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Mechanical Waves

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1. Introduction

One of the physical phenomena used in medical diagnostics and therapy is ultrasound. We know, for example, ultrasonography, ultrasonic lithotripsy, ultrasonic massages. One of the basic communication channels of humans is sound. The human body is also affected by infrasound. To understand the basic principles of these phenomena, we explain here the origin, properties, and detection of mechanical waves in a matter.

Every continuous medium consists of individual particles (atoms and molecules) that interact with each other. If we do not perceive the microscopic structure, we only see the resulting continuous substance—a *continuum*. The forces of interaction among microscopic particles are observed from outside as macroscopic forces of a different character. The first group represents elastic forces, connected with elastic deformation, which disappears when the force ceases to act. These forces have a *conservative character*, which means they do not cause any heat losses. Elastic forces occur in solids and liquids. Among the elastic forces can be included gas pressure forces if the deformation of the gaseous body is adiabatic (without thermal exchange). The application of the elastic forces is a condition for the transmission of mechanical waves.

Besides, there act the forces connected with losses of mechanical energy in substances, related to plastic deformation, irradiation, heating, etc. These forces are non-conservative and cause the damping of mechanical waves.

1.1 Propagation of longitudinal deformation in an elastic material

Consider a semi-infinite homogeneous elastic medium with a planar surface. On this surface, we cause mechanical deformation by force with area density $f_0(t)$ perpendicular to the surface. This deformation is described by the time function $u_0(t)$ of the surface displacement perpendicular to the surface. The deformation propagates in the medium due to its elasticity in the direction of the z -axis perpendicular to the surface. The displacement of the medium particles is a function $u(z, t)$ of the coordinate z and time t , **Figure 1**. Suppose that surface deformation is the same over the entire surface of the medium and does not depend on transversal x and y coordinates. Therefore, even the propagating deformation does not depend on the transverse coordinates and represents a *plane wave*.

In the medium, we define an elementary cylinder with an axis perpendicular to the surface, the cross-section S , and length dz . When compiling the physical model of wave propagation, we replace individual particles of a substance by volume elements with a volume $dV = S dz$, where dz is the length of the cylindrical element. The mass of the element is $dm = \rho dV$, where ρ is the density of the medium. A discrete spring model of the previous chapter, we replace with the elastic forces F acting on the given element from left and right. During the movement, the mass element is also subjected

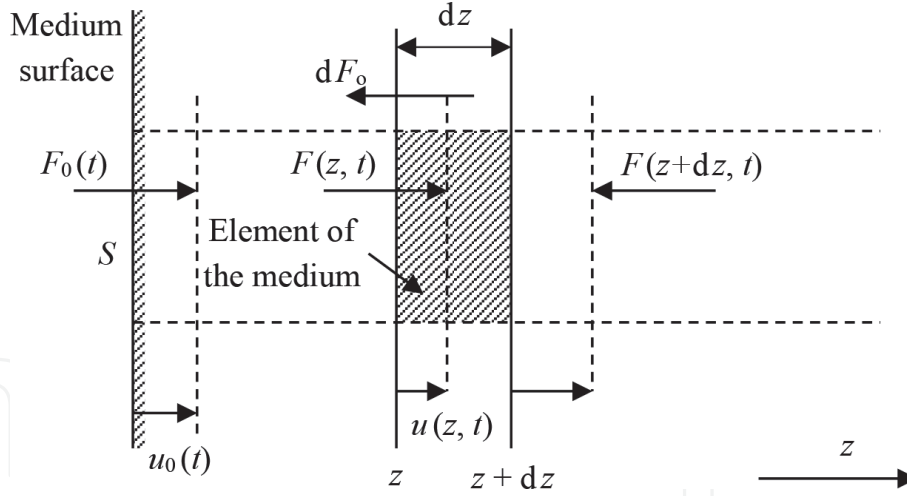


Figure 1.
Dynamics of the volume element of the elastic medium.

to internal friction, which is represented by a resistive force dF_o dependent on the speed of the element movement.

Due to the propagation of the deformation of the medium, the bases of the elements move in the longitudinal direction z . The deflection of the given cross-section with the coordinate z from the balanced position is denoted as $u(z, t)$.

The motion of an element is described by the equation of mechanical motion

$$dm \ a = F(z, t) - F(z + dz, t) - dF_o, \quad (1)$$

where $a = \frac{\partial^2 u(z, t)}{\partial t^2}$ is the acceleration of the element.

The compressive forces F cause a linear deformation of the element, which is described by Hooke's law for tensile stress

$$\sigma = k \ \varepsilon = \frac{F}{S} = k \ \frac{u(z, t) - u(z + dz, t)}{dz}, \quad (2)$$

where σ is the tensile stress, k is the coefficient of elasticity of the environment and ε is the relative compression of the element.

The resistive force usually has the character of a viscous resistance, which is characterised by a linear dependence on the speed of movement of the element. The bulk density of this force is

$$\frac{dF_o}{dV} = -r \frac{\partial u(z, t)}{\partial t}, \quad (3)$$

Where $v = \frac{\partial u(z, t)}{\partial t}$ is the speed of movement of the element and r is the coefficient of resistance of the medium.

We express slight differences ΔF and Δu using their gradient

$$F(z, t) - F(z + dz, t) = -\frac{\partial F}{\partial z} dz, \quad u(z, t) - u(z + dz, t) = -\frac{\partial u}{\partial z} dz.$$

The compressive forces F cause a linear deformation of the element, which is described by Hooke's law for tensile stress, then takes the form

$$\sigma = -k \ \frac{\partial u(z, t)}{\partial z} \quad (4)$$

and represents the tensile (mechanical) stress in the deformed medium caused by the propagation of the mechanical deformation.

The motion of an element is described by the equation of mechanical motion (1), which thus has the form

$$\rho \frac{\partial^2 u(z,t)}{\partial t^2} = k \frac{\partial^2 u(z,t)}{\partial z^2} - r \frac{\partial u(z,t)}{\partial t},$$

or

$$\frac{\partial^2 u(z,t)}{\partial z^2} - \frac{r}{k} \frac{\partial u(z,t)}{\partial t} - \frac{\rho}{k} \frac{\partial^2 u(z,t)}{\partial t^2} = 0. \tag{5}$$

The dynamics of motion of the elements of a continuous elastic medium being described by this partial differential equation.

1.1.1 Longitudinal mechanical waves in a lossless elastic material

The space–time distribution of oscillations of particles (volume elements) of the medium is obtained by solving the differential Eq. (5) under specific initial and boundary conditions.

Let us first consider a simple case of oscillation propagation in a medium with negligible losses (ideal elastic medium). Eq. (5) is simplified by omitting the mean term for $r = 0$. The differential equation takes the form

$$\frac{1}{c^2} \frac{\partial^2 u(z,t)}{\partial t^2} = \frac{\partial^2 u(z,t)}{\partial z^2}, \tag{6}$$

where $c^2 = \frac{k}{\rho}$.

The solution of this equation is a function

$$u(z,t) = f(z \pm ct), \tag{7}$$

which we can be convinced by direct substitution into the Eq. (6).

The shape of the function f is shown in **Figure 2**.

The elements of the medium oscillate around their equilibrium positions and their movement is described by the displacement u from the equilibrium position.

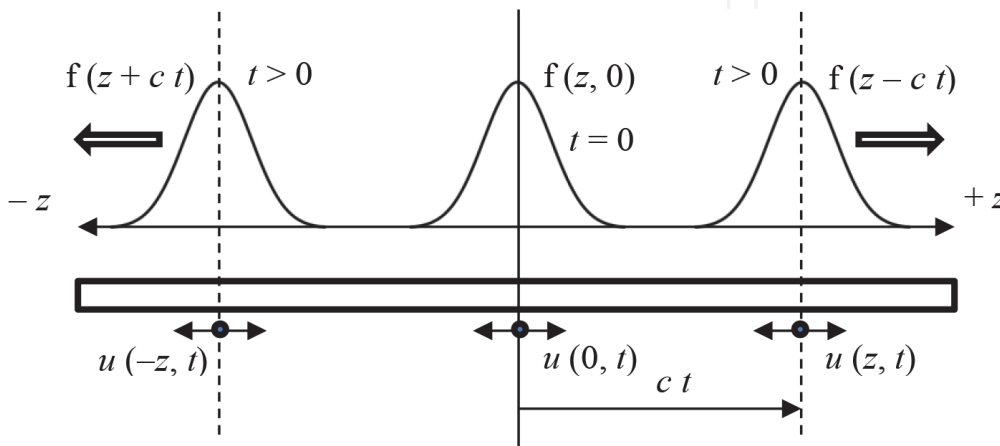


Figure 2.
Wave propagation in the medium.

The distribution of the displacement in the medium at the beginning ($t = 0$) is shown by the function $f(z, 0)$ —an *initial deformation* (source of deformation). At time $t > 0$ the waveform is shifted along the z -axis by a length $\pm ct$. For a solution with a sign $(-)$, the shift is to the right, for a sign $(+)$ to the left, if the medium allows it. The initial deformation thus propagates along the z -axis to both sides at a speed $\Delta z/t = c$ without attenuation and distortion. Thus, a *mechanical wave* occurs that propagates in the medium at the speed, see relation [Eq. (6)],

$$c = \sqrt{\frac{k}{\rho}}. \quad (8)$$

Such a wave both transmits the energy associated with the oscillations of the elements of the substance and transmits information. If we enter a certain disturbance (source signal) at one end of the rod with length L , this disturbance is transmitted with a delay L/c to the other end, where it can be detected by a suitable detection device.

If the medium is solid, the elastic constant k has the meaning of the modulus of elasticity E , and the speed of propagation of the longitudinal wave is $c = \sqrt{\frac{E}{\rho}}$.

Example 1 Propagation of longitudinal mechanical wave in steel.

Steel, as well as other flexible solids, allows the propagation of longitudinal mechanical waves, for example, ultrasound. For modulus of elasticity value $E = 200 \text{ GPa}$ and density $\rho = 7.8 \times 10^3 \text{ kg}\cdot\text{m}^{-3}$ the propagation speed is $c \approx 5.1 \times 10^3 \text{ m}\cdot\text{s}^{-1}$.

In other solids, the speed of longitudinal ultrasound is usually lower, around $(3 \div 5) \times 10^3 \text{ m}\cdot\text{s}^{-1}$.

1.1.2 Transverse mechanical waves in an elastic medium

In the previous part, we solved an example of the propagation of *longitudinal* deformation in an elastic medium. This section shows how the *transversal* (shear) deformation propagates in an elastic medium. The situation is shown in **Figure 3**. The medium is deformed so that the displacement u of its elements is perpendicular to the direction of propagation z .

Eq. (1) remains valid without change. The only difference is that the longitudinal deformation is replaced by shearing strain, and then Eq. (2) gets the form

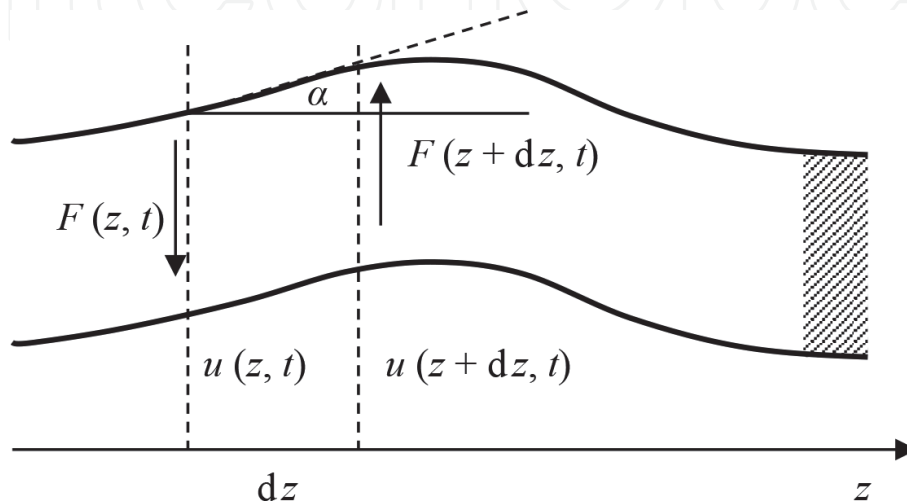


Figure 3.
Propagation of shear deformation in a material.

$$\frac{F}{S} = -G \frac{u(z + dz, t) - u(z, t)}{dz}. \quad (9)$$

where G is the shear modulus of a given medium.

The deformation of the medium associated with the propagating wave causes shear stress in the elastic environment

$$\tau = -G \frac{\partial u(z, t)}{\partial z}. \quad (10)$$

If the wave is undamped or weakly damped, the transverse wave propagates in the medium at a speed

$$c = \sqrt{\frac{G}{\rho}}. \quad (11)$$

Example 2 Propagation of transverse mechanical wave in steel.

Shear modulus of steel is $G = (79 \div 89)$ GPa and density $\rho = 7.8 \times 10^3 \text{ kg}\cdot\text{m}^{-3}$. For the value of $G = 80$ GPa, according to Eq. (11), the speed of the wave is $c \approx 3200 \text{ m}\cdot\text{s}^{-1}$.

Because applies to solid materials $E/3 < G < E/2$, transverse wave propagation speed is 60% ÷ 70% of the longitudinal wave speed, for example, for steel $c_L \approx 5.1 \text{ km}\cdot\text{s}^{-1}$, $c_T \approx 3.2 \text{ km}\cdot\text{s}^{-1}$.

As is clear from the physical origin of the wave, the transverse wave *cannot* propagate in liquid and gaseous media, as these media cannot be deformed by shear stress ($G = 0$). When shear is applied in a liquid medium, creep occurs, and the transverse stress has a highly viscous (lossy) character. A transverse wave can only propagate in *elastic solid materials*.

1.1.3 Propagation of mechanical waves along an elastic string

Another example of a common cause of wave motion is the propagation of transverse excitation along an elastic flexible string.

A string element with a mass dm moves under the effect of the tensile forces of the string, **Figure 4**. The longitudinal motion equation has the form

$$dm a_z = F(z + dz, t) \cos \alpha(z + dz, t) - F(z, t) \cos \alpha(z, t)$$

and in the transverse direction

$$dm a_y = F(z + dz, t) \sin \alpha(z + dz, t) - F(z, t) \sin \alpha(z, t).$$

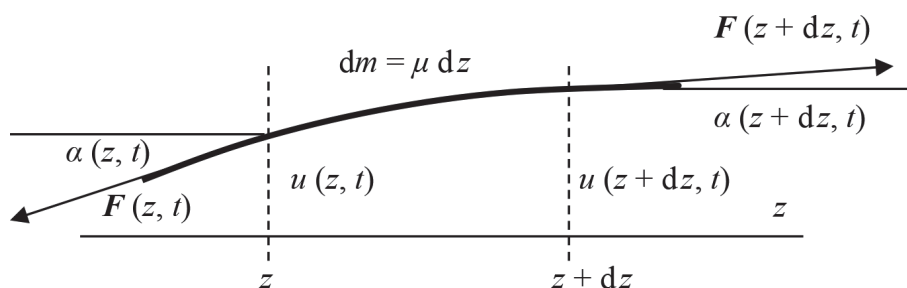


Figure 4.
Transverse movement of a section of a string.

For very small deviations, it is valid: $\cos \alpha \approx 1$ and $a_z \approx 0$.
From the first equation we get

$$F(z + dz, t) \approx F(z, t) = F.$$

The fibre is stretched with the same force F along its entire length.

