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# Spectroscopic Study of Baryons

*Zalak Shah and Ajay Kumar Rai*

## Abstract

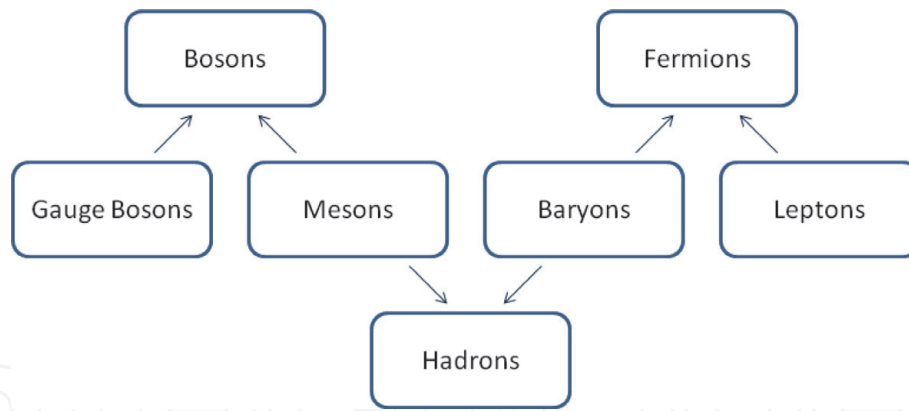
Baryons are the combination of three quarks(antiquarks) configured by  $qqq(\bar{q}\bar{q}\bar{q})$ . They are fermions and obey the Pauli's principal so that the total wave function must be anti-symmetric. The SU(5) flavor group includes all types of baryons containing zero, one, two or three heavy quarks. The Particle Data Group (PDG) listed the ground states of most of these baryons and many excited states in their summary Table. The radial and orbital excited states of the baryons are important to calculate, from that the Regge trajectories will be constructed. The quantum numbers will be determined from these slopes and intersects. Thus, we can help experiments to determine the masses of unknown states. The other hadronic properties like decays, magnetic moments can also play a very important role to emphasize the baryons. It is also interesting to determine the properties of exotic baryons nowadays.

**Keywords:** baryons, potential model, mass spectra, Regge trajectories, decays

## 1. Introduction

The particle physics has a remarkable track record of success by discovering the basic building blocks of matter and their interactions. Everything in the observed universe is found to be made from a few basic building blocks called *fundamental particles*, governed by four fundamental forces. All of these are encapsulated in the **Standard Model (SM)**. The Standard Model has been established in early 1970s and so far it is the most precise theory ever made by mankind. In the Standard Model elementary particles are considered to be the constituents of all observed matter. These elementary particles are quarks and leptons and the force carrying particles, such as gluons and W, Z bosons. All stable matter in the universe is made from particles that belong to the first generation; any heavier particles quickly decay to the next most stable level. The concept of the quark was first proposed by Murray Gell-Mann and George Zweig in 1937. The quarks have very strong interaction with each other, that is a reason, they always stuck inside composite system. But quarks interact with leptons weakly, the best example of this is protons, neutrons and electrons of atomic nuclei. The flavored quarks combine together in various aggregates called hadrons. The Standard Model includes 12 elementary particles of half-spin known as fermions. They follow the Pauli exclusion principle and each of them have a corresponding anti-particle. There are also twelve integer-spin particles which mediate interactions between these particles known as Bosons. They obey the Bose-Einstein statistics. The classification of bosons and fermions along with hadrons are drawn in **Figure 1**.

The Quantum Chromodynamics (QCD) as the theory of strong interactions was successfully used to explore spectroscopic parameters and decay channels of hadrons during last five decades. The interaction is governed by massless spin 1 objects



**Figure 1.**  
The classifications of particles.

called gluons. Quarks inside the hadrons exchange gluons and create a very strong color force field. To conserve color charge, quarks constantly change their color by exchanging gluons with other quarks. As the quarks within a hadron get closer together, the force of containment gets weaker so that it asymptotically approaches zero for close confinement. The quarks in close confinement are completely free to move about. This condition is referred to as “asymptotic freedom”. An essential requirements for the progress in hadronic physics is the full usage of present facilities and development of new ones, with a clear focus on experiments that provide genuine insight into the inner workings of QCD.

Hadron spectroscopy is a tool to reveal the dynamics of the quark interactions within the composite systems. The short-lived hadrons and missing excited states could be identified through the possible decays of the resonance state. The experimentally discovered states are listed in summary tables of Particle Data Group [1]. The worldwide experiments such as LHCb, BELLE, BARBAR, CDF, CLEO are main source of identification of heavy baryons so far and especially LHCb and Belle experiments have provided the new excited states in heavy baryon sector recently [2]. The various phenomenological approaches for spectroscopy is all about to use the potential and establish the excited resonances. These approaches are, relativistic quark model, HQET, QCD Sum rules, Lattice-QCD, Regge Phenomology, and many Phenomenological models [3–8]. An overview to the current status of research in the field of baryon physics from an experimental and theoretical aspects with a view to provide motivation and scope for the present chapter. The present study covers the baryons with one heavy and two light quarks [9–13]; two heavy and one light quark [14, 15] as well as three heavy quarks [16–18]. We also like to discuss the spectroscopy of nucleons [19, 20]. The decay properties, magnetic moments and Regge Trajectories are also discussed.

## 2. Light, heavy flavored baryons and exotics

In the case of baryons, when three same quark combines, definitely their electric charge, spin, orbital momentum would be same. This might violates the Pauli exclusion principal stated “*no two identical fermions can be found in the same quantum state at the same time*”. However, the color quantum number of each quark is different so that the exclusion principle would not be violated. One of the most significant aspects of the baryon spectrum is the existence of almost degenerate levels of different charges which have all the characteristics of isospin multiplets, quartets, triplets, doublets and singlets. A more general charge formula that encompasses all these nearly degenerate multiplets is

$$Q = I_3 + \frac{1}{2}Y, \quad Y = B + S + C/B' \tag{1}$$

where  $Q, I_3, Y, B, C, S, B'$  are referred as charge, isospin, hypercharge, baryon number, charm, strangeness and bottomness, respectively. The strangeness of baryon is always negative.

The baryons are strongly interacting fermions made up of three quarks and have  $\frac{1}{2}$  integer spin. They obey the Pauli exclusion principle, thus the total wave function must be anti-symmetric under the interchange of any two quarks. Since all observed hadrons are color singlets, the color component of the wave function must be completely anti-symmetric.

$$\begin{aligned} \text{For Octet,} \quad & \begin{cases} \vec{s}_u \cdot \vec{s}_s + \vec{s}_d \cdot \vec{s}_s = -1 & \text{symmetric} \\ \vec{s}_u \cdot \vec{s}_s + \vec{s}_d \cdot \vec{s}_s = 0 & \text{antisymmetric} \end{cases} \\ \text{For Decuplet,} \quad & \begin{cases} \vec{s}_u \cdot \vec{s}_s + \vec{s}_d \cdot \vec{s}_s = \frac{1}{2} & \text{symmetric} \\ \vec{s}_u \cdot \vec{s}_s + \vec{s}_d \cdot \vec{s}_s = \frac{3}{2} & \text{antisymmetric} \end{cases} \end{aligned}$$

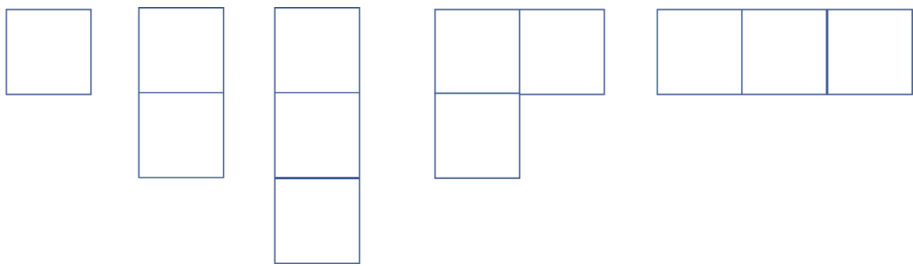
Here,  $\vec{s}_u, \vec{s}_d$  and  $\vec{s}_s$  are spin of  $u, d, s$  quarks.

