Chapter

The Fourier Transform Method for Second-Order Integro-Dynamic Equations on Time Scales

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Abstract

In this chapter we introduce the Fourier transform on arbitrary time scales and deduct some of its properties. In the chapter are given some applications for second-order integro-dynamic equations on time scales.

Keywords: time scale, Fourier transform, generalized shift problem, integro-dynamic equation

1. Introduction

Starting with the pioneering work of Hilger [1], the measure chains and in particular, the time scales have gained a great attention in the last decades. Especially, theoretical studies on dynamic equations on general time scales, which can be regarded as generalization of the differential equations, achieved big progress [2, 3].

The main aim of this chapter is to introduce the Fourier transform on arbitrary time scales and to deduct some of its properties. We give applications for solving of second-order integro-dynamic equations on time scales.

The chapter is organized as follows. In the next section we give some basic definitions and facts from time scale calculus, Laplace, bilateral Laplace transform. In Section 3 we define the Fourier transform and deduct some of its properties. In Section 4 we give applications for second-order integro-dynamic equations on time scales.

2. Preliminaries and auxiliary results

2.1 Time scales

Throughout this paper, we will assume that the reader is familiar with the basics of the time scale calculus. A detailed introduction to the time scale calculus is given in [2, 3]. Here, we collect the definitions and theorems that will be most useful in this paper.

Definition 2.1. *A time scale, denoted by* $\mathbb{T}$*, is a nonempty, closed subset of* $\mathbb{R}$*. For* $a, b \in \mathbb{T}$*, we let* $[a, b]$ *denote the set* $[a, b] \cap \mathbb{T}$*.*

Definition 2.2. *Let* $\mathbb{T}$ *be a time scale. For* $t \in \mathbb{T}$*, we define the forward jump operator* $\sigma : \mathbb{T} \to \mathbb{R}$ *by* $\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}$*, and the backward jump operator* $\rho : \mathbb{T} \to \mathbb{T}$ *is given by* $\rho(t) = \sup\{x \in \mathbb{T} : s < t\}$*.*

By convention, we take $\inf \varnothing = \sup\mathbb{T}$, $\sup\varnothing = \inf \mathbb{T}$. For a function $f : \mathbb{T} \to \mathbb{R}$, we will use the notation $f^{\sigma}(t)$ for the composition $f(\sigma(t))$.

Definition 2.3. *The graininess function* $\mu : \mathbb{T} \to [0, \infty)$ *is defined by* $\mu(t) = \sigma(t) - t$*,* $t \in \mathbb{T}$*.*

Definition 2.4. *Let* $t \in \mathbb{T}$*. If* $\sigma(t) = t$ *and* $t < \sup\mathbb{T}$*, then t is right-dense. If* $\sigma(t) > t$*, then t is right-scattered. Similarly, if* $\rho(t) = t$ *and* $t > \inf \mathbb{T}$*, then t is left-dense. If* $\rho(t) < t$*, then t is left-scattered.*

Definition 2.5. *If* $\sup\mathbb{T} = m$ *such that m is left-scattered, then define* $\mathbb{T}^{\kappa} = \mathbb{T} \backslash \{m\}$*, otherwise, define* $\mathbb{T}^{\kappa} = \mathbb{T}$*.*

Definition 2.6. *A function* $f : \mathbb{T} \to \mathbb{R}$ *is rd-continuous provided it is continuous at right-dense points in* $\mathbb{T}$ *and its left-sided limits exist and are finite at all left-dense points in* $\mathbb{T}$*. A function* $p : \mathbb{T} \to \mathbb{R}$ *is regressive provided* $1 + \mu(t)p(t) \neq 0$*,* $t \in \mathbb{T}^{\kappa}$*. The set of all regressive and rd-continuous functions on a time scale* $\mathbb{T}$ *is denoted by* $\mathcal{R} = \mathcal{R}(\mathbb{T})$*. We use the notation* $\mathcal{R}^{+}$ *to denote the subgroup of those* $p \in \mathcal{R}$ *for which* $1 + \mu(t)p(t) > 0$ *for all* $t \in \mathbb{T}^{\kappa}$*.*

Definition 2.7. *The delta derivative of* $f : \mathbb{T} \to \mathbb{R}$ *at* $t \in \mathbb{T}^{\kappa}$*, is defined to be*

$$f^{\Delta}(t) = \lim_{s \to t} \frac{f(\sigma(t)) - f(s)}{\sigma(t) - s} \tag{1}$$

provided this limit exists.