For small angles α is valid $\sin \alpha \approx \tan \alpha = \frac{\partial u}{\partial z}$. The second equation thus takes the form

$$dm \frac{\partial^2 u}{\partial t^2} = F \left[\frac{\partial u(z + dz, t)}{\partial z} - \frac{\partial u(z, t)}{\partial z} \right] = F \frac{\partial^2 u(z, t)}{\partial z^2} dz$$

If we express the mass of the element $dm = \mu dz$, where μ is the line mass density of the string, the equation gets the form of a wave equation

$$\frac{\partial^2 u(z, t)}{\partial z^2} - \frac{\mu}{F} \frac{\partial^2 u}{\partial t^2} = 0,$$

where $c = \sqrt{\frac{F}{\mu}}$ is the speed of propagation of the transverse wave along the string.

The speed of the wave can be changed by changing the line mass density μ and the force F that stretches the string. As we will show later, the speed c determines the oscillation frequency of a string with a finite length l , for example, the strings of a musical instrument, and as we know, the pitch can be changed by a tuning pin that changes the tensioning force, and the strings for lower tones are thicker than the strings for higher tones.

The *vocal cords* work on the principle of string oscillation. The length of the vocal cords varies between men and women and changes with age. Therefore, women and children have higher voices (higher frequencies) and men have deeper voices (lower frequencies). The person can control the mechanical tension of the vocal cords by means of the fine muscles of the larynx, and thus change the height of the emitted sound. This ability is used for singing.

1.1.4 Mechanical waves in gases

Compression wave propagation takes place in gases. If the pressure increases at a certain point, the change of the pressure gradually propagates through the gas. As the pressure change process is very fast, the adiabatic process is applied (without heat exchange)

$$p V^\kappa = p_0 V_0^\kappa, \quad (12)$$

where κ is the adiabatic constant (for air $\kappa = 1.4$).

If the volume changes according to **Figure 1**, we get

$$V = V_0 + S [u(z + dz, t) - u(z, t)] = S dz + S \frac{\partial u}{\partial z} dz = \left[1 + \frac{\partial u}{\partial z} \right] S dz.$$

Substituting into Eq. (12) we have

$$p = p_0 \frac{1}{\left(1 + \frac{\partial u}{\partial z}\right)^\kappa} \approx p_0 \left(1 - \kappa \frac{\partial u}{\partial z}\right),$$

and the pressure change is

$$\Delta p = -p_0 \kappa \frac{\partial u}{\partial z}. \quad (13)$$

The gas elasticity constant for adiabatic waves is then $k = \kappa p_0$.
 Force on the element is

$$dF = S[p(z, t) - p(z + dz, t)] = -S \frac{\partial p}{\partial z} dz.$$

The equation of motion has the form

$$\rho \frac{\partial^2 u}{\partial t^2} = \kappa p_0 \frac{\partial^2 u}{\partial z^2}.$$

It is the wave equation of a wave with a speed

$$c = \sqrt{\frac{\kappa p_0}{\rho}}.$$

Example 3 Sound propagation in the air.

The sound propagates well in the air and thanks to this fact we can communicate and perceive sounds from the surroundings. Under normal conditions (pressure $p_0 = 101 \text{ kPa}$, temperature $t = 20^\circ\text{C}$ or $T = 293.15 \text{ K}$) is according to the gas state equation for molar mass $M_m = 29 \times 10^{-3} \text{ kg}\cdot\text{mol}^{-1}$ and gas constant $R = 8.314 \text{ J}\cdot\text{K}^{-1}\cdot\text{mol}^{-1}$ air density $\rho = \frac{p_0 M_m}{RT} \approx 1.20 \text{ kg}\cdot\text{m}^{-3}$. For the air adiabatic constant, $\kappa = 1.4$ is the speed of sound propagation in the air $c = \sqrt{\frac{\kappa RT}{M_m}} \approx 340 \text{ m}\cdot\text{s}^{-1}$.

As we can see from the theoretical result, the speed of sound in the air does not depend on the air pressure, but only on the thermodynamic temperature T . At the temperature $t = 40^\circ\text{C}$ is $c \approx 355 \text{ m}\cdot\text{s}^{-1}$ and at the temperature $t = -10^\circ\text{C}$ is $c \approx 325 \text{ m}\cdot\text{s}^{-1}$. Usually, we do not notice this difference, but it is significantly manifested, for example, when tuning wind instruments. The speed of sound propagation is manifested, for example, by the delay of the sound thunder after the lightning during a storm or by the reverberation or echo when the sound reflects from an obstacle.

1.1.5 Mechanical waves in liquids

Liquids represent a fluid medium in which there is zero shear elasticity. Therefore, liquids *do not transmit* transverse mechanical waves. The mechanical stress thus has a longitudinal orientation and causes a change in the volume (compression) of the liquid. If we consider a plane wave in which deformation occurs only in one direction z , the deformation can be described by the relation

$$p = -\frac{1}{\gamma} \frac{\partial u(z, t)}{\partial z},$$

where γ is the compressibility of the liquid.

The speed of propagation of a mechanical wave in a liquid is $c = \sqrt{\frac{1}{\rho\gamma}}$.

For water with values $\gamma = 4.8 \times 10^{-10} \text{ Pa}^{-1}$ and $\rho = 1.0 \times 10^3 \text{ kg}\cdot\text{m}^{-3}$ we have the speed of sound (ultrasound) $c \approx 1.44 \times 10^3 \text{ m}\cdot\text{s}^{-1}$.

Material	Density $\text{kg}\cdot\text{m}^{-3}$	Phase velocity $\text{m}\cdot\text{s}^{-1}$	Specific attenuation $\text{dB}/(\text{m}\cdot\text{MHz})$	Acoustic resistance $\times 10^6 \text{ kg}\cdot\text{m}^4\cdot\text{s}^{-1}$
Air at 20°C	340	1.2	—	0.0004
Water	1000	1480	0.22	1.48
Bone	1975	3476	690	7.38
Blood	1060	1570	20	1.68
Brain	1040	1560	60	1.62
Fat	950	1478	48	1.40
Breast	1020	1510	75	1.54
Muscle	1050	1547	109	1.62
Tendon	1100	1670	470	1.84
Myocardium	1060	1576	52	1.67
Lungs	1060	1595	50	1.69

Source: http://www.science.mcmaster.ca/medphys/images/files/courses/772/Ultrasound/Pressure_attenuation_for_tissues_and_materials.pdf.

Table 1.
Characteristic acoustic parameters of selected substances and tissues at frequency $f_o = 1 \text{ MHz}$.

As the soft tissues of the human body have high water content (up to 70%), their mechanical properties are close to water. In such tissues, only ultrasound with longitudinal polarisation propagates and the propagation speed does not differ very much from the value for water, see **Table 1**.

1.1.6 Wave polarisation

As shown in the previous section, the waves propagate in space at a speed c . The individual particles or volume elements of the medium oscillate around their equilibrium positions, but they do not propagate in space together with the wave. The particles can oscillate in different directions with respect to the direction of wave propagation. The direction of oscillation of the particles is called the *polarisation of the wave*. One of the basic polarizations is *longitudinal polarisation*, where the particles oscillate in the same direction as the direction of the wave propagation. Typical examples are in paragraphs 1.1.5 or 1.1.4. The second basic polarisation is the *transverse polarisation*, where the particles oscillate in a direction perpendicular to the direction of the wave propagation. If the direction of oscillation does not change, we speak of *linear polarisation*. An example is a transverse wave in a solid medium, paragraph 1.1.2, or on an elastic string, paragraph 1.1.3. By suitable excitation, it is also possible to generate a harmonic wave with transverse polarisation, in which the displacement vector follows an ellipse in the transverse plane or a circle—*elliptical* and *circular* transverse polarisation. Such polarisation arises from the superposition of two waves with linear polarisation in mutually perpendicular directions, perpendicular to the propagation direction, and with a phase shift of $\pi/2$ rad. In addition, waves with combined transverse and longitudinal polarisation can be generated, when the total displacement of the particles is a superposition of the displacements in the longitudinal and transverse directions. A typical example is waves on the water surface. The Rayleigh surface wave is elliptically polarised in a plane perpendicular to the material surface and parallel to the direction of wave propagation (particles on the material surface describe an ellipse in a plane perpendicular to the surface and parallel to the direction of wave propagation).

Material	Speed of wave c [$\text{m}\cdot\text{s}^{-1}$]		
Polarisation	Longitudinal	Transversal	Elliptical
Steel	5100	3200	
Concrete	3000	2000	
Water	1440	—	
Air (100 kPa, 32°F)	340	—	
Seismic Rayleigh waves	—	—	3000
Tsunami waves on the open seas	—	—	30
Guitar string—chamber A	—	500	
Vacuum	—	—	

Table 2.
Mechanical waves speed in selected media.

Table 2 shows some typical values of wave propagation speed with different polarizations in different materials. In fact, the values have a considerable variance for different conditions—the values given are only indicative for creating a basic representation. In fluids (liquids and gases), mechanical waves with transverse polarisation cannot propagate, and in a vacuum, the mechanical wave cannot propagate at all.

1.2 Mechanical behaviour of mechanical waves

1.2.1 Particle displacement and particle velocity

As shown, the elements (particles) of the medium oscillate around their equilibrium positions. The first of the quantities describing their motion is the particle displacement $\mathbf{u}_a(\mathbf{r}, t)$. In general, it is a vector quantity that is a function of the position \mathbf{r} and time t .

The movement of medium elements is described by *particle velocity* $\mathbf{v}_a = \partial \mathbf{u}_a / \partial t$. It is the velocity of oscillating motion of particles.

The particle velocity \mathbf{v}_a is not related to the speed c of the wave. While the particle velocity \mathbf{v}_a is a function of position \mathbf{r} and time t , the propagation speed c is constant and depends only on the properties of the medium.

1.2.2 Acoustic pressure and acoustic power

The waves are accompanied by local deformation of the elastic medium. As shown in the analysis of the propagation of individual types of waves, for example, Eqs. (4), (10), or (13), mechanical stress or pressure, are directly proportional to the gradient of the particle displacement and their magnitude can be expressed by a quantity of *acoustic pressure*

$$p_a = \rho \, c^2 \, \frac{\partial u_a(z, t)}{\partial z}, \tag{14}$$

where z is the coordinate in the direction of wave propagation and t is time.
As a result of this acoustic pressure, the strength limit of the material may be exceeded and thus the material is destroyed, or cavitation (formation of vacuum bubbles) may occur in the liquid. These phenomena are used in ultrasonic cleaning of objects, for example, fine mechanics, surgical instruments, in which the particles

of impurities are separated from the cleaned body by the action of ultrasound. The action of intense ultrasound also achieves the spraying of liquids, which use, for example, ultrasonic humidifiers. In technical practice, ultrasonic drills, and cutters suitable for working especially of hard and brittle materials such as ceramics, glass, and porcelain, are used.

In medicine, intensive ultrasound is used in *ultrasound lithotripsy*—the destruction of kidney stones, bladder stones, or gallstones. The ultrasonic wave of power ultrasound device using an acoustic lens (condenser) is focused on the place with the stone. The effect of a series of ultrasonic pulses (tens to thousands of pulses with a repetition frequency of several Hz) crushes the stone into the form of sand, which is then washed away by the urinary or bile ducts.

The device shown in **Figure 5** is used for extracorporeal lithotripsy (ultrasound device is outside of the patient body). It also contains an X-ray positioning device, which precisely targets the location of the stone. Due to the very different acoustic impedance of the stone and the surrounding tissue, the ultrasonic pulse will not cause tissue damage. The main advantage is that the procedure is *non-invasive*.

Intracorporeal lithotripsy is also currently used. In **Figure 6** is a device that allows lithotripsy by ultrasonic or pneumatic pulse. The thin metallic probe transmitting the ultrasound is introduced through a body orifice (urinary) or a small body cut, tightly to the stone, and large and hard stones are disrupted by intense pressure or pneumatic shock waves with the energy of up to 100 mJ. An ultrasonic probe with a pulse power of up to 150 W and a frequency of approximately 25 kHz crashes the stone and removes soft stones and residues after crushing. This method is very



Figure 5.
Extracorporeal lithotripsy.



Figure 6.
Intracorporeal lithotripter.

effective, relatively patient-friendly, and requires only minimal surgery to create an incision for the probe.

Another application is in ophthalmic surgery in the process of phacoemulsification. Ultrasound converts the eye lens into an emulsion, which is then aspirated. After this removal of the lens, a new artificial lens is implemented.

Cavitation using intensive ultrasound is also used by cavitation ultrasonic liposuction. Ultrasound disrupts the membranes of fat cells, whereby the liquid contents of the cells enter the intercellular space and from there are washed away by the lymphatic system. It serves for the non-invasive removal of local fat and cellulite and serves to strengthen the subcutaneous muscles.

The exertion of acoustic pressure on moving particles of the medium represents mechanical power, which expresses the *acoustic power density* (power per unit of transverse area)

$$\pi_a = -p_a \quad v_a = -\rho c^2 \frac{\partial u_a}{\partial z} \frac{\partial u_a}{\partial t}. \quad (15)$$

This power is supplied to the mechanical wave by a wave source, and the power propagates in the medium along with the wave. In this way, the mechanical wave can be used to transfer energy, for example, by ultrasonic heating of the tissue. In medical ultrasound applications, for example, in ultrasonic sonography, care must be taken to limit the power used to avoid burns, especially on the surface layers of the body. Mean value of acoustic power

$$I = \langle \pi_a \rangle = -\rho c^2 \left\langle \frac{\partial u_a}{\partial z} \frac{\partial u_a}{\partial t} \right\rangle \quad (16)$$

represents the *wave intensity*.

The *wave intensity* is decisive in the formation of auditory perception. The sound volume is directly proportional to the $\log I$ (logarithm of intensity).

In biomedicine, the power of a focused ultrasound beam is used to destroy a target tissue in the focal point (usually a tumour) by ‘burning’ it. Ultrasonic ablation controlled by magnetic resonance is used, for example, for non-invasive removal of uterine fibroids. Magnetic resonance imaging serves to accurately target the site of the tumour into which the ultrasound beam is focused.

Ultrasound with a frequency of about 20 kHz uses an *ultrasonic scalpel* in surgery. Its application causes the cutting surface to heat to a temperature of 50–100°C, which causes coagulation of soft tissues during cutting and reduces bleeding from small vessels.

Ultrasound therapy is used, for example, in tissue regeneration, promoting the formation of new cells, stimulating blood flow to the tissue, etc. It is used in the treatment of joints, supports the dissolution of fibroblasts, the formation of collagen, reduces tension in the tissue, has an analgesic and anti-inflammatory effect, and positively stimulates wound healing after injuries. It is also used in dermatology and cosmetics.

1.3 Plane harmonic mechanical wave

The function $f(z, t)$, see Eq. (7), was not specified in more detail and can have a very diverse course. There may exist pulse waves, for example, lithotripsy uses a series of high-energy ultrasound pulses, or various modulated waves are used to transmit information, for example, human voice, or a series of pulses in the digital transmission of information. A special case represents a harmonic wave, which is

generated by a source with harmonic time dependence. Harmonic waves are often used, for example, in medical diagnostics. We know that any course of the periodic function $f(z, t)$ can be considered as a superposition of harmonic waves, using the Fourier transform. The analysis of harmonic waves is, therefore, of considerable theoretical and practical importance. We will especially appreciate the harmonic dependence of the waves in cases of more complex transmission properties of the medium, for example, propagation of waves in a lossy or dispersive medium.

1.3.1 Harmonic mechanical waves

Harmonic waves are generated by a source of harmonic oscillations on the surface of the transmission medium. We express the particle displacement on the surface ($z = 0$) by a harmonic function

$$u(0, t) = U_0 \sin \omega t.$$

The symbolic-complex method is preferably used to solve time-harmonically dependent functions. In linear systems, all steady-state harmonic response functions have the same time dependence.

$$f(z \pm ct) = F(z) e^{j\omega t}, \text{ e.g., } \mathbf{u}(z, t) = \mathbf{U}(z) e^{j\omega t},$$

where \mathbf{u} is a complex particle displacement and \mathbf{U} is its complex amplitude (phasor).

