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Double Pole Method in QCD Sum Rules for Vector Mesons

Mikael Souto Maior de Sousa and Rômulo Rodrigues da Silva

Abstract

The QCD Sum Rules approach had proposed by Shifman, Vaishtein Zakharov Novikov, Okun and Voloshin (SVZNOV) in 1979 and has been used as a method for extracting useful properties of hadrons having the lowest mass, called as ground states. On the other hand, the most recent experimental results make it clear that the study of the excited states can help to solve many puzzles about the new XYZ mesons structure. In this paper, we propose a new method to study the first excited state of the vector mesons, in particular we focus our attention on the study of the ρ vector mesons, that have been studied previously by SVZNOV method. In principle, the method that we used is a simple modification to the shape of the spectral density of the SVZNOV method, which is written as “pole + continuum”, to a new functional form “pole + pole + continuum”. In this way, We may obtain the ρ and the $\rho(2S)$ masses and also their decay constants.

Keywords: QCD Sum Rules, double pole, light quarks, vector mesons

1. Introduction

The successful QCD sum rules was created in 1977 by Shifman, Vainshtein, Zakharov, Novikov, Okun and Voloshin [1–4], and until today is widely used. Using this method, we may obtain many hadron parameters such as: hadrons masses, decay constant, coupling constant and form factors, all they giving in terms of the QCD parameters, it means, in terms of the quark masses, the strong coupling and nonperturbative parameters like quark condensate and gluon condensate.

The main point of this method is that the quantum numbers and content of quarks in hadron are presented by an interpolating current. So, to determine the mass and the decay constant of the ground state of the hadron, we use the two-point correlation function, where this correlation function is introduced in two different interpretations. The first one is the OPE's interpretation, where the correlation function is presented in terms of the operator product expansion (OPE).

On the phenomenological side we can be written the correlation function in terms of the ground state and several excited states. The usual QCDSR method uses an ansatz that the phenomenological spectral density can be represented by a form “pole + continuum”, where it is assumed that the phenomenological and OPE spectral density coincides with each other above the continuum threshold. The continuum is represented by an extra parameter called s_0 , as being correlated with the onset of excited states [5].

In general, the resonances occurs with $\sqrt{s_0}$ lower than the mass of the first excited state. For the ρ meson spectrum, for example, the ansatz “pole + continuum” is a

good approach, due to the large decay width of the $\rho(2S)$ or $\rho(1450)$, which allows to approximate the excited states as a continuum. For the ρ meson [6], the value of $\sqrt{s_0}$ that best fit the mass and the decay constant is $\sqrt{s_0} = 1.2 \text{ GeV}$ and for the $\varphi(1020)$ meson the value is 1.41 GeV . We note that the values quoted above for $\sqrt{s_0}$ are about 250 MeV below the poles of $\rho(1450)$ and $\varphi(1680)$. One interpretation of this result is due to the effect of the large decay width of these mesons.

Novikov et al. [1], in a pioneering paper, proposed, for the charmonium sum rule, that the phenomenological side with double pole (“pole + pole + continuum”) and $\sqrt{s'_0} = 4 \text{ GeV}$, where s'_0 is the new parameter that takes in to account the second “pole” in this ansatz. In this way, this value is correlated with the threshold of pair production of charmed mesons. Using this $\sqrt{s'_0}$ value and the Sum Rule Momentum at $Q^2 = 0$, they presented the first estimate for the gluon condensate and a very good estimated value for η_c meson, that is about 3 GeV , while the experimental data had shown 2.83 GeV for this meson [1, 2, 4].

