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Chapter

Cosmology and Cosmic Rays Propagation in the Relativity with a Preferred Frame

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Abstract

In this chapter, cosmological models and the processes accompanying the propagation of the cosmic rays on cosmological scales are considered based on particle dynamics, electrodynamics and general relativity (GR) developed from the basic concepts of the 'relativity with a preferred frame'. The 'relativity with a preferred frame', designed to reconcile the relativity principle with the existence of the cosmological preferred frame, incorporates the preferred frame at the fundamental level of special relativity (SR) while retaining the fundamental space-time symmetry which, in the standard SR, manifests itself as Lorentz invariance. The cosmological models based on the modified GR of the 'relativity with a preferred frame' allow us to explain the SNIa observational data without introducing the dark energy and also fit other observational data, in particular, the BAO data. Applying the theory to the photo pion-production and pair-production processes, accompanying the propagation of the Ultra-High Energy Cosmic Rays (UHECR) and gamma rays through the universal diffuse background radiation, shows that the modified particle dynamics, electrodynamics and GR lead to measurable signatures in the observed cosmic rays spectra which can provide an interpretation of some puzzling features found in the observational data. Other possible observational consequences of the theory, such as the birefringence of light propagating in vacuo and dispersion, are discussed.

Keywords: general relativity, FRW models, late-time cosmic acceleration, dark energy, UHECR, gamma rays, photo pion-production, pair-production

1. Introduction

Lorentz symmetry is arguably the most fundamental symmetry of physics, at least in its modern conception. Physical laws are Lorentz-covariant among inertial frames; namely, the form of a physical law is invariant under the Lorentz group of space-time transformations. Therefore, the Lorentz symmetry sets a fundamental constraint for physical theories. Nevertheless, modifications of special relativity (SR) and possible violations of Lorentz invariance have recently obtained increased attention. Although, the success of general relativity (GR) to describe all observed gravitational phenomena proves the fundamental importance of Lorentz invariance in our current understanding of gravitation, some of the modern theories (unification theories, extensions of the standard model and so on) suggest a violation of

special relativity. The aim of most of the Lorentz violating theories is to modify a Lorentz invariant theory by introducing small phenomenological Lorentz-violating terms into the basic relations of the theory (Lagrangian density, dispersion relation and so on) and predict what can be expected from it. Reviews of the most popular approaches [1–26] to parameterizing Lorentz violating physics in the context of their relation to the 'relativity with a preferred frame' can be found in [27, 28]. Some of those studies are discussed in the following sections about the results obtained in the present paper.

The theory termed 'relativity with a preferred frame' developed in [27–29] represents a very special type of a Lorentz violating theory that is conceptually different from others found in the literature. It is not even a preferred frame that makes a difference—all violations of Lorentz invariance, made by distorting Lorentz-invariant relations of the theory, imply the existence of a preferred frame for the formulation of the physical laws, the one in which all the calculations need to be carried out, since breaking relativistic invariance also invalidates the transformations that allow us to change reference frame. The first major difference of the present analysis from the above-mentioned studies is that the Lorentz violation *is not introduced* into the theory but it is a result of using freedom in formulation one of two basic principles of special relativity, the principle of universality of the speed of light. In other terms, Lorentz's violation is ingrained into the framework of the theory at some fundamental level. The second major difference is that the relativistic invariance, in the sense that the form of a physical law is invariant under the space-time transformations between inertial frames, is not violated—it is a *Lorentz violation without violation of relativistic invariance*.

To outline the framework of the theory named 'relativity with a preferred frame' one has to start from the definition of the preferred frame. In the 'relativity with a preferred frame', the preferred frame is defined as the only frame where propagation of light is isotropic, while it is anisotropic in all other frames moving relative to the preferred one (it is a common definition in the studies investigating the fundamentals of special relativity and its potential breaking).[‡] Discussing the anisotropy of propagation of light one has to distinguish between the *two-way* speed of light, i.e. the average speed from source to observer and back, and the *one-way* speed which is a speed of light in one direction—*either* from source to observer *or* back. In the 'relativity with a preferred frame', it is the *one-way* speed of light that is assumed to be anisotropic in all the frames except the preferred frame, while the *two-way* speed of light is isotropic and equal to c in all inertial frames.[§] The analysis is based on the invariance of the equation of (anisotropic) light propagation for the space-time transformations between inertial frames and the group structure of the transformations plays a central role in the analysis. Although, the existence of the preferred frame seems to be in contradiction both with the basic principles of special relativity and with the group property of the transformations, in the framework of the 'relativity with a preferred frame', those principles are retained. The crucial element, which allows retaining the relativistic invariance and the group

[‡] It is worth noting that, although the anisotropy of speed of light is one of the central features of the present analysis, this theory stands apart from the ample literature on the conventionality of simultaneity and clock synchronization. A discussion of those issues in the context of the 'relativity with a preferred frame' can be found in [29, 30].

[§] In the modern versions of the experiments designed to test special relativity and the so-named 'test theories' (e.g., [31, 32], see a discussion in [27, 29, 30]), the tests are meant to detect the anisotropy of the *two-way* speed of light.

property of the space-time transformations, is that the anisotropy parameter k, figuring in the equation of the anisotropic light propagation, is treated as a variable that takes part in the group transformations (for more details, see Section 2). Then the preferred frame, in which $k = 0$, enters the analysis on equal footing with other frames since nothing distinguishes the transformations to/from that frame from the transformations between two frames with $k \neq 0$. The space-time symmetry underlying the group of transformations between inertial frames, which in the standard SR is expressed by the existence of the combination invariant under the transformations (interval), in the 'relativity with a preferred frame', reveals itself also in the form of the invariant combination, a counterpart of the interval of the standard SR. Such a 'modified space-time symmetry' paves the way to extensions of the kinematics of the 'relativity with a preferred frame' to free-particle dynamics, general relativity and electromagnetic field theory.

The above-described generalization of special relativity cannot be validated by experiments measuring the speed of light since only the two-way speed of light, the same in all the frames, can be measured. For creating a physical theory, predictions of which can be compared with observational data, it is needed to identify the preferred frame of the present analysis, which is defined by the property of isotropy of the one-way speed of light, with a frame possessing the property that velocity of any other frame relative to it can be measured using some physical phenomena. In the present analysis, that preferred frame is a comoving frame of cosmology or the CMB frame (note that identifying the preferred frame with the CMB frame is a common feature of practically all Lorentz-violating theories). It is the only frame possessing the property, that motion of any other frame relative to it is distinguishable, and, in addition, this frame, like the preferred frame of the present analysis, is defined based on the isotropy property. As a result of specifying the preferred frame, all the relations of the 'relativity with a preferred frame', as well as of its extensions, contain only one universal constant b which is a parameter to be adjusted for fitting the results of the theory to observational data.

Identifying the preferred frame with the cosmological comoving frame implies that the theory should be applied to phenomena on cosmological scales. Studying different phenomena requires extensions of the modified SR kinematics to different areas of physics. The purpose of this chapter is to present a unified view of the extensions and their applications based on the concept of the modified space-time symmetry. This includes extension to general relativity (Section 4.1) and constructing cosmological models based on the modified general relativity (Section 4.2); extension to the dynamics of the free particles (Section 3.1) and its application to the processes accompanying the Ultra High Energy Cosmic Rays (UHECR) and the gamma-rays propagation (Sections 5.1 and 5.2); extension to electromagnetic field (Section 3.2) and studying electromagnetic waves based on the modified electrodynamics (Section 3.3) with application to the gamma-rays propagation (Section 5.3).

2. Special relativity kinematics

Kinematics of the 'relativity with a preferred frame' will be only outlined in this section, for a detailed presentation see [27–29].

The transformations between two arbitrary inertial reference frames S and S', with the coordinate systems $\{X, Y, Z, T\}$ and $\{x, y, z, t\}$ in the standard configuration (with the y- and z-axes parallel to the Y- and Z-axes and S' moving relative to S with the velocity v in the positive direction of the common x-axis), are considered. In the subsequent analysis, the group property of the space-time transformations is

used as a primary tool. Groups of transformations are sought using the condition of invariance of the equation of anisotropic light propagation [30]

$$ds^2 = c^2dt^2 - 2kc\ dtdx - \left(1-k^2\right)dx^2 - dy^2 - dz^2 = 0 \tag{1}$$

where k is the anisotropy parameter such that speeds of light in the positive and negative x -directions are

$$c^{(+)} = \frac{c}{1+k}, \quad c^{(-)} = \frac{c}{1-k} \tag{2}$$

Eq. (1) incorporates both the anisotropy of the *one-way* speed of light as equation (2) shows and the universality of the *two-way* speed of light in the sense that it is equal to c in all inertial frames (see, e.g., [33, 34]).[‖] The transformations involve both the space and time coordinates (x,y,z,t) and the anisotropy parameter k so that the equations of light propagation in the frames S and S' are

$$c^2dT^2 - 2Kc\ dTdX - \left(1-K^2\right)dX^2 - dY^2 - dZ^2 = 0, \tag{3}$$

$$c^2dt^2 - 2kc\ dtdx - \left(1-k^2\right)dx^2 - dy^2 - dz^2 = 0 \tag{4}$$

where K and k are the values of the anisotropy parameter in the frames S and S' respectively. The one-parameter (a) group of transformations of variables from $\{X,Y,Z,T,K\}$ to $\{x,y,z,t,k\}$, which converts (3) into (4), is sought in the form

$$\begin{aligned} &x = f(X,T,K;a), \quad t = q(X,T,K;a); \\ &y = g(Y,Z,K;a),\ z = h(Y,Z,K;a); \quad k = p(K;a) \end{aligned} \tag{5}$$

where, based on the symmetry arguments, it is assumed that the transformations of the variables x and t do not involve the variables y and z and vice versa. According to the Lie group method (see, e.g., [35, 36])., the infinitesimal transformations corresponding to (5) are introduced, as follows

$$\begin{aligned} &x \approx X + \xi(X,T,K)a, \quad t \approx T + \tau(X,T,K)a, \\ &y \approx Y + \eta(Y,Z,K)a, \quad z \approx Z + \zeta(Y,Z,K)a, \quad k \approx K + \kappa(K)a \end{aligned} \tag{6}$$

Proceeding by the usual Lie group technique (see [27–29] for details) one can define the form of the transformations in (x,y,z,t,k) variables. Calculating invariants of the group one can define a combination (a counterpart of the interval of the standard relativity) that is invariant under the transformations, namely

$$d\tilde{s}^2 = \frac{1}{\lambda(k)^2}\left(c^2dt^2 - 2kc\ dtdx - \left(1-k^2\right)dx^2 - dy^2 - dz^2\right) \tag{7}$$

[‖] Although the form (1) seems to be attributed to the one-dimensional formulation, in the three-dimensional case, the equation has the same form if the anisotropy vector $\mathbf{k}$ is directed along the x-axis [30]. In the present analysis, the x-axis defines also the line of relative motion of the two frames but it does not lead to any ambiguity. The assumption, that the anisotropy vector $\mathbf{k}$ is along the direction of relative motion of the frames S' and S, is justified by that one of the frames in a set of frames with different values of k is a preferred frame. Since the anisotropy is attributed to the motion with respect to the preferred frame, it is expected that the axis of anisotropy is either in the direction of motion or opposite to it.

where

$$\lambda(k) = exp\left[-\int_0^k \frac{p}{\kappa(p)}dp\right] \tag{8}$$

with $\kappa(k)$ being the group generator for the variable $k(a)$, see equation (6). Furthermore, introducing the new variables

$$\widetilde{t} = \frac{1}{c\lambda(k)}(ct - kx), \quad \widetilde{x} = \frac{1}{\lambda(k)}x, \quad \widetilde{y} = \frac{1}{\lambda(k)}y, \quad \widetilde{z} = \frac{1}{\lambda(k)}z \tag{9}$$

converts the invariant combination (7) into the Minkowski interval

$$d\widetilde{s}^2 = c^2 d\widetilde{t}^2 - d\widetilde{x}^2 - d\widetilde{y}^2 - d\widetilde{z}^2 \tag{10}$$

while the transformations take the form of rotations in the $(\widetilde{x}, \widetilde{t})$ space (Lorentz transformations). However, in the calculation of physical effects, the 'true' time and space intervals in the 'physical' variables (t, x, y, z), obtained from $(\widetilde{t}, \widetilde{x}, \widetilde{y}, \widetilde{z})$ by the transformation inverse to (9), are to be used.