Definition 2.8. *For* $p \in \mathcal{R}$*, the generalized exponential function* $e_p : \mathbb{T} \times \mathbb{T} \to \mathbb{R}$ *is defined by*

$$e_p(t, s) = \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\right), \tag{2}$$

for $s, t \in \mathbb{T}$*, where the cylinder transformation,* $\xi_h(z)$*, is defined by*

$$\xi_h(z) = \begin{cases} \dfrac{1}{h} Log(1 + zh), & h > 0, \\ z, \quad h = 0. \end{cases} \tag{3}$$

Definition 2.9. *For* $p, q \in \mathcal{R}$*, we define the operation* $\oplus$ *and* $\ominus$ *as follows*

$$(p \oplus q)(t) = p(t) + q(t) + \mu(t)p(t)q(t), \quad (\ominus p)(t) = -\frac{p(t)}{1 + \mu(t)p(t)}. \tag{4}$$

The proof of the next theorem is given in [2, 3].

Theorem 2.1. *If* $p, q \in \mathcal{R}$ *and* $t, s, r \in \mathbb{T}$*, then*

1. $e_0(t, s) = 1$, $e_p(t, t) = 1$.

2. $e_p^{\sigma}(t, s) = (1 + \mu(t)p(t))e_p(t, s)$.

3. $e_p(s, t) = \frac{1}{e_p(t, s)} = e_{\ominus p}(t, s)$.

4. $e_p(t,s)e_p(s,r) = e_p(t,r)$.

5. $e_p(t,s)e_q(t,s) = e_{p\oplus q}(t,s)$.

6. $e_p(t,t_0) > 0$ *for any* $t_0, t \in \mathbb{T}$ *if* $p \in \mathcal{R}$ *and* $1+\mu(t)p(t) > 0$ *for any* $t \in \mathbb{T}^\kappa$.

Definition 2.10. *For* $h > 0$, *the Hilger complex plane is defined by* $\mathbb{C}_h = \mathbb{C}\backslash\left\{-\frac{1}{h}\right\}$ *and we take* $\mathbb{C}_0 = \mathbb{C}$ *and* $\mathbb{C}_\infty = \mathbb{C}\backslash\{0\}$.

Definition 2.11. *For given* $h \in [0,\infty)$, *the Hilger real part of a number* $z \in \mathbb{C}$ *is given by the formula*

$$Re_h(z) = \begin{cases} Re(z), & h = 0, \\ \dfrac{|1+hz|-1}{h}, & 0 < h < \infty, \\ |z|, & h = \infty. \end{cases} \tag{5}$$

It is known, see [4], that for a fixed z and $0 < h < \infty$, $\mathrm{Re}_h(z)$ is a nondecreasing function of h. This relationship extends to $h = \infty$ because for any $0 < h < \infty$,

$$\mathrm{Re}_h(z) = \frac{|1+hz|-1}{h} \leq \frac{1+h|z|-1}{h} = |z| = \mathrm{Re}_\infty(z). \tag{6}$$

2.2 The Laplace transform

Here we suppose that $\sup\mathbb{T} = \infty$ and $s \in \mathbb{T}$.

Definition 2.12. *For* $0 \leq h \leq \infty$ *and* $\lambda \in \mathbb{R}$, *we define*

$$\mathbb{C}_h(\lambda) = \{z \in \mathbb{C}_h : Re_h(z) > \lambda\} \tag{7}$$

and

$$\bar{\mathbb{C}}_h(\lambda) = \{z \in \mathbb{C}_h : 0 < Re_h(z) < \lambda\}. \tag{8}$$

Definition 2.13. *Define minimal graininess as follows* $\mu_*(s) = \inf_{t\in[s,\infty)}\mu(t)$.