In general, by QCDSR, the excited states are studied in “pole + pole + continuum” ansatz with $Q^2 = 0$ [1, 2], as we can see in the spectral sum [7], the Maximum Entropy [8] and Gaussian Sum Rule with “pole + pole + continuum” ansatz [9] approaches. There are studies on the $\rho(1S, 2S)$ mesons [8, 10, 11], nucleons [7, 12], $\eta_c(1S, 2S)$ mesons [2], $\psi(1S, 2S)$ mesons [1, 13] and $Y(1S, 2S)$ mesons [14]. In this paper, we obtain the $\rho(1S, 2S)$ mesons masses and their decay constants taking the “pole + pole + continuum” ansatz in QCD sum rules,

2. The two point correlation function

As it is known, to determinate the hadron mass and the decay constant in QCD sum rules, we may use the two-point correlation function [3], that is given by

$$\Pi_{\mu\nu} = i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu(x) j_\nu^\dagger(0) | 0 \rangle, \quad (1)$$

where, on the OPE side, this current density for $q\bar{q}$ vector mesons has the following form:

$$j_\mu(x) = \delta_{ab} \bar{q}_a(x) \gamma_\mu q_b(x), \quad (2)$$

where the subscribe index a and b represents the color index. Now, using Eq. (2) in Eq. (1) we have

$$\Pi_{\mu\nu} = i \delta_{ab} \delta_{cd} \int d^4x e^{iqx} \langle 0 | T \{ \bar{q}_a(x) \gamma_\mu q_b(x) [\bar{q}_c(0) \gamma_\nu q_d(0)]^\dagger | 0 \rangle. \quad (3)$$

Evaluating Eq. (3) in terms of the OPE [15], which can be written by a dispersion relation, where this relation depends on the QCD parameters, the correlation function takes the form:

$$\Pi_{\mu\nu}^{\text{OPE}}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{\text{OPE}}(q^2), \quad (4)$$

with:

$$\Pi^{\text{OPE}}(q^2) = \int_{s_0^{\min}}^{\infty} ds \frac{\rho^{\text{OPE}}(s)}{s - q^2} + \Pi^{\text{nonPert}}(q^2), \quad (5)$$

Note that

$$\rho^{Pert}(s) = \frac{\text{Im}[\Pi^{OPE}(s)]}{\pi}, \quad (6)$$

and $\Pi^{nonPert}(q^2)$ is the term that add the condensates contributions, besides, s_0^{min} is the minimum value of the s parameter to have an imaginary part of the $\Pi^{Pert}(s)$.

On the phenomenological side, the interpolating current density may be written considering just hadronic freedom degrees, it means, inserting a complete set of intermediate states among the operator, where they are the creation and annihilation describes by the interpolating current density. In this way we can use the following operator algebra

$$\langle 0 | j_\mu(0) | V(q) \rangle = f m \epsilon_\mu(q), \quad (7)$$

where f and m are, respectively, the decay constant and the mass of the meson and $\epsilon_\mu(q)$ is a unitary polarizing vector. So, substituting Eq. (7) in Eq. (1) after some intermediate mathematical steps we get:

$$\Pi_{\mu\nu}^{Phen}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{Phen}(q^2). \quad (8)$$

The Invariant part of Eq. (8), $\Pi^{Phen}(q^2)$, is given by the following dispersion relation:

$$\Pi^{Phen}(q^2) = \int_{s_0^{min}}^{\infty} ds \frac{\rho^{Phen}(s)}{s - q^2}, \quad (9)$$

where $\rho^{Phen}(s) = f^2 \delta(s - m^2) + \rho^{cont.}(s)$. Thus, we have the Eq. (9) written as follow:

$$\Pi^{Phen}(q^2) = \frac{f^2}{m^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho^{cont.}(s)}{s - q^2}. \quad (10)$$

Note that, we can introduce a minimum number of parameters in the calculus by the approach $\rho^{cont.}(s) = \Theta(s - s_0) \rho^{OPE}(s)$, using this in Eq. (10) we get:

$$\Pi^{Phen}(q^2) = \frac{f^2}{m^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}, \quad (11)$$

so, s_0 can be understood as a parameter indicating that for s values greater than s_0 there is only contribution from the continuum, it means, s_0 is called a continuum threshold.