The differential Eq. (5), including the damping element, takes the form

$$\frac{d^2 \mathbf{U}(z)}{dz^2} + \left(\frac{\omega^2}{c^2} - j\omega\eta \right) \mathbf{U}(z) = 0, \quad (17)$$

Where $\eta = r/E$ is the coefficient of losses due to internal friction in the medium. We got a regular differential equation that has an exponential solution

$$\mathbf{u}(z) = \mathbf{U}_0 e^{\pm j \mathbf{k} z}, \text{ and thus } \mathbf{u}(z, t) = \mathbf{U}_0 e^{j(\omega t \pm \mathbf{k} z)}, \quad (18)$$

where

$$\mathbf{k} = \sqrt{\frac{\omega^2}{c^2} - j\omega\eta} = \alpha + j\beta \text{ is a complex wave number.} \quad (19)$$

From relation [Eq. (19)] we determine the components of the complex wavenumber

$$\alpha = \frac{\omega}{\sqrt{2}c} \sqrt{1 + \sqrt{1 + \left(\frac{\eta c^2}{\omega} \right)^2}}, \quad (20)$$

$$\beta = \frac{\omega}{\sqrt{2}c} \sqrt{-1 + \sqrt{1 + \left(\frac{\eta c^2}{\omega} \right)^2}}. \quad (21)$$

For low-loss medium is valid $\frac{\eta c^2}{\omega} \ll 1$, or for angular frequency $\omega \gg \eta c^2$,

we will use simplification $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ for $x \ll 1$ and adjust the coefficients to a simplified form.

$$\alpha \approx \frac{\omega}{c}, \text{ and } \beta \approx \frac{1}{2}\eta c.$$

Using the components of a complex wavenumber, we get a complex deflection form, which is a linear combination of both solutions, (Eq. 18),

$$\mathbf{u}(z, t) = \mathbf{U}_{01} e^{-\beta z} e^{j(\omega t - \alpha z)} + \mathbf{U}_{02} e^{\beta z} e^{j(\omega t + \alpha z)},$$

or real form of the equation

$$u(z, t) = U_{01} e^{-\beta z} \sin(\omega t - \alpha z + \psi_1) + U_{02} e^{\beta z} \sin(\omega t + \alpha z + \psi_2).$$

The first wave function describes an exponentially damped wave propagating in the z -axis direction, the second is an exponentially damped wave propagating in the opposite direction z .

The real component of a complex wave number is the *wave attenuation coefficient*, which provides information about the *effective wave propagation length*

$$\delta = 1/\beta, \quad (22)$$

which indicates the distance over which the amplitude of the particle displacement drops to $1/e \approx 37\%$ of the initial amplitude at the surface.

The harmonic component of the wave function is a periodic function with a period of 2π rad. The expression in parentheses is the phase of the wave, ψ is the phase constant. At $z = \text{constant}$, the function is periodic in time with period T . The reciprocal value of the period is the frequency $f = 1/T$.

For $t = \text{constant}$, the function is periodic in space with a period λ , which is called the *wavelength*.

For the temporal and spatial periods of the wave we have

$$T = \frac{2\pi}{\omega} \text{ and } \lambda = \frac{2\pi}{\alpha}. \quad (23)$$

A certain place of medium with a constant wave phase $\omega t \pm \alpha z + \psi = \text{const.}$ travels with time. For the time Δt , the place of the constant phase is shifted by Δz , while for zero phase change, we have $\omega \Delta t \pm \alpha \Delta z = 0$, from where

$$c = \frac{\omega}{\alpha} = \frac{\mp \Delta z}{\Delta t} \text{ or } \lambda = \frac{c}{f}. \quad (24)$$

The quantity c is the *phase velocity* of the harmonic wave.

The attenuation of ultrasound relates to various physical mechanisms as the intrinsic viscosity, dispersion on inhomogeneities, turbulent losses in gases, etc. The individual influences are frequency dependent, which is described by the frequency dependence of the attenuation coefficient β . It can be approximated by the power dependence $\beta = \beta_0 (f/f_0)^\kappa$, where f_0 is the reference frequency and κ a suitable approximation factor. Medical sonography uses ultrasound in the frequency range of units up to tens of MHz. Viscous losses predominate for most tissues in this frequency range, in which a linear approximation ($\kappa = 1$) is satisfactory. A few typical values are in **Table 1**. The values in the table, valid for biological tissues, are only indicative; they may differ for tissues of the same species from different

sources. For fat or bone, the variety is especially great. For water and tissues, the attenuation coefficient increases approximately in direct proportion to frequency. For air, however, the dependence is more progressive ($\kappa \rightarrow 2$) due to another mechanism of wave energy loss, among which turbulent losses dominate.

For air, the characteristic values are $\beta = 0.22 \text{ dB}\cdot\text{m}^{-1}/22 \text{ dB}\cdot\text{m}^{-1}/236 \text{ dB}\cdot\text{m}^{-1}$ for frequencies 1 MHz/10 MHz/33 MHz.

Ultrasound attenuation is significant at the imaging of internal organs using sonography. Since the ultrasound beam reflected from a given organ is used for imaging, the level of the received signal depends on the depth of the organ under the body surface and thus of the total attenuation. The frequency of the ultrasound, therefore, is adopted to the possibilities of signal detection. Frequencies above 10 MHz are used for ophthalmic sonography. In the sonography of obese patients, a lower frequency of about 2 MHz is chosen to reduce the signal attenuation.

1.3.2 Particle displacement and particle velocity of a harmonic wave

Consider a wave with harmonic time dependence with a complex particle displacement

$$\mathbf{u}_a(z, t) = \mathbf{U}_0 e^{j(\omega t - kz)}. \quad (25)$$

The direction of the phasor \mathbf{U}_0 of the surface particle displacement with respect to the direction of wave propagation determines the direction of polarisation of the wave.

Complex particle velocity is

$$\mathbf{v}_a(z, t) = \frac{\partial \mathbf{u}_a(z, t)}{\partial t} = j\omega \mathbf{U}_0 e^{j(\omega t - kz)} = j\omega \mathbf{u}_a(z, t). \quad (26)$$

The particle velocity is also described by a complex harmonic function and its phase shift relative to the particle displacement is $\pi/2$ rad.

1.3.3 Acoustic pressure and acoustic power of a harmonic wave

Following the relation [Eq. (14)], we express the acoustic pressure of a harmonic wave

$$\mathbf{p}_a(z, t) = -\rho c^2 \frac{\partial \mathbf{u}_a(z, t)}{\partial z} = j\mathbf{k} \rho c^2 \mathbf{U}_0 e^{j(\omega t - kz)} = j\mathbf{k} \rho c^2 \mathbf{u}_a(z, t).$$

Acoustic pressure is related to particle displacement and particle velocity

$$\mathbf{p}_a(z, t) = j\mathbf{k} \rho c^2 \mathbf{u}_a(z, t) = \mathbf{k} \frac{\rho c^2}{\omega} \mathbf{v}_a(z, t). \quad (27)$$

The ratio of acoustic pressure and particle velocity is one of the characteristic wave properties of the medium and is called *acoustic impedance*

$$\mathbf{Z}_a = \frac{\mathbf{p}_a}{\mathbf{v}_a} = \frac{\rho c^2}{\omega} \mathbf{k} = \rho c \sqrt{1 - j \frac{\eta c^2}{\omega}} = R_a + j X_a. \quad (28)$$

The complex acoustic impedance has a real component *acoustic resistance* and an imaginary component *acoustic reactance*.

For low loss or lossless medium, $\frac{\eta c^2}{\omega} \ll 1$ an approximate relationship is valid

$$Z_a = \rho c \sqrt{1 - j \frac{\eta c^2}{\omega}} \approx \rho c \left(1 - j \frac{\eta c^2}{2\omega} \right).$$

Acoustic resistance and acoustic reactance are

$$R_a = \frac{\rho c^2}{\omega} \quad \alpha = \rho c, \text{ and } X_a = -j \frac{\eta c^2}{2\omega} \rho c = -j \frac{\eta c^2}{2\omega} R_a. \quad (29)$$

The acoustic properties of selected substances and tissues are in **Table 1**. The complex power density, see Eq. (15), is expressed by the relation

$$\pi_a = \mathbf{p}_a \mathbf{v}_a^* = \mathbf{k} \omega \rho c^2 \mathbf{u}_a(z, t) \mathbf{u}_a^*(z, t) = \omega^2 \frac{\rho c^2}{\omega} \mathbf{k} u_{am}^2 = Z_a v_{am}^2 = \frac{1}{Z_a^*} p_{am}^2. \quad (30)$$

The real part represents an active power, the imaginary part a reactive power of the wave. The energy transfer by a wave is expressed by the mean value of the active component of the complex power. It is the *acoustic flux intensity* of the harmonic wave

$$I = R_a \frac{v_{am}^2}{2} = R_a v_{aef}^2 = \frac{p_{aef}^2}{R_a}, \quad (31)$$

where $v_{aef} = \frac{v_{am}}{\sqrt{2}}$, and $p_{aef} = \frac{p_{am}}{\sqrt{2}}$ are the effective values of particle velocity and acoustic pressure.

The acoustic wave intensity in the medium decreases exponentially after the formula

$$I = R_a \frac{v_{am0}^2}{2} e^{-2\beta z} = I_0 e^{-2\beta z}.$$

Attenuation is the result of the conversion of acoustic wave energy into heat. The volume density of the heat created by the attenuation of a wave passing the medium and causing its heating is

$$q = -\frac{\partial I}{\partial z} = 2\beta I_0 e^{-2\beta z} = q_0 e^{-2\beta z},$$

where $q_0 = 2\beta I_0 [\text{W} \cdot \text{m}^{-3}]$ is the heat density at the surface (at the wave source).

The highest heat density is in the surface layer of the medium with a thickness of $\delta/2$ (half the effective wave propagation length).

Ultrasound is used in medicine to heat the surface layers of tissues. The thickness of the heated layer can be adjusted by changing the frequency of the ultrasound. Warming the tissue increases blood flow and supports regeneration processes, the breakdown of muscle metabolites, etc. Focused ultrasound is also used for hyperthermia and tissue ablation, for example, in the brain in the treatment of Parkinson's disease. The mechanical pressure, induced by the ultrasonic wave, is also used in *sonophoresis*, which is the support of drug transport into muscles and tendons through the skin. In dermatology, sonophoresis is used to transport cosmetics or pharmaceuticals, such as hyaluronic acid, various proteins, and moisturisers. Ultrasound massage increases blood flow, and thus oxygenation

of the skin tissue reduces fatigue of cellular structures and partially breaks down subcutaneous fat.

1.4 Wave reflection and dispersion

In a homogeneous medium, the wave proceeds in one direction as a *travelling wave*. If inhomogeneity occurs in the path of the wave (body, change of substance properties, etc.), the wave is reflected or dispersed. Sound reflection is known as echo or reverberation. We speak of *wave reflection* if the dimensions of the reflecting area are significantly larger than the wavelength of the wave, for example, sound reflection from room walls or in nature from a rock wall or water surface. If the dimensions of the inhomogeneity are comparable or less than the wavelength, the usual imagination cannot be used, and diffraction on the inhomogeneities must be considered. In such a case, we speak of *wave dispersion* at dispersing centres, for example, scattering of ultrasound on blood erythrocytes.

1.4.1 Reflection of the wave from a planar interface

1.4.1.1 Laws of reflection and refraction

The reflection of waves from the rigid interface is complicated. To explain the phenomenon, we use a simple case of wave reflection from the planar interface of two homogeneous media.

To display the propagation of the waves, we use a geometric representation of the rays (**Figure 7**). In a homogeneous medium, the rays are lines, in a non-homogeneous medium, the rays are curves.

The laws of reflection and refraction are satisfied with the reflection of waves from the interface and the transition through it. For the angle of incidence α , angle of reflection α' and refractive angle β , relative to the normal to the interface (**Figure 7**), are valid following formulas

$$\alpha' = \alpha, \text{ and } \sin \beta = \frac{c_2}{c_1} \sin \alpha, \quad (32)$$

where c_1 and c_2 are the speeds of waves in both media.

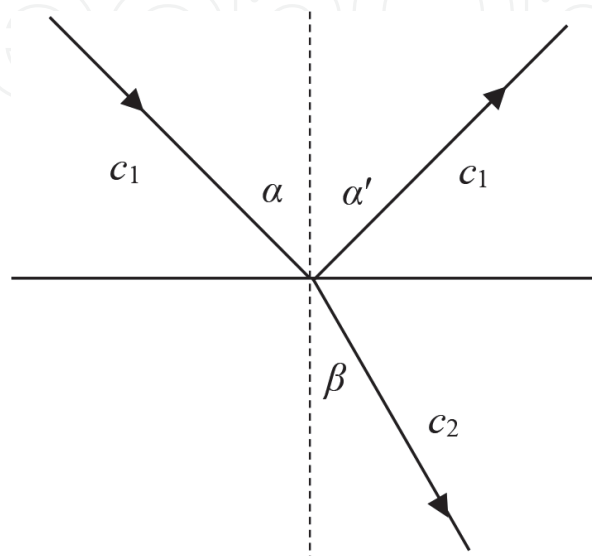


Figure 7.
Reflection and refraction of the wave at the interface of two media.

1.4.1.2 Total reflection

If the wave passes from a medium with a lower speed to a medium with a higher speed, for example, $c_2 > c_1$, there are angles of incidence for which is

$$\sin \beta = \frac{c_2}{c_1} \sin \alpha > 1, \text{ or } \sin \alpha > \frac{c_1}{c_2} = \sin \alpha_m. \tag{33}$$

Since $\sin \beta \leq 1$, for $\alpha > \alpha_m$ there cannot occur transition of the wave to the second medium, and there is a complete (*total*) reflection. The angle α_m is called the *limiting angle of total reflection*.

Example 4 Total sound reflection at the water/air interface.

The sound propagates in the air at a speed of approximately $c_1 = 340 \text{ m}\cdot\text{s}^{-1}$, and in water at a speed of $c_2 = 1440 \text{ m}\cdot\text{s}^{-1}$. The edge angle α_m of total reflection is $\alpha_m \approx 14^\circ$, according to Eq. (33). The sound that hits from the air the surface of the water at angle $\alpha > 14^\circ$ will not penetrate under the water level.

1.4.1.3 Energy transfer of wave across the interface

To assess the intensity of the reflected wave and the wave penetrating the second medium, we will use the case of the perpendicular impact of the wave on the interface to simplify the calculations (**Figure 8**).

Consider the perpendicular incidence of a longitudinally polarised wave at the interface of two media with acoustic impedances Z_1 and Z_2 . In the first medium propagate the incident wave (i - incident), and the reflected wave (r - reflected). In the second medium, only the penetrating wave (t - transmitted) propagates.

The total sound pressure at the left and right interfaces is the same

$$p_i + p_r = p_t,$$

the particle velocity at the interface must be the same from the left and the right

$$v_i + v_r = v_t.$$

According to Eq. (28), there is $p_i = Z_1 v_i$, $p_r = -Z_1 v_r$ and $p_t = Z_2 v_t$, and after substitution and editing we get

$$\begin{aligned} v_t &= \frac{2Z_1}{Z_1 + Z_2} v_i & p_t &= \frac{2Z_2}{Z_1 + Z_2} p_i \\ v_r &= \frac{Z_1 - Z_2}{Z_1 + Z_2} v_i & p_r &= \frac{Z_2 - Z_1}{Z_1 + Z_2} p_i \end{aligned} \tag{34}$$

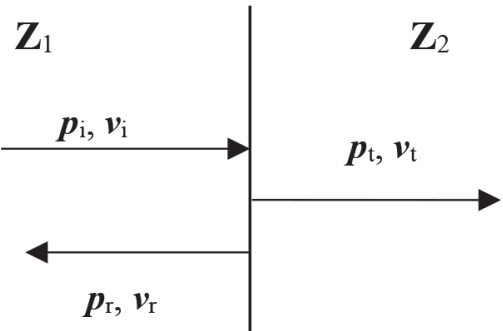


Figure 8.
Reflection of the wave from the interface and transition of the wave across the interface.

