Chapter

Path Integral Two Dimensional Models of P– and D–Wave Superconductors and Collective Modes

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Abstract

The main parameter, which describes superfluids and superconductors and all their main properties is the order parameter. After discovery the high temperature superconductors (HTSC) and heavy fermion superconductors (HFSC) the unconventional pairing in different superconductors is studied very intensively. The main problem here is the type of pairing: singlet or triplet, orbital moment of Cooper pair value L, symmetry of the order parameter etc. Recent experiments in Sr_2RuO_4 renewed interest in the problem of the symmetry of the order parameters of the HTSC. The existence of CuO_2 planes – the common structural factor of HTSC – suggests we consider two-dimensional (2D) models. A 2D– model of *p*–pairing using a path integration technique has been developed by Brusov and Popov. A 2D model of *d*–pairing within the same technique has been developed by Brusov et al. All properties of 2D–superconductors (for example, of CuO_2 planes of HTSC) and, in particular, the collective excitations spectrum, are determined by these functionals. We consider all superconducting states, arising in symmetry classification of p-wave and d-wave 2D–superconductors, and calculate the full collective modes spectrum for each of these states. This will help to identify the type of pairing and the symmetry of the order parameter in HTSC and HFSC.

Keywords: path integral, two-dimensional models, P- and D-wave superconductors, collective modes

1. Introduction

The main parameter, which describes superfluids and superconductors and all their main properties is the order parameter, which is equal to zero above transition temperature T_c into superconducting (superfluid) state and becomes nonzero below T_c. In Bose–systems the transition is caused by Bose–Einstein condensation of bosons while in case of Fermi–systems first the pairing of fermions with creation of Bose particles (Cooper pairs) takes place with their subsequent condensation. Besides the ordinary superconductors (where traditional s–pairing takes place) after discovery the high temperature superconductors (HTSC) and heavy fermion superconductors (HFSC) the unconventional pairing in different superconductors is studied very intensively [1–5]. The main problem here is the type of pairing: singlet or triplet, orbital moment of Cooper pair value L, symmetry of the order parameter etc.

Recent experiments in Sr_2RuO_4 [2–4] renewed interest in the problem of the symmetry of superconducting order parameters of the high temperature superconductors (HTSC).

Sr_2RuO_4 has been the candidate for a spin–triplet superconductor for more than 25 years. Recent NMR experiments have cast doubt on this candidacy. Symmetry–based experiments are needed that can rule out broad classes of possible superconducting order parameters. In Ref. 3 authors use the resonant ultrasound spectroscopy to measure the entire symmetry–resolved elastic tensor of Sr_2RuO_4 through the superconducting transition. They observe a thermodynamic discontinuity in the shear elastic modulus c_{66}, which implies that the superconducting order parameter has two components. A two–component p–wave order parameter, such as $p_x + ip_y$, satisfies this requirement. As this order parameter appears to have been precluded by recent NMR experiments, the alternative two–component order parameters of Sr_2RuO_4 are as following $\{d_{xz}, d_{yz}\}$ and $\{d_{x2-y2}, g_{xy(x2-y2)}\}$.

Authors of Ref. 4 have come to similar conclusions. They use ultrasound velocity to probe the superconducting state of Sr_2RuO_4. This thermodynamic probe is sensitive to the symmetry of the superconducting order parameter. Authors observe a sharp jump in the shear elastic constant c_{66} as the temperature is increased across the superconducting transition. This supposes that the superconducting order parameter is of a two–component nature.

The existence of CuO_2 planes [6] – the common structural factor of HTSC – suggests we consider 2D models. A 2D– model of p–pairing using a path integration technique has been developed by Brusov and Popov [7, 8]. A 2D model of d–pairing within the same technique has been developed by Brusov et al. [9–14]. The models use the hydrodynamic action functionals, which have been obtained by path integration over "fast" and "slow" Fermi–fields. All properties of 2D–superconductors (for example, of CuO_2 planes of HTSC) and, in particular, the collective excitations spectrum, are determined by these functionals. We consider all superconducting states, arising in symmetry classification of 2D–superconductors and calculate the full collective modes spectrum for each of these states. Current study continue our previous investigation [5], where we consider the problem of distinguish the mixture of two d–wave states from pure d–wave state of HTSC.

2. Two–dimensional models of p– and d–pairing in unconventional superconductors

2.1 *p*–Pairing

Below we develop 2D–model of p–pairing starting with the 3D scheme considered by Brusov et al. [9–14].

Two main distinctions between 3D–case and 2D–case are as follows:

a. The Cooper pair orbital moment $\boldsymbol{l}$ ($\boldsymbol{l}$ = 1) should be perpendicular to the plane and can have only two projections on the $\hat{z}$–axis: ± 1. The p– pairing is a triplet, thus the total spin of the pair is equal to 1, and in the case of 2D p–pairing we have $3 \times 2 \times 2 = 12$ degrees of freedom. In this case one can describe the superconducting state by complex 2×3 matrices $c_{ia}(p)$. The number of the collective modes in each phase is equal to the number of degrees of freedom. Just remind that in the 3D case this number is equal to 18.

b. Vector $\mathbf{x}$ is a 2D vector and square "volume" will be $S = L^2$ (instead of $V = L^3$ in 3D case).

2.1.1 Two–dimensional p*–wave superconducting states*

Effective action In case of two–dimensional p–wave superconductivity effective action takes a form (see the case of two–dimensional superfluidity of ^{3}He in Chapter XIX of Ref. 1)

$$S_{eff} = -\beta V \frac