The expression (7) for the modified interval and the transformations (9) contain the function $\lambda(k)$ which depends on the unspecified function $\kappa(k)$, the infinitesimal group generator for the variable k. This uncertainty reflects the fact that, within the above-developed framework, there is no possibility to determine the value of the anisotropy parameter k or, in other terms, to determine which frame is the preferred one, since only the two-way speed of light, equal to c in all the frames, can be measured. To specify the theory, such that its predictions could be compared with observations, there should exist a possibility to measure the frame velocity relative to a preferred frame using some other physical phenomena. Under the assumption that it is possible, the argument, that anisotropy of the one-way speed of light in an arbitrary inertial frame is due to its motion for a preferred frame, combined with group properties of the transformations, leads to the conclusion that the anisotropy parameter k in a frame moving relative to a preferred frame with velocity $\overline{\beta} = \bar{v}/c$ should be given by some universal function of that velocity, as follows

$$k = F(\bar{\beta}) \quad \text{or} \quad \bar{\beta} = f(k) \tag{11}$$

where $\bar{\beta} = f(k)$ is a function inverse to $F(\bar{\beta})$. Then the group generator $\kappa(k)$ is calculated by (see [27–29] for details)

$$\kappa(k) = \frac{1 - f^2(k)}{f'(k)} \tag{12}$$

which allows to calculate the factor $\lambda(k)$ from (8). Next, with the expression (11) for k introduced into (8), the factor $\lambda(k)$ becomes a function $B(\bar{\beta})$ of the frame velocity $\bar{\beta}$ relative to a preferred frame, as follows

$$\lambda(k(\bar{\beta}) \;\Rightarrow\; B(\bar{\beta}) = exp\left[-\int_0^{\bar{\beta}} \frac{F(m)}{1 - m^2}dm\right] \tag{13}$$

In the subsequent analysis, those general relations are specified using an approximation for $F(\bar{\beta})$ based on the following argument. An expansion of the function $F(\bar{\beta})$ in series in $\bar{\beta}$ should not contain even powers of $\bar{\beta}$ since it is expected

that a direction of the anisotropy vector changes to the opposite if a direction of motion for a preferred frame is reversed: $F(\bar{\beta}) = -F(-\bar{\beta})$. Thus, with accuracy up to the third order in $\bar{\beta}$, the dependence of the anisotropy parameter on the velocity for a preferred frame can be approximated by

$$k = F(\bar{\beta}) \approx b\bar{\beta}, \quad \bar{\beta} = f(k) \approx k/b \tag{14}$$

With this approximation, the group generator $\kappa(k)$ calculated using (12) takes the form

$$\kappa(k) = b - \frac{k^2}{b} \tag{15}$$

and, correspondingly, the factors $\lambda(k)$ and $B(\bar{\beta})$ calculated from equations (8) and (13) become

$$\lambda(k) = \left(1 - \frac{k^2}{b^2}\right)^{b/2} \tag{16}$$

$$B(\bar{\beta}) = \left(1 - \bar{\beta}^2\right)^{b/2} \tag{17}$$

Thus, after the specification, all the equations contain only one undefined parameter, a universal constant b. It is worth reminding that, even though the specified law (14) is linear in β, it does include the second-order term which is identically zero. Therefore describing the anisotropy effects, which are of the order of β^2, by the above equations, is legitimate. In particular, the expression (17) for $B(\bar{\beta})$ is valid up to the second-order in β and, with the same order of approximation, it can be represented as

$$B(\bar{\beta}) = 1 - \frac{b}{2}\bar{\beta}^2 \tag{18}$$

3. Extensions to other areas of physics

3.1 Free particle dynamics

In this section, the free particle dynamics of the 'relativity with a preferred frame' developed in [28] is presented in a shortened form. The modified dynamics is developed based on the existence of the invariant combination $d\tilde{s}$ (a counterpart of the interval of the standard SR) defined by equation (7). Then the action integral for a free material particle is [37]

$$S = -mc\int_a^b d\tilde{s} = \int_{t_a}^{t_b} L dt \tag{19}$$

where the integral is along the world line between two given world points and L represents the Lagrange function. The invariant $d\tilde{s}$ defined by (7) can be represented in the form

$$d\tilde{s} = \frac{cdtQ(k, \beta_x, \beta)}{\lambda(k)} \tag{20}$$

where

$$Q(k,\beta_x.\beta)=\sqrt{(1-k\beta_x)^2-\beta^2};\qquad \beta_x=\frac{v_x}{c},\quad \beta^2=\frac{v_x^2+v_y^2+v_z^2}{c^2} \tag{21}$$

and

$$v_x=\frac{dx}{dt},\ v_y=\frac{dy}{dt},\ v_z=\frac{dz}{dt} \tag{22}$$

are components of the velocity vector. Then the Lagrange function is defined by

$$L=-mc^2\frac{Q(k,\beta_x.\beta)}{\lambda(k)} \tag{23}$$

which is used to obtain expressions for the momentum $\boldsymbol{P}$ and energy E of a particle, as follows

$$\begin{aligned}&P_x=\frac{1}{c}\frac{\partial L}{\partial\beta_x}=mc\frac{k+\beta_x\left(1-k^2\right)}{\lambda(k)Q(k,\beta_x.\beta)},\quad P_y=\frac{1}{c}\frac{\partial L}{\partial\beta_y}=mc\frac{\beta_y}{\lambda(k)Q(k,\beta_x.\beta)},\\&P_z=\frac{1}{c}\frac{\partial L}{\partial\beta_z}=mc\frac{\beta_z}{\lambda(k)Q(k,\beta_x.\beta)}\end{aligned} \tag{24}$$

and

$$E==P_xv_x+P_yv_y+P_zv_z-L=mc^2\frac{1-k\beta_x}{\lambda(k)Q(k,\beta_x.\beta)} \tag{25}$$

Proceeding with the four-dimensional formulation, we will use the variables $(\tilde{t},\tilde{x},\tilde{y},\tilde{z})$ defined by (9) which allows converting the invariant combination (7) into the form (10) of the Minkowski interval. Introducing the four-dimensional contrainvariant radius vector by

$$\left(x^0,x^1,x^2,x^3\right)=(c\tilde{t},\tilde{x},\tilde{y},\tilde{z})=\frac{1}{\lambda(k)}(ct-kx,x,y,z) \tag{26}$$

we define the contrainvariant four-velocity vector as

$$u^i=\frac{dx^i}{d\tilde{s}} \tag{27}$$

where the superscript i runs from 0 to 3. Using (26) and (20) in (27) yields

$$\left(u^0,u^1,u^2,u^3\right)=\frac{1}{Q(k,\beta_x.\beta)}\left(1-k\beta_x,\beta_x,\beta_y,\beta_z\right) \tag{28}$$

where $Q(k,\beta_x.\beta)$ is defined by (21). Correspondingly, covariant four-dimensional radius-vector and velocity vector are defined by

$$(x_0,x_1,x_2,x_3)=(c\tilde{t},-\tilde{x},-\tilde{y},-\tilde{z}), \tag{29}$$

$$(u_0,u_1,u_2,u_3)=\frac{1}{Q(k,\beta_x.\beta)}\left(1-k\beta_x,-\beta_x,-\beta_y,-\beta_z\right) \tag{30}$$

and the following relations hold

$$dx_i dx^i = d\tilde{s}^2 \tag{31}$$

$$u^i u_i = 1 \tag{32}$$

where a common rule of summation over repeated indexes is assumed. Next, recalling that the momentum four-vector is defined by

$$p_i = -\frac{\partial S}{\partial x^i} \tag{33}$$

and using the principle of the least action [37] we find (see [28] for details) that

$$p_i = mcu_i \tag{34}$$

while the contravariant components of the four-momentum vector are

$$p^i = mcu^i \tag{35}$$

Then from the identity (32) we get

$$p_i p^i = m^2 c^2 \tag{36}$$

Recalling that

$$P_x = \frac{\partial S}{\partial x}, \quad P_y = \frac{\partial S}{\partial y}, \quad P_z = \frac{\partial S}{\partial z}, \quad E = -\frac{\partial S}{\partial t} \tag{37}$$

with allowance for (26) and (33), we have

$$\begin{aligned} P_x &= \frac{1}{\lambda(k)}\left(\frac{\partial S}{\partial x^1} - k\frac{\partial S}{\partial x^0}\right) = \frac{kp_0 - p_1}{\lambda(k)}, \quad P_y = \frac{1}{\lambda(k)}\frac{\partial S}{\partial x^2} = -\frac{p_2}{\lambda(k)}, \\ P_z &= \frac{1}{\lambda(k)}\frac{\partial S}{\partial x^3} = -\frac{p_3}{\lambda(k)}, \quad E = -\frac{c}{\lambda(k)}\frac{\partial S}{\partial x^0} = \frac{cp_0}{\lambda(k)} \end{aligned} \tag{38}$$

which, upon using (34) and (30), yields the relations (24) and (25) for the three-momentum and energy. Solving equations (38) for the components of the four-momentum vector we get

$$p_0 = \frac{E\lambda(k)}{c}, \quad p_1 = \lambda(k)\left(\frac{kE}{c} - P_x\right), \quad p_2 = -\lambda(k)P_y, \quad p_3 = -\lambda(k)P_z \tag{39}$$

Then using (39) in (36) yields a dispersion relation for a free particle which can be represented in the form

$$\left(\frac{E}{c^{(+)}} - P_x\right)\left(\frac{E}{c^{(-)}} + P_x\right) = P_y^2 + P_z^2 + \frac{m^2 c^2}{\lambda(k)^2} \tag{40}$$

where the speeds of light $c^{(+)}$ and $c^{(-)}$ in the positive and negative x -directions are defined by equation (2). It follows from (40) that for massless particles moving along the x -axis in the positive x direction

$$P_x = \frac{E}{c^{(+)}} = \frac{E(1+k)}{c} \tag{41}$$

while for massless particles moving in the negative x direction

$$P_x = -\frac{E}{c^{(-)}} = -\frac{E(1-k)}{c} \tag{42}$$

3.2 Electromagnetic field equations

The invariant action integral for a charged material particle in the electromagnetic field is made up of two parts: the action for the free particle defined by (19) and a term describing the interaction of the particle with the field. The invariance is provided by using the combinations that are invariant in the Minkowskian variables (26) so that the action integral takes the form [37]

$$S = \int_a^b \left(-mcd\tilde{s} - \frac{e}{c}A_i dx^i\right) \tag{43}$$

where the coordinates x^i are related to physical coordinates (t, x, y, z) by (26) and A_i are components of the (covariant) four-potential vector expressed through the contravariant components A^i by

$$(A_0, A_1, A_2, A_3) = \left(A^0, -A^1, -A^2, -A^3\right) \tag{44}$$

Upon representing the four-potential as

$$\left(A^0, A^1, A^2, A^3\right) = \left(\tilde{\phi}, \widetilde{\boldsymbol{A}}\right) = \left(\tilde{\phi}, \widetilde{A}_x, \widetilde{A}_y, \widetilde{A}_z\right) \tag{45}$$

where $A^0 = \tilde{\phi}$ is a scalar potential and the three-dimensional vector $\widetilde{\boldsymbol{A}}$ is the vector potential of the field, the electromagnetic part of the action integral can be written in the form

$$S = \int_{\tilde{t}_1}^{\tilde{t}_2} \left(\frac{e}{c}\tilde{\boldsymbol{A}} \cdot \tilde{\boldsymbol{v}} - e\tilde{\phi}\right) d\tilde{t} \tag{46}$$

Here and in what follows, 'tilde' indicates that variables and operations are in Minkowskian space-time variables (26). Note that, while scalars and components of three-dimensional vectors in the Minkowskian formulation appear with 'tilde', four-dimensional Minkowskian variables are not supplied with 'tilde'. It does not lead to any confusion since the four-dimensional notation does not applicable to the formulation in physical variables.