Since the impedances are generally complex quantities, the corresponding wave quantities are also complex - phasors.

In the case of a wave incident perpendicularly to the interface, the intensities of the passing and reflected waves are

$$I_r = \left| \frac{Z_2 - Z_1}{Z_1 + Z_2} \right|^2 I_i, \quad I_t = I_i - I_r = \frac{4 \operatorname{Re} \{ Z_1 Z_2^* \}}{|Z_1 + Z_2|^2} I_i, \quad (35)$$

where $I_i = \operatorname{Re} \{ \mathbf{p}_i \mathbf{v}_i^* \}$, $I_r = \operatorname{Re} \{ \mathbf{p}_r \mathbf{v}_r^* \}$ and $I_t = \operatorname{Re} \{ \mathbf{p}_t \mathbf{v}_t^* \}$ are the intensities of the incident, reflected, and penetrating wave, respectively.

The result shows that the reflection of the wave from the interface occurs only in the case of a change in acoustic impedance. If the magnitude of the impedance ratio is several orders of magnitude, there is a practically total reflection (with the same or opposite phase).

The principle of wave reflection is used to investigate the structure of bodies. In nature, the reflection of sound is observed as an echo. The use of reflected ultrasound, for example, bats, flaw detectors, or sonar works on the principle of reflection from the inhomogeneous medium. In medicine, ultrasonography works on the principle of ultrasound reflection. Ultrasound is transmitted by a probe into the body, and the signal reflected from individual organs is received. Mirror reflection takes place from the planar interfaces. The flat surface of the reflecting body needs not to be perfectly smooth. If the surface is rough, and the irregularities are considerably smaller than the wavelength of the wave, the surface behaves as planar. If the irregularities are randomly distributed and larger than the wavelength, the waves are reflected in different directions concerning the surface plane. It is called *diffuse reflection* and *diffuse dispersion*. If the diffuse dispersion is ideal, the waves are scattered, in all directions. An example of diffuse reflection is the reflection of sound from a ragged wall of a room or a waving water level. An example of diffuse dispersion is, for example, sound dispersion from broken walls in a music hall.

1.4.1.4 Transmission of wave through an acoustic layer

A significant case represents the transition of a wave through an acoustic layer. The wave propagation through a layer is proper for impedance matching, the formation of reflective or non-reflective layers, or acoustic resonators. The simple model illustrates (**Figure 9**), which utilises a perpendicular wave impact on the plane-parallel acoustic layer.

The conditions on the interfaces are described in the previous paragraph.

$$\begin{aligned} \mathbf{p}_i + \mathbf{p}_{r1} &= \mathbf{p}_{t1} + \mathbf{p}_{r21} & \mathbf{p}_{t12} + \mathbf{p}_{r2} &= \mathbf{p}_{t3} \\ \mathbf{v}_i + \mathbf{v}_{r1} &= \mathbf{v}_{t1} + \mathbf{v}_{r21} & \mathbf{v}_{t12} + \mathbf{v}_{r2} &= \mathbf{v}_{t3}, \end{aligned}$$

where

$$\begin{aligned} \mathbf{p}_i &= Z_1 \mathbf{v}_i, & \mathbf{p}_{r1} &= -Z_1 \mathbf{v}_{r1}, & \mathbf{p}_{t1} &= Z_2 \mathbf{v}_{t1}, & \mathbf{p}_{r21} &= -Z_2 \mathbf{v}_{r21}, & \mathbf{p}_{t12} &= Z_2 \mathbf{v}_{t12}, \\ \mathbf{p}_{t3} &= Z_3 \mathbf{v}_{t3}, & \mathbf{p}_{t12} &= \mathbf{p}_{t1} e^{-j\mathbf{k}d}, & \mathbf{p}_{r21} &= \mathbf{p}_{r2} e^{-j\mathbf{k}d}, \\ \mathbf{v}_{t12} &= \mathbf{v}_{t1} e^{-j\mathbf{k}d}, & \mathbf{v}_{r21} &= \mathbf{v}_{r2} e^{-j\mathbf{k}d}. \end{aligned}$$

After substituting and editing, we get relationships for complex transition and reflection coefficients

$$\mathbf{t} = \frac{\mathbf{v}_{t3}}{\mathbf{v}_i} = \frac{4Z_1Z_2}{(Z_1 + Z_2)(Z_2 + Z_3) e^{jk d} + (Z_2 - Z_1)(Z_3 - Z_2)e^{-jk d}} \quad (36)$$

$$\mathbf{r} = \frac{\mathbf{v}_{r1}}{\mathbf{v}_i} = \frac{(Z_1 - Z_2)(Z_3 + Z_2)e^{jk d} + (Z_1 + Z_2)(Z_2 - Z_3)e^{-jk d}}{(Z_1 + Z_2)(Z_2 + Z_3) e^{jk d} + (Z_1 - Z_2)(Z_2 - Z_3)e^{-jk d}}. \quad (37)$$

The complex amplification \mathbf{a} of the acoustic wave inside the layer describes the ratio of the particle velocity on the input surface inside the layer and the resulting particle velocity on the outside surface of the layer from the side of the wave source

$$\mathbf{a} = \frac{\mathbf{v}_{t1}}{\mathbf{v}_i + \mathbf{v}_r} = \frac{(Z_2 + Z_3)}{(Z_2 + Z_3) + (Z_2 - Z_3)e^{-j2kd}}. \quad (38)$$

The input impedance of the layer is

$$Z_{in} = \frac{\mathbf{p}_i + \mathbf{p}_{r1}}{\mathbf{v}_i + \mathbf{v}_{r1}} = Z_1 \frac{\mathbf{v}_i - \mathbf{v}_{r1}}{\mathbf{v}_i + \mathbf{v}_{r1}} = \frac{Z_2 + Z_3 - (Z_2 - Z_3)e^{-j2kd}}{Z_2 + Z_3 + (Z_2 - Z_3)e^{-j2kd}} Z_2. \quad (39)$$

From the Eqs. (37) and (39), we get the relation for the reflection factor

$$\mathbf{r} = \frac{Z_1 - Z_{in}}{Z_1 + Z_{in}}.$$

As can be seen from the resulting relations, the quantities \mathbf{r} , \mathbf{t} , \mathbf{a} , and Z_{in} depend on the thickness d of the layer and of the phase thickness $k d$.

We will identify two extreme cases:

For the case (+) is $k d = \pi$ rad, $d = \lambda/2$, and $e^{jk d} = e^{j\pi} = -1$, we have

$$\mathbf{a}^+ = \frac{Z_2 + Z_3}{2Z_2}, \quad Z_{in}^+ = Z_3, \quad (40)$$

$$\mathbf{r}^+ = \frac{Z_1 - Z_3}{Z_1 + Z_3}, \quad \mathbf{t}^+ = -\frac{2Z_1}{Z_1 + Z_3}. \quad (41)$$

For the case (−) is $k d = \pi/2$ rad, $d = \lambda/4$, and $e^{jk d} = j$.

$$\mathbf{a}^- = \frac{Z_2 + Z_3}{2Z_3}, \quad Z_{in}^- = Z_3 \left(\frac{Z_2}{Z_3} \right)^2, \quad (42)$$

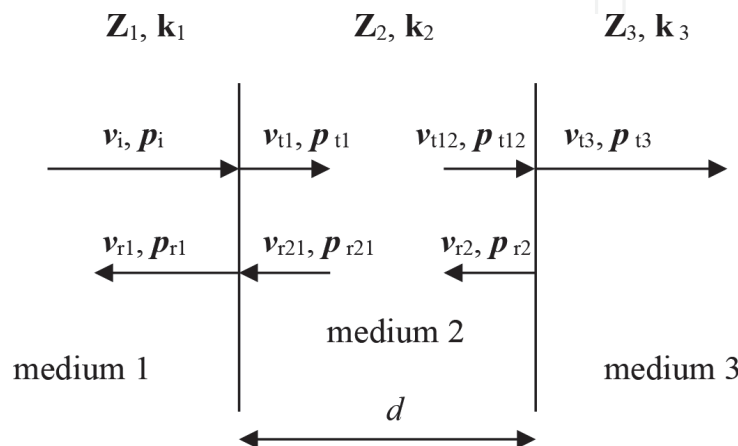


Figure 9.
Transition of wave through an acoustic layer.

$$\mathbf{r}^- = \frac{\mathbf{Z}_1\mathbf{Z}_3 - \mathbf{Z}_2\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_2}, \quad \mathbf{t}^- = -j \frac{2\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_2}. \quad (43)$$

For $\mathbf{Z}_2 \ll \mathbf{Z}_3$ is $\mathbf{a}^+ \gg 1$, it means in the layer, the acoustic field is considerably amplified in the first case (+). This is the *half-wave resonator*, for example, air layer limited by rigid walls (room with concrete walls) or some musical wind instruments.

A room without dispersion elements (bare parallel walls) is a half-wave resonator (for $d = 12$ m is $\lambda = 24$ m and $f \approx 14$ Hz). A very gentle vibration of the walls, caused by, for example, traffic or a rotating machine in the cellar, results in a resonantly greatly amplified infrasound, which is not audible but harms humans.

If $\mathbf{Z}_2 \gg \mathbf{Z}_3$ an amplification occurs in the second case (−), $\mathbf{a}^- \gg 1$. This is the *quarter-wave resonator*. An example is a solid plate placed in water or air, for example, a piezoelectric transducer generating ultrasound into soft tissue during ultrasonography. The ear canal in the outer ear also acts as a quarter-wave resonator: length $l \sim 25$ mm, $\lambda \approx 10$ cm, $f \approx 3.3$ kHz—this frequency is best perceived by the ear due to resonant amplification (see **Figure 12**, p. 31).

Next, we see that the layer transforms the impedance from the value \mathbf{Z}_3 to \mathbf{Z}_{in} . When the structure meets impedances $\mathbf{Z}_1 \neq \mathbf{Z}_3$, the waves are reflected at their interface. If we insert the layer between them with the proper thickness so that the impedances are $\mathbf{Z}_{\text{in}} = \mathbf{Z}_1$ and $\mathbf{Z}_{\text{out}} = \mathbf{Z}_3$, there are no reflections, and the *non-reflective matching* occurs. From Eq. (43) follows that non-reflective matching occurs by inserting a layer with a thickness $\lambda/4$ and impedance $\mathbf{Z}_2 = \sqrt{\mathbf{Z}_1\mathbf{Z}_3}$.

The layer acts as an *impedance transformer*. The transformer can be single or multi-layered, where the impedance transformation (impedance matching) takes place gradually in finer steps—a *stepped transformer*. It can also be continuous, where the impedance changes continuously along with the transformer. A typical example is a funnel transformer that was mounted on historical gramophones to transfer the sound from a phonograph needlepoint into space. The funnel-shaped are some wind instruments with embouchure like a trumpet, French horn, or speakers, alarm siren, etc. An inverted funnel transformer is used for listening, for example, an older funnel stethoscope. A similar principle also utilises a modern stethoscope (see **Figure 10**).

The auricle of the outer ear also acts as an adjusting element (sometimes we increase its effect by placing the palm to the ear) (see **Figure 10**).

The acoustic transformer is also used as a power concentrator (**Figure 10(a)**). If we generate a wave with intensity $I_1 = P/S_1$ on the area S_1 of the source side of the funnel, then the power propagates through the funnel without reflection to the other end. The output intensity is $I_2 = P/S_2$. It increases the intensity (and the acoustic pressure), $I_2 = I_1 (S_1/S_2)$. The output of the concentrator can serve as a working tool, for example, a drill for very hard and brittle materials, or a tool for surface treatment and engraving of hard bodies (glass, ceramics, crystals), etc.

1.4.2 Wave dispersion

When there are small objects compared to the wavelength in the transmission path, the previous idea of reflection from a large area is not applicable. In this case, we consider a wave reflection on the elements of the object surface and the subsequent superposition on elementary reflected waves. It is the diffraction of waves. This phenomenon is called *wave dispersion*. The dispersion theory is relatively complicated, but for small particles with radius R , for which applies the condition $(2\pi R/\lambda) \ll 1$, where λ is the wavelength of the wave, the intensity I of the wave dispersed by the angle ϑ related to the original direction of propagation of the wave expresses the Rayleigh relation

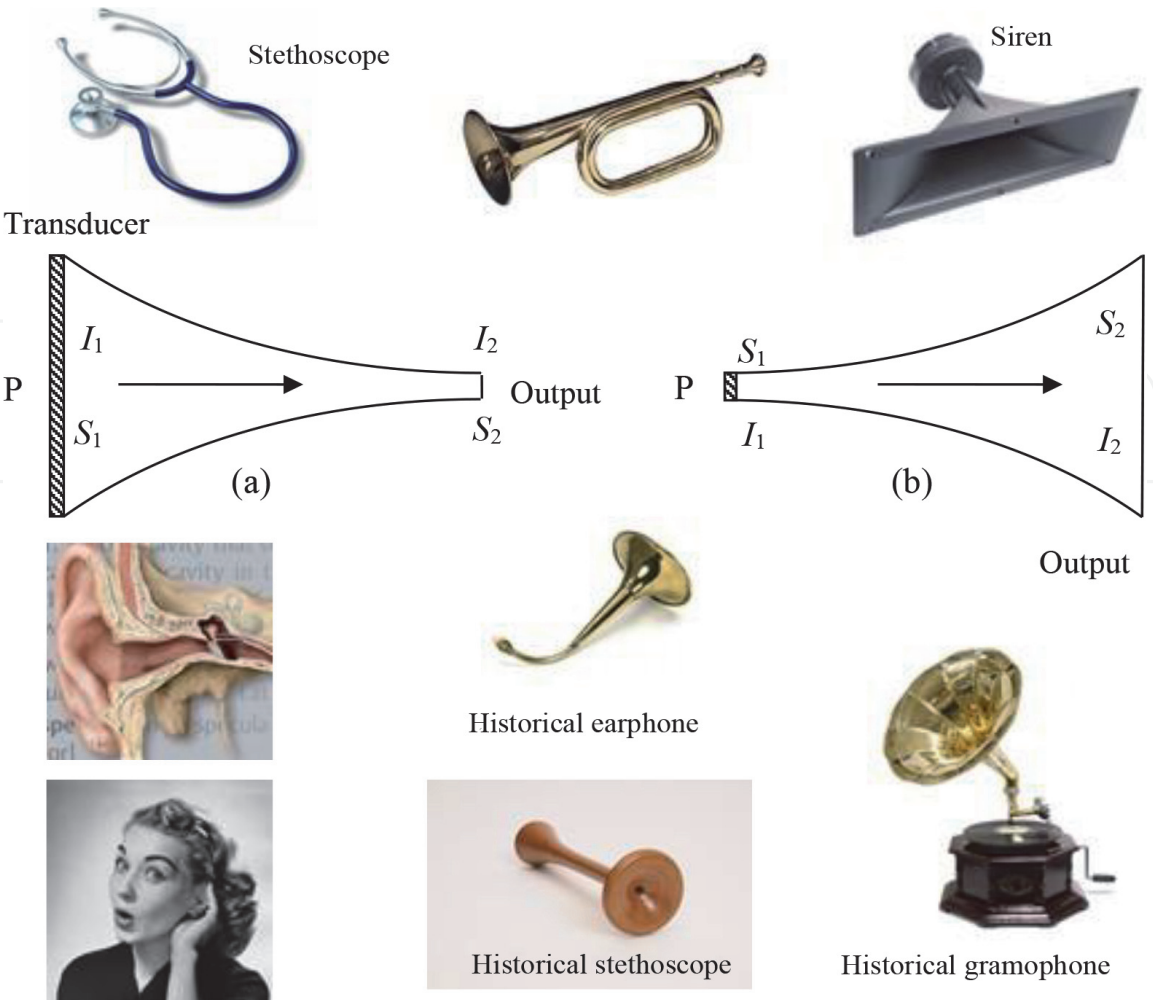


Figure 10.
Several methods of acoustic impedance matching used in sound transmission and reception, (a) sound amplification, (b) sound matching to the open air.

$$I = I_0 k R^2 \left(\frac{R}{\lambda} \right)^4 (1 + \cos^2 \vartheta) \frac{1}{r^2}, \quad (44)$$

where k is the proportionality constant that depends on the particle concentration, ϑ angle of dispersion, r distance from the dispersing particle, and I_0 intensity of the incident wave.