If λ is positively regressive, then for any $z \in \mathbb{C}_{\mu_*(s)}(\lambda)$, it is known (see [4]) that

$$\begin{gathered} |e_{\lambda\ominus z}(t,s)| \leq e_{\lambda\ominus \mathrm{Re}_{\mu_*(s)}(z)}(t,s), \quad t \in [s,\infty), \\ \lim_{t\to\infty} \mathrm{Re}_{\mu_*(s)(z)}(t,s) = 0 \quad \text{and} \quad \lim_{t\to\infty} e_{\lambda\ominus z}(t,s) = 0. \end{gathered} \tag{9}$$

Definition 2.14. *If* $X \subset \mathbb{T}$ *and* $\alpha \in \mathcal{R}^+$ *is a constant, then we say that* $f \in C_{rd}(\mathbb{T})$ *is of exponential order* α *on* X *if there exists a constant* K *such that for all* $t \in X$, *the bound* $|f(t)| \leq Ke_\alpha(t,s)$ *holds.*

If $f \in C_{rd}([s,\infty))$ is of exponential order α, then for any $z \in \mathbb{C}_{\mu_*(s)}(\alpha)$ (see [4]) $\lim_{t\to\infty} f(t)e_{\ominus z}(t,s) = 0$.

Definition 2.15. *If* $f : \mathbb{T} \to \mathbb{C}$ *and* $z \in \mathbb{C}$ *is a complex number such that for all* $t \in [s,\infty)$ *we have* $1+\mu(t)z \neq 0$, *then the Laplace transform is defined by the improper integral*

$$\mathcal{L}(f)(z,s) = \int_s^\infty f(\tau)e_{\ominus z}(\sigma(\tau),s)\Delta\tau, \tag{10}$$

whenever the integral exists.

Significant work has been conducted in [4, 5] and references therein to understand the analytical properties of the Laplace transform.

2.3 The bilateral Laplace transform

Here we suppose that $\sup\mathbb{T} = \infty$, $\inf \mathbb{T} = -\infty$ and $s \in \mathbb{T}$. Denote $\mu^*(s) = \sup_{t\in(-\infty,s]}\mu(t)$, $\bar{\mu}(s) = \inf_{t\in(-\infty,s]}\mu(t)$. For $\lambda \in \mathcal{R}$, define

$$M_\lambda(t,s) = \int_t^s \frac{1}{1+\lambda\mu(\tau)}\Delta\tau. \tag{11}$$

For $\lambda \in \mathcal{R}^+((-\infty,s])$, $\lambda \in \mathbb{R}$, it is known (see [6])

1. $M_\lambda^\Delta(t,s) < 0$ for all $t \in (-\infty,s)$, where the differentiation is with respect to t.

2. $\lim_{t\to-\infty} M_\lambda(t,s) = \infty$.

3. $|e_{\lambda\ominus z}(t,s)| \le e_{\lambda\ominus \mathrm{Re}_{\mu^*(s)}(z)}(t,s)$.

4. $\lim_{t\to-\infty} e_{\lambda\ominus \mathrm{Re}_{\mu^*(s)}(z)}(t,s) = 0$.

5. $\lim_{t\to-\infty} e_{\lambda\ominus z}(t,s) = 0$.

Definition 2.16. *Suppose that $f : \mathbb{T} \to \mathbb{R}$ is regulated. Then the bilateral Laplace transform of f is defined by*

$$\mathcal{L}^b(f)(z,s) = \int_{-\infty}^{\infty} f(t)e_{\ominus z}(\sigma(t),s)\Delta t, \tag{12}$$

for regressive $z \in \mathbb{C}$ where the improper integral exists.

Definition 2.17. *Let $\alpha, \gamma \in \mathbb{R}$. We say that a function $f \in C_{rd}(\mathbb{T})$ has double exponential order (α,γ) on $\mathbb{T}$ if the restrictions $f|_{(-\infty,s]}$ and $f|_{[s,\infty)}$ are of exponential order α and γ, respectively.*

If $f \in C_{rd}(\mathbb{T})$ is of double exponential order (α,γ), in [6], they are proved the following properties

1. for any $z \in \mathbb{C}_{\mu_*(s)}(\gamma)$, $\lim_{t\to\infty} f(t)e_{\ominus z}(t,s) = 0$.

2. for any $z \in \bar{\mathbb{C}}_{\mu^*(s)}(\alpha)$, $\lim_{t\to-\infty} f(t)e_{\ominus z}(t,s) = 0$.