It was considered that  $u, d$  and  $s$  are the sole elementary quarks. The symmetry group to consider three flavors of quark is done by  $SU(3)$  symmetry group.  $SU(3)$  flavor symmetry of light quarks. Each of these symmetries refers to an underlying threefold symmetry in strong interaction physics.  $SU(3)$  is the group symmetry transformations of the 3-vector wavefunction that maintain the physical constraint that the total probability for finding the particle in one of the three possible states equals 1. A Young diagram is the best way to represent the symmetries consists of an array of boxes arranged in one or more left-justified rows, with each row being at least as long as the row beneath. The correspondence between a diagram and a multiplet label is: The top row juts out  $\alpha$  boxes to the right past the end of the second row, the second row juts out  $\beta$  boxes to the right past the end of the third row, etc. The representation is shown in **Figures 2** and **3**.

The flavor wave functions of baryon states can then be constructed to be members of  $SU(3)$  multiplets as [5].

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

Murray Gell-Mann introduced the Eightfold geometrical pattern for mesons and baryons in 1962 [21]. The eight lightest baryons fit into the hexagonal array with two particle in center are called baryon octet. A triangular array with 10 particles are called the baryon decuplet. Moreover, the antibaryon octet and decuplet also exist



**Figure 2.**  
The representation of the multiplets  $(1,0), (0,1), (0,0), (1,1),$  and  $(3,0)$  in  $SU(3)$  diagrams.

1	2	3	1	3	1	2	1
			2		3		2
							3

**Figure 3.**  
A standard young tableaux for  $N = 3$ .

with opposite charge and strangeness [22]. Baryons having only  $u$  and  $d$  quarks are called nucleons  $N$  and  $\Delta$  resonances. The *proton* and *neutron* have spin ( $I_3 = \frac{1}{2}$ ) and  $\Delta$  particles have spin ( $I_3 = \frac{3}{2}$ ). The four possible four combinations of the symmetric wave function gives four  $\Delta$  particles;  $\Delta^{++}$  ( $uuu$ ,  $I_3 = 3/2$ ),  $\Delta^+$  ( $uud$ ,  $I_3 = 1/2$ ),  $\Delta^0$  ( $udd$ ,  $I_3 = -1/2$ ) and  $\Delta^-$  ( $ddd$ ,  $I_3 = -3/2$ ). Particles with combination of  $u$ ,  $d$  and  $s$  quarks are called hyperons;  $\Lambda$ ,  $\Sigma$ ,  $\Xi$  and  $\Omega$ . While discussing heavy sector baryons, we need baryons having heavy quark(s) combination. Any  $s$  quark(s) of hyperon baryons can be replaced by heavy quark ( $c$ ,  $b$ ) in heavy baryon particles. The added heavy quark(s) will be added to the suffix of the particular baryon. The particles are also depend on isospin quantum number, such that  $\Sigma$  and  $\Xi$  baryons have isospin triplets and doublets, respectively.  $\Lambda_c$  and  $\Sigma_c$  ( $\Lambda_b$  and  $\Sigma_b$ ) are formed by replacing one  $s$  quark. For  $\Xi$  baryon, replacement of one  $s$  quark gives  $\Xi_c$  ( $\Xi_b$ ) and the particles  $\Xi_{cc}$ ,  $\Xi_{bb}$  and  $\Xi_{bc}$  found while replacing two  $s$  quarks. The biggest family is found for  $\Omega$  particle. Replacement of one  $s$  quark gives  $\Omega_c$  ( $\Omega_b$ ); two  $s$  quarks replace to provide  $\Omega_{cc}$ ,  $\Omega_{bb}$  and  $\Omega_{bc}$ ; all three quark replacement with  $s$  quark give  $\Omega_{ccc}$ ,  $\Omega_{bbb}$ ,  $\Omega_{bbc}$  and  $\Omega_{ccb}$  particles.

$SU(4)$  group includes all of the baryons containing zero, one, two or three heavy  $Q$  (charm or beauty) quarks with light  $u$ ,  $d$  and  $s$  quarks. The number of particles in a multiplet,  $N=N(\alpha, \beta, \gamma)$  is

$$N = \frac{(\alpha + 1)}{1} \cdot \frac{(\beta + 1)}{1} \cdot \frac{(\gamma + 1)}{1} \cdot \frac{(\alpha + \beta + 2)}{2} \cdot \frac{(\beta + \gamma + 2)}{2} \cdot \frac{(\alpha + \beta + \gamma + 3)}{3} \quad (2)$$

It is clear from Eq. (2) that multiplets that are conjugate to one another have the same number of particles, but so can other multiplets. The multiplets  $(3,0,0)$  and  $(1,1,0)$  each have 20 particles. This multiplet structure is expected to be repeated for every combination of spin and parity which provides a very rich spectrum of baryonic states. The multiplet numerology of the tensor product of three fundamental representation is given as:

$$4 \otimes 4 \otimes 4 = 20 \oplus 20'_1 \oplus 20'_2 \oplus \bar{4}. \quad (3)$$

Representation shows totally symmetric 20-plet, the mixed symmetric 20'-plet and the total anti-symmetric  $\bar{4}$  multiplet. The charm baryon multiplets are presented in **Figure 4**. The ground levels of  $SU(4)$  group multiplets are  $SU(3)$  decuplet, octet, and singlet, respectively. These baryon states can be further decomposed according to the heavy quark content inside. According to the symmetry, the heavy baryons belong to two different  $SU(3)$  flavor representations: the symmetric sextet  $6_s$  and anti-symmetric anti-triplet  $\bar{3}_A$ . It can also be represented by young tableaux (refer **Figure 2**). The observed resonances of all light and heavy baryons are listed in PDG (2020) baryon summary **Table 1**.





(contains two-quarks and two anti-quarks) and *pentaquarks* (contains four-quarks plus an anti-quark) states with active gluonic degrees of freedom (hybrids), and even states of pure glue (glueballs) so far. Many experiments Belle, Barabar, CLEO, BESIII, LHCb, ATLAS, CMS and DO collaborations are working on the investigation of these exotic states. The theoretical approaches such as effective field theories of QCD, various quark models, Sum rules, Lattice QCD, etc. also predicted many states of the exotic states.

The two pentaquarks  $P_c^+(4380)$  and  $P_c^+(4450)$ , discovered in 2015 by the LHCb collaboration, in the  $J/\psi p K$  invariant mass distribution [23]. The newly observed states,  $P_c^+(4440)$ ,  $P_c^+(4457)$ ,  $P_c^+(4312)$  were investigated via different methods in 2019 by LHCb. These states are considered in various recent studies and the majority suggested as negative spin parity quantum number [24]. The investigations of pentaquark states resulted in support of different possibilities for their sub-structures leaving their structures still ambiguous. Therefore to discriminate their sub-structure we need further theoretical and experimental investigations.

R. Jaffe obtained six-quark states built of only light u, d, and s quarks called as *dibaryon* or *hexaquark* that belong to flavor group  $SU_f(3)$ . Using for analysis the MIT quark-bag model, Jaffe predicted existence of a H-dibaryon, i.e., a flavor-singlet and neutral six-quark uuddss bound state with isospin-spin-parity  $I(J^P) = 0(0^+)$  [25]. In the past fifteen years, new states have been observed called the XYZ states, different from the ordinary hadrons. Some of them, like the charged states, are undoubtedly exotic. Theoretical study include the phenomenological quark model to exotics, non-relativistic effective field theories and lattice QCD calculations and enormous experimental studies we can see on XYZ states.