In the electrodynamics of the standard special relativity (which, in our case, is electrodynamics in Minkowskian variables), the electric and magnetic field intensities are defined based on equations of motion of a charged particle obtained from the Lagrange equations.

$$\frac{d}{d\tilde{t}}\left(\frac{\partial \tilde{L}}{\partial \tilde{\boldsymbol{v}}}\right) = \frac{\partial \tilde{L}}{\partial \tilde{\boldsymbol{r}}} \tag{47}$$

where, in the Lagrange function $\tilde{L}$, a part related to the electromagnetic field is given by the integrand of (46). Then the electric and magnetic field intensities $\widetilde{\boldsymbol{E}}$

and $\widetilde{\boldsymbol{H}}$ are introduced by separating the right-hand side of the vector equation of motion (the force) into two parts, one of which does not depend on the velocity of the particle and the second depends on the velocity, being proportional to the velocity and perpendicular to it, as follows.

$$\frac{d\tilde{\boldsymbol{p}}}{d\tilde{t}} = e\widetilde{\boldsymbol{E}} + \frac{e}{c}\tilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{H}} \tag{48}$$

where $\tilde{\boldsymbol{p}}$ is the momentum vector. The electric and magnetic field intensities are related to the potentials by

$$\widetilde{\boldsymbol{E}} = -\frac{1}{c}\frac{\partial \widetilde{\boldsymbol{A}}}{\partial \tilde{t}} - \widetilde{\text{grad}}\ \tilde{\phi}; \quad \widetilde{\boldsymbol{H}} = \widetilde{\text{curl}}\ \widetilde{\boldsymbol{A}} \tag{49}$$

The same line of arguments is used to derive equations describing the electromagnetic field in physical variables (t, x, y, z). The action integral is represented in the form

$$S = \int_{t_a}^{t_b} L dt \tag{50}$$

where t is the 'physical' time related to the Minkowskian variables via (26) and L is the Lagrangian in physical variables. The free particle part of L is defined by Eqs. (21)–(23). To obtain the electromagnetic field part of the Lagrangian, the right-hand side of (46) is transformed to physical space-time variables and then the new variables (ϕ, A_x, A_y, A_z) (modified potentials) are introduced by the relations

$$\begin{aligned} &A^0 = \widetilde{\phi} = \lambda(k)\phi,\ A^1 = \widetilde{A}_x = \lambda(k)(A_x - k\phi), \\ &A^2 = \widetilde{A}_y = \lambda(k)A_y,\ A^3 = \widetilde{A}_z = \lambda(k)A_z \end{aligned} \tag{51}$$

As the result, the Lagrangian function L in the action integral (50) takes the form

$$L = L_p + \frac{e}{c}\left(v_x A_x + v_y A_y + v_z A_z\right) - e\phi \tag{52}$$

where L_p is the free particle part of L defined by Eqs. (21)–(23). Substituting (52) into the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{v}}\right) = \frac{\partial L}{\partial \boldsymbol{r}} \tag{53}$$

yields

$$\frac{d\boldsymbol{p}}{dt} = -\frac{e}{c}\frac{\partial \boldsymbol{A}}{\partial t} - e\ \text{grad}\ \phi + \frac{e}{c}\boldsymbol{v} \times \text{curl}\ \mathbf{A} \tag{54}$$

Thus, upon using the modified potentials, equations of motion in physical variables have the same form as in the standard relativity and the physical electric and magnetic field intensities are expressed through the modified potentials by the relations

$$\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} - \text{grad}\ \phi; \quad \mathbf{H} = \text{curl}\ \mathbf{A} \tag{55}$$

of the same form (49) as in the standard relativity.

It is evident that the first pair of the Maxwell equations in physical variables, which is derived from Eq. (55), have the same form as in the standard relativity

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t}; \quad \text{div } \mathbf{H} = 0 \tag{56}$$

To obtain the second pair of Maxwell equations in physical variables let us calculate the components of the electromagnetic field tensor F_{ik} defined by

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k} \tag{57}$$

Expressing A_i in the right-hand side of (57) through the modified potentials by (51) and then transforming the result to physical space-time variables using (26), with subsequent use of Eq. (55), yields the expressions for the components F_{ik} of the electromagnetic field tensor in terms of physical electric and magnetic field intensities. The result can be written as a matrix in which the index $i = 0, 1, 2, 3$ labels the rows, and the index k the columns, as follows

$$F_{ik} = \lambda(k)^2 \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -H_z + kE_y & H_y + kE_z \\ -E_y & H_z - kE_y & 0 & -H_x \\ -E_z & -H_y - kE_z & H_x & 0 \end{pmatrix} \tag{58}$$

while

$$F^{ik} = \lambda(k)^2 \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -H_z + kE_y & H_y + kE_z \\ E_y & H_z - kE_y & 0 & -H_x \\ E_z & -H_y - kE_z & H_x & 0 \end{pmatrix} \tag{59}$$

Note that the terms with k in the expressions (58) and (59) spoil the property, that $F^{ik} \to F_{ik}$ when $\boldsymbol{E} \to -\boldsymbol{E}$, of the standard relativity electrodynamics.

The electromagnetic field equations are obtained with the aid of the principle of least action [37] in the form

$$\frac{\partial F^{ik}}{\partial x^k} = 0 \tag{60}$$

(only fields in a vacuum, that are relevant to the subject of this paper, are considered). Substituting (59) into (60) and transforming the equations to physical space-time variables, upon combining equations with different 'i' and using the first pair of the Maxwell Eq. (56), yields the second pair of the Maxwell equations in the three-dimensional form

$$\text{div } \mathbf{E} = -\frac{k}{c}\frac{\partial E_x}{\partial t}; \quad \text{curl } \mathbf{H} = \left(1 - k^2\right)\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - 2k\frac{\partial \boldsymbol{E}}{\partial x} + k \text{ grad } E_x \tag{61}$$

An important feature of Eq (61) is their linearity in $\boldsymbol{E}$ and $\boldsymbol{H}$ and hence in A^i. The Lorentz-violating terms thereby avoid the complications of nonlinear modifications to the Maxwell equations, which are known to occur in some physical situations such as nonlinear optics or when vacuum polarization effects are included. Another

feature is that the extra Lorentz-violating terms involve only the electric field, as well as its derivatives.

Note the existence of an alternative way of the derivation of the modified Maxwell Eqs. (56) and (61). Based on Eqs. (49), (51), and (55), the electric and magnetic field intensities $\widetilde{\boldsymbol{E}}$ and $\widetilde{\boldsymbol{H}}$ in Minkowskian formulation can be related to the physical electric and magnetic field intensities $\boldsymbol{E}$ and $\boldsymbol{H}$, as follows

$$\begin{aligned} &\widetilde{E_x} = \lambda(k)^2 E_x,\ \widetilde{E_y} = \lambda(k)^2 E_y,\ \widetilde{E_z} = \lambda(k)^2 E_z, \\ &\widetilde{H_x} = \lambda(k)^2 H_x,\ \widetilde{H_y} = \lambda(k)^2 \big(H_y + kEz\big),\ \widetilde{H_z} = \lambda(k)^2 \big(H_z - kE_y\big) \end{aligned} \tag{62}$$

The same relations are seen in the expressions (58) for the components of the electromagnetic field tensor. It is readily verified that substituting the relations (62) into the Maxwell equations of the standard relativity

$$\widetilde{\text{curl}}\ \widetilde{\boldsymbol{E}} = -\frac{1}{c}\frac{\partial \widetilde{\boldsymbol{H}}}{\partial \widetilde{t}},\ \widetilde{\text{div}}\ \widetilde{\boldsymbol{H}} = 0, \quad \widetilde{\text{curl}}\ \widetilde{\boldsymbol{H}} = \frac{1}{c}\frac{\partial \widetilde{\boldsymbol{E}}}{\partial \widetilde{t}},\ \widetilde{\text{div}}\ \widetilde{\boldsymbol{E}} = 0 \tag{63}$$

as

$$\widetilde{E_x}(\tilde{t},\tilde{x},\tilde{y},\tilde{z}) = \lambda(k)^2 E_x(t(\tilde{t},\tilde{x}), x(\tilde{x}), y(\tilde{y}), z(\tilde{z})),\ \ \ldots \tag{64}$$

where

$$t(\tilde{t},\tilde{x}) = \lambda(k)\left(\tilde{t} + \frac{k}{c}\tilde{x}\right),\ x(\tilde{x}) = \lambda(k)\tilde{x}.,\ y(\tilde{y}) = \lambda(k)\tilde{y},\ z(\tilde{z}) = \lambda(k)\tilde{z} \tag{65}$$

yields the modified Maxwell Eqs. (56) and (61).

3.3 Electromagnetic waves

Like the electromagnetic wave equation of the standard relativity electrodynamics, the equation describing electromagnetic waves in the electrodynamics of the relativity with a preferred frame can be derived straight from the modified Maxwell equations (reproduced below for convenience)

$$\text{div}\ \mathbf{H} = 0, \quad \text{curl}\ \mathbf{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t} \tag{66}$$

$$\text{div}\ \mathbf{E} = -\frac{k}{c}\frac{\partial E_x}{\partial t}; \quad \text{curl}\ \mathbf{H} = \big(1-k^2\big)\frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} - 2k\frac{\partial \boldsymbol{E}}{\partial x} + k\ \text{grad}\ E_x \tag{67}$$

Eliminating $\boldsymbol{H}$ by taking 'curl' from the second equation of (66) and substituting curl $\mathbf{H}$ from the second equation of (67), with the subsequent use of differential consequences of the first equation of (67) for eliminating mixed space derivatives, yields

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} - \big(1-k^2\big)\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} + 2k\frac{1}{c}\frac{\partial^2 f}{\partial t\ \partial x} = 0 \tag{68}$$

where $f(t,x,y,z)$ stands for any component of $\boldsymbol{E}$. It is readily verified that the wave equation for $\boldsymbol{H}$ obtained from the modified Maxwell equations in a similar way has the same form (68).

Alternatively, the wave Eq. (68) can be derived from (60) expressed in terms of the potentials using (57) while imposing the Lorentz gauge condition

$$\frac{\partial A^k}{\partial x^k} = 0 \tag{69}$$

Converting the derivatives in the resulting equation

$$\frac{\partial^2 A^k}{\partial x_k \ \partial x^k} = 0 \tag{70}$$

into derivatives in physical space-time variables yields equations of the form (68) with f being any component of A^k. Given the fact, that equations (51) and (55) relating A^k to the modified potentials $(\phi, \boldsymbol{A})$ and then to $\boldsymbol{E}$ and $\boldsymbol{H}$ are linear, it is evident that any of those variables obeys Eq. (68).