Note that, by the Quark-Hadron duality we can develop the two-point correlation function in both different interpretation that are equivalent each other. I.e., we can match the correlation function by de OPE, Eq. (5), with the correlation function by the Phenomenological side, Eq. (10), through the Borel transformation.

3. Borel transformation

To match the Eqs. (5) and (10) is not that simple, because in the OPE side the calculations of the all OPE terms is almost impossible, in this way, at someone moment

we must truncate the series and, beyond this, guaranties its convergence. However, for the truncation of the series to be possible, the contributions of the terms of higher dimensions must be small enough to justify to be disregarded in the expansion.

Thereby, for both descriptions to be in fact equivalent, we must suppress both the contributions of the highest order terms of the OPE and the contributions of the excited states on the phenomenological side. It is can be done by the Borel transformation that is define as follow:

$$B[\Pi(Q^2) = \Pi[M^2] = \lim_{Q^2, n \rightarrow \infty} \frac{(Q^2)^n}{n!} \left(-\frac{\partial}{\partial Q^2} \right)^n \Pi(Q^2), \quad (12)$$

$$\frac{Q^2}{n} = M^2$$

where $Q^2 = -q^2$ is the momentum in the Euclidian space and M^2 is a variable rising due to Borel transformation application and it is called *Borel Mass*.

Because of this, we can determine a region of the M^2 space in which both the highest order contributions from OPE and those from excited states are suppressed, so that the phenomenological parameters associated with the hadron fundamental state can be determined. Therefore, we must determine an interval of M^2 where this comparison is adequate, enabling the determination of reliable results. This interval is called Borel Window.

At the Phenomenological side, we introduce some approximations when we assume that the spectral density can be considering as a polo plus a continuum of excited states. So, we must suppress the continuum contributions for the result to be sufficiently dominated by de pole.

4. The double pole method

This method is consisted by the assumption that the spectral density at the phenomenological side can be given like [16]:

$$\rho^{Phen}(s) = (f_1)^2 \delta[s - (m_1)^2] + (f_2)^2 \delta[s - (m_2)^2] + \rho^{OPE}(s) \Theta(s - s'_0), \quad (13)$$

where m_1 and f_1 are, respectively, the ground state meson mass and decay constant, m_2 and f_2 are, respectively, the first excited state meson mass and decay constant, beyond this, we include a new parameter s'_0 marking the onset of the continuum states. As we can see in **Figure 1**, the parameters Δ and Δ' consists in a gap among the ground and first excited states and among the first excited and the continuum states respectively. They are defined by the decay width of these states.

Note that, inserting Eq. (13) in Eq. (9) we get the following two-point phenomenological correlation function:

$$\Pi^{Phen}(q^2) = \frac{(f_1)^2}{(m_1)^2 - q^2} + \frac{(f_2)^2}{(m_2)^2 - q^2} + \int_{s'_0}^{\infty} ds \frac{\rho^{OPE}(s)}{s - q^2}, \quad (14)$$

Applying the Borel transformation in Eqs. (14) and (5) we get:

$$\Pi^{Phen}(M^2) = (f_1)^2 e^{-(m_1)^2/M^2} + (f_2)^2 e^{-(m_2)^2/M^2} + \int_{s'_0}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2}, \quad (15)$$