It follows from Rayleigh's relationship that small particles disperse waves in all directions $\vartheta \in (0, \pi)$ rad. The intensity of the dispersed wave depends on the concentration of the particles.

Dispersion of ultrasound with a frequency of about 15 MHz (wavelength of about 100 μm) by red blood cells with a diameter of about 10 μm is used to imaging and measure the speed of blood movement using the Doppler effect.

Ultrasound dispersed on small particles is also used in ultrasonography in the examination of cavities using a contrast medium, which is a liquid with small (micron) particles or microscopic bubbles whose size is smaller than the wavelength of the wave.

1.5 Doppler effect

Several significant applications are related to the *Doppler effect*, which is manifested by a change of the frequency of the detected waves when the source and receiver are in relative motion.

1.5.1 Moving source and static receiver

If the transmitter (source) moves towards the stationary receiver, during time T_0 of one wave period transmitted by the source, the distance between the source and receiver changes by $\Delta l = v T_0 \cos \alpha$, where v is the velocity of the source and α the angle between the direction of the vector of source velocity and connecting line source-receiver. The beginning of the wave period comes to the receiver at the speed c of the wave for a time $t_1 = l/c$. The end of the period comes in the time $t_2 = T_0 + (l - v T_0 \cos \alpha)/c$. The receiver thus records the time between the end and the beginning of the period $T = t_2 - t_1 = T_0 - T_0 v \cos \alpha / c$. It means that the received wave period is

$$T = \frac{T_0}{1 + \frac{v}{c} \cos \alpha}.$$

The receiver thus records the frequency of the incoming wave $f = 1/T$

$$f = f_0 \left(1 + \frac{v}{c} \cos \alpha \right). \quad (45)$$

The relative frequency change is

$$\frac{\Delta f}{f_0} = \frac{v}{c} \cos \alpha. \quad (46)$$

As the source approaches the receiver, the frequency increases. If it moves away, the frequency decreases. If the source moves perpendicularly to the source-receiver line, the frequency does not change. By measuring the change in frequency, the velocity $v \cos \alpha$ of the receiver relative to the source can be measured.

1.5.2 Static source and moving receiver

Now, the source moves towards a stationary receiver at speed $u \cos \beta$, where β is the angle between the velocity vector u and the connecting line source-receiver. If the source sends the beginning of the period T_0 of the wave, it comes to the receiver in time $t_1 = l/c$. Before sending the end of the period, the source moves to the receiver by $T_0 u \cos \beta$. The end of the period goes to the receiver in time $t_2 = (l - T_0 u \cos \beta)/c$. The time between receiving the end and the beginning of the period is $T = T_0 + t_2 - t_1$

$$T = T_0 + \frac{l - T_0 u \cos \beta}{c} - \frac{l}{c} = T_0 \left(1 - \frac{u}{c} \cos \beta \right).$$

The frequency of the received wave is as follows

$$f = \frac{f_0}{1 - \frac{u}{c} \cos \beta}. \quad (47)$$

The relative frequency change is

$$\frac{\Delta f}{f_0} = \frac{\frac{u}{c} \cos \beta}{1 - \frac{u}{c} \cos \beta}, \text{ for } u \ll c \text{ we have } \frac{\Delta f}{f_0} \approx \frac{u}{c} \cos \beta. \quad (48)$$

If the source approaches the receiver, the received frequency increases, if the receiver moves away, the received frequency decreases. The frequency of the received wave changes only if the distance between the source and receiver changes. If the source moves perpendicularly to the source-receiver connecting line, the received frequency remains the same as that of the source. From the change of received frequency, it is possible to determine the speed $u \cos \beta$ of the source relative to the receiver.

1.5.3 Wave reflection from a moving object

Let us consider the source of the wave with frequency f_0 propagating towards an object moving in direction of the connecting line at speed $u \cos \beta$. Frequency f_1 of the wave caught by the moving object is given by the Eq. (47). The wave with frequency f_1 reflects from the object back to the source. The object now represents a moving transmitter with the speed $u \cos \beta$ and the source a stationary receiver. The frequency f_1 changes to f according to Eq. (45). The frequency of the received reflected wave is thus

$$f = \frac{f_1}{1 - \frac{u}{c} \cos \beta} = f_0 \frac{1 + \frac{u}{c} \cos \beta}{1 - \frac{u}{c} \cos \beta},$$

for $u \ll c$ we have $f \approx f_0 (1 + 2 \frac{u}{c} \cos \beta)$ or $\frac{\Delta f}{f_0} \approx 2 \frac{u}{c} \cos \beta$.

The relative change of the wave frequency is thus directly proportional to the speed of the object approaching or moving away from the acoustic wave source.

Other acoustic devices utilise the Doppler principle, for example, SONAR (sound navigation and ranging) for the identification of moving objects under the sea level (submarines, flocks of fish), or when using electromagnetic waves to measure the speed of road vehicles, flying aircraft, clouds, etc.

In biomedicine, the Doppler effect is mainly used in ultrasonography, or to measure the velocity of blood flow in blood vessels. Doppler effect is a common phenomenon of all waves irrespective of their physical nature.

Example 5 LIDAR.



LIDAR (Light Radar) is a device that uses the EM waves of a laser beam reflected from a monitored object. RADAR has a wide range of uses, for example, in air traffic, meteorology, etc. LIDAR is used, for example, to control speed in road transport. Older types used microwave radiation; today's modern devices use optical waves. A narrow beam (3 mrad) of radiation with a wavelength of 905 nm (infrared radiation) is transmitted towards the vehicle in pulses of length $\tau = 30$ ns.

From a vehicle distance of 1 km, the pulse returns in $6.7 \mu\text{s}$. Due to the Doppler effect, at the speed of the vehicle, $u = 50 \text{ km/s}$, period T_0 between pulses changes by $\Delta T/T_0 \approx 10^{-7}$. At the repetition period, $T_0 = 1 \text{ ms}$ and sending of $N = 500$ pulses is the measurement time $t = 0.5 \text{ s}$. During this time is the total Doppler shift $\Delta t = t (\Delta T/T_0) \approx 50 \text{ ns}$. This shift can be safely evaluated using digital analysis. The laser beam is precisely pointed on the oncoming vehicle, up to about 1 km (normally 300 m). The measured speed is documented, and with built-in binoculars or a photo camera, the vehicle is addressable.

The same principle is used by meteorological radars, which use EM waves of the wavelength of 2–10 cm (microwaves). The result is, for example, maps of clouds movement and precipitation. The radar can also distinguish the type of precipitation (rain, snow).

1.6 Spectral bands of mechanical waves

The effects and practical use of mechanical waves depend on their frequency. The frequency bands of mechanical waves are defined concerning the frequency band of sound audible by the human ear. For a healthy hearing organ, an audible sound interval is 20 Hz–20 kHz. The music reference tone is *chamber A* with a frequency of $f = 440 \text{ Hz}$, which is also the fundamental tone of the human voice. It is the tone according to which tune musical instruments. The technical reference frequency is 1 kHz. The human ear is most sensitive to a frequency range of approximately 300 Hz–5.5 kHz with a maximum sensitivity at about 3 kHz.

Single-frequency or *monochromatic* sound is a harmonic one with only one frequency. Most sounds are *multi-frequency* or *polychromatic* ones, which means that they contain waves with more frequencies. If the sound frequency spectrum is continuous, then we speak about *sound noise*. If the sound spectrum is discrete and composed of the fundamental frequency and higher harmonic components, the sound is called *tone*. The fundamental frequency determines the *pitch of tone* and content of the higher harmonics *colour of tone* (*timbre*). If one plays a tone of the same pitch on different instruments, it sounds different on each of them. The difference is in the colour of the tone, it means the content of higher harmonics.

Sound affects the human psyche. The relaxing effects of classical music or, conversely, the excitatory effects of aggressive forms of music are known. These effects are used in sound therapy, sometimes to manipulate masses of people.

The hearing organs of various animals differ and have a different frequency range of sensitivity. For example, the dog hears a sound frequency range of 40 Hz–40 kHz, like a horse. Elephants, on the contrary, perceive vibrations with a frequency of up to 16 Hz. The bats perceive ultrasound with a frequency of up to 150 kHz.

Waves with a frequency $f < 20 \text{ Hz}$ are not heard by humans but can be perceived as mechanical vibrations. Waves with these frequencies are called *infrasound*. The wavelengths of infrasound in the air are $\lambda > 17 \text{ m}$. It is comparable with the dimensions of building structures. Infrasound occurs in halls, offices, and living spaces. Low-frequency vibrations produce the operation of various rotating machines, traffic, or seismic processes. Infrasound has a significant effect on humans, mostly negative with long-term exposure. It can lead to mental disorders and reduced immunity, which is an open path to secondary diseases. Controlling the level of infrasound in living and working areas is an important task of hygienic inspection.

The influence of low-frequency infrasound on the human psyche is used to create specific moods. A typical example is the deep tones of organ pipes, which are supposed to evoke a deeper spiritual experience in cathedrals. On the contrary, the

deep tones with high intensity at discotheques intend to put participants into a trance.

Waves with a frequency $f > 20$ kHz are not heard by humans and represent *ultrasound* (US). Due to a wide spectrum of ultrasound, some bands have a special name, for example, 100 kHz US, MHz US, *hypersound* ($f > 1$ GHz), etc. Ultrasound is used in medical diagnostics—ultrasonography (2–20 MHz), or in physical ultrasound therapy—massage, warming of muscles and tendons (about 1 MHz), ultrasound lithotripsy (crushing kidney, bladder, or gallstones by a series of intense ultrasound pulses 25–30 kHz), etc.

The MHz-US probe can be inserted into a vessel using a catheter to dissolve the thrombus by ultrasound support of the thrombolytic drugs.

An important application is transdermal sonophoresis (drug transfer through the top layer of the skin), in which the barrier of the top layer of the skin (*stratum corneum*) to the transfer of large organic drug molecules with a relative molecular weight greater than 500 is overcome by ultrasound of frequency of 150 kHz.

1.7 Sources and detectors of mechanical waves

There is a direct interaction between the oscillations of the particles and the wave. Particle oscillations (dynamic deformation of the medium) are a source of the wave. On the other hand, waves in the medium cause oscillations of particles. It represents the principle of generation and detection of mechanical waves (sound, infrasound, and ultrasound).

A mechanical wave is generated by any time-varying (dynamic) deformation of the elastic or quasi-elastic medium. The sources can be divided according to the physical principle or according to the properties of the waves. According to the physical principle, they can be mechanical, thermal, electro-dynamic, optical, etc. The sources are coherent or incoherent, or from the geometric point of view: dot, line, surface, specially structured, etc. In this part, the sources of mechanical waves are discussed in terms of physical principles. To create a wave in a medium, the initial disturbance of the medium is necessary. According to the mechanism of this disturbance, we know the following sources of mechanical waves, regardless of the time course or geometric structure of the generated field of particle displacement.

In the following section, on the contrary, we describe the physical principles of mechanical wave detection. The wave cause oscillations that can be detected mechanically, electro-dynamically, optically, etc. Accordingly, we distinguish among diverse ways of detection of mechanical waves (infrasound, sound, and ultrasound).

1.7.1 Mechanical sources of mechanical waves

The mechanical excitation of mechanical waves occurs during mechanical excitation of the surface by an external force. A classic example is the collision of two rigid bodies, for example, the impact of a hammer on the anvil. In this way, for example, the sound of musical instruments such as piano or drums is generated. The impact sound is incoherent, but using a resonator, a coherent harmonic component (appropriate tone) can be selected from a wide range of sounds.

The excitation of mechanical waves also occurs due to friction. Sometimes we hear the whistling of vehicle brakes or the sound of a wheel rubbing against the road during heavy braking. Children know the sound of ‘whistling chalk’ on the blackboard, which also results from friction between the chalk and the blackboard. Regarding musical instruments, it is about creating sound in string instruments by rubbing the bow against a string. The vibration of the body can also occur by the

friction of the flowing gas. Such oscillates the tongue of reed musical instruments, for example, clarinet or saxophone. We observe this principle even when sound is produced in the *vocal cords*. The air flowing through the vocal cord openings, their flexible walls oscillate, generating sound. By changing the tension in the vocal cords, the pitch of the generated tone is controlled. Mechanical wave also occurs due to the turbulent flow of a liquid or gas. In laminar flow, the movement of the individual particles of the medium is smooth and there is no excitation of the disturbances. Laminar flow is also called *silent flow* (e.g., water flowing in a smooth straight trough). However, in the case of eddy (vortex) flow, due to the irregular movement of individual elements of the fluid, mechanical excitations occur. They generate mechanical waves propagating inside and outside the medium. The turbulence arises in fluid when passing a sharp obstacle (bypassing water around stones in a stream). We know, for example, the excitation of the sound in the bottle by blowing on her throat. This principle is used by musical instruments such as various flutes.

Turbulent flow occurs in the tube when the critical value of the Reynolds number is exceeded. This number gives the formula $Re = d v \rho / \eta$ (d is tube diameter, v flow rate, ρ density, η fluid viscosity) and describes the ratio between viscous and dynamic resistance. The critical value, about $Re \sim 2300$, determines the critical flow rate of the fluid at which the flow changes from laminar to turbulent. In the case of a low-density gas, this velocity is low, and therefore, we hear hissing when blowing through narrowed lips. This method of sound excitation occurs when playing the trumpet. As turbulence arises during the flow of air through the respiratory organs, the flow of air in the trachea is heard using a *stethoscope*. Medical doctors use this phenomenon for the diagnosis of respiratory disease.

There is also turbulent blood flow in the cardiovascular system, audible with a stethoscope. Typical is, for example, examining the heart by listening to the murmur of blood passing through the valves, or the flow of blood in the aorta. Besides the heart and aorta, the flow in healthy vessels is laminar (quiet). However, turbulence can occur when any obstacle occurs. In this way, the arising sound can discover thrombus in a vessel or sclerotic or pressure narrowing an artery. Medical diagnostics use this phenomenon in the *auscultation method of measuring blood pressure*. The inflatable cuff constricts the artery and thus the flow of blood through the place of constriction vanishes or gets turbulent. One can hear it using a stethoscope. The sounds accompanying the turbulent flow are called *Korotkoff sounds*. They occur when the cuff pressure ranges from *diastolic* to *systolic* blood pressure. By measuring the pressure in the cuff and listening to the Korotkoff sounds of flowing blood, one determines the values of both limit pressures.

The principle of mechanical excitation of oscillations and mechanical detection of sound use the human hearing organ. The membrane represented by the eardrum oscillates due to the pressure modulation of the air (sound waves). Fine auditory bones (hammer, anvil, and stirrups) in the middle ear transmit the oscillations to the detection system of the inner ear (*cochlea*—snail).

1.7.2 Thermic sound excitation

Thermic excitation of mechanical waves arises due to the thermal expansion of the medium during a sudden change of temperature. If a place heats up quickly, it expands due to thermal expansion, causing the medium particles to move. This movement represents excitation, which then propagates like a mechanical wave due to the elasticity of the medium. Depending on the frequency, it is infrasound, sound, or ultrasound. In practice, we observe this phenomenon, for example, in electric discharges in gases. An electric discharge causes a sudden heating of the gas

at the point of discharge and thus its expansion. The resulting sound is audible as a crack accompanying the discharge, for example, crackling the very high voltage insulators—400 kV, or thunder accompanying lightning during a storm. This sound is a non-coherent wave. However, there are also coherent spark sources in which the waves are excited by a pulse generator with regularly repeating sparks.