Much of the propagation behavior of the electromagnetic wave is encoded in its dispersion relation, which provides spectral information for the modes. To find the dispersion relation the ansatz in the form of monochromatic plane waves is used, as follows

$$f(t,x,y,z) = f_a(\omega, \boldsymbol{q}))\, exp\left[i\ \left(q_x x + q_y y + q_z z - \omega t\right)\right] \tag{71}$$

where ω and $\boldsymbol{q} = \left(q_x, q_y, q_z\right)$ can be regarded as the frequency and wave vector of the mode or as the associated energy and momentum (taking the real part is understood, as usual). Substituting (71) into (68) yields the dispersion relation

$$c^2 q^2 - 2ckq_x\omega - \left(1-k^2\right)\omega^2 = 0 \quad \text{where } q^2 = q_x^2 + q_y^2 + q_z^2 \tag{72}$$

The dispersion relation (72) can be also represented in the form

$$\left(\frac{\omega}{c^{(+)}} - q_x\right)\left(\frac{\omega}{c^{(-)}} + q_x\right) = q_y^2 + q_z^2 \tag{73}$$

where $c^{(+)}$ and $c^{(-)}$ are defined by (2). The form (73) adheres to the dispersion relation (40) for free massless particles with E and $\boldsymbol{P}$ replaced by ω and $\boldsymbol{q}$. In the standard relativity, the polynomial (72) determining ω reduces to one with two quadruply degenerate roots $\omega = \pm cq$ which correspond to the opposite directions of the group velocity. In the modified electrodynamics, the polynomial also has two roots

$$\omega = c\frac{-kq_x + \sqrt{\left(1-k^2\right)q^2 + k^2 q_x^2}}{1-k^2}, \quad \omega = c\frac{-kq_x - \sqrt{\left(1-k^2\right)q^2 + k^2 q_x^2}}{1-k^2} \tag{74}$$

Like as in the standard relativity case, the two roots (74) are obtained from each other by changing the sign of ω but, in the case of $k \neq 0$, it is accompanied by a change of sign of the anisotropy parameter k.

More insight about the wave motion implied by Eq. (68) can be gained from the modified Maxwell Eqs. (66) and (67). Eq. (66), which are unaffected by the modifications, reduce with the ansatz (71) to

$$\boldsymbol{q} \cdot \boldsymbol{H} = 0, \quad \omega\, \boldsymbol{H} = -\boldsymbol{q} \times \boldsymbol{E} \tag{75}$$

The first of these equations shows that the magnetic field remains transverse to $\boldsymbol{q}$ despite the Lorentz violation. The second equation shows that the magnetic field $\boldsymbol{H}$ is perpendicular to the electric field $\boldsymbol{E}$. The first equation of (67) reduces to

$$\boldsymbol{q}\cdot\boldsymbol{E}=\omega\frac{k}{c}E_x \tag{76}$$

Eq. (76) implies the existence of two modes.

The first one corresponds to the electric field with $E_x=0$. Then it follows from (76) that the electric field is perpendicular to $\boldsymbol{q}$. Further, the condition $E_x=0$ implies that the vector $\boldsymbol{E}$ lies in the plane (y,z) and so the vector $\boldsymbol{q}$ is directed along the x -axis (the direction of the anisotropy vector $\boldsymbol{k}$). Therefore $H_x=0$ and also, based on rotational symmetry in the plane (y,z), it can be set $H_z=0$ which implies $E_y=0$. In such a case, the first equation of (66) shows that $q_y=0$ and the first equation of (66) shows that $q_z=0$. Then the second equation of (66) and the second equation of (67) reduce to the system of equations for the two nonzero components of the electric and magnetic field intensities E_z and H_y while the requirement of vanishing the determinant of the system yields the dispersion relation (72). Thus, the mode with $E_x=0$ represents a usual electromagnetic plane wave with the magnetic and electric fields transverse to the direction of propagation of the wave $\boldsymbol{q}$ and perpendicular to each other, which propagates along the direction of the anisotropy vector (but with the modified dispersion relation).

The second mode corresponds to the case $E_x\neq 0$. Then it follows from (76) that the electric field vector is not normal to $\boldsymbol{q}$. Since, according to the second equation of (75), $\boldsymbol{H}$ is normal to the plane of $\boldsymbol{E}$ and $\boldsymbol{q}$, one can choose, without loosing generality, the direction of $\boldsymbol{H}$ to be along the y -axis and the plane of the vectors $\boldsymbol{q}$ and $\boldsymbol{E}$ to be the (x,z) -plane. Then the first equation of (75) gives $q_y=0$ and it is readily verified that the remaining equations of (66) and (67) can be satisfied only if $q_z\neq 0$ with ω, q_x and q_z obeying the dispersion relation (72) where it is set $q_y=0$. Note the particular case, when $\boldsymbol{E}$ is directed along the x -axis ($E_z=0$), in which the dispersion relation degenerates to

$$\omega=\frac{cq_x}{k},\quad q_z=\pm\frac{q_x}{k} \tag{77}$$

Thus, the second mode represents electromagnetic wave, in which the magnetic field $\boldsymbol{H}$ is transverse to direction of propagation $\boldsymbol{q}$ and perpendicular to the electric field $\boldsymbol{E}$, like as in the regular wave, but, as distinct from the regular wave, the electric field is not normal to $\boldsymbol{q}$. Another characteristic feature of such a wave, that distinguishes it from the first mode, is that the direction of propagation is not along the anisotropy vector $\boldsymbol{k}$ and so not along with the velocity of relative motion of the source and the observer. It implies that in the case when the relative motion velocity is only the cosmological recession velocity, such a wave propagates not along a line of sight.

It is worthwhile to note a distinguishing feature of the above analysis as compared with other studies of electromagnetic waves in the presence of the Lorentz violation. Typically, different modes arising due to the Lorentz violation correspond to different roots of the modified dispersion relation (see, e.g., [6, 38–40]). The present analysis provides an unusual example when two different modes correspond to the same root of the dispersion relation (for the waves propagating to the observer. it is the second root of (74)). The existence of two modes is revealed only when one studies the corresponding solutions of the modified Maxwell equations. It is worth also noting that the present analysis is performed solely in terms of field intensities $\boldsymbol{E}$ and $\boldsymbol{H}$ while most studies of electromagnetic waves in the presence of the Lorentz violation involve also the electromagnetic field potentials A^k which are accompanied by extensive discussions of different gauge choices and their influence on the results.

4. Cosmology

4.1 General relativity

The basic principle of general relativity, the Equivalence Principle (see, e.g. [41]), which asserts that at each point of spacetime it is possible to choose a 'locally inertial' coordinate system where objects obey Newton's first law, is valid independently of the law of propagation of light assumed. In other terms, it can be applied when the processes in the locally inertial frame are governed by the laws of 'relativity with a preferred frame'. Based on that there exists the invariant combination (7), which by the change of variables (9) is converted into the Minkowski interval, one can state that the general relativity equations in arbitrary coordinates (x^0, x^1, x^2, x^3) are valid if the locally inertial coordinates $(\xi^0, \xi^1, \xi^2, \xi^3)$ are

$$\xi^0 = c\tilde{t},\ \xi^1 = \tilde{x},\ \xi^2 = \tilde{y},\ \xi^3 = \tilde{z} \tag{78}$$

where $\tilde{t}, \tilde{x}, \tilde{y}$ and $\tilde{z}$ are defined by (9). In these variables, the invariant spacetime distance squared $ds^2 = g_{ik}dx^i dx^k$ is equal to $d\tilde{s}^2 = \eta_{ik}d\xi^i d\xi^k$ (the notation η_{ik} is used for the Minkowski metric and the rule of summation over repeated indices is implied). Thus, the apparatus of general relativity is applied in the coordinates (x^0, x^1, x^2, x^3) while, in the calculation of the 'true' time and space intervals, the 'physical' variables (t^*, x^*, y^*, z^*) (it is the new notation for what was before (t, x, y, z)) are to be used. Eq. (9) relating the physical coordinates to the 'locally inertial' coordinates, rewritten with allowance for (78) and (9), are

$$t^* = \frac{1}{c}\lambda(k)\left(\xi^0 + k\xi^1\right),\ x^* = \lambda(k)\xi^1,\ y^* = \lambda(k)\xi^2,\ z^* = \lambda(k)\xi^3 \tag{79}$$

The 'true' time and space intervals can be determined using a procedure similar to that described in [37]. Applying that procedure (see [27] for details) yields the following relations for the 'true' proper time interval dt^* and the element dl^* of 'the true' spatial distance:

$$dt^* = \frac{1}{c}\lambda(k)\sqrt{g_{00}}dx^0 \tag{80}$$

$$dl^* = \lambda(k)\sqrt{\gamma_{\alpha\beta}dx^\alpha dx^\beta},\quad \gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}} \tag{81}$$

where g_{ik} $(i, k = 0, 1, 2, 3)$ are components of the space-time metrical tensor and $\gamma_{\alpha\beta}$ $(\alpha, \beta = 1, 2, 3)$ are components of the space metrical tensor. It is important to note, that the expression for the proper velocity of a particle $v = dl^*/dt^*$ is not modified, since the time and the distance intervals are modified by the same factor $\lambda(k)$.

4.2 Cosmological models

Modern cosmological models assume that, at each point of the universe, the 'typical' (freely falling) observer can define the (preferred) Lorentzian frame in which the universe appears isotropic. The metric derived based on isotropy and homogeneity (the *Robertson-Walker metric*) has the form [41, 42]

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1 - K_c r^2} + r^2 d\Omega\right),\quad d\Omega = d\theta^2 + \sin^2\theta d\phi^2 \tag{82}$$

where a comoving reference system, moving at each point of space along with the matter located at that point, is used. This implies that the coordinates (r, θ, ϕ) are unchanged for each typical observer. In (82), and further throughout this section, the system of units in which the speed of light is equal to unity, is used. The time coordinate $x^0 = t$ is the synchronous proper time at each point of space. The constant K_c (this notation is used, instead of common k or K, to avoid confusion with the symbols for the anisotropy parameter) by a suitable choice of units for r can be chosen to have the value $+1$, 0, or -1.