For $z \in \mathbb{C}$, we define

$$\bar{\bar{\mu}}(s,z) = \begin{cases} \mu^*(s), & \mathrm{Re}_{\bar{\mu}(s)}(z) \le 0, \\ \bar{\mu}(s), & \mathrm{Re}_{\bar{\mu}(s)}(z) > 0. \end{cases} \tag{13}$$

Definition 2.18. *Let $\alpha \in \mathcal{R}^+((-\infty,s])$ and $\gamma \in \mathcal{R}^+([s,\infty))$, α, $\gamma \in \mathbb{R}$. We say that (s,α,γ) is an admissible triple if*

$$\mathbb{C}_{s,\alpha,\gamma} = \left\{z \in \mathbb{C} : \mathit{Re}_{\mu^*(s)}(z) < \alpha, \quad \mathit{Re}_{\mu_*(s)}(z) > \gamma, \quad 1 + \bar{\bar{\mu}}(s,z)\,\mathit{Re}_{\bar{\mu}(s)}(z) \ne 0\right\} \ne \varnothing. \tag{14}$$

If (s, α, γ) is an admissible triple and if $f \in \mathcal{C}_{rd}(\mathbb{T})$ is of double exponential order (α, γ), then in [6] it is proved that $\mathcal{L}^b(\cdot, s)$ exists on $\mathbb{C}_{s,\alpha,\gamma}$, converges absolutely and uniformly, and

$$\lim_{|z|\to\infty} \mathcal{L}^b(f)(z, s) = 0. \tag{15}$$

3. The Fourier transform

Suppose that $\mathbb{T}$ is a time scale so that $\inf \mathbb{T} = -\infty$, $\sup\mathbb{T} = \infty$ and $s \in \mathbb{T}$.

Definition 3.1. *Suppose that $f : \mathbb{T} \to \mathbb{R}$ is regulated. Then the Fourier transform of the function f is defined by*

$$\mathcal{F}(f)(x, s) = \int_{-\infty}^{\infty} f(t) e^{\sigma}_{\ominus ix}(t, s)\Delta t \tag{16}$$

for $x \in \mathbb{R}$ for which $1 + ix\mu(t) \neq 0$ for any $t \in \mathbb{T}^{\kappa}$ and the improper integral exists.

Definition 3.2. *Let $\alpha \in \mathcal{R}^{+}([s, \infty))$, $\gamma \in \mathcal{R}^{+}((-\infty, s])$. We say that (s, γ, α) is a real admissible triple if*

$$\begin{gathered} R_{s,\gamma,\alpha} = \{x \in \mathbb{R} : \ Re_{\mu^*(s)}(ix) < \gamma, \quad Re_{\mu_*(s)}(ix) > \alpha, \\ 1 + \bar{\bar{\mu}}(s)\, Re_{\bar{\bar{\mu}}(s)}(ix) \neq 0\} \neq \emptyset. \end{gathered} \tag{17}$$

If $f \in \mathcal{C}_{rd}(\mathbb{T})$, then the triple (s, γ, α) is a real admissible triple and f is of double exponential order (α, γ), then $\mathcal{F}(f)(\cdot, s)$ exists on $R_{s,\gamma,\alpha}$ and converges absolutely and uniformly on $R_{s,\gamma,\alpha}$. Below we will list some of the properties of the Fourier transform.

Theorem 3.1. *Let $f, g : \mathbb{T} \to \mathbb{R}$, $\alpha, \beta \in \mathbb{C}$. Then*

$$\mathcal{F}(\alpha f + \beta g)(x, s) = \alpha\mathcal{F}(f)(x, s) + \beta\mathcal{F}(g)(x, s) \tag{18}$$

for those $x \in \mathbb{R}$ for which $1 + x\mu(t) \neq 0$, $t \in \mathbb{T}^{\kappa}$, and the respective integrals exist.

Proof. We have

$$\begin{aligned} \mathcal{F}(\alpha f + \beta g)(x, s) &= \int_{-\infty}^{\infty} (\alpha f + \beta g)(t) e^{\sigma}_{\ominus ix}(t, s)\Delta t \\ &= \alpha \int_{-\infty}^{\infty} f(t) e^{\sigma}_{\ominus ix}(t, s)\Delta t + \beta \int_{-\infty}^{\infty} g(t) e^{\sigma}_{\ominus ix}(t, s)\Delta t = \alpha\mathcal{F}(f)(x, s) + \beta\mathcal{F}(g)(x, s). \end{aligned} \tag{19}$$

