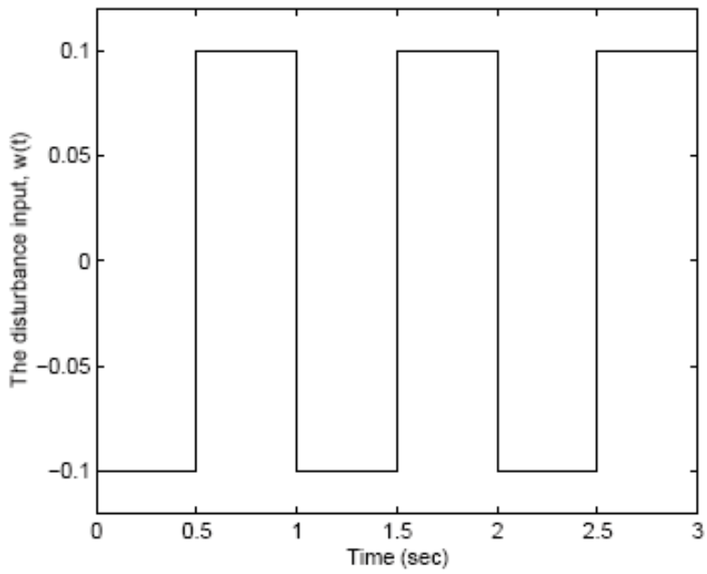


Fig. 4. The disturbance input, w(t).

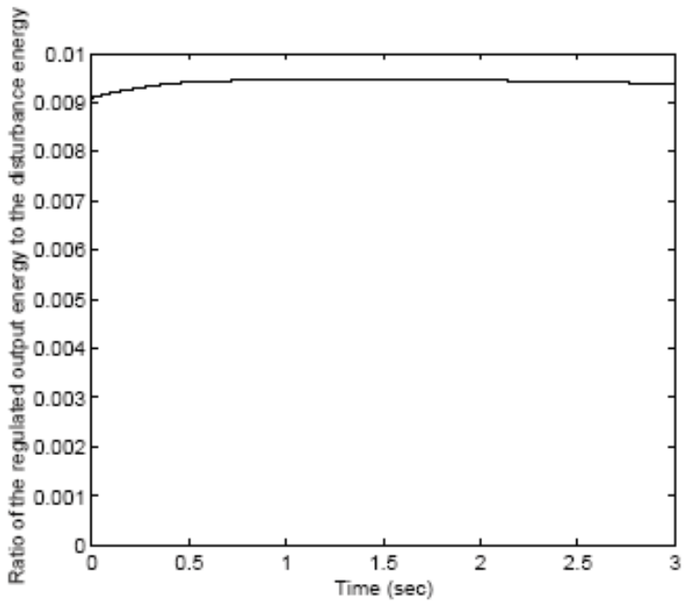


Fig. 5. The ratio of the regulated output energy to the disturbance noise energy,

$$\left(\frac{\int_0^{T_f} z^T(t) z(t) dt}{\int_0^{T_f} w^T(t) w(t) dt} \right) \text{ with } \varepsilon = 0.005.$$

	The performance index γ
ε	State-feedback control design
0.005	0.0970
0.10	0.4796
0.30	0.8660
0.40	0.9945
0.41	> 1

Table 2. The performance index γ for different values of ε.

5. Conclusion

This chapter has investigated the problem of designing a robust $\mathcal{H}_\infty$ controller for a class of uncertain nonlinear two time-scale dynamic systems with Markovian jumps that guarantees the $\mathcal{L}_2$- gain from an exogenous input to a regulated output to be less or equal to a prescribed value. First, we approximate this class of uncertain nonlinear two time-scale dynamic systems with Markovian jumps by a class of uncertain Takagi-Sugeno fuzzy models with Markovian jumps. Then, based on an LMI approach, sufficient conditions for the uncertain nonlinear two time-scale dynamic systems with Markovian jumps to have an $\mathcal{H}_\infty$ performance are derived. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard, but also to nonstandard uncertain nonlinear two time-scale dynamic systems. An illustrative example is used to illustrate the effectiveness of the proposed design techniques.

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