Introducing, instead of r, the radial coordinate χ by the relation $r = S(\chi)$ with

$$S(\chi) = \begin{cases} \sin\chi & \text{for } K_c = 1 \\ \sinh\chi & \text{for } K_c = -1 \\ \chi & \text{for } K_c = 0 \end{cases} \tag{83}$$

and replacing the time t by the *conformal time* η defined by

$$dt = a(t)d\eta \tag{84}$$

converts (82) into the form

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2 - S^2(\chi)\, d\Omega\right] \tag{85}$$

The information about the scale factor $a(t)$ in the Robertson-Walker metric can be obtained from observations of shifts in the frequency of light emitted by distant sources. The frequency shift can be calculated by considering the propagation of a light ray in isotropic space with the metric (85) adopting a coordinate system in which we are at the center of coordinates $\chi = 0$ and the source is at the point with a coordinate $\chi = \chi_1$. A light ray propagating along the radial direction obeys the equation $d\eta^2 - d\chi^2 = 0$. For a light ray coming toward the origin from the source, that equation gives

$$\chi_1 = -\eta_1 + \eta_0 \tag{86}$$

where η_1 corresponds to the moment of emission t_1 and η_0 corresponds to the moment of observation t_0. The *red-shift parameter* z is defined by

$$z = \frac{\nu_1}{\nu_0} - 1 \tag{87}$$

where ν_0 is the observed frequency and ν_1 is the frequency of the emitted light which coincides with the frequency of a spectral line observed in terrestrial laboratories. Calculations within the framework of the relativity with a preferred frame (see details in [27]) lead to the relation

$$z = \frac{a(\eta_0)}{a(\eta_0 - \chi_1)B(\bar{\beta}_1)} - 1 \tag{88}$$

The relation expressing the *Luminosity Distance* d_L of a cosmological source in terms of its redshift z is one of the fundamental relations in cosmology. It has been exploited to get information about the time evolution of the expansion rate. In a matter-dominated cosmological model of the universe (Friedman-Robertson-Walker model) based on the standard GR, solving the gravitational field equations yields the luminosity distance-redshift relation of the form

$$d_L = H_0^{-1}\left(z + \frac{1}{2}\left(1 - q_0^{(D)}\right)z^2 + \cdots\right) \tag{89}$$

where the deceleration parameter $q_0^{(D)}$ is positive for all three possible values of the curvature parameter K_c which means that, in that model, the expansion of the universe is decelerating. However, recent observations of Type Ia supernovae (SNIa), fitted into the luminosity distance versus redshift relation of the form (89), corresponding to the deceleration parameter $q_0^{(D)} < 0$ which indicates that the expansion of the universe is accelerating. This result is interpreted as that the time evolution of the expansion rate cannot be described by a matter-dominated cosmological model. To explain the discrepancy within the context of general relativity and fit the theory to the SNIa data, the dark energy, a new component of the energy density with strongly negative pressure that makes the universe accelerate, is introduced (see, e.g., [42]).

In the relativity with a preferred frame, solving the modified GR equations for a matter-dominated model lead to the luminosity distance-redshift relation of the form, which allows fitting the results of observations with supernovae so that the acceleration problem can be naturally resolved—there is no acceleration and so no need in introducing the dark energy. Below, the calculations leading to the modified luminosity distance-redshift relation are outlined (for more details see [27]).

In the relativity with a preferred frame, the expression for d_L is obtained in the form [27]

$$d_L = a(\eta_0)(1+z)S(\chi_1) \tag{90}$$

which coincides with a common form of the relation for d_L [37, 42]. Nevertheless, even though it does not contain the factor $B(\bar{\beta}_1)$, the dependence of d_L on z obtained by eliminating χ_1 from Eqs. (90) and (88) will differ from the common one since the relation (88) for z does contain the factor $B(\bar{\beta}_1)$. To derive the dependence $d_L(z)$ in a closed-form using Eqs. (90) and (88), the function $a(\eta)$ determining the dynamics of the cosmological expansion it to be defined by solving the gravitational field equations of Einstein which requires to make some tentative assumptions about the cosmic energy density ρ and the form of equation of state giving the pressure p as a function of the energy density. The energy density $\rho(t)$ is usually assumed to be a mixture of non-relativistic matter with equation of state $p = 0$ and dark energy with equation of state $p = w\rho$ while ignoring the relativistic matter (radiation). In the commonly accepted ΛCDM model, the dark energy obeys the equation of state with $w = -1$ (vacuum) which is equivalent to introducing into Einstein's equation a cosmological constant Λ. Then the fundamental Friedmann equation, which is obtained as a consequence of the Einstein field equations, can be written in the form (see, e.g., [42])

$$(x')^2 = H_0^2 x^2\left(\Omega_\Lambda + \Omega_M x^{-3} + \Omega_K x^{-2}\right) \tag{91}$$

where

$$x(t) = \frac{a(t)}{a_0}, \quad a_0 = a(t_0) \tag{92}$$

and the parameters Ω_Λ, Ω_M and Ω_K are defined by

$$\Omega_\Lambda = \frac{\rho_{V0}}{\rho_c}, \quad \Omega_M = \frac{\rho_{M0}}{\rho_c}; \qquad \rho_c = \frac{3H_0^2}{8\pi G}, \quad \Omega_K = -\frac{K_c}{a_0^2 H_0^2} \tag{93}$$

where G is Newton's gravitational constant, ρ_{V0} and ρ_{M0} are the present energy densities in the vacuum and non-relativistic matter and ρ_c is the critical energy density. Being evaluated at $t = t_0$ Eq. (91) becomes

$$\Omega_\Lambda + \Omega_M + \Omega_K = 1 \tag{94}$$

The Friedmann Eq. (91) allows us to calculate the radial coordinate χ_1 of an object of a given redshift z. Eq. (86) defining χ_1 can be represented in the form

$$\chi_1 = \eta_0 - \eta_1 = \int_{\eta_1}^{\eta_0} d\eta = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0}\int_{x_1}^{1} \frac{dx}{x'x} \tag{95}$$

where x' is a function of x defined by the Friedmann Eq. (91) and $x_1 = a(t_1)/a_0$. Then using Eq. (91) in (95) yields

$$\chi_1 = \int_{x_1}^{1} \frac{dx}{a_0 H_0 x^2 \sqrt{\Omega_\Lambda + \Omega_M x^{-3} + \Omega_K x^{-2}}} \tag{96}$$

In the standard cosmology, Eq. (88) (with $B(\bar{\beta}_1) = 1$) provides a simple relation

$$x_1 = \frac{1}{1+z} \tag{97}$$

so that (96) becomes a closed-form relation for $\chi_1(z)$. For a 'concordance' model, which is the flat space ΛCDM model, $\Omega_K = 0$ and $\Omega_\Lambda = 1 - \Omega_M$, Eq. (96) can be represented in the form

$$\chi^c_{1m}(z_1) = \int_0^{z_1} \frac{dz}{\sqrt{1 - \Omega_M + \Omega_M (1+z)^3}}, \quad \chi^c_{1m} = \chi^c_1 a_0 H_0 \tag{98}$$

Here and in what follows, quantities with a superscript "c" refer to the concordance model, with the original notation secured for the corresponding quantities of the present model. Then the luminosity distance is calculated as

$$d^c_L(z_1) = \frac{1}{H_0}(1+z_1)\chi^c_{1m}(z_1) \tag{99}$$

In the framework of the present analysis, expressing χ_1 as a function of z_1 by combining Eqs. (96) and (88) becomes more complicated in view of the fact that $\bar{\beta}_1$, and so the factor $B(\bar{\beta}_1)$, depend on χ_1. We will outline the calculations for the case of a flat universe, $\Omega_K = 0$, which is also the assumption of the concordance model.[¶] With that assumption and the presumption, that in the cosmology based on the relativity with a preferred frame there is no need in introducing dark energy,

[¶] In the present model, this assumption is not obligatory. It is worthwhile to note that, despite what is frequently claimed, a flatness of the universe is not stated in modern cosmology. Given the fact, that there is no direct measurement procedure of the curvature of space independent of the cosmological model assumed, the flatness of the space is the result valid only within the framework of the ΛCDM model.

$\Omega_\Lambda = 0$, the relation $\chi_1(x_1)$ is obtained from (96) in an analytical form, which allows finding the dependence $x_1(\chi_1)$ by inverting the result, as follows

$$\chi_{1m} = 2(1 - \sqrt{x_1}) \quad \Rightarrow \quad x1 = \frac{1}{4}(\chi_{1m} - 2)^2, \quad \chi_{1m} = a_0 H_0 \chi_1 \tag{100}$$

The dependence $B(\chi_1)$, with the accuracy up to third order in χ_1, is given by [27]

$$B(\bar{\beta}_1(\chi_1)) = 1 - \frac{b}{2}\left(\chi_{1m}^2 + \Omega_M \chi_{1m}^3\right) \tag{101}$$

Substituting (101) and (100) into (88) reduces the problem to a transcendental equation for $\chi_{1m}(z_1)$, as follows

$$\frac{1}{4}(\chi_{1m}(z_1) - 2)^2(z_1 + 1)\left(1 - \frac{b}{2}\left(\chi_{1m}(z_1)^2 + \chi_{1m}(z_1)^3\right)\right) = 1 \tag{102}$$

Representing the solution of (102) as a series in z_1 yields

$$\chi_{1m}(z_1) = z_1 + \frac{1}{4}(-3 - 2b)z_1^2 + \frac{1}{8}(5 + 4b + 4b^2)z_1^3 \tag{103}$$

Then the relation $d_L(z_1)$, calculated from (90) with $S(\chi_1) = \chi_1$, is

$$d_L(z_1) = \frac{1}{H_0}\left(z_1 + \frac{1}{4}(1 - 2b)z_1^2 + \frac{1}{8}(4b^2 - 1)z_1^3\right) \tag{104}$$

To compare the results produced by the model with those, obtained from an analysis of type Ia supernova (SNIa) observations, one needs some fitting formulas for the dependence $d_L(z)$ derived from the observational data. It is now common, in an analysis of the SNIa data, to fit the Hubble diagram of supernovae measurements to the ΛCDM model (mostly, to the concordance model) and represent the results as constraints on the model parameters (see, e.g. [43]). Therefore, in what follows, a comparison of the results with the SNIa data is made by comparing the dependence $d_L(z)$ produced by the present model with $d_L^c(z)$ for the concordance model while using constraints on the parameter Ω_M^c from the SNIa data analysis. It is found that, for every value of Ω_M^c from the interval, defined by fitting the SNIa data to the concordance model, the parameter b can be chosen such that the dependence $d_L(z)$ coincided with $d_L^c(z)$ with a quite high accuracy (were graphically undistinguishable). An example is given in **Figure 1** where the dependence $d_L(z)$ for $\Omega_M = 1$ (flat universe), defined by Eq. (104), is plotted for three different values of b together with $d_L^c(z)$ of the concordance model. It demonstrates that there exists a value of b (in the present case it is $b = 0.672$) for which the deviation is negligible. As it was mentioned above, in the present model the assumption of the flat universe is not obligatory. Calculations for other values of Ω_M (remind that $\Omega_K = 1 - \Omega_M$) show that for every value of $\Omega_M > 0$ there exists the value of b, for which the deviation $d_L(z)$ from $d_L^c(z)$ is negligible. It is worth clarifying again that the above is intended to be a comparison of the dependence $d_L(z)$ yielded by the present model with that derived from the SNIa observations so that the dependence $d_L^c(z)$ for the 'concordance' model plays a role of a fitting formula for the SNIa data.