As a hadronic molecule [26, 27], deuteron has been well-established loosely bound state of a proton and a neutron. Ideally, the large masses of the heavy baryons reduce the kinetic energy of the systems, which makes it easier to form bound states. Such a system is approximately non-relativistic. Therefore, it is very interesting to study the binding of two heavy baryons *dibaryon* and a combination of heavy baryon and an antibaryon *baryonium*.

### 3. Spectroscopic properties

Hadron spectroscopy is a key to strong interactions in the region of quark confinement and very useful for understanding the hadron as a bound state of quarks and gluons. Any system within a standard model becomes difficult to deal considering all the interaction of quark-quark, quark-gluon and gluon-gluon. This is the reason for using constituent quark mass incorporating all the other effects in the form of some parameters. The bound state heavy baryons can be studied in the QCD motivated potential models treating to the non relativistic Quantum mechanics. A Constituent Quark Model is a modelization of a baryon as a system of three quarks or anti-quarks bound by some kind of confining interaction. The present study deals with the Hypercentral Constituent Quark Model(hCQM), an effective way to study three body systems is through consideration of Jacobi coordinates.

The hypercentral approach has been applied to solve bound states and scattering problems in many different fields of physics and chemistry. The basic idea of the hypercentral approach to three-body systems is very simple. The two relative coordinates are rewritten into a single six dimensional vector and the non-relativistic *Schrödinger* equation in the six dimensional space is solved. The potential expressed in terms of the hypercentral radial co-ordinate, takes care of the three body interactions effectively [28]. The ground states and some of the excited states of light

baryons has also been affirmed theoretically by this scheme. It is interesting to identify the mass spectrum of heavy baryons (singly, doubly and triply) in charm as well as bottom sector and then to the light sector. We consider a nonrelativistic Hamiltonian given by

$$H = \frac{P_x^2}{2m} + V(x) \quad (4)$$

where,  $m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}$ , is the reduced mass and  $m_\rho$  and  $m_\lambda$  are reduced masses of Jacobi co-ordinates  $\vec{\rho}$  and  $\vec{\lambda}$ .  $x$  is the six dimensional radial hyper central coordinate of the three body system. Non-relativistically, this interaction potential,  $V(x)$  consists of a central term  $V_c(r)$  and spin dependent part  $V_{SD}(r)$ . The central part  $V_c(r)$  is given in terms of vector (Coulomb) plus scalar (confining) terms as

$$V_c(r) = V_V + V_S = -\frac{2\alpha_s}{3r} + \beta r^\nu \quad (5)$$

The short-distance part of the static three-quark system, arising from one-gluon exchange within baryon, is of Coulombic shape. Here, we can observe that the strong running coupling constant ( $\alpha_s$ ) becomes smaller as we decrease the distance, the effective potential approaches the lowest order one-gluon exchange potential given in Eq. (4) as  $r \rightarrow 0$ . So, for short distances, one can use the one gluon exchange potential, taking into account the running coupling constant  $\alpha_s$ . We employ the Coulomb plus power potential (CPP $_\nu$ ) with varying power index  $\nu$ , as there is no definite indication for the choices of  $\nu$  that works at different hadronic sector. The values of potential index  $\nu$  is varying from 0.5 to 2.0; in other words, S.R (1/2), linear (1.0), 3/2 power-law (1.5) and quadratic (2.0) potentials are taken into account in case of singly heavy baryon. The hypercentral potential  $V(x)$  as the color coulomb plus power potential with first order correction and spin-dependent interaction are written as,

$$V(x) = V^0(x) + \left( \frac{1}{m_\rho} + \frac{1}{m_\lambda} \right) V^{(1)}(x) + V_{SD}(x) \quad (6)$$

where  $V^0(x)$  is given by the sum of hyper Coulomb (hC) interaction and a confinement term

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x^\nu \quad (7)$$

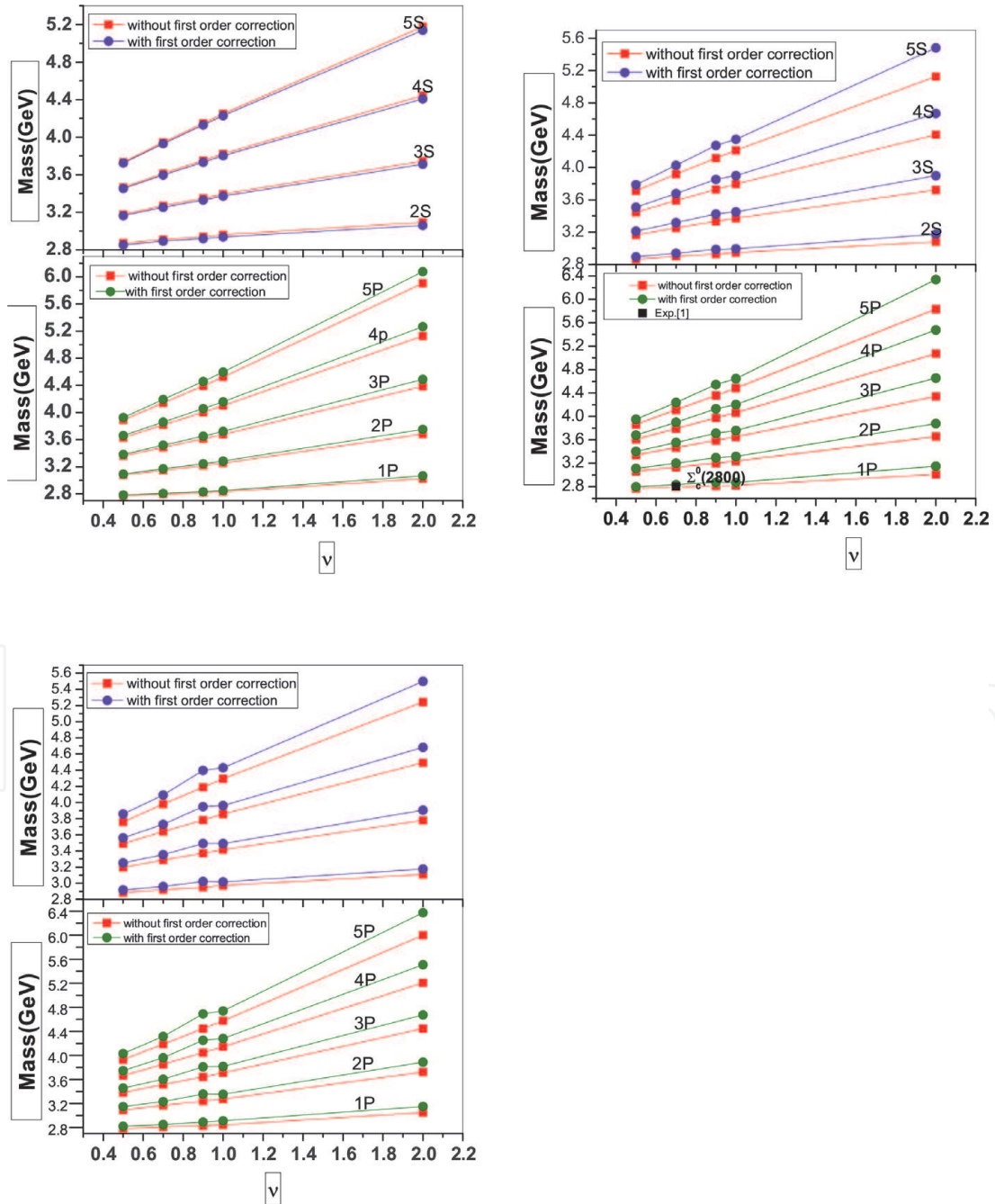
and first order correction as similar to the one given by [29],

$$V^{(1)}(x) = -C_F C_A \frac{\alpha_s^2}{4x^2} \quad (8)$$

Here,  $C_F$  and  $C_A$  are the Casimir charges of the fundamental and adjoint representation. For computing the mass difference between different degenerate baryonic states, we consider the spin dependent part of the usual one gluon exchange potential (OGE). The spin-dependent part,  $V_{SD}(x)$  contains three types of the interaction terms, such as the spin-spin term  $V_{SS}(x)$ , the spin-orbit term  $V_{\gamma S}(x)$  and tensor term  $V_T(x)$ . Considering all isospin splittings, the ground and excited state masses are determined for all heavy baryon system. The radial excited states from 2S-5S and orbital excited states from 1P-5P, 1D-4D and 1F-2F are calculated using the hCQM formalism. These mass spectra can be found in our Refs. [10-18].