Thermal excitation can also be achieved by absorbing light radiation. In practice, the excitation of mechanical waves by power pulsed laser radiation is used. If the focused high-power laser beam impacts a certain place on or below the surface of the body, the medium absorbs the radiation, and thus it is locally heated. By the action of a laser beam with pulse modulation, it is possible to excite in the exposed body a mechanical wave with a frequency equal to the repetition rate of the pulse modulation. This phenomenon uses, for example, *photoacoustic tomography* (PAT) or *photoacoustic microscopy* (PAM).

In medical practice, *laser laparoscopic lithotripsy* uses the photoacoustic excitation of power ultrasound. The laser radiation propagates by an optical fibre to the surface of a kidney, bladder, or gall stone. Series of power optical pulses excites an ultrasound with intensity, at which the acoustic pressure exceeds the edge strength of the material. In such a way, the stone is mechanically broken into small pieces, which are then naturally washed out of the relevant organ. The advantage of this procedure is that it is only slightly invasive. It requires only small cuts for laparoscopic probes. The targeting of the laser pulse to the stone is more accurate and easier compared to extracorporeal ultrasound lithotripsy.

1.7.3 Electrodynamic sources and detectors of mechanical waves

Electrodynamic sources (speakers) and sound detectors (microphones) use the interaction between a moving conductor and a magnet in whose magnetic field it moves. A force $d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$ exerts on the element $d\mathbf{l}$ of the length of the conductor in the magnetic field of magnetic induction \mathbf{B} , and through which passes the electric current I . The conductor has a shape of a cylindrical coil that moves in the direction of its axis. Changes of the current cause changes of the force \mathbf{F} , and thus a change of the coil acceleration. If a membrane is attached to the coil, the movement of the coil cause movement of the membrane. Its oscillations then generate a mechanical wave (sound). Electrodynamic loudspeakers have various configurations. They appear in radio and TV receivers, audio sets, headphones, etc. Their advantage is the high fidelity of the transformation of the electrical signal into sound. Electrodynamic loudspeakers provide a wide range of power from μW to kW and a great dimensional variability from tiny electroacoustic transducers with dimensions of the order of 1 mm to large loudspeakers with dimensions of about 1 m for generating intensive sound or infrasound.

If the coil is forced to move in the magnetic field of the magnet, or if the magnet moves inside the coil, an electric voltage induces in the coil according to Faraday's law, $u = d\Phi/dt$, where Φ is the magnetic flux of the coil. The movement of the coil follows the movement of the connected mechanical part of the system, which oscillates due to the alternating pressure of the incident sound wave. In such an arrangement, electrodynamic microphone, or electrodynamic phonograph pickups are constructed. In the case of a microphone, the sound wave drives the membrane connected with the coil, and thus the acoustic wave is transformed into an electric signal.

Electrodynamic microphones enable precise recording of sound in a very wide frequency range from infrasound to low-frequency ultrasound (tens of kHz). In addition to their normal use in sound technology, they are also used to measure the level of 'acoustic pollution' of the environment, especially in the field of infrasound,

which negatively affects the quality of the environment. Similarly, an ultrasound that is inaudible to the human ear can be sensed. Sensitive microphones are also used in the detection of ultrasound, used, for example, by bats or dolphins for spatial orientation.

From a technical point of view, a moving coil system is mainly used in loudspeakers (sound sources), while detectors use a moving magnet and fixed coil arrangement.

If current flows through the coil, the turns of the winding are attracted to each other. During the flowing of alternating current or current pulses, the coil vibrates and generates sound. For example, exciting coils in a magnetic resonance device are very noisy, which is a negative aspect of the MRI investigation.

1.7.4 Electrostatic source and detector of mechanical waves

The electrostatic loudspeaker uses the dependence of the force between two parallel electrodes of a capacitor on the electrical voltage between them. If one electrode is fixed and the other is a fine movable membrane, the effect of the time-varying voltage causes deflection of the membrane. In this way, the membrane oscillations generate a mechanical wave in the surrounding medium. This type of speaker is suitable only for special purposes, for example, as simple sound indicators.

Electrostatic microphones are used more often. The incident mechanical wave deflects the movable flexible electrode (membrane) of the capacitor and changes the capacitance of the capacitor. Keeping the voltage constant, it modulates the current $i = u \, dC/dt$ passing the capacitor. This current represents the output electrical signal. Electrostatic microphones have a balanced frequency response, high sensitivity, and low distortion. They are utilised in studio technology and for precise acoustic measurements.

1.7.5 Magnetostrictive transducer

Magnetostriction is observed in ferromagnetic materials. The external magnetic field changes the orientation of the spontaneous magnetization domains, which is accompanied by a small change in the size of the material sample. In the direction of the magnetic field, the sample dilates in the direction perpendicular to the magnetic field contracts. The alternating magnetic field generated by the current coil, the rod of ferromagnetic material is in, causes the rod to oscillate. The oscillations are transmitted by acoustic coupling to the surrounding environment. The mainly used ferromagnetic material is nickel and its alloys.

Magnetostrictive transducers are used as ultrasound sources in a wide frequency range, or as actuators in automation.

There exists also an inverse magnetoelastic phenomenon. The mechanical deformation of a ferromagnetic rod changes its magnetization. It results in induced voltage in the coil wound around the ferromagnetic rod.

Magnetostrictive transducers are not used in medicine.

1.7.6 Piezoelectric transducer

Piezoelectric transducers are important technical sources and detectors of mechanical waves with lots of applications. The principle is based on a direct or inverse piezoelectric effect. The piezoelectric transducer consists of a plate of piezoelectric material with deposited metallic electrodes, like a parallel-plate capacitor. After connecting the voltage to the electrodes and raising an electric field in the

piezoelectric plate, the thickness of the plate changes (inverse piezoelectric effect). By applying a time-varying voltage, the plate mechanically vibrates and generates a mechanical wave in the surrounding medium. Most applications of the piezoelectric transducers are *ultrasound sources*. The transducer efficiency increases mechanical resonance. The thickness of the plate is then $\lambda/2$ (half-wavelength resonator), where λ is the wavelength of the ultrasound in the plate at a given frequency. At a thickness of under 1 mm, the resonant frequency is above 1 MHz. Piezoelectric transducers are advantageous for generating and detecting mechanical waves in a frequency range from Hz to tens of GHz.

In the case of mechanical deformation of the plate, for example, by compression, an electric voltage appears directly between the electrodes in direct proportion to the relative change in thickness (direct *piezoelectric effect*). When a mechanical wave hits the transducer, it mechanically oscillates and thus deforms over time. An electrical voltage appears between the electrodes with a time dependence corresponding to the time dependence of the deformation. The piezoelectric transducer thus serves as an *ultrasound detector*. In practice, one transducer is often used as a wave generator and a wave detector with a time separation of transmission and receiving modes (single-probe pulse method).

The piezoelectric phenomenon occurs in some types of anisotropic crystals, for example, SiO_2 , LiNbO_3 , etc. In technical practice are cheaper piezoelectric ceramics $\text{Pb}(\text{Zr}_x\text{Ti}_{1-x})\text{O}_3$, referred to as PZT (lead zirconium titanate) ceramics, or organic piezoelectric foils, for example, polyvinylidene dihydrochloride (PVDF). Ceramics or organic foils can be easily shaped during production and allow to make various structured transducers. A great advantage of piezoelectric transducers is their very small size, if necessary. It is possible to incorporate them directly into microchips as part of electronic integrated circuits. Structured piezoelectric probes can be composed of piezoelectric segments. It is used, for example, in *ultrasonographic* medical instruments. Organic films are mainly used as low-power ultrasound generators or sensitive ultrasound detectors. Ceramic transducers, on the other hand, can generate acoustic power of up to $10^7 \text{ W}\cdot\text{m}^{-2}$ at frequencies of tens of kHz. Intensive ultrasound is used, for example, in ultrasonic cleaners, ultrasonic machining, ultrasonic welding, etc.

Medicine utilises ultrasound very widely. In addition to ultrasonography, which is an important diagnostic tool, ultrasound is used in sonophoresis (incorporation of nutrients or drugs into the skin), deep micro-massage, lipolysis (fat dissolution), deep heating, removal of dental plaque, etc. Intensive ultrasound uses lithotripsy (crushing of kidney, bladder, or gall stones) or ultrasonic scalpels.

1.8 Perception of sound by human hearing

Sound represents periodic fluctuations of air pressure. The eardrum hitting a sound wave oscillates. The oscillations are transmitted through a system of fine bones (*hammer, anvil, stirrup*) to the entrance of the ear canal of the inner ear (*snail*), oval window (**Figure 11(a)**). It represents the entrance of the sound channel in the inner snail and generates a wave propagating inside the snail. Along with the snail, there are nerve endings. The nerve fibres getting out of them, create the auditory nerve. It transmits a signal to the auditory centre in the brain, where auditory perception arises. The auditory organ distinguishes not only the intensity of the sound but also the pitch (frequency) of the tone. **Figure 11(b)** schematically shows the snail and the places of maximum sensitivity to different frequencies of sound. **Figure 11(c)** shows the width of the widening snail sound channel from the *cochlear base* (left) to the *cochlear apex* (right).

The snail channel is a complex acoustic resonator in which the maxima of oscillations with different frequencies are at different places along the channel, as

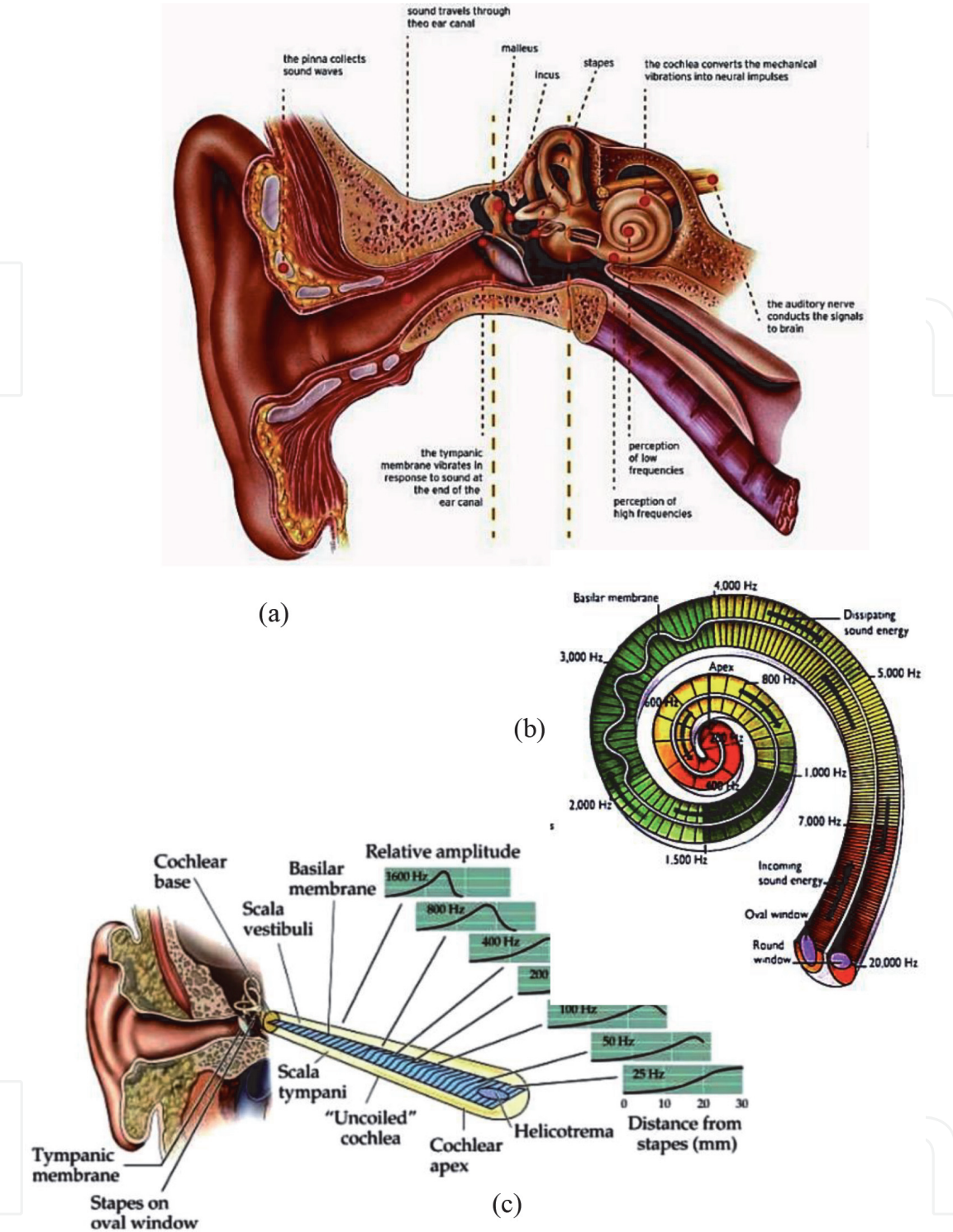


Figure 11. Human ear apparatus involve: (a) human ear structure, (b) the sound detecting snail, (c) uncoiled cochlea with points of sensitivity to different frequencies.

indicated in **Figure 11(b)** and (c). The snail expands from the oval window, which increases the resonant wavelength and thus decreases the resonant frequency. Since the sound attenuation increases with frequency, the smallest transverse dimension is at the beginning (high tones) and the largest at the end (deep tones). The nerve endings on the perimeter of the snail are sensitive to resonant vibrations at given locations. Each nerve fibre thus transmits a different frequency. The snail is a complex frequency analyser of sound. Bundle of nerve fibres—*cochlear nerve*, transmits stimuli to the auditory centre in the brain. In this way, we perceive different tones.

A certain minimum of wave intensity is required for nerve excitation. In a healthy ear, the minimum sound pressure at the eardrum required for perception of the sound at a frequency of 1 kHz is on average $p_0 = 20 \text{ }\mu\text{Pa}$. In the air with an impedance of $Z \approx 400 \text{ kg} \cdot \text{m}^{-2} \cdot \text{s}^{-1}$ it corresponds to the acoustic intensity $I_0 \approx 1 \text{ pW} \cdot \text{m}^{-2}$ ($10^{-12} \text{ W} \cdot \text{m}^{-2}$). This value at 1 kHz determines the *threshold of audibility*.

The sound perception of the human auditory organ is a *logarithmic* function of the sound intensity. It provides its enormous range of perception. Therefore, a *logarithmic acoustic intensity level* I_{dB} (in dB-decibels) is used to evaluate the sound level. It is given by the relation

$$I_{\text{dB}} = 10 \log \frac{I}{I_0}.$$

Typical values of the sound intensity level are given in **Table 3**.

Intensive sound threatens the ear not only by its intensity but also by the time of its action. For example, noise with the intensity level of 80 dB damages the cells of the ear when exposed for more than 8 hours, of the level of 90 dB for only 1 hour, and the level of 120 dB for only 10–16 seconds. Sound of the level of 140 dB and above, damages cells of the ear immediately, and the changes are irreversible. Damage of the cells often accompanies ‘phantom’ sound (tinnitus), such as humming or whistling in the ear. Sound of an intensity level $I_{\text{dB}} > 120 \text{ dB}$ is accompanied by pain. The intensity level of 120 dB, means $I = 1 \text{ W} \cdot \text{m}^{-2}$, therefore, determines the *threshold of pain*. Sound with an intensity level $I_{\text{dB}} > 150 \text{ dB}$ paralyses a person, causing unconsciousness or even death.