The Baryon Acoustic Oscillations (BAO) data are commonly considered as confirming the accelerated expansion and imposing constraints on the dark energy parameters. Applying the cosmological models based on the 'relativity with a preferred frame' to the interpretation of the BAO data provides an alternative view on

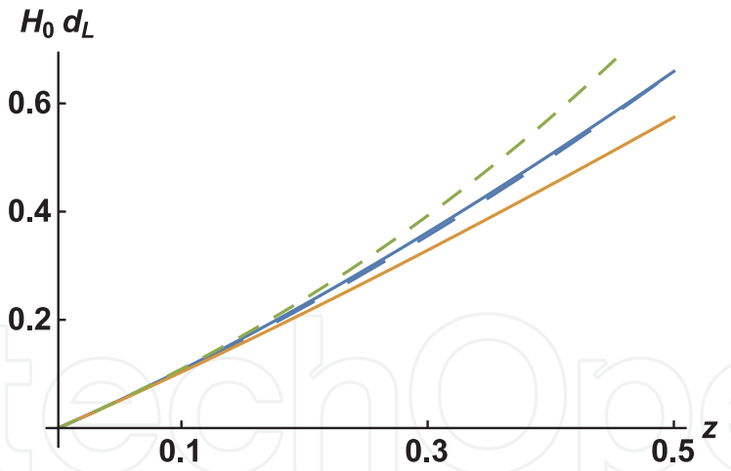


Figure 1.
Dependence of the luminosity distance d_L on the red-shift z: thin solid line for the concordance model with $\Omega_M^c = 0.31$; short-dashed for the present model with $\Omega_M = 1$, $b = -1.2$; long-dashed for the present model with $\Omega_M = 1$, $b = -0.672$; thick solid for the present model with $\Omega_M = 1$, $b = -0.2$.

the role of the BAO observations in cosmology. Comparing the predictions of the present model with the recently released galaxy clustering data set of the Baryon Oscillation Spectroscopic Survey (BOSS), part of the Sloan Digital Sky Survey III (SDSS III), shows that the BAO data can be well fit to the present cosmological model. The BAO data include two independent sets of data: the BAO scales in transverse and line-of-sight directions which can be interpreted to yield the comoving angular diameter distance $D_M(z)$ and the Hubble parameter $H(z)$ respectively. In [44], the results of several studies studying the sample provided by the BOSS data with a variety of methods are combined into a set of the final consensus constraints on $D_M(z)$ and $H(z)$ that optimally capture all of the information. It is found (see [27] for details) that the results yielded by the present model are consistent with the consensus constraints of [44] on both $D_M(z)$ and $H(z)$. The two regions in the plane (Ω_M, b) defined by constraints on these two sets are overlapped such that the overlapping area corresponds to the values of the model parameters for which the results on $H(z)$ and $D_M(z)$ are consistent both with the BAO data and with each other. And what can be considered as a very convincing proof of the robustness of the present model is that a line in the plane $(\Omega_M.b)$, on which the results produced by the present model fit also the SNIa observational data, passes inside that quite narrow overlapping region defined by the BAO data. Thus, the results produced by the present model fit three different sets of data by adjusting (together with the matter density parameter Ω_M) only one universal parameter b. It is worth noting again that, as distinct from the concordance model to which the SNIa and BAO data are commonly fitted by adjusting the dark energy parameters, the present model fits well all the data without introducing dark energy.

5. Propagation of cosmic rays

5.1 Attenuation of the UHECR due to the pion photoproduction process

In this section, the application of the theory to the description of the effects due to the interactions of the Ultra-High Energy Cosmic Rays (UHECR) with universal diffuse background radiation in the course of the propagation of cosmic rays from their sources to Earth over long distances (see, e.g., review articles [45–47]) is considered. The interactions of the UHECR with the CMB photons are characterized by a

well-defined energy threshold for the energy suppression due to pion photoproduction by UHECR protons—the Greisen-Zatsepin-Kuzmin (GZK) cutoff [48, 49]. The fluxes of cosmic ray protons with energies above this threshold would be strongly attenuated over distances of a few tens of Mpc so that the cosmic ray protons from the sources at a larger distance, even if they were accelerated to energies higher than the threshold, would not be able to survive the propagation. The energy position of the GZK cutoff can be predicted based on special relativity as a theoretical upper limit ('GZK limit') on the energy of UHECR set by pion photoproduction in the interactions of cosmic ray particles with the microwave background radiation. Calculating the GZK limit based on the particle dynamics of the special relativity with a preferred frame developed in Section 3.1 (see [28] for details) yields

$$\frac{Eth}{Est} = \left(1 - z^2\right)^{-b}; \quad Est = \frac{\varepsilon_\pi\left(2\varepsilon_p + \varepsilon_\pi\right)}{4E^{(\gamma)}} \tag{105}$$

where Eth is the threshold value of the UHECR protons energy calculated using equations of the relativity with a preferred frame, Est is the standard value of the GZK threshold calculated using equations of the standard relativity, $\varepsilon_p = m_p c^2$ and $\varepsilon_\pi = m_\pi c^2$ are the proton and pion rest energies and $E^{(\gamma)}$ is the CMB photon energy.

It is seen that the expression (105) for the threshold energy of the proton differs from the common one by the factor $\left(1 - z^2\right)^{-b}$. The universal constant b is negative, both as it is expected from intuitive arguments and as it is found by fitting the cosmological model developed in the framework of the 'relativity with a preferred frame' to the observational data (Section 4.2). Therefore the threshold energy decreases as the distance to the source of the particles (the redshift z) increases (**Figure 2**, left panel).

This effect may contribute to the interpretation of the data on the mass composition of UHECR which is a key observable in the context of the physics of UHECR as it fixes few fundamental characteristics of the sources. The mass composition of UHECR became a matter of active debate after that the Pierre Auger Collaboration (Auger) reported on its recent observations [50, 51]. The observations of Auger, far the largest experiment set-up devoted to the detection of UHECR, have shown that the UHECR mass composition is dominated by protons only at energies around and below 10^{18} eV and then the fraction of protons is progressively decreasing up to

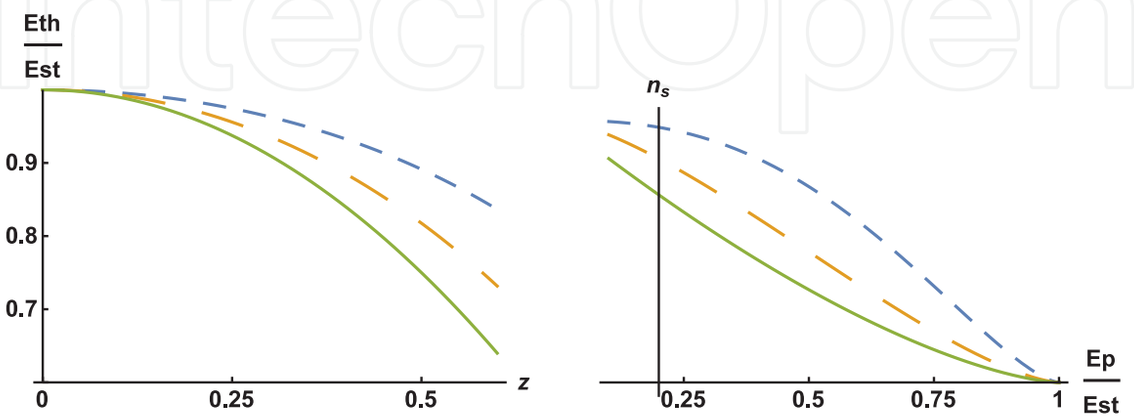


Figure 2.
Left panel: dependence of the correction factor to the GZK threshold on the source redshift z for different values of the parameter b: shot-dashed for $b = -0.4$; long-dashed for $b = -0.7$; solid for $b = -1$. Right panel: Number of sources n_s (in arbitrary units), that may contribute to the observed flux of protons at the energy Ep, versus $\frac{Ep}{Est}$, where Est is the standard GZK threshold value, for different values of b: shot-dashed for $b = -0.4$; long-dashed for $b = -0.7$; solid for $b = -1$.

energies of $10^{19.6}$ eV. It seemed to be not consistent with the general consensus, that UHECRs are mostly protons and that sources should accelerate them to $>10^{20}$ eV. At the same time, the Telescope Array (TA) experiment, even if with 1/10 of the Auger statistics, collected data seemed to confirm the pre-Auger scenario [52]. A common effort of the Auger and TA collaborations allowed to reconcile the interpretations of the Auger and TA observations so that the evidence for a composition becoming gradually heavier towards higher energies is now considered to be well established. It implies that the primary UHECR flux at the sources includes both protons and heavy nuclei which are to be accelerated with very high maximum injection energies. This imposes severe constraints on the parameters of the acceleration models and has served as a stimulus to build new acceleration models or reanimate the previously developed models that can potentially explain the phenomenology of the UHECR mass composition data. The models are characterized by a complex scenario and/or include some exotic assumptions.

The complexity of the scenario and the severe constraints on the model parameters, required in the case of a composition with heavy nuclei, are not present in the case if the UHECR mass composition is dominated by protons. In the latter case, the scenario is much simpler, only protons are accelerated with very high maximum injection energies. The view that the UHECR are mostly protons is, theoretically, a natural possibility. Proton is the most abundant element in the universe and several different astrophysical objects, at present and past cosmological epochs could provide efficient acceleration even if it requires very high luminosities and maximum acceleration energies. The models of interaction of UHECR with the astrophysical background are also much simpler if the UHECR are mostly protons. In this case, the only relevant astrophysical background is the CMB [53, 54]. This fact makes the propagation of UHE protons free from the uncertainties related to the background, being the CMB exactly known as a pure black body spectrum that evolves with red-shift through its temperature.

The results of the present study allow reconciling (at least, partially) the view, that, the primary UHECR flux at the sources is dominated by protons accelerated with very high maximum injection energies, with the observational evidence that the fraction of protons in the UHECR is decreasing towards higher energies. The apparent contradiction can be resolved by taking into account the effect, predicted by the present analysis, that the number of sources, which may contribute to the observed flux of protons at a given energy, is progressively decreasing with the energy increases. This effect is a consequence of the threshold condition (105) which implies that, among protons produced by a source at some z, only those having the energies lower than the threshold energy for that z, can reach the Earth. In other terms, for a given value Ep of the proton energy, there exists a value z_{th} of the redshift (distance D_{th}) such that, for the UHECR sources with $D > D_{th}$, the GZK threshold Eth is less than Ep and so the protons with the energy Ep injected by the sources at the distances $D > D_{th}$ cannot reach the Earth. Thus, the sources, that may contribute to the observed flux at the energy Ep, are confined within the sphere of the radius D_{th}, with D_{th} decreasing when the proton energy Ep is increasing. If the distribution of the sources in space is more or less uniform, the number of sources Ns, that may contribute to the observed flux at the energy Ep, decreases with Ep (**Figure 2**, right panel). Thus, reducing the fraction of protons in the observed UHECR flux towards the higher energies can be considered as the result of reducing the number of sources contributing to the flux.

5.2 Attenuation due to the pair-production process

Gamma rays (γ) propagating from distant sources to Earth interact with the photons of the extragalactic background light (γ_b) being able to produce e^+e^- through the process of pair production

$$\gamma \ + \ \gamma_b \rightarrow e^+ \ + \ e^- \tag{106}$$

which has the effect of a significant energy attenuation in the flux of high-energy gamma rays. Such interaction takes place for gamma rays with energies (E_γ) above the threshold of pair production. The existence of a threshold can be also expressed as the minimum energy ($E_{\gamma_b}^{th}$) that a γ_b needs to produce a e^+e^-.

The following assumptions should be made if we intend to calculate the threshold value of the energy of the gamma-rays photons:

i. It is needed to take the lowest energy the high-energy photon can have to react with the background photon to yield the two particles which correspond to the situation when they both are produced at rest in their center of mass frame after the collision.

ii. To maximize the energy available from the collision, the initial momenta of the two particles in the lab frame should be pointing in opposite directions.

Let us equate the square of the total 4—momentum $p^{(L)} = p^{(\gamma)} + p^{(\gamma_b)}$ in the lab frame before the collision with the square of the total 4—the momentum of the outgoing particles $p^{(CM)} = p^{(+)} + p^{(-)}$ in their center of mass frame after the collision

$$\left(p^{(\gamma)} + p^{(\gamma_b)}\right)^2 = \left(p^{(+)} + p^{(-)}\right)^2 \tag{107}$$

The right-hand side of (107) is calculated, as follows

$$\left(p^{(+)} + p^{(-)}\right)^2 = \left(p_0^{(+)} + p_0^{(-)}\right)^2 - \left(p_1^{(+)} + p_1^{(-)}\right)^2 - \left(p_2^{(+)} + p_2^{(-)}\right)^2 - \left(p_3^{(+)} + p_3^{(-)}\right)^2 \tag{108}$$

where Eq. (39) are to be substituted into (108), with the three-momentum and energy defined by equations (24), (25) and (21) in which it is set $\beta_x = \beta_y = \beta_z = 0$ for both particles. As the result, we obtain the following expression for the square of the total 4—momentum of outgoing particles

$$\left(p^{(+)} + p^{(-)}\right)^2 = c^2(m_e + m_e)^2 \tag{109}$$

Note that, although P_x does not vanish for $\beta_x = \beta_y = \beta_z = 0$, the component p_1 of the four-momentum does vanish since, in the expression (39) for p_1, the first term compensates the non-vanishing part of P_x.