The correction used in the potential have its dominant effect on potential energy. In our calculation, the effect of the correction to the potential energy part is decreasing as mass of the system increasing. For example, if noticed the maximum effect of heavy  $\Xi$  baryon family then, for the radial excited states of  $\Xi$  baryons are  $\Xi_c(3.0\%) > \Xi_{cc}(3.0\%) > \Xi_b(0.41\%) > \Xi_{bc}(0.3\%) > \Xi_{bb}(0.16\%)$ . The orbital excited states are (1.4–3.5%) for  $\Xi_c$ , (0.1–0.4%) for  $\Xi_b$  and in case of doubly heavy region the error is rising in order of baryons  $\Xi_{bb} > \Xi_{bc} > \Xi_{cc}$ . Singly heavy baryons show the effect from (0.1–1.7%), Doubly heavy baryons show the effect from (0.1–1.0%) and Triply heavy baryons show the effect from (0.2–0.9%) in orbital excited states. For better idea, we shown the effect of masses with and without first order corrections in case of  $\Sigma_c$  baryon (See, **Figure 5**). In the Figure, we plotted the graph of potential index vs. mass. The radial excited states 2S–5S and the orbital excited states 1P–5P are shown for  $\Sigma_c^{++}$ ,  $\Sigma_c^0$  and  $\Sigma_c^+$  baryons.



**Figure 5.**  
Spectra of  $\Sigma$  triplets in S and P state with and without first order correction [13].

Recently, we calculate the masses of  $N^*$  and  $\Delta$  resonances using the hCQM model by adding the first-order correction to the potential. The complete mass spectra with individual states graphically compared with the experimental states in **Figure 6**. We can observe that, many states are in accordance with the experimental resonances. We also predicted the  $J^P$  values of unknown states.

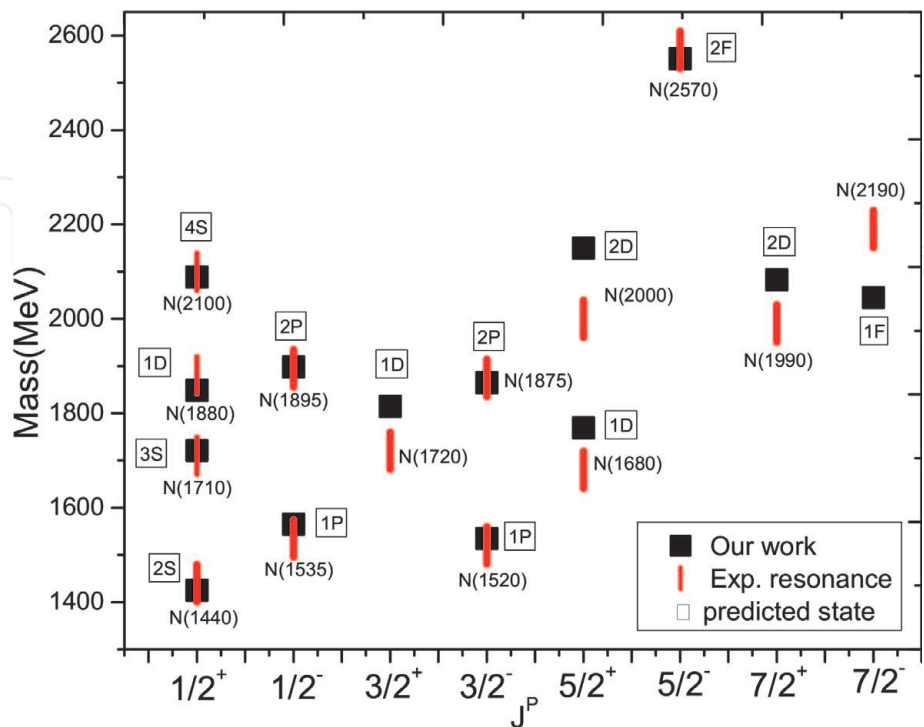
### 3.1 Regge trajectory

We can say that, the mass spectra of hadrons can be conveniently described through Regge trajectories. These trajectories will aid in identifying the quantum number of particular resonance states. The important three properties of Regge Trajectories are: Linearity, Divergence and Parallelism. The higher excited radial and orbital states mass calculation enable to construct the Regge trajectories in the  $(n, M^2)$  and  $(J, M^2)$  planes. These graphical representation is useful in assigning  $J^P$  value to the experimental unknown states. We are getting almost linear, parallel and equidistance lines in each case of the baryons.

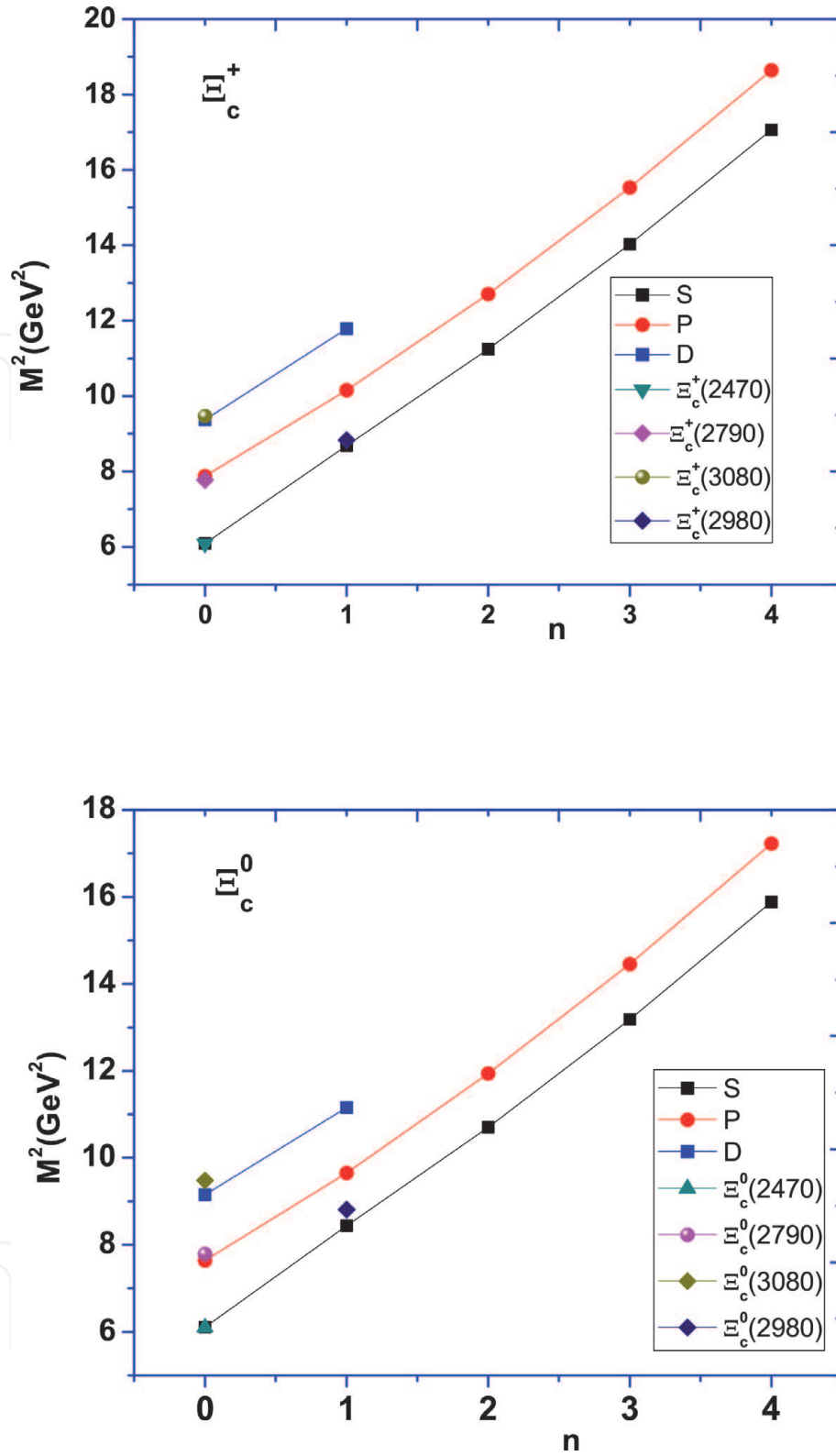
Some of the obtained masses are plotted in accordance with quantum number as well as with natural and unnatural parities. For the singly heavy baryons, the trajectory is shown for  $\Xi_c$  doublets baryons (See, **Figure 7**). For the doubly heavy baryons, the spectra of  $\Xi_{bc}$  and  $\Omega_{bc}$  are less determined till the date. Thus, we plotted their trajectories in **Figure 8**. The natural and unnatural parities are shown in  $(J, M^2)$  for all triply heavy baryons;  $\Omega_{ccc}$ ,  $\Omega_{bbb}$ ,  $\Omega_{bbc}$  and  $\Omega_{ccb}$  (See, **Figures 9 and 10**). The rest trajectories of all heavy baryon families in both planes can be found in our articles.