A person evaluates sound subjectively by its perception by an auditory organ. One perceives the sound with the same intensity differently at different frequencies. The measure of sound perception is *loudness*. The loudness H of the sound of a given frequency is equal to the intensity I of the sound of a frequency of 1 kHz with the same subjective impression. The *level of loudness (volume)* is defined similarly to the level of intensity I_{dB} by a logarithmic relationship

$$H_{\text{dB}}(f) = 10 \log \frac{H}{H_0},$$

where $H_0 = H_0(f)$ is the *threshold of audibility* at the sound frequency f . The unit of measure of the loudness level H_{dB} is Ph (Phon). A graphical representation of the loudness level spectrum is in **Figure 12**. The lowest curve corresponds to the

Sound	Intensity level [dB]
Audibility threshold	0
Leaf murmur	10
Whispering	30
Loud call	60
Scream, symphonic orchestra	80
Rock concert, disco	110
Jet aircraft take-off, from the distance of 1 m	120
Pain threshold	120
Firecracker, flash grenade	170

Table 3.
Sound energy flux density level of typical sounds.

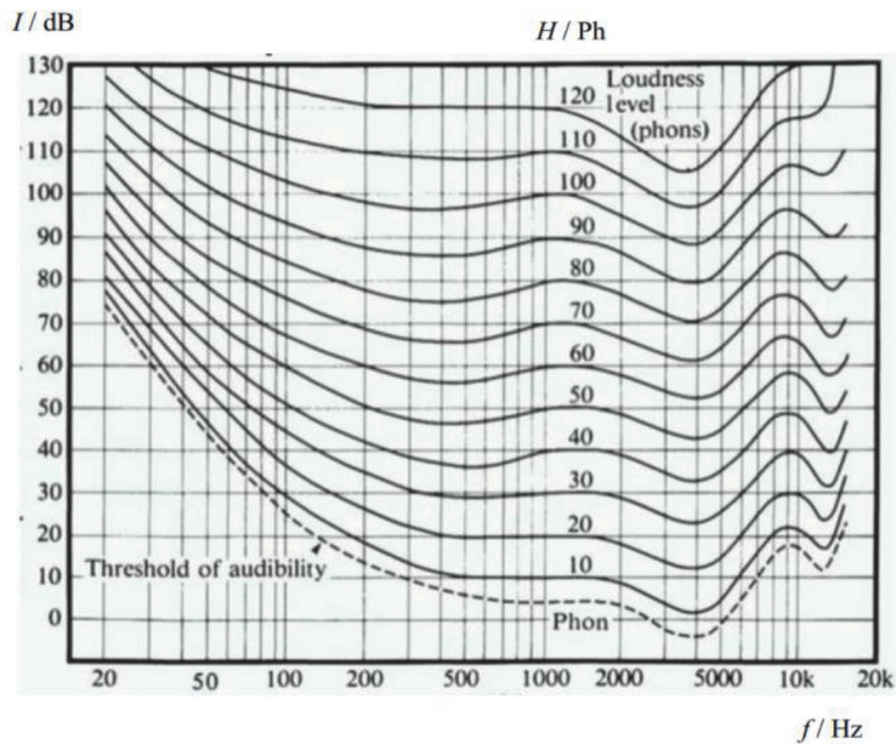


Figure 12.

Loudness level spectrum (Study library: <https://studylib.net/doc/5632183/lecture14>).

threshold value H_0 as a function of the sound frequency f . It was determined experimentally as the average value of the data from many tested subjects with normal hearing.

The thick lines correspond to the constant loudness with the values marked on the middle scale (at 1 kHz the values of loudness level are the same as the intensity level on the right scale). The threshold of audibility is lowest at around 3 kHz and increases towards extreme sound frequencies. We hear sound with a frequency of 30 Hz only at an intensity level of around 60 dB. The graph is standardised and corresponds to the average hearing organ of a healthy young person. With increasing age, the ability to hear sound generally decreases, mostly at extreme frequencies.

The volume is also affected by the external auditory canal and the auricle. The auricle is an acoustic transformer that adjusts the ambient impedance to the impedance of the external auditory canal. The effect of the ear can be supported by its effective enlargement using the palm or a funnel, **Figure 12**.

The auditory meatus is a $\lambda/4$ resonator, which amplifies the sound around the resonant frequency. With its length of approximately 2.5 cm and sound speed of 340 m/s, the resonant frequency is $f_r \approx 3.4$ kHz. In **Figure 13**, we can see the minimum of curves in the vicinity of this frequency. We also see that at low loudness, the sensitivity to bass and treble significantly reduces. If you want to 'enjoy' the entire frequency range of a song while listening to music, we need to choose a loudness level of around 80 dB. To understand the spoken word, it is also necessary to capture the edge frequencies of the spectrum, which also leads to the requirement to speak loudly enough.

1.9 Medical diagnostics using sound and ultrasound

1.9.1 Audiometry

The diagnostic method of *audiometry* deals with the investigation of the spectral sensitivity of the auditory organ of individual people, **Figure 13**. The patient is in an



Figure 13.
 Audiometric chamber.

acoustically isolated chamber and from the technician (left) receives into the headphones sound with different frequencies and gradually increasing intensity. The patient signals the moment when he begins to hear a sound. The line of resulting lowest levels of the sound intensities versus frequency is an *audiogram*.

The sound propagates to the inner ear (snail) through the eardrum or a skull bone, **Figure 14**. The air-conduction hearing test is performed using headphones, bone-conduction by placing the transducer directly on the bone behind the ear. The audiogram shows measurements by air (solid curve) and bone (dashed curve). The audiograms on the right show the airway for the right (red curve) and the left (blue curve) ear. The yellow area shows the intensity and frequency typical for hearing individual sounds of speech. The healthy hearing audiogram (above) provides a comfortable understanding of speech. The audiogram of hearing loss (below) shows hearing that is unable to understand speech—to distinguish the sounds.

Hearing correction is enabled by various instruments, see **Figure 15**. We often observe that for improving the hearing perception, we tend to put the palm to the ear (left picture). In old films from the 1920s, we can see listening to ‘funnels’. Today, hearing loss is solved by electronic listening aids inserted into the ear canal, the picture on the right.

1.9.2 Auscultation

The classical method of examining body sounds is listening—*auscultation*. The method utilises sounds (murmurs) which produce a turbulent flow of fluids (liquids and gases) in the cardiovascular or respiratory system. For example, the murmurs of the blood flow through the heart valves or aorta are audible through the walls of the chest. By listening to them, the doctor can assess the normal functioning of the heart or its disorders. Murmurs also arise due to the obstacles (stenosis, thrombus, etc.) in vessels. By listening to these murmurs, it is possible to identify problem areas. Similarly, auscultation of sounds that produces air flowing through the trachea and bronchial tubes allows investigating respiratory diseases. Another case is an examination of a foetus in the body of a mother. Various sound concentrators

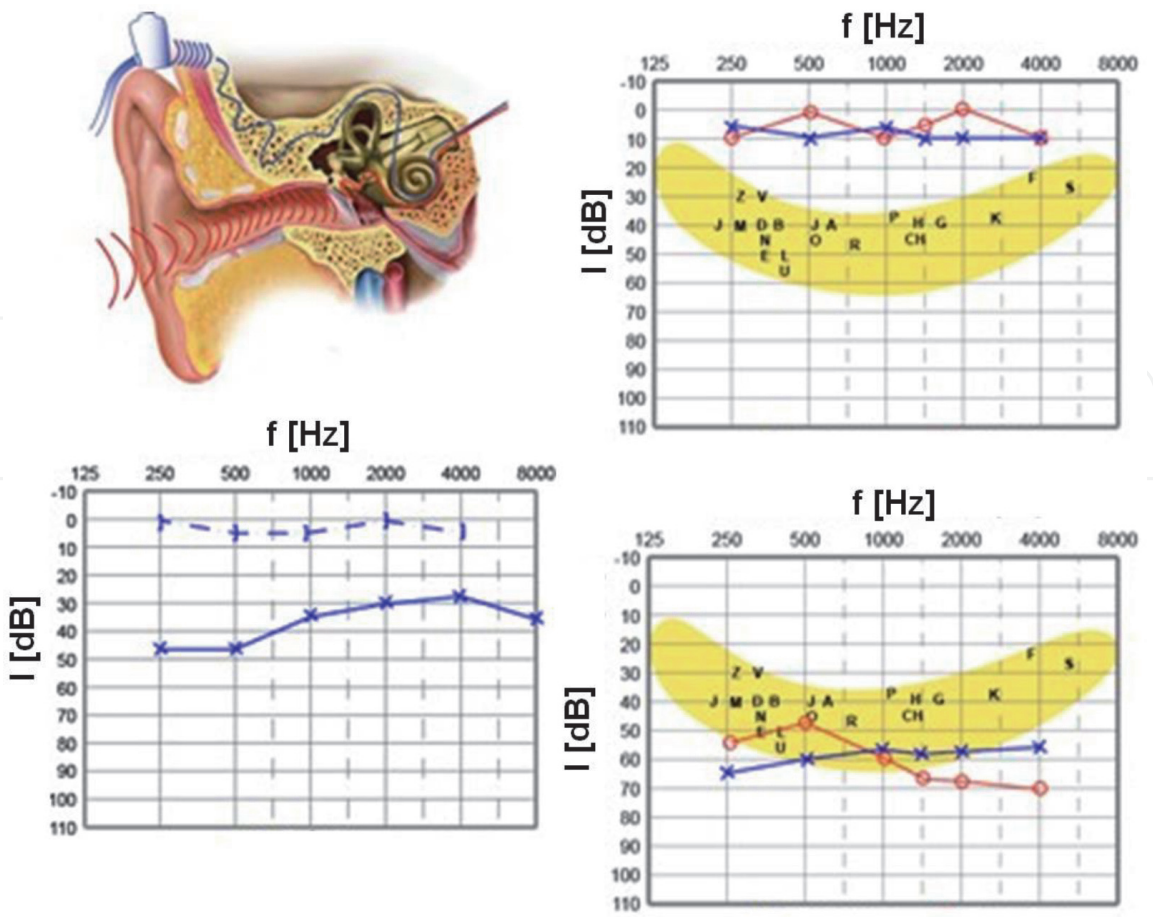


Figure 14. Audiogram and its evaluation.



Figure 15. Hearing aids.

help to amplify the murmurs. The historical tool was a funnel. Modern medicine utilises a *stethoscope*.

Obstruction as a source of murmurs can also be the narrowing of the vessel by the application of lateral pressure. This uses the auscultation method of measuring blood pressure, **Figure 16**. The cuff around the limb is inflated with a balloon to a pressure higher than the systolic one. The pressure stops flowing blood in the artery, and no sound is heard in the stethoscope applied under the cuff. If the pressure in the cuff drops below the systolic (SYS) one after the cuff slow deflating, blood begins to flow through the constricted artery. The flow is turbulent and is accompanied by murmurs—*Korotkov's sounds* (*Nikolay Korotkov 1874 - 1937*), heard in a stethoscope. The volume of the murmurs at first increases to its maximum value and then gradually decreases. The murmur disappears when the pressure drops to the diastolic (DIA) one, and the blood flow becomes laminar (silent). The doctor monitors the pressure in the cuff with a connected pressure gauge, and at the

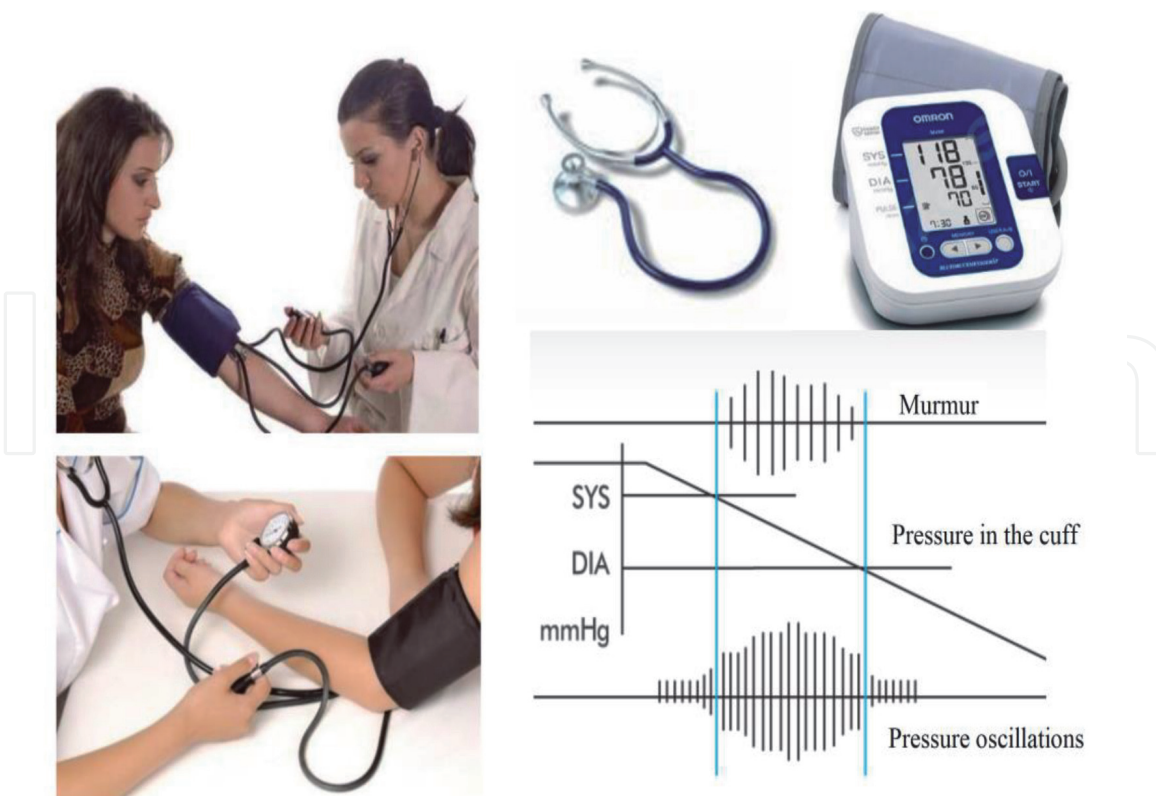


Figure 16.
 Auscultation and oscillometric measurement of blood pressure.

same time listens to the murmurs with a stethoscope. In such a way, the doctor determines systolic pressure at the first hearing of murmurs and diastolic pressure at their extinction.

In addition to this auscultation method, there also exists an oscillometric method. Because of the blood pressure pulsation, small periodic pressure changes occur in the inflated cuff. The device has a pressure sensor connected with the tube inflating the cuff, which picks up these pressure pulses. The initial cuff inflation realises a built-in compressor. The pressure pulses in the cuff are similar time-course of the murmurs (**Figure 16**). After the digital processing of the signal of the pressure sensor, the display of the device shows numerical values of the systolic and diastolic pressure and the heart rate. Oscillometric instruments are simple and do not require any special operation. Therefore, they are used for home blood pressure control or continuous pressure monitoring. In the case of a one-time medical examination, doctors prefer the auscultation method because of its better accuracy.

1.9.3 Ultrasonography

1.9.3.1 Basic ultrasonographic imaging

Ultrasonography is a diagnostic method that uses the propagation of ultrasound in substances and its reflection on the impedance interfaces, for example, LABUDA [1], SHUNG [2]. In technical practice, there exists ultrasonic flaw detection (engineering, construction), which allows the examination of cracks or inhomogeneities in a material. Medical ultrasonography (USG) enables imaging of the internal organs of the body with different acoustic impedance. The ultrasonic transducer generates a short ultrasonic pulse on the surface of the body. It proceeds into the depth of the body, reflects from the impedance interfaces of the organ's tissue, and

returns to the transducer. It now serves as a detector of the reflected pulses. The detected signal is processed by a receiver and then displayed on the screen of the USG device (**Figure 17**).