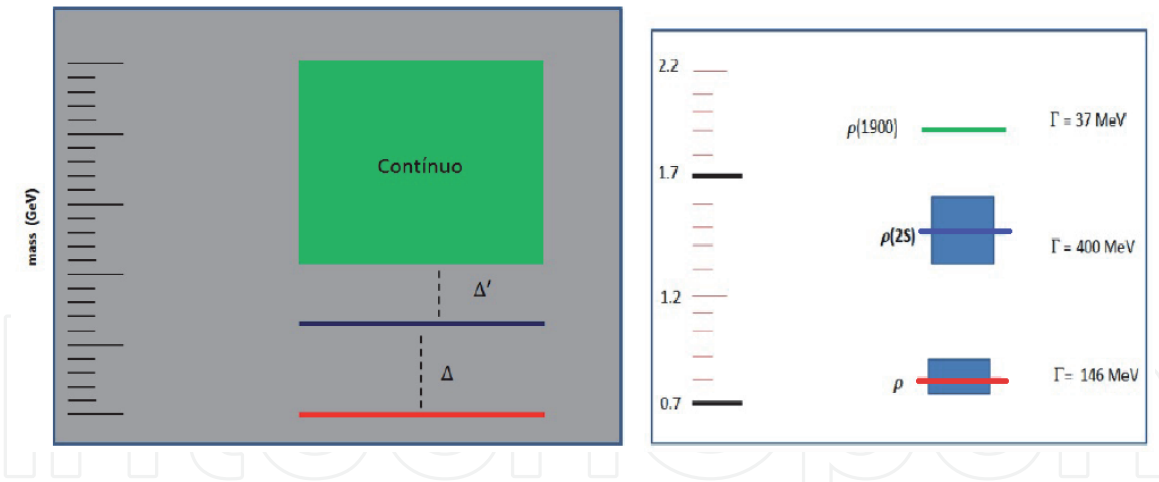


Figure 1. On the left side it is seen the double pole ansatz, Δ and Δ' represent the Gaps among the ground, first excited and continuum states. On the right side it is seen the mass spectra for the ρ meson and its resonances [16–18].

$$\Pi^{OPE}(M^2) = \int_{s'_0}^{s'_0} ds \rho^{OPE}(s) e^{-s/M^2} + \int_{s'_0}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2} + \Pi^{cond}(M^2). \quad (16)$$

By the Quark-Hadron duality we have $\Pi^{Phen}(M^2) = \Pi^{OPE}(M^2)$, thus:

$$(f_1)^2 e^{-(m_1)^2/M^2} + (f_2)^2 e^{-(m_2)^2/M^2} = \int_{s'_0}^{s'_0} ds \rho^{OPE}(s) e^{-s/M^2} + \Pi^{cond}(M^2). \quad (17)$$

The contribution of the resonances is given by:

$$CR = \int_{s'_0}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2}. \quad (18)$$

To develop Eq. (17) let us make a variable change taking $M^{-2} = x$, so we write:

$$(f_1)^2 e^{-(m_1)^2 x} + (f_2)^2 e^{-(m_2)^2 x} = \int_{s'_0}^{s'_0} ds \rho^{OPE}(s) e^{-sx} + \Pi^{cond}(x). \quad (19)$$

Now, taking the derivative of Eq. (19) with respect to x we get:

$$-(m_1 f_1)^2 e^{-(m_1)^2/M^2} - (m_2 f_2)^2 e^{-(m_2)^2/M^2} = \frac{d}{dx} \Pi^{OPE}(x), \quad (20)$$

where, now, we are considering

$$\Pi^{OPE}(x) = \int_{s'_0}^{s'_0} ds \rho^{OPE}(s) e^{-sx} + \Pi^{cond}(x). \quad (21)$$

We observe that the Eqs. (19) and (21) form an equations system in x variable. In this system we can make a new change of variables as follows:

$$A(x) = (f_1)^2 e^{-(m_1)^2 x} \text{ and } B(x) = (f_2)^2 e^{-(m_2)^2 x}, \quad (22)$$

this way we get the following system:

$$A(x) + B(x) = \Pi^{OPE}(x). \quad (23)$$

$$-(m_1)^2 A(x) - (m_2)^2 B(x) = \frac{d}{dx} \Pi^{OPE}(x) \quad (24)$$

Solving the above system of equation for $A(x)$ and $B(x)$ we have:

$$A(x) = \frac{\frac{d}{dx} \Pi^{OPE}(x) + \Pi^{OPE} m_2^2}{m_2^2 - m_1^2}. \quad (25)$$

$$B(x) = \frac{\frac{d}{dx} \Pi^{OPE}(x) + \Pi^{OPE} m_1^2}{m_1^2 - m_2^2}. \quad (26)$$