This completes the proof. □

Theorem 3.2. *Let $f : \mathbb{T} \to \mathbb{R}$ be enough times Δ-differentiable. For any $k \in \mathbb{N}$, we have*

$$\mathcal{F}\left(f^{\Delta^k}\right)(x, s) = (ix)^k \mathcal{F}(f)(x, s) \tag{20}$$

for those $x \in \mathbb{R}$ for which $1 + x\mu(t) \neq 0$, $t \in \mathbb{T}^{\kappa}$, and the respective integrals exist and

$$\lim_{t\to\pm\infty} f^{\Delta^l}(t) e_{\ominus ix}(t, s) = 0, \quad l \in \{0, \dots, k-1\}. \tag{21}$$

Proof. We will use the principle of mathematical induction.

1. For $k = 1$, we have

$$\begin{aligned}\mathcal{F}(f^{\Delta})(x,s) &= \int_{-\infty}^{\infty} f^{\Delta}(t)e^{\sigma}_{\ominus ix}(t,s)\Delta t = \lim_{t\to\infty} f(t)e_{\ominus ix}(t,s) - \lim_{t\to-\infty} f(t)e_{\ominus ix}(t,s)\\ &\quad -\int_{-\infty}^{\infty}(\ominus ix)(t)f(t)e_{\ominus ix}(t,s)\Delta t = ix\int_{-\infty}^{\infty} f(t)e^{\sigma}_{\ominus ix}(t,s)\Delta t\\ &= ix\mathcal{F}(f)(x,s).\end{aligned} \tag{22}$$

2. Assume that

$$\mathcal{F}\left(f^{\Delta^k}\right)(x,s) = (ix)^k\mathcal{F}(f)(x,s) \tag{23}$$

for some $k \in \mathbb{N}$.

3. We will prove that

$$\mathcal{F}\left(f^{\Delta^{k+1}}\right)(x,s) = (ix)^{k+1}\mathcal{F}(f)(x,s). \tag{24}$$

Really, we have

$$\mathcal{F}\left(f^{\Delta^{k+1}}\right)(x,s) = ix\mathcal{F}\left(f^{\Delta^k}\right)(x,s) = (ix)^{k+1}\mathcal{F}(f)(x,s). \tag{25}$$

This completes the proof. □

Theorem 3.3. *Let $f : \mathbb{T} \to \mathbb{R}$. Then*

$$\overline{\mathcal{F}(f)(x,s)} = \mathcal{F}(f)(-x,s) \tag{26}$$

for those $x \in \mathbb{R}$ for which $1 \pm x\mu(t) \neq 0$, $t \in \mathbb{T}^{\kappa}$, and the respective integrals exist.

Proof. From the definition of the Fourier transform, we have

$$\begin{aligned}\overline{\mathcal{F}(f)(x,s)} &= \overline{\int_{-\infty}^{\infty} e^{\int_s^{\sigma(t)}\frac{1}{\mu(\tau)}\mathrm{Log}(1+\mu(\tau)(\ominus(ix))(\tau))\Delta\tau} f(t)\Delta t}\\ &= \int_{-\infty}^{\infty} \overline{e^{\int_s^{\sigma(t)}\frac{1}{\mu(\tau)}\mathrm{Log}(1+\mu(\tau)(\ominus(ix))(\tau))\Delta\tau} f(t)}\Delta t\\ &= \int_{-\infty}^{\infty} e^{\int_s^{\sigma(t)}\frac{1}{\mu(\tau)}\mathrm{Log}(1+\mu(\tau)(\ominus(i(-x)))(\tau))\Delta\tau} f(t)\Delta t\\ &= \mathcal{F}(f)(-x,s).\end{aligned} \tag{27}$$