### 3.2 Decays: strong, radiative and weak

The particles which are known to us decay by a similar sort of dissipation. Those who decay rapidly are unstable and take a long time are metastable. Some particles, like electron, three lightest neutrinos (and their antiparticles), the photon are stable

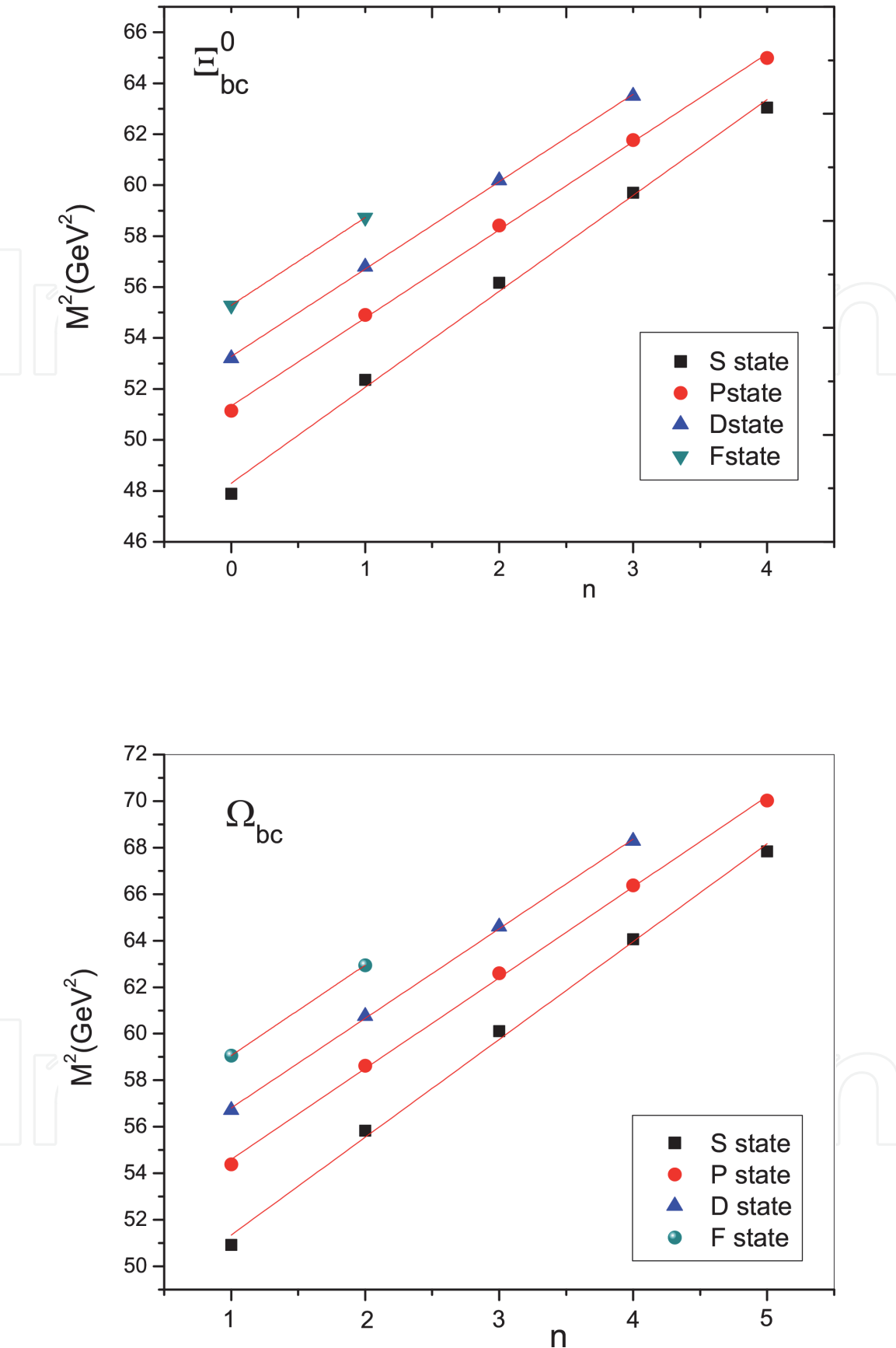


**Figure 6.**  
The resonances are predicted from the first radial excited state (2S) to the orbital excited state (2F) for  $N^*$  baryons [19].