Note that, Eqs. (25) and (26) presents information about the hadron masses and their coupling constants, to eliminate de coupling constants we have to take the derivative of Eq. (25) and then dividing the result by the own Eq. (25) and the same procedure with Eq. (26). Thus, we have:

$$m_1 = \sqrt{-\frac{\frac{d}{dx} \Pi^{OPE}(x) m_2^2 + \frac{d^2}{dx^2} \Pi^{OPE}}{\frac{d}{dx} \Pi^{OPE}(x) + \Pi^{OPE} m_2^2}}, \quad (27)$$

$$m_2 = \sqrt{-\frac{\frac{d}{dx} \Pi^{OPE}(x) m_1^2 + \frac{d^2}{dx^2} \Pi^{OPE}}{\frac{d}{dx} \Pi^{OPE}(x) + \Pi^{OPE} m_1^2}}. \quad (28)$$

This way we have the both solutions coupling each other. On the other hand, what we are looking for are mass solutions for the ground state and its first excited state independent each other. To do so, we take the second derivative of the Eq. (19) with respect to x and the result we divide by Eq. (23) for decoupling of the m_1 mass. Note that the same procedure can be done for Eqs. (19) and (24) for decoupling of the m_2 mass. So, for the m_2 mass we have:

$$m_2^4 = \frac{\frac{d^3}{dx^3} \Pi^{OPE}(x) + m_1^2 \frac{d^2}{dx^2} \Pi^{OPE}(x)}{\frac{d}{dx} \Pi^{OPE}(x) + \Pi^{OPE}(x) m_1^2}. \quad (29)$$

Substituting Eq. (29) in Eq. (28) we obtain the following polynomial equation:

$$m_2^4 a + m_2^2 b + c = 0, \quad (30)$$

where a , b and c are respectively:

$$a = -\left(\frac{d}{dx} \Pi^{OPE}(x)\right)^2 + \Pi^{OPE}(x) \frac{d^2}{dx^2} \Pi^{OPE}(x), \quad (31)$$

$$b = -\left(\frac{d^2}{dx^2} \Pi^{OPE}(x)\right) \frac{d}{dx} \Pi^{OPE}(x) + \Pi^{OPE}(x) \frac{d^3}{dx^3} \Pi^{OPE}(x), \quad (32)$$

$$c = \left(\frac{d^3}{dx^3} \Pi^{OPE}(x)\right) \frac{d}{dx} \Pi^{OPE}(x) - \frac{d^2}{dx^2} \Pi^{OPE}(x). \quad (33)$$

Note that, for the m_1 mass, following the same procedure we get the other polynomial equation like Eq. (30). Thus, solving the polynomial equation, given by Eq. (30), and the same for m_1 mass, the physical solutions the represent m_1 like the ground state mass and m_2 like the first excited state mass are given by:

$$m_1 = \sqrt{-\frac{b + \sqrt{\Delta}}{2a}}, \quad (34)$$

$$m_2 = \sqrt{-\frac{b - \sqrt{\Delta}}{2a}}. \quad (35)$$

These results can be developed to obtain the masses of the ground state and its first excited state for any $\bar{q}q$ vector meson, also, we can calculate their coupling constants using the masses estimated in the Eqs. (25) and (26).