The left-hand side of Eq. (107) is to be expressed in terms of the high-energy and background photons energies using the relations between the particle's momentum and energies obtained from the dispersion relation (40). The high-energy photons move to the observer, in the direction opposite to the direction the velocity of the lab frame relative to the observer (relative to the preferred frame) which is chosen to be a positive direction of the x-axis. So, the high-energy photon moves along the x-axis in the negative x direction while the background photon moves, according to the threshold assumption (ii), in the positive x direction. Thus, the momenta of the photons are related to their energies using Eq. (41), as follows

$$P_x^{(\gamma)} = -\frac{E_\gamma(1-k)}{c}, \quad P_x^{(\gamma_b)} = \frac{E_{\gamma_b}(1+k)}{c} \tag{110}$$

where k is the anisotropy parameter in the lab frame. Then the left-hand side of (107) is calculated as follows (head-on collision)

$$\left(p^{(\gamma)}+p^{(\gamma_b)}\right)^2=\left(p_0^{(\gamma)}+p_0^{(\gamma_b)}\right)^2-\left(p_1^{(\gamma)}+p_1^{(\gamma_b)}\right)^2$$
$$=\left(\frac{E_\gamma\lambda(k)}{c}+\frac{E_{\gamma_b}\lambda(k)}{c}\right)^2-\left(\lambda(k)\left(\frac{kE_\gamma}{c}-P_x^{(\gamma)}\right)+\lambda(k)\left(\frac{kE_{\gamma_b}}{c}-P_x^{(\gamma_b)}\right)\right)^2 \tag{111}$$

Substituting (110) for $P_x^{(\gamma)}$ and $P_x^{(\gamma_b)}$ into (111) yields

$$\left(p^{(\gamma)}+p^{(\gamma_b)}\right)^2=4\lambda(k)^2\frac{E_\gamma E_{\gamma_b}}{c^2} \tag{112}$$

Then using Eqs. (112) and (109) in (107) and solving the resulting equation for E_γ one obtains the expression for the threshold energy of the high-energy photon

$$E_\gamma^{th}=\frac{m_e^2c^4}{\lambda(k)^2E_{\gamma_b}} \tag{113}$$

or the expression for the threshold energy of the background photon (minimum energy to produce e^+e^-)

$$E_{\gamma_b}^{th}=\frac{m_e^2c^4}{\lambda(k)^2E_\gamma} \tag{114}$$

The factor $\lambda(k)$ can be represented as a function $B(\bar{\beta})$ of the frame velocity $\bar{\beta}$ relative to a preferred frame which, with an accuracy up to $(\bar{\beta})^3$, is given by the expression (see (17))

$$B(\bar{\beta})=\left(1-\bar{\beta}^2\right)^{b/2} \tag{115}$$

In a cosmological context, where $\bar{\beta}$ is a recession velocity of a source, $\bar{\beta}$ depends on the cosmological redshift of an object z. Although the expansion of $\bar{\beta}(z)$ in series, besides the leading term z, includes terms of the order z^2 and higher, they do not contribute to the expression for $\bar{\beta}^2$ up to the terms of the order z^3 and so, with the accuracy of the expression (115), $\bar{\beta}^2$ can be replaced by z^2. Then the threshold equation takes the form

$$\frac{E_{\gamma_b}^{th}}{E_{\gamma_b}^{Sth}}=\left(1-z^2\right)^{-b};\quad E_{\gamma_b}^{Sth}=\frac{m_e^2c^4}{E_\gamma} \tag{116}$$

where $E_{\gamma_b}^{th}$ is the modified value of the threshold and $E_{\gamma_b}^{Sth}$ is the standard value of the threshold. It is seen that the expression (116) for the threshold energy of the background photon differs from the standard one by the factor $(1-z^2)^{-b}$. The universal constant b is negative, both as it is expected from intuitive arguments and as it is found by fitting the cosmological model developed in the framework of the 'relativity with a preferred frame' to the observational data [27]. Thus, the threshold energy of the background photon decreases with the distance to the source (the redshift z).

Attenuation of gamma rays with the energy E_γ from the source at redshift z_s due to the pair production process is characterized by the optical depth $\tau_\gamma\left(E_\gamma, z_s\right)$. For z_s not too large one typically has $\tau_\gamma(E_0, z_s) < 1$ so that the Universe is optically thin along the line of sight of the source and if it happens that $\tau_\gamma(E_0, z_s) > 1$ the Universe becomes optically thick at some point along the line of sight. The value z_h such that $\tau_\gamma(E_0, z_s) = 1$ defines the γ-ray horizon for a given E_0, and sources beyond the horizon tend to become progressively invisible as z_s further increases. The optical depth is evaluated by

$$\tau_\gamma\left(E_\gamma, z_s\right) = \int_0^{l_s(z_s)} dl \; K_{\gamma\gamma_b}\left(E_\gamma, l(z)\right) \tag{117}$$

where $K_{\gamma\gamma_b}\left(E_\gamma, l(z)\right)$ is the γ-ray absorption coefficient, which represents the probability per unit path length, l, that a γ-ray will be destroyed by the pair-production process. The absorption coefficient is calculated by convolving the spectral number density $n_b\left(E_{\gamma_b}, z\right)$ of background photons at a redshift z with the cross section of the pair production process $\sigma\left(\left(E_\gamma, E_{\gamma_b}, \theta, z\right)\right.$ (θ is the angle between the direction of propagation of both photons) for fixed values of E_{γ_b} and θ and next integrating over these variables [55], as follows

$$K_{\gamma\gamma_b}\left(E_\gamma, z\right) = \int_{-1}^{1} d(\cos\theta) \; \frac{1 - \cos\theta}{2} \int_{E_{\gamma_b}^{th}}^{\infty} dE_{\gamma_b} \; n_b\left(E_{\gamma_b}, z\right) \sigma\left(\left(E_\gamma, E_{\gamma_b}, \theta, z\right)\right. \tag{118}$$

Then the integral over distance l in (117) is represented as an integral over z to arrive at the expression for the optical depth in the form

$$\tau_\gamma\left(E_\gamma, z_s\right) = \int_0^{z_s} dz \; \frac{dl(z)}{dz} \; K_{\gamma\gamma_b}\left(E_\gamma, z\right) \tag{119}$$

The threshold energy of background photons $E_{\gamma_b}^{th}$ taking part in the expressions (118) and (119) is corrected according to (116) such that $E_{\gamma_b}^{th}$ decreases with the distance to the source (the redshift z). The cumulative outcome of this phenomenon may result in measurable variations in the expected attenuation of the gamma rays flux reducing the expected flux.

The preferred frame effects may influence the optical depth also via the cosmological part of the expression (119). In the Robertson-Walker metric (82) (or (85)), the distance element dl is defined as $dl = a(t)d\chi$ where $a(t)$ is the scale factor and χ is the radial distance element defined by (83). These quantities are calculated based on the GR equations (more specifically, Friedman equations) which leads to the expression (96) for the radial distance χ where the parameters are to be specified according to the cosmological model accepted. Commonly the quantity $\frac{dl(z)}{dz}$ is calculated within the standard 'concordance' ΛCDM cosmological model, where the expression (96) is specified to $\Omega_K = 0$, $\Omega_\Lambda = 1 - \Omega_M$ and x_1 given by (97), which yields

$$\frac{dl(z)}{dz} = \frac{1}{H_0} \; \frac{1}{(z+1)\sqrt{1 - \Omega_M + \Omega_M(1+z)^3}} \tag{120}$$

In the cosmology of the relativity with a preferred frame, $\Omega_\Lambda = 0$ and $\Omega_K = 1 - \Omega_M$ and, upon using these values in (96), one has for $\frac{dl(z)}{dz}$ the following

$$\frac{dl(z)}{dz} = \frac{1}{H_0}\,\frac{a(t)}{a(t_0)}\,\frac{1}{\sqrt{1-\Omega_M+\Omega_M(1+z)}} \tag{121}$$

where the quantity $\frac{a(t)}{a(t_0)}$ is to be calculated using several other equations as it is done (for the particular case $\Omega_M = 1$) in equations from (100) to (104). Similar calculations for the general case $\Omega_M \neq 1$ lead to the expression for $\frac{dl(z)}{dz}$ represented as series in z, as follows

$$\frac{dl(z)}{dz} = \frac{1}{H_0}\left(1+\left(-2-b-\frac{\Omega_M}{2}\right)z+\left(3+3b+\frac{3b^2}{2}+\Omega_M+\frac{3\Omega_M^2}{8}\right)z^2 + \left(-3-4b-\frac{5b^2}{2}-\Omega_M-\frac{3\Omega_M^2}{8}\right)z^3\right) \tag{122}$$

In the concordance model relation (120), the value $\Omega_M = 0.31$. obtained from the observational data (see [27] for references), is used. In the present model, there is an interval of allowed values of Ω_M and the corresponding values of b, within which the results fit both the SNIa and BAO data [27]. The curvature K_c in the present model is not obligatory zero but the value of $\Omega_M = 1$ corresponding to the flat universe is within the interval of allowed values of Ω_M. Although Eqs. (120) and (122) defining dependence $\frac{dl(z)}{dz}$ on z in the concordance model and in the present model look completely different, the corresponding dependencies practically coincide as it is seen from **Figure 3**. Thus, the preferred frame effects influence $\tau_\gamma\left(E_\gamma, z_s\right)$ only via the threshold value $E_{\gamma_b}^{th}$ in (118), like in other Lorentz-violating theories (see, e.g., [56–58]).

5.3 Astrophysical tests for vacuum dispersion and vacuum birefringence

In the literature on Lorentz violation, as major features of the behavior of electromagnetic waves in vacuum in the presence of Lorentz violation, vacuum dispersion and vacuum birefringence are considered. Astrophysical tests for vacuum dispersion of light from astrophysical sources seek differences in the velocity of light at different wavelengths due to Lorentz violation which should result in observed arrival-time differences. For differences in the arrival times of different wavelengths to be interpreted as caused by differences in the light velocities, explosive or pulsed sources of radiation that produce light over a wide range of wavelengths in a short period, such as gamma-ray bursts, pulsars, or blazars, are to be used. All those are point sources, which have the disadvantage (to impose constraints on Lorentz violation) that a single line of sight is involved, which provides sensitivity to only a restricted portion of space for free coefficients of the Lorentz violating models.