**Figure 7.**

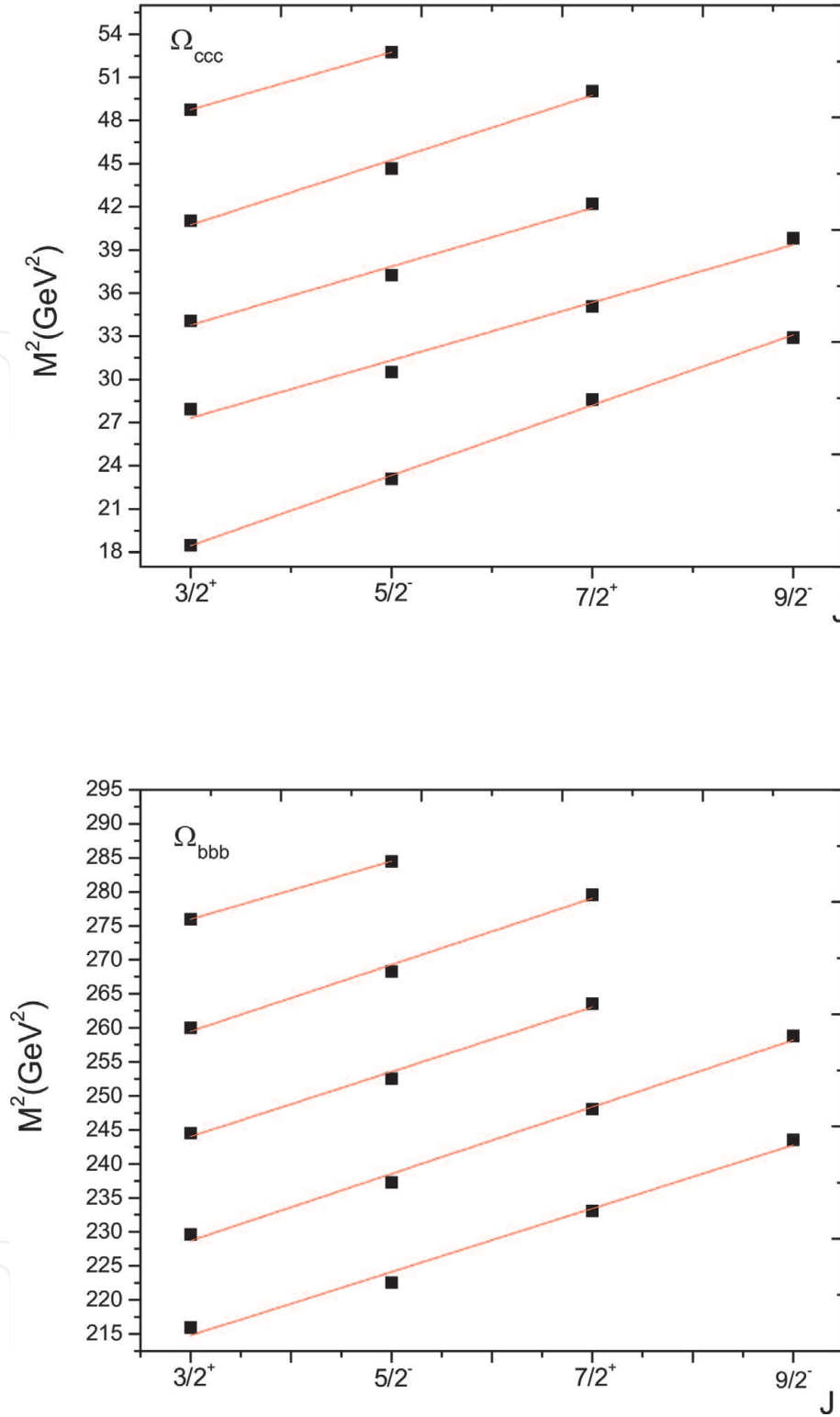
Variation of mass for  $\Xi_c^0$  and  $\Xi_c^+$  with different states. The  $(M^2 \rightarrow n)$  Regge trajectories for  $J^P$  values  $\frac{1}{2}^+$ ,  $\frac{1}{2}^-$  and  $\frac{5}{2}^+$  are shown from bottom to top. Available experimental data are also given with particle names [13].

partices (*never decays*). The observations of decays of baryon and meson resonances afford a valuable guidance in assigning to the correct places in various symmetry schemes. The correct isotopic spin assignment is likely to be implied by the experimental branching ratio into different charge states of particles produced by the decay, while experimental decay widths provide a means of extracting phenomenological coupling constants. For the success of a particular model, it is required to produce not only the mass spectra but also the decay properties of these baryons.



**Figure 8.**  
The doubly charm-beauty baryons in  $(M^2 \rightarrow n)$  plane [14, 15].

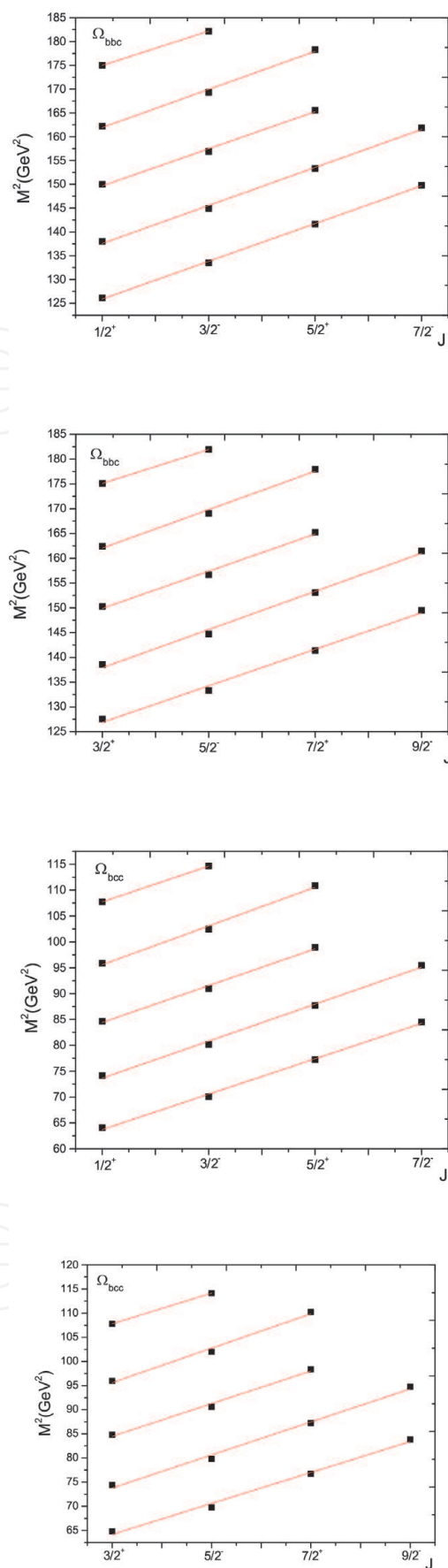
For the success of a particular model, it is required to produce not only the mass spectra but also the decay properties of these baryons. The masses obtained from the hypercentral Constituent Quark Model (hCQM) are used to calculate the radiative and the strong decay widths. Such calculated widths are reasonably close to



**Figure 9.**  
The triply charm and triply bottom Regge trajectories in  $(J, M^2)$  plane [16].

other model predictions and experimental observations (where available) [30]. The effective coupling constant of the heavy baryons is small, which leads to their strong interactions perturbatively and makes it easier to understand the systems containing only light quarks. The Heavy Hadron Chiral Perturbation Theory (HHcPT) describes the strong interactions in the low-energy regime by an exchange of light Goldstone boson. Some of the strong P -wave couplings among the s-wave baryons and S-wave couplings between the s-wave and p-wave baryons are shown in **Table 2** with PDG values.





**Figure 10.** Parent and daughter ( $J, M^2$ ) Regge trajectories for triply heavy charm-beauty baryons with natural (first) and unnatural (second) parities [17, 18].

The electromagnetic properties are one of the essential key tools in understanding the internal structure and geometric shapes of hadrons. In the present study, the magnetic moments of heavy flavor and light flavor baryons are computed based on

	Decay mode	Present	PDG
$P$ -wave transitions	$\Sigma_c^{++} (1^2S_{\frac{1}{2}}) \rightarrow \Lambda_c^+ \pi^+$	1.72	$1.89^{+0.09}_{-0.18}$
$P$ -wave transitions	$\Sigma_c^+ (1^2S_{\frac{1}{2}}) \rightarrow \Lambda_c^+ \pi^0$	1.60	$< 4.6$
$P$ -wave transitions	$\Sigma_c^0 (1^2S_{\frac{1}{2}}) \rightarrow \Lambda_c^+ \pi^-$	1.17	$1.83^{+0.11}_{-0.19}$
$S$ -wave transitions	$\Sigma_c^{++} (1^2P_{\frac{1}{2}}) \rightarrow \Lambda_c^+ \pi^+$	68.19	$75^{+22}_{-17}$
$S$ -wave transitions	$\Sigma_c^+ (1^2P_{\frac{1}{2}}) \rightarrow \Lambda_c^+ \pi^0$	62.92	$62^{+60}_{-40}$
$S$ -wave transitions	$\Sigma_c^0 (1^2P_{\frac{1}{2}}) \rightarrow \Lambda_c^+ \pi^-$	66.44	$72^{+22}_{-15}$
$S$ -wave transitions	$\Lambda_c^+ (1^2P_{\frac{1}{2}}) \rightarrow \Sigma_c^0 \pi^+$	4.45	$2.6 \pm 0.6$

**Table 2.**

Several strong one-pion decay rates (in MeV) [30].

the non-relativistic hypercentral constituent quark model using the spin-flavor wave functions of the constituent quark and their effective masses [11, 13]. Generally, the meaning of the constituent quark mass corresponds to the energy that the quarks have inside the color singlet hadrons, we call it as the effective mass. The magnetic moment of baryons are obtained in terms of the spin, charge and effective mass of the bound quarks. The study has been performed for all singly, doubly and triply heavy baryon systems for positive parity  $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ .

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_{iz} | \phi_{sf} \rangle \quad (9)$$

where

$$\mu_i = \frac{e_i \sigma_i}{2m_i^{eff}} \quad (10)$$

$e_i$  is a charge and  $\sigma_i$  is the spin of the respective constituent quark corresponds to the spin flavor wave function of the baryonic state. The effective mass for each of the constituting quark  $m_i^{eff}$  can be defined as [31].

$$m_i^{eff} = m_i \left( 1 + \frac{\langle H \rangle}{\sum_i m_i} \right) \quad (11)$$

where,  $\langle H \rangle = E + \langle V_{spin} \rangle$ . Using these equations, we calculate magnetic moments of singly, doubly and triply heavy baryons. The spin flavor wave function [32]  $\phi_{sf}$  of all computed heavy flavor baryons are given in **Table 3**.

The electromagnetic radiative decay width is mainly the function of radiative transition magnetic moment  $\mu_{B'_c \rightarrow B_c}$  (in  $\mu_N$ ) and photon energy ( $k$ ) [12] as

$$\Gamma_\gamma = \frac{k^3}{4\pi} \frac{2}{2J+1} \frac{e}{m_p^2} \mu_{B_c \rightarrow B'_c}^2 \quad (12)$$

where  $m_p$  is the mass of proton,  $J$  is the total angular momentum of the initial baryon ( $B_c$ ). Some radiative decays are mentioned below:

- $\Sigma_c^{*0} \rightarrow \Sigma_c^0$ : 1.553
- $\Xi_c^* \rightarrow \Xi_c^0$ : 0.906
- $\Omega_c^{*0} \rightarrow \Omega_c^0$ : 1.441
- $\Sigma_c^{*0} \rightarrow \Lambda_c^+$ : 213.3

Weak decays of heavy hadrons play a crucial role to understand the heavy quark physics. In these decays the heavy quark acts as a spectator and the light quark inside heavy hadron decays in weak interaction [33]. The transition can be  $s \rightarrow u$  or  $d \rightarrow u$  depending on the available phase space. Since the heavy quark is spectator in such case, one can investigate the behavior of light quark system. These kind of small phase space transition could be possible in semi-electronic, semi-muonic and

Baryons	function	Our	Baryons	function	Our
$n(udd)$	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_u$	-1.997	$\Delta$	$2\mu_u + \mu_d$	2.28
$\Sigma_c^{++}(uuc)$	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_c$	1.834	$\Sigma_c^{*++}$	$2\mu_u + \mu_c$	3.263
$\Sigma_c^0(udc)$	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_c$	-1.091	$\Sigma_c^{*+}$	$\mu_u + \mu_d + \mu_c$	1.1359
$\Sigma_c^+(ddc)$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_d - \frac{1}{3}\mu_c$	0.690	$\Sigma_c^{*0}$	$2\mu_d + \mu_c$	-1.017
$\Xi_c^0(dsc)$	$\frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	-1.011	$\Xi_c^{*0}$	$\mu_d + \mu_s + \mu_c$	-0.825
$\Xi_c^+(usc)$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_c$	0.559	$\Xi_c^{*+}$	$\mu_u + \mu_s + \mu_c$	1.329
$\Omega_c^0(ssc)$	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_c$	-0.842	$\Omega_c^{*0}$	$2\mu_s + \mu_c$	-0.625
$\Sigma_b^+(uub)$	$\frac{4}{3}\mu_u - \frac{1}{3}\mu_b$	2.288	$\Sigma_b^{*+}$	$2\mu_u + \mu_b$	3.343
$\Sigma_b^-(ddb)$	$\frac{4}{3}\mu_d - \frac{1}{3}\mu_b$	-1.079	$\Sigma_b^{*-}$	$2\mu_d + \mu_b$	-1.709
$\Xi_b^0(usb)$	$\frac{2}{3}\mu_u + \frac{2}{3}\mu_s - \frac{1}{3}\mu_b$	0.798	$\Xi_b^{*0}$	$\mu_u + \mu_s + \mu_b$	1.072
$\Xi_b^-(dsb)$	$\frac{2}{3}\mu_d + \frac{2}{3}\mu_s - \frac{1}{3}\mu_b$	-0.943	$\Xi_b^{*-}$	$\mu_d + \mu_s + \mu_b$	-1.471
$\Omega_b^-(ssb)$	$\frac{4}{3}\mu_s - \frac{1}{3}\mu_b$	0.761	$\Omega_b^{*-}$	$2\mu_s + \mu_b$	-1.228
$\Xi_{cc}^+(dcc)$	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_d$	0.784	$\Xi_{cc}^{*+}$	$2\mu_c + \mu_d$	0.068
$\Xi_{cc}^{++}(ucc)$	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_u$	0.031	$\Xi_{cc}^{*++}$	$2\mu_c + \mu_u$	2.218
$\Xi_{bb}^-(dbb)$	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_d$	0.196	$\Xi_{bb}^{*-}$	$2\mu_b + \mu_d$	-1.737
$\Xi_{bb}^0(ubb)$	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_u$	-0.662	$\Xi_{bb}^{*0}$	$2\mu_b + \mu_u$	1.6071
$\Xi_{bc}^0(dbc)$	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_d$	0.527	$\Xi_{bc}^{*0}$	$\mu_b + \mu_c + \mu_d$	-0.448
$\Xi_{bc}^+(ubc)$	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_u$	-0.304	$\Xi_{bc}^{*+}$	$\mu_b + \mu_c + \mu_u$	2.107
$\Omega_{cc}^+(ccs)$	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_s$	0.692	$\Omega_{cc}^{*+}$	$2\mu_c + \mu_s$	0.285
$\Omega_{bb}^-(bbs)$	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_s$	0.108	$\Omega_{bb}^{*-}$	$2\mu_b + \mu_s$	-1.239
$\Omega_{bc}^0(bcs)$	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_s$	0.439	$\Omega_{bc}^{*0}$	$\mu_b + \mu_c + \mu_s$	-0.181
$\Omega_{bcc}^+(bcc)$	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_b$	0.606	$\Omega_{bcc}^{*+}$	$\mu_b + 2\mu_c$	0.8198
$\Omega_{bbc}^0(bbc)$	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_c$	-0.233	$\Omega_{bbc}^{*+}$	$2\mu_b + \mu_c$	0.228

**Table 3.** Magnetic moments (in nuclear magnetons) with spin-flavor wavefunctions of  $J^P = \frac{1}{2}^+, \frac{3}{2}^+$  are listed for nucleons (light) and singly, doubly, triply (heavy) baryons.