This completes the proof. □

Theorem 3.4. *Let $f : \mathbb{T} \to \mathbb{R}$ be regulated and*

$$F(t) = \int_a^t f(\tau)\Delta\tau, \quad t \in \mathbb{T}, \tag{28}$$

for some fixed $a \in \mathbb{T}$. Then

$$\mathcal{F}(F)(x,s) = -\frac{i}{x}\mathcal{F}(f)(x,s) \tag{29}$$

for those $x \in \mathbb{R}$, $x \neq 0$, for which

$$\lim_{t \to \pm\infty} F(t) e_{\ominus ix}(t, s) = 0. \tag{30}$$

Proof. We have

$$\begin{aligned}
\mathcal{F}(F)(x,s) &= \int_{-\infty}^{\infty} F(t) e^{\sigma}_{\ominus ix}(t,s)\Delta t = \int_{-\infty}^{\infty} F(t)(1+\mu(t)(\ominus(ix))(t)) e_{\ominus ix}(t,s)\Delta t \\
&= \int_{-\infty}^{\infty} F(t) \frac{1}{1+i\mu(t)x} e_{\ominus ix}(t,s)\Delta t = -\frac{1}{ix}\int_{-\infty}^{\infty} F(t)\frac{-ix}{1+i\mu(t)x} e_{\ominus ix}(t,s)\Delta t \\
&= \frac{i}{x}\int_{-\infty}^{\infty} F(t)(\ominus ix)(t) e_{\ominus ix}(t,s)\Delta t = \frac{i}{x}\int_{-\infty}^{\infty} F(t) e^{\Delta}_{\ominus ix}(t,s)\Delta t \\
&= \frac{i}{x}\left(\lim_{t\to\infty} F(t)e_{\ominus ix}(t,s) - \lim_{t\to-\infty} F(t) e_{\ominus ix}(t,s)\right) - \frac{i}{x}\int_{-\infty}^{\infty} f(t) e^{\sigma}_{\ominus ix}(t,s)\Delta t \\
&= -\frac{i}{x}\mathcal{F}(f)(x,s)
\end{aligned} \tag{31}$$

for those $x \in \mathbb{R}$, $x \neq 0$, for which

$$\lim_{t \to \pm\infty} F(t) e_{\ominus ix}(t, s) = 0. \tag{32}$$

This completes the proof. □

4. Applications to second-order integro-dynamic equations

Consider the equation

$$y^{\Delta^2} + a_1 y^{\Delta} + a_2 y = \int_a^t f(s)\Delta s, \tag{33}$$

where $a_1, a_2 \in \mathbb{R}, f \in \mathcal{C}_{rd}(\mathbb{T}), f : \mathbb{T} \to \mathbb{R}$. Let $s \in \mathbb{T}$ be fixed. Let also, $x \in \mathbb{R}$ be such that

$$x^2 - ia_1 x - a_2 \neq 0 \tag{34}$$

and

$$\lim_{t\to\pm\infty} y^{\Delta^l}(t) e_{\ominus ix}(t,s) = 0, \quad l = 0, 1, \tag{35}$$

and

$$\lim_{t\to\pm\infty} F(t) e_{\ominus ix}(t,s) = 0, \tag{36}$$

where

$$F(t) = \int_a^t f(s)\Delta s, \quad t \in \mathbb{T}. \tag{37}$$

Here $a \in \mathbb{T}$ is a fixed constant. Set

$$Y(x) = \mathcal{F}(y)(x,s). \tag{38}$$

Then

$$\begin{aligned} \mathcal{F}\left(y^{\Delta}\right)(x,s) &= ix\mathcal{F}(y)(x,s) \\ &= ixY(x), \\ \mathcal{F}\left(y^{\Delta^2}\right)(x,s) &= (ix)^2\mathcal{F}(y)(x,s) \\ &= -x^2Y(x) \end{aligned} \tag{39}$$

and

$$\mathcal{F}(f)(x,s) = -\frac{i}{x}\mathcal{F}(x,s). \tag{40}$$

Then the Eq. (33) takes the form

$$-x^2Y(x) + ia_1xY(x) + a_2Y(x) = -\frac{i}{x}\mathcal{F}(x,s), \tag{41}$$

or

$$\left(x^2 - ia_1x - a_2\right)Y(x) = \frac{i}{x}\mathcal{F}(f)(x,s), \tag{42}$$

or

$$Y(x) = \frac{i}{x(x^2 - ia_1x - a_2)}\mathcal{F}(f)(x,s). \tag{43}$$

Consequently

$$y(t) = \mathcal{F}^{-1}\left(\frac{i}{x(x^2 - ia_1x - a_2)}\mathcal{F}(f)(\cdot,s)\right)(t), \quad t \in \mathbb{T}, \tag{44}$$

provided that $\mathcal{F}^{-1}$ exists.

Additional classifications

AMS Subject Classification: 39A10, 39A11, 39A12

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