5. Results for the ρ meson

For the ρ meson we use the $\Pi^{OPE}(x)$ given by the Feynman diagrams that to be seen in [citar o greiber]. In this way we have:

$$\rho^{OPE}(s) = \frac{1}{4\pi^2} \left(1 + \frac{\alpha_s}{\pi} \right) \quad (36)$$

and

$$\Pi^{cond}(x) = x \left(\frac{1}{12} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle + 2 m_q \langle \bar{q}q \rangle \right) - x^2 \frac{112}{18} \pi \alpha_s \langle \bar{q}q \rangle^2, \quad (37)$$

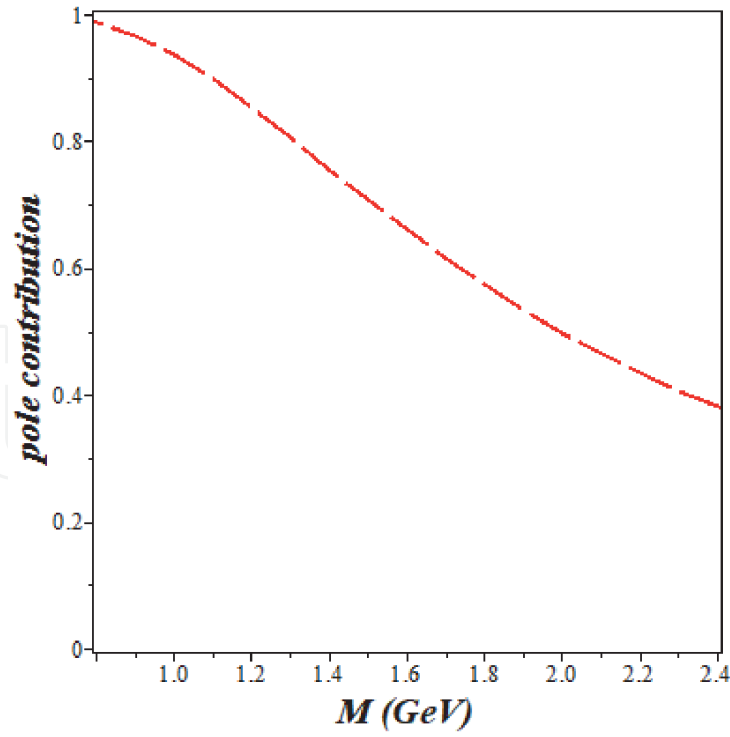
where α_s is the strong coupling constant, m_q is the light quark mass, $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ is the gluon condensate, $\langle \bar{q}q \rangle$ is the quark condensate and $s_0^{min} = 4 m_q^2$.

Following [19], for the ρ meson we use the parameters: $\alpha_s = 0.5$, $m_q = (6.4 \pm 1.25) \text{ MeV}$, $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{ GeV}^3$, $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0012 \pm 0.004) \text{ GeV}^4$ at the renormalization scale $\mu = 1 \text{ GeV}$.

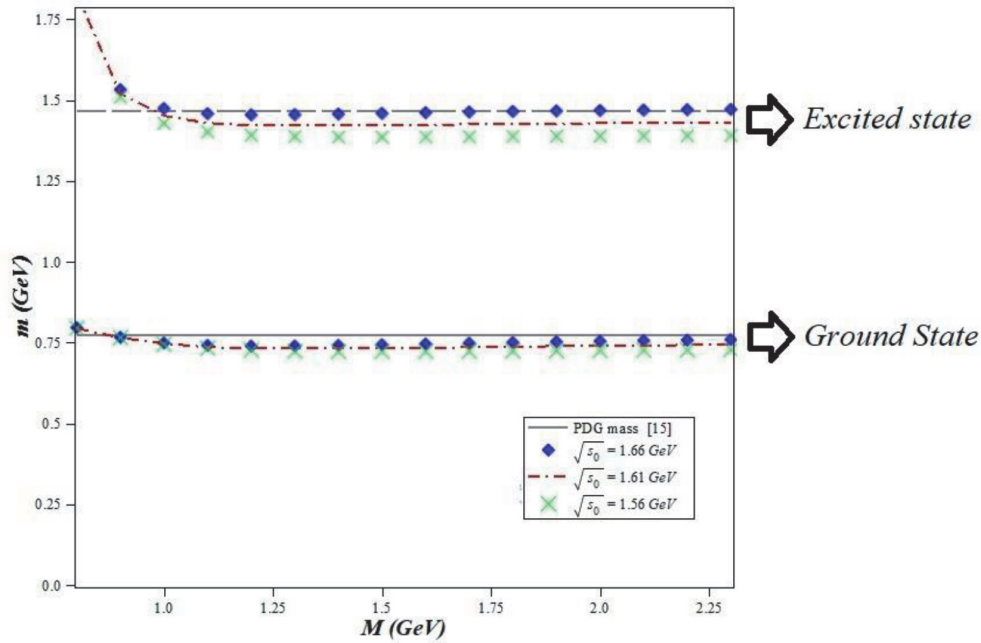
Using the mass of the $\rho(3S) = 1.9 \text{ GeV}$ [16], we get $\sqrt{s_0'} = 1.9 \text{ GeV}$, but in this case, the decay constant of the excited state is bigger than the decay constant of the ground state, that way the sum rules fails. Furthermore, the maximum value of the $\sqrt{s_0'}$ parameter is 1.66 GeV . That way, the excited state decay constant is a bit lower than the ground state decay constant. The minimum value for $\sqrt{s_0'}$ is 1.56 GeV , because $\sqrt{s_0'} - m_{\rho(2S)}$ reaches the value of 100 MeV .