The same is valid for the present theory leading to the dispersion relation (72). In the case of the waves propagating along the x-axis (aligned with the anisotropy vector k), when $q_y = q_z = 0$ and $q_x = q$, the two routes (74) become

$$\omega = \pm\frac{c}{1\pm k}q \tag{123}$$

which corresponds to the waves propagating in the opposite directions. For a wave propagating to the observer from a cosmological source, with the x-axis directed from the observer to the source, the group velocity is

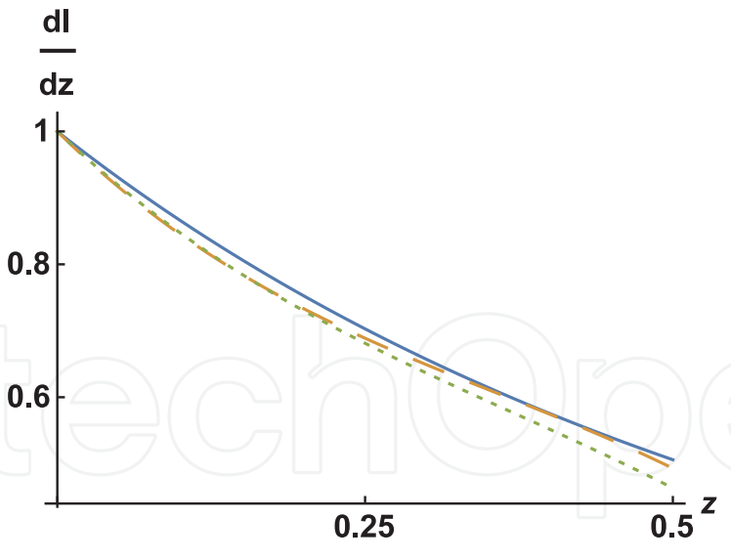


Figure 3.
The dependence of $\frac{dl(z)}{dz}$ (multiplied by H_0) on z for the concordance model with $\Omega_M = 0.31$ (solid) and for the cosmological model, based on 'relativity with a preferred frame' [27], with $\Omega_M = 1$, $b = -0.672$ (dashed) and $\Omega_M = 0.5$, $b = -0.495$ (dotted) where the values of the parameters Ω_M and b are chosen from those consistent with both the SNIa and BAO data (see [27]).

$$\frac{\partial \omega}{\partial q} = -\frac{c}{1-k} \tag{124}$$

It does not depend on q and so there is no place for vacuum dispersion.

Another test, that is commonly used for setting constraints on the parameters of the Lorentz-violating theories in electrodynamics, is the vacuum birefringence test. In birefringent scenarios, the two eigenmodes propagate at slightly different velocities. This implies that the superposition of the modes is altered as light propagates in free space. Since the two modes differ in polarization, the change in superposition causes a change in the net polarization of the radiation. However, it does not apply to the present theory leading to the dispersion relation (72). The two roots of the dispersion relation correspond to the waves propagating in different directions. Thus, no two eigenmodes are propagating in the same direction and so there is no possibility for vacuum birefringence. Thus, neither tests for vacuum dispersion nor tests for vacuum birefringence can impose restrictions, additional to those imposed by cosmological data, on the values of the only parameter of the theory b.

The vacuum birefringence and vacuum dispersion are widely discussed in the literature as astrophysical tests of Lorentz violation in the pure photon sector of the standard-model extension (e.g., [6, 38, 59–62]). Therefore it is of interest, in that context, to compare the Lorentz-violating terms, appearing in the Lagrangian due to the preferred frame effects in the present study, with those introduced as a formal SME extension. Extracted from the SME, the Lorentz-violating electrodynamics can be written in terms of the usual field strength F_{ik} defined by (57) and the potentials A^k, as follows

$$L = -\frac{1}{4}F_{ik}F^{ik} - \frac{1}{4}(k_F)_{nmik}F^{nm}F^{ik} + \frac{1}{2}(k_{AF})^n \varepsilon_{nmik}A^m F^{ik} \tag{125}$$

In what follows, we calculate the Lagrangian of the electrodynamics with a preferred frame and compare the Lorentz violating terms in that Lagrangian with those in (125). Calculating $L = -\frac{1}{4}F_{ik}F^{ik}$ using equations (58) and (59) yields

$$L = \lambda(k)^4\left(\frac{1}{2}\left(\boldsymbol{E}^2 - \boldsymbol{H}^2\right) + k\left(E_y H_z - E_z H_y\right) - k^2\frac{1}{2}\left(E_y^2 + E_z^2\right)\right) \tag{126}$$

It is seen that the form (126) is in a sense more general than (125) because of the Lorentz violating multiplier $\left(\lambda(k)^4\right)$. However, since the multiplier does not depend on the field variables and so does not influence the form of the field equations, it can be disregarded. Then the Lorentz-violating terms in (126) can be written based on (59) in terms of the field strength, as follows

$$L_{add} = k\left(F^{02}F^{12} + F^{13}F^{03}\right) - \frac{1}{2}k^2\left(F^{02}F^{02} + F^{03}F^{03}\right) \tag{127}$$

which fits the form (125) with the coefficients

$$(k_F)_{0212} = -4k,\ (k_F)_{0313} = -4k,\ (k_F)_{0202} = 2k^2,\ (k_F)_{0303} = 2k^2 \tag{128}$$

while other $(k_F)_{nmik}$ as well as all k_{AF} are zeros. The second term on the right-hand side of (125), not contributing to the Lagrangian of the present theory, could be disregarded from the beginning because it has theoretical difficulties associated with negative contributions to the energy [6, 38]. The Lagrangian defined by (127) (or (128)) provides an example of the Lorentz-violating SME (in a pure photon sector) which leads to equations of the electromagnetic wave propagation not exhibiting the vacuum birefringence and vacuum dispersion effects.

6. Discussion

The 'relativity with a preferred frame' incorporates the existence of the cosmological preferred frame into the framework of the theory while preserving fundamental principles of the SR: the principle of relativity and the principle of universality of the light propagation. The relativistic invariance is preserved in the sense, that the physical laws are covariant (their form does not change) under the group of transformations between inertial frames, and the relativistic symmetry is preserved (although modified) in the sense that there exists a combination, a counterpart of the interval of the standard relativity theory, which is invariant under the transformations. The existence of the modified symmetry provides an extension of the theory to general relativity such that the general covariance is also preserved. Thus, the 'relativity with a preferred frame' is a relativity theory, both in the special relativity and in the general relativity parts. Except for identifying the preferred frame with a comoving frame of cosmology, the theory does not include any assumptions. No approximations are involved besides approximating the universal function $k = F(\bar{\beta})$, defining dependence of the anisotropy parameter on the frame velocity relative to the preferred frame, by the expression $F(\bar{\beta}) = b\bar{\beta}$ valid up to the third order in $\bar{\beta}$. As the result, all the relations of the theory include only one universal parameter b.

The problem of defining allowed values of b is to be considered in the context of verification of the theory by observations since nothing in the theory itself imposes constraints on the values of b. Discussing the results of the application of the theory to natural phenomena, one can separate the conceptual and quantitative aspects. In the conceptual aspect, the cosmological models, developed using the modified general relativity, are of the most importance. First, it is related to the interpretation of the luminosity distance versus redshift relation deduced from the SNIa data, which

has played a revolutionary role in the development of modern cosmology concepts. That relation, corresponding to the negative deceleration parameter, cannot be explained using cosmological matter-dominated models (Friedman-Robertson-Walker models) based on the standard general relativity. To explain the data, in modern cosmology, dark energy, a new type of energy with a peculiar equation of state corresponding to negative pressure, is introduced. In the cosmology of the 'relativity with a preferred frame', the luminosity distance versus redshift relation for the matter-dominated cosmological model contains corrections, such that the effective deceleration parameter can be negative. As the result, neither the acceleration of the universe expansion nor the dark energy providing the acceleration is needed. The consistency of the cosmological models, based on the 'relativity with a preferred frame', is supported by that, for any reasonable value of the parameter Ω_M, there exists a value of b such that the luminosity distance versus redshift relation fits with high accuracy the SNIa data.

In the applications of the theory to the BAO data, the conceptual and quantitative aspects go together. The BAO observations provide two different sets of data: BAO scales in transverse and line-of-sight directions. Measurements of the angular distribution of galaxies yield the quantity $D_M(z)$ which is the comoving angular diameter distance. Measurements of the redshift distribution of galaxies yield the value of the Hubble parameter $H(z)$. The fact that the two regions in the plane (Ω_M, b), within which the predictions of the present theory fit the D_M data and the $H(z)$ data, are overlapped, both provides a support for the theory and places quite tight constraints on the values of the parameters Ω_M and b since they should be confined within a quite narrow overlapping region. An additional (and quite strong) argument in favor of both consistency of the theory and estimates for the parameter b is that the line in the plane (Ω_M, b), on which the results of the present model fit the SNIa data, lies within that narrow region. Thus, the results fit well three different sets of observational data with the values of the theory parameter b confined within a quite narrow interval (approximately from $b = -0.4$ to $b = -0.8$).

Next, it might be expected that some constraints on allowed values of b could arise as the result of applying the theory to the cosmic rays data. In the propagation of the Ultra-High Energy Cosmic Rays from distant sources to Earth, the most remarkable effect is the attenuation due to pion photoproduction by UHECR protons which is characterized by the GZK threshold. Applying the 'relativity with a preferred frame' to the calculation of the energy threshold for the attenuation process results in the correction factor to the GZK limit. Although a comparison of that prediction of the theory with the data on the UHECR flux does not straightforwardly lead to constraints on the values of b, another issue, namely the data on the mass composition of UHECR, provides indirect confirmation of the theory. Those data, showing that the UHECR mass composition is dominated by protons only at energies around and below 10^{18} eV and then the fraction of protons is progressively decreasing up to energies of $10^{19.6}$ eV, contradict the previous consensus that UHECRs are mostly protons accelerated in the sources to $>10^{20}$ eV. The prediction of the 'relativity with a preferred frame', that the GZK threshold energy decreases with the distance to the source of the particles (with the values of the parameter b defined by the cosmological data) allows to resolve, at least, partially, the contradiction between the view, that the primary UHECR flux is mostly protons accelerated to very high energies, and the observational data showing that the fraction of protons in the UHECR is decreasing towards higher energies. The explanation lies in that, because of decreasing the energy threshold with the distance to the source, the number of sources, contributing to the observed flux of protons at a given energy, should be progressively decreasing with the energy increasing.

Applying the modified particle dynamics to the pair-production process, which is responsible for attenuation of the gamma-rays flux, does not provide quantitative constraints on the values of the parameter b or indirect confirmations of the theory. At the same time, the results of applying the modified electromagnetic field dynamics to the behavior of electromagnetic waves in a vacuum maybe counted as a kind of indirect confirmation of the theory. The vacuum birefringence and vacuum dispersions are the features present in the popular Lorentz-violating theories (e.g., [6, 38, 59–62]) and the fact, that no indications of the existence of those phenomena are found in observations, imposes constraints on the values of numerous parameters of those theories. On the contrary, the electromagnetic field equations and based on them the electromagnetic wave equation of the present theory, although modified such that the Lorentz invariance is violated, does not predict such features as the vacuum birefringence and vacuum dispersion. Thus the absence of observational evidence for the existence of those phenomena may be considered as an argument in favor of the theory.

In general, the fact, that applying the theory containing only one universal parameter to several different phenomena does not lead to any contradictions, proves a consistency of its basic principles. The presence of only one parameter in the theory is a consequence of the fact that, as distinct from the popular Lorentz-violating theories, where Lorenz violation is introduced phenomenologically by adding Lorentz-violating terms to the Lorentz invariant relations, the 'relativity with a preferred frame' starts from the physically reasonable modification of the basic postulates of the SR. The generalized relativistic invariance, and so the Lorentz invariance violation, are ingrained in the theory at the most fundamental level being imbedded into the metric. It is also worth to emphasize that the conceptual basis of the theory has been developed without having in mind possible applications. It is aimed at designing the framework which would allow to incorporate the preferred frame into special relativity while retaining the relativity principle and the fundamental space-time symmetry. Nevertheless, the theory provides explanations of some observational data, that were regarded as puzzling after their discovery (like the SNIa luminosity distance-redshift relation indicating the acceleration of the universe and the absence of high energy protons in the UHECR flux). As the result, the concepts (among which dark energy is the most striking one), introduced to explain those puzzling features, become redundant. All the above justifies treating the 'relativity with a preferred frame' as an alternative to some currently accepted theories.

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