The imaging indicated in the figure, in which the magnitude of the reflected signal is displayed by the height of the pulse is called the A-mode (amplitude). Each pulse displays the impedance interface. The time delay t corresponds to the depth $d = c t/2$ of the interface under the surface, where c is the speed of ultrasound inside the body.

Structured piezoelectric transducers are the main part of the probes with shapes adapted to specific applications. Several examples of USG probes are in **Figure 18**. The USG probes consist of an array of many small transducers, which allows forming the ultrasound beam.

In the case of 2D imaging, the B-mode (brightness) is used. In this mode, the signal modulates the brightness of the track. An example of comparing A-mode and B-mode is in **Figure 19**.

If a series of parallel or diverging ultrasonic beams are successively sent into the object under investigation, brightness modulated lines are obtained for each of them. By displaying these lines on the monitor, a 2D brightness modulated profile of the internal structure of the object occurs, **Figure 20**. **Figure 20(a)** shows an image of a vessel provided by a series of parallel beams (rectangular image). **Figure 20(b)**

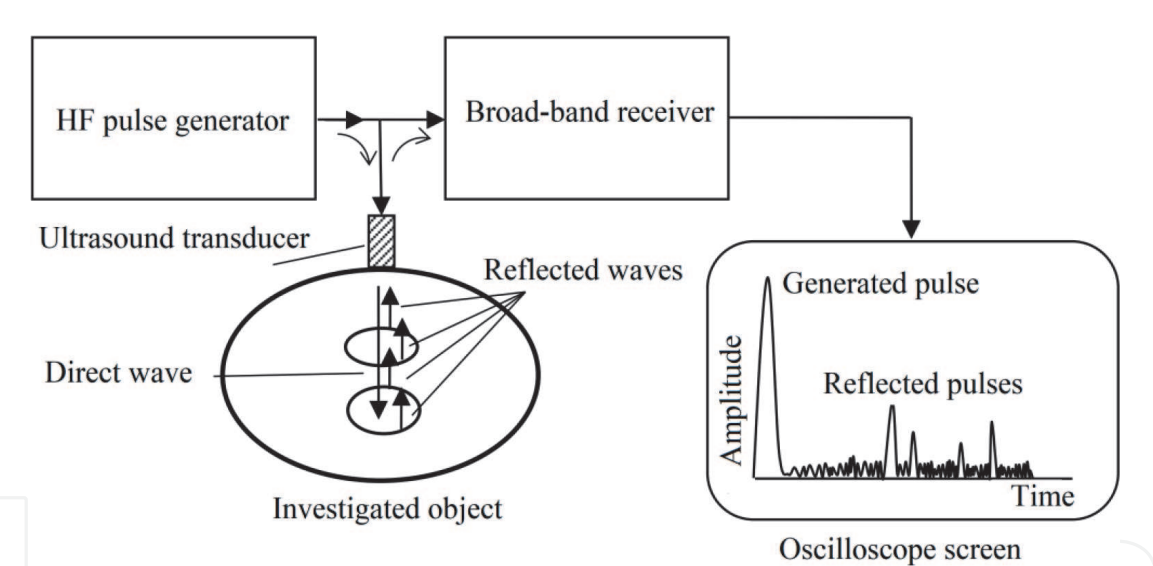


Figure 17.
Displaying of reflected ultrasonic signals in the A-mode.

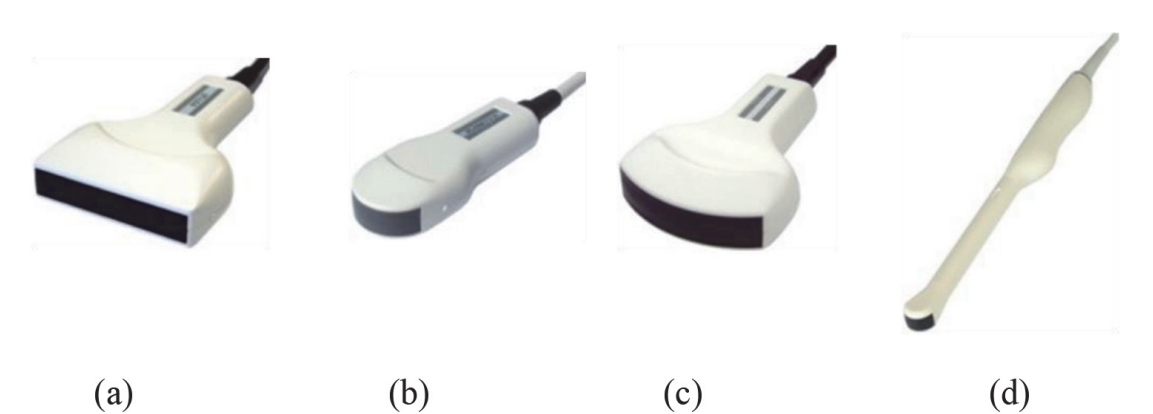


Figure 18.
Several types of USG probes - linear (a), convex (b) and (c), endoscopic (d).

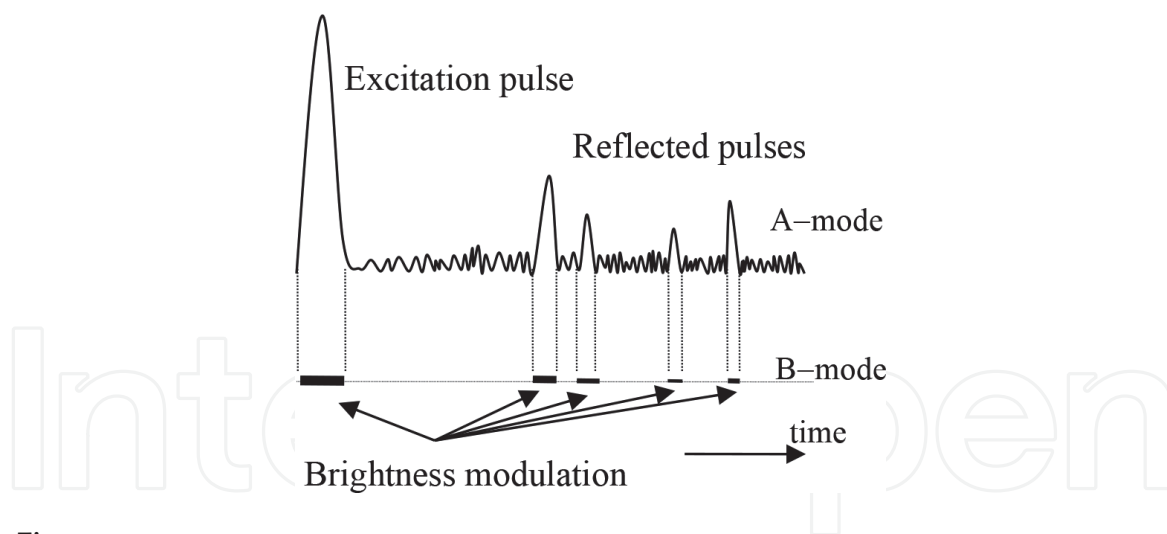


Figure 19.
Comparison of A-mode and B-mode of imaging.

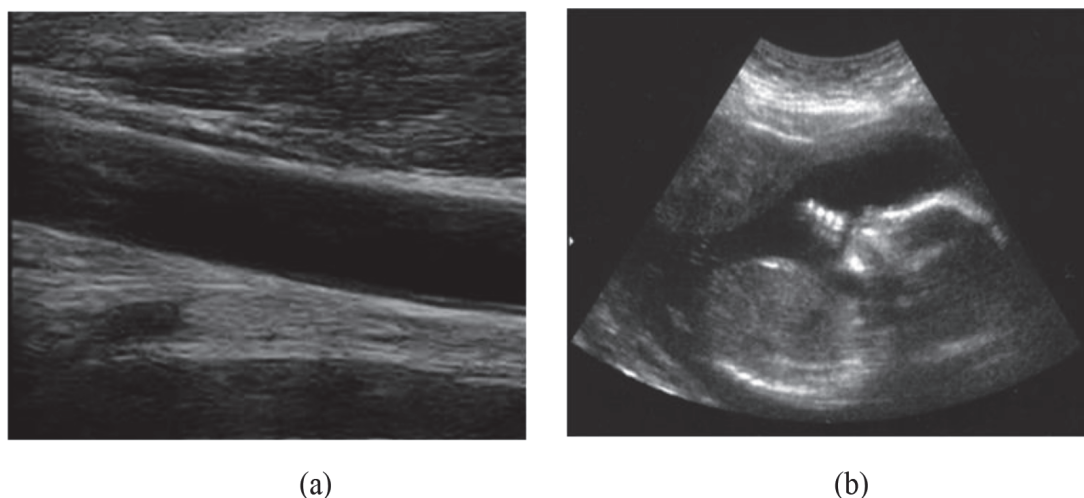


Figure 20.
Ultrasonographic 2D-imaging: (a) rectangular imaging of a vessel with a linear probe, (b) sectoral imaging of a foetus in the mother's body by a convex probe. (royalty free stock photos: <https://www.123rf.com>).

shows a foetus in the body of a mother by a series of diverging beams, arranged in a certain angular sector.

As ultrasonography utilises the principle of ultrasound reflection, it is not suitable for imaging structural parts with an extreme value of the acoustic impedance, gas-filled cavities with a very low impedance, or solid tissues (bones) with a very high impedance. Ultrasound is practically completely reflected from the surface of such objects, and therefore, it is not possible to obtain information about the structures behind these surfaces. The method is suitable for imaging soft tissues, for example, imaging of the heart, kidney, liver, digestive tract, etc. Since ultrasonography has no adverse effects on the human body, it is even used to examine the foetus in the mother's body, **Figure 20(b)**.

1.9.3.2 Doppler sonography

The Doppler effect is used in acoustic diagnostics to track moving objects, for example, heart, blood flow, etc. When ultrasound reflects from the object that moves in the direction of the ultrasound beam, the frequency of the reflected ultrasound is changed due to the Doppler effect. Doppler sonography device has an electrical circuit for separating signals with the shifted frequency (frequency

discriminator), which have been generated by reflection from moving parts of organs, for example, from the beating heart, or from flowing blood. These signals are digitally colour-coded and displayed in a sonogram. Doppler detection also allows suppressing reflections with the original frequency from stationary parts, thus improving the contrast of the image of moving organs. Since the processing of the frequency-shifted signal is time and memory-consuming due to every pixel of the image colour-coding, the Doppler mode is not used for the whole image. The doctor chooses Doppler mode only for a selected demanded part of the image, see **Figure 21**.

The left image is a picture of blood flow through *stenosis* (narrowing of a blood vessel) from the left side rightwards. The ultrasound image is approximately parallel to the vessel, that is, laminar blood flow before stenosis (left side) is red-coloured (see coding on the left side of the image). Behind the stenosis (right side), the blood flow is complex—turbulent, while some blood flowing towards the transducer is yellow-coloured, and blood flowing from the transducer is blue-coloured.

The right image is a picture of the blood flow through the ventricle. Current technical means, especially fast computers, allow obtaining an image in a few tenths of a second so that it is possible to observe the motion of the object online. An example of an online Doppler image of the ventricle in motion can be seen at the page http://cs.wikipedia.org/wiki/Soubor:Doppler_mitral_valve.gif.

1.9.4 Ultrasonic measurement of blood flow in vessels

The Doppler effect allows measuring blood flow in vessels. A principle of the device is in **Figure 22**. It uses a continuous harmonic ultrasonic wave with a frequency of 4–8 MHz, which generates a transmitting transducer T of the probe. Erythrocytes in the blood (the largest blood particles) disperse the ultrasonic wave, and the Doppler-shifted wave returning to the probe detects the receiving transducer R.

The device evaluates the Doppler shift of frequency and displays it in units of blood flow velocity. If we know the cross-section of the vessel, for example, from ultrasonographic measurement, one can determine the volume flow of blood in the vessel. The specific shape of the course of the blood velocity versus time, (**Figure 22** right), the leading and trailing edges of the pulses, various maxima, and minima provide information about the state of the vascular system.

The method is relatively simple and therefore is suitable for indicative angiological investigation. The disadvantage of the method is that the probe reads a response from all vessels, and it is problematic to evaluate only the single one. On the other hand, it is advantageous for an examination of vessels with high blood velocity and of subsurface vessels, for example, in dermatology and phlebology.

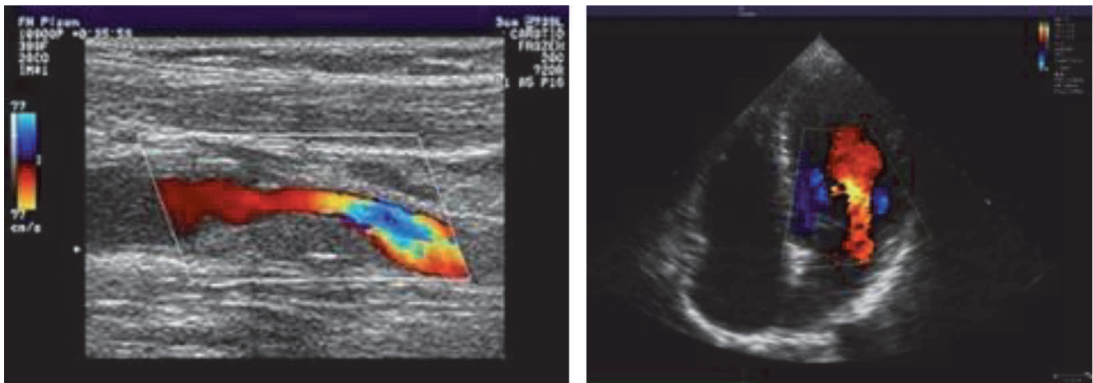


Figure 21.
Doppler ultrasonography images.

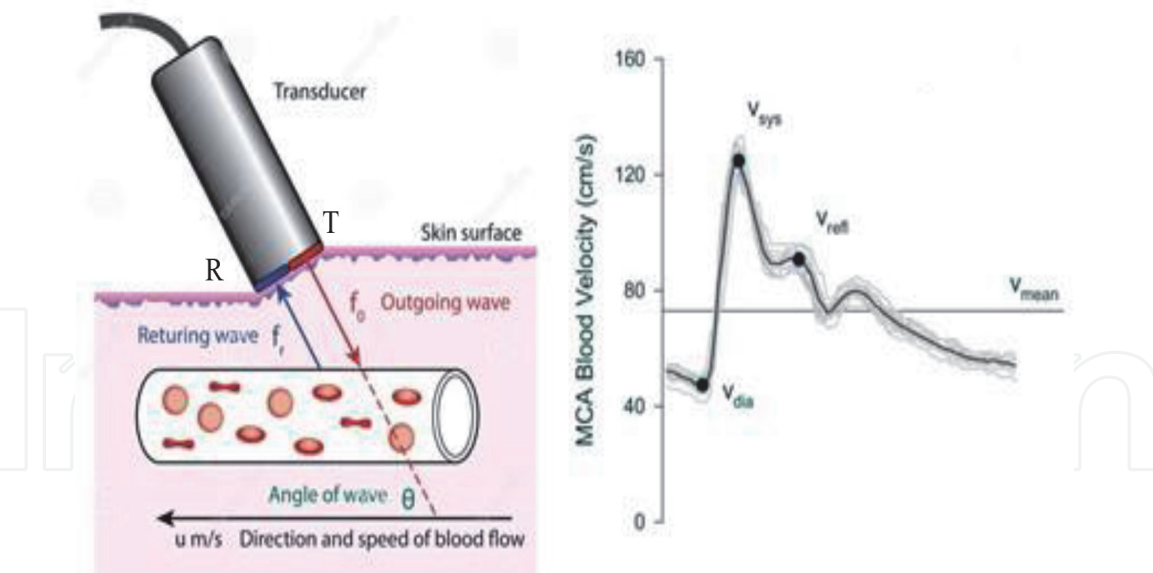


Figure 22.
Blood flow rate probe.


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