In this way, we can find the Borel window where the QCDSR is valid. In this case, the Borel window is shown in **Figure 2** and it is calculated by the ratio between Eqs. (17) and (18) for a given $\sqrt{s_0'}$ value and considering the $\rho(1S, 2S)$ masses given by [16]. We can see that to have a good accuracy on our results we have to evaluate the QCDSR in a range of $0.8 \leq M \leq 2.3$, where the pole contribution is bigger than 40%.

In **Figure 3** we display the masses of the $\rho(1S, 2S)$ mesons as function of the Borel mass for three different values of the $\sqrt{s_0'}$ parameter that are: 1.66 GeV (polygonal blue point), 1.61 GeV (red dot-dashed line), 1.56 GeV (diagonal green cross point) and the grey lines representing the masses of the ground state and the first excited state for the ρ meson according to [16]. We can see that for the first


Figure 2.

The red dashed line represents the pole contribution as function of the Borel mass. Note that, for $\sqrt{s'_0} = 1.66 \text{ GeV}$ and the $\rho(1S, 2S)$ meson masses given by [16], the range where the QCDSR is $8\text{GeV} \leq M \leq 2.3 \text{ GeV}$, where the polo contribution is bigger than 40%.


Figure 3.

The masses of the $\rho(1S, 2S)$ mesons as function of the Borel mass for three different values of the $\sqrt{s_0}$ parameter where 1.66 GeV is given by polygonal blue point, 1.61 GeV by red dot-dashed line, 1.56 GeV by diagonal green cross point and the grey lines representing the masses of the ground state and the first excited state for the ρ meson according to [16].

excited state the mass average is closely to the experimental data for the $\rho(2S)$, where its mass is about 1.45 GeV [16].

For the ground state, in **Figure 3**, we show that the mass average of the $\rho(1S)$ is about 750 MeV, also pretty close to that one seen in [16], that is about 775 MeV for the experimental data.