Mode	$J^P \rightarrow J'^{P'}$	Decay Rates(GeV)		Ref. [33]
$\Xi_c^0 \rightarrow \Lambda_c^+ e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$	$7.839 \times 10^{-19}$	$7.839 \times 10^{-19}$	$7.91 \times 10^{-19}$
$\Xi_c^0 \rightarrow \Sigma_c^+ e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$	$4.416 \times 10^{-23}$	$7.023 \times 10^{-24}$	$6.97 \times 10^{-24}$
$\Omega_c^0 \rightarrow \Xi_c^+ e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$	$2.143 \times 10^{-18}$	$2.290 \times 10^{-18}$	$2.26 \times 10^{-18}$
$\Omega_c^0 \rightarrow \Xi_c^{*+} e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$	$2.057 \times 10^{-28}$	$2.436 \times 10^{-28}$	$1.49 \times 10^{-29}$
$\Xi_b^- \rightarrow \Lambda_b^0 e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$	$5.928 \times 10^{-19}$		$6.16 \times 10^{-19}$
$\Omega_b^- \rightarrow \Xi_b^0 e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$	$4.007 \times 10^{-18}$		$4.05 \times 10^{-18}$
$\Omega_b^- \rightarrow \Xi_b^{*0} e^- \bar{\nu}$	$\frac{1}{2}^+ \rightarrow \frac{3}{2}^+$	$1.675 \times 10^{-26}$		$3.27 \times 10^{-28}$

**Table 4.**

Semi-electronic decays in  $s \rightarrow u$  transition for charm baryons are listed [13].

non leptonic decays of the heavy baryons and mesons. We calculate here, the semi-electronic decays for strange-charm heavy flavor baryons  $\Omega_c$ ,  $\Xi_c$ ,  $\Xi_b$  and  $\Omega_b$  using our spectral parameters.

The differential decay rates for exclusive semi-electronic decays are given by [33],

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M^5 |V_{CKM}|^2}{192\pi^3} \sqrt{w^2 - 1} P(w) \quad (13)$$

where  $P(w)$  contains the hadronic and leptonic tensor. Assuming that the form factors are slowly varying functions of the kinematic variables, we may replace all form factors by their values at variable  $w=1$ . The calculated semi-electronic decays for  $\Xi_c$ ,  $\Omega_c$ ,  $\Xi_b$  and  $\Omega_b$  baryons are listed in **Table 4**. We can observe that our results are in accordance with ref. [33] for singly heavy baryons.

#### 4. Current cenario in the field of baryons

Baryons with heavy quarks provide a beautiful laboratory to test our ideas of QCD. As the heavy quarks mass increases its motion decreases and the baryons properties are increasingly governed by the dynamics of the light quark and approach a universal limit. As we discussed, many theoretically approaches are calculating and obtaning the masses and decay widths of heavy baryons. We study the mass spectroscopy of ligt and heavy baryons and their properties. A few number of excited states for the singly heavy baryons have also been reported along with their ground states. The singly charmed baryons are,  $\Lambda_c(2286)^+$ ,  $\Lambda_c(2595)^+$ ,  $\Lambda_c(2625)^+$ ,  $\Lambda_c(2880)^+$ ,  $\Lambda_c(2940)^+$ ,  $\Lambda_c(2765)^+$ ,  $\Lambda_c(2860)^+$ ,  $\Sigma_c(2455)^{++,+,0}$ ,  $\Sigma_c(2520)^{++,+,0}$ ,  $\Sigma_c(2800)^{++,+,0}$ ,  $\Xi_c(2468)^{+,0}$ ,  $\Xi_c'(2580)^{+,0}$ ,  $\Xi_c'(2645)^{+,0}$ ,  $\Xi_c(2790)^{+,0}$ ,  $\Xi_c(2815)^{+,0}$ ,  $\Xi_c'(2930)^0$ ,  $\Xi_c(2980)^{+,0}$ ,  $\Xi_c(3055)^+$ ,  $\Xi_c(3080)^{+,0}$ ,  $\Xi_c'(3123)^+$ ,  $\Omega_c(2695)^0$ ,  $\Omega_c(2770)^0$ . The singly beauty baryons are  $\Lambda_b(5619)^0$ ,  $\Lambda_b(5912)^0$ ,  $\Lambda_b(5920)^0$ ,  $\Sigma_b(5811)^+$ ,  $\Sigma_b(5816)^-$ ,  $\Sigma_b(5832)^+$ ,  $\Sigma_b(5835)^-$ ,  $\Xi_b(5790)^{-,0}$ ,  $\Xi_b(5945)^0$ ,  $\Xi_b(5955)^-$ ,  $\Xi_b(5935)^{-'}$ . And now, LHCb experiment has identified new resonances(excited states) for singly heavy baryons (See, **Table 5**). The experimental state and masses are in first two columns. The third column shows our predicted masees and in the forth column we assign the states with  $J^P$  values. We can observe that, apart from first radial and orbital excited states, we also have D state resonances in heavy

Names	Exp. Mass (MeV) [2]	Predicted Mass (MeV)	Baryon State
$\Lambda_c^+(2860)$	$2756.1 \pm 0.5$	2842	$(1^2D_{\frac{3}{2}})$
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1_{-0.5}^{0.3}$	2976,2993	$(1^2P_{\frac{3}{2}}), (1^4P_{\frac{3}{2}})$
$\Omega_c(3050)^0$	$3050.4 \pm 0.1 \pm 0.1_{-0.5}^{0.3}$	3011,3028	$(1^2P_{\frac{1}{2}}), (1^4P_{\frac{1}{2}})$
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3_{-0.5}^{0.3}$	2947	$(1^4P_{\frac{5}{2}})$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5_{-0.5}^{0.3}$	3011,3028	$(1^2P_{\frac{1}{2}}), (1^4P_{\frac{1}{2}})$
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9_{-0.5}^{0.3}$	3100,3126	$(2^2S_{\frac{1}{2}}), (2^2S_{\frac{3}{2}})$
$\Xi_{cc}^{++}(3620)$	$3521.4 \pm 0.99$	3511	$(1^2S_{\frac{1}{2}})$
$\Xi_c^0(2965)$	$2964.88 \pm 0.26$	2903,2940	$(1^2S_{\frac{1}{2}})$
$\Lambda_b^+(6072)$	$6072.3 \pm 2.9$	6066	$(2^2S_{\frac{1}{2}})$
$\Lambda_b^+(6146)$	$6146.17 \pm 0.33$	6121	$(1^2D_{\frac{3}{2}})$
$\Lambda_b^+(6152)$	$6152.51 \pm 0.26$	6119	$(1^2D_{\frac{5}{2}})$
$\Sigma_b^+(6097)^-$	$6098 \pm 1.7$	6122	$(1^2P_{\frac{1}{2}})$
$\Sigma_b(6097)^+$	$6095.8 \pm 1.7$	6131	$(1^2P_{\frac{1}{2}})$
$\Omega_b(6316)^-$	$6315.64 \pm 0.31 \pm 0.07$	6313	$(1^4P_{\frac{5}{2}})$
$\Omega_b(6330)^-$	$6330.3 \pm 0.28 \pm 0.07$	6331	$(1^2P_{\frac{1}{2}}), (1^4P_{\frac{1}{2}})$
$\Omega_b(6340)^-$	$6339.71 \pm 0.26 \pm 0.05$	6321	$(1^2P_{\frac{3}{2}})$
$\Omega_b(6350)^-$	$6349.88 \pm 0.35 \pm 0.05$	6326	$(1^4P_{\frac{3}{2}})$
$\Xi_b(6227)^0$	$6227.1_{1.5}^{1.4} \pm 0.5$	6193,6309	$(2^2S_{\frac{1}{2}}), (2^4S_{\frac{3}{2}})$

**Table 5.**  
The newly observed baryonic states are listed with the observed mass in column 1 & 2. Our predicted baryonic states are compared with  $J^P$  value and masses.

sector. According to the SU(3) symmetry we also have doubly and triply baryons in charm as well as bottom sector. Among which, the evidence of 1S state for doubly heavy baryons  $\Xi_{cc}^+$  and  $\Xi_{cc}^{++}$  are observed by the SELEX and the LHCb experiments respectively. The Belle and CDF collaboration had also observed some of the particles. The future experiments like Panda, Belle-II and BES-II are expected to give more results soon.



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