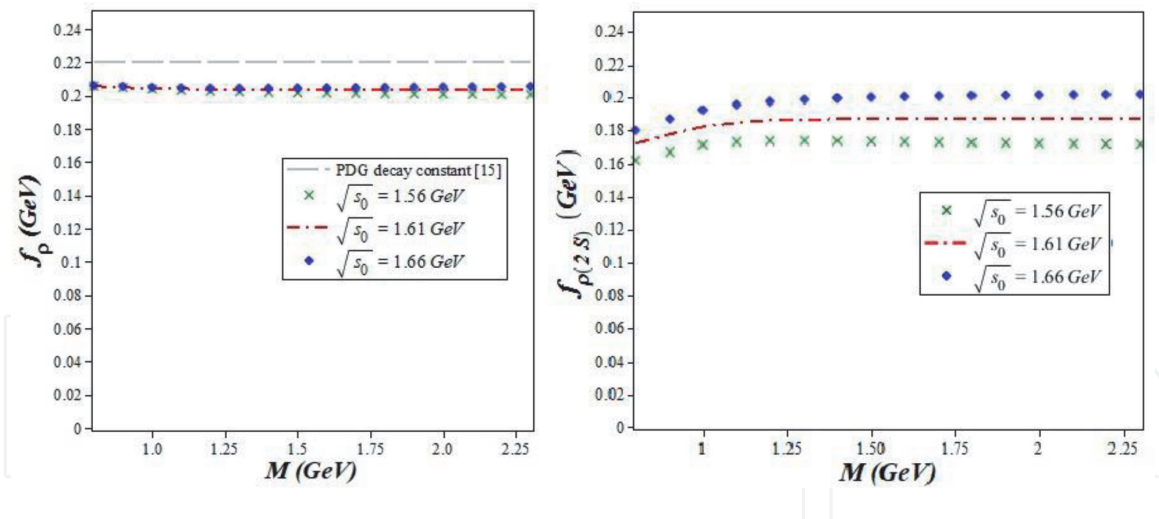


Figure 4. On the left side, we have the value of the decay constant for the $\rho(1S)$ meson about (203 ± 5) MeV. The values of the $\sqrt{s_0}$ parameter are: 1.66 GeV (polygonal blue dot line), 1.61 GeV (red dash-dotted line) and 1.56 GeV (diagonal green cross dot line), the grey dashed line is the experimental value [16] that is 220 MeV. On the right side, we have the value of the decay constant for the $\rho(2S)$ meson about (186 ± 14) MeV for the same values of the $\sqrt{s_0}$ parameter are: 1.66 GeV (polygonal blue dot line), 1.61 GeV (red dash-dotted line) and 1.56 GeV (diagonal green cross dot line).

To evaluate the decay constants, we use the experimental masses given by [16]. For the $\rho(1S)$ we use 0.775 GeV and for the $\rho(2S)$ we use 1.46 GeV. In **Figure 4**, we display the decay constant for the $\rho(1S)$ and $\rho(2S)$ mesons as function of the Borel mass for three different values of the $\sqrt{s_0}$ parameter.

In **Figure 4**, on the left side, we have an average for the decay constant of the $\rho(1S)$ about (203 ± 5) MeV, note that the maximum value for the decay constant is that one where $\sqrt{s_0} = 1.66$ GeV (polygonal blue point), the grey dashed line represents the experimental data [16] for the $\rho(1S)$ decay constant that is 220 MeV. On the right side, we have an average for the decay constant of the $\rho(2S)$ about (186 ± 14) MeV, where we considered uncertainty with respect to $\sqrt{s_0}$ parameter at $M = 2$ GeV.

Furthermore, it is interesting note that in Ref. [20] we can see another way to extract the experimental decay constant of the ρ^\pm from semileptonic decay, $\tau^\pm \rightarrow \rho^\pm \nu_\tau$. Note that in Ref. [19].

6. Conclusions

In this work, we made a little revision about the QCD Sum rules method and presented a new method for calculation of the hadronic parameters like mass and decay constant [19] as function of the Borel mass.

We show that the double pole method on QCDSR consists in a fit on the interpretation of the correlation function by the phenomenological side, where the relations dispersion is now presented with two poles plus a continuum of excited states, being these two poles representing the ground state and the first excited state.

For the $\rho(1S, 2S)$ mesons we had a good approximation for the calculations of these masses comparing with the experimental data on the literature. Beyond that, for the decay constant of the $\rho(2S)$ meson we had a good prediction like it is seen in [19] where $f_{\rho(2S)} = (182 \pm 10)$ MeV.

Our intention with this work consists on the studying of the vector mesons testing the accuracy of the double pole method and apply this method to analyze others kind of mesons such as scalar mesons.

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Conflict of interest

The authors declare no conflict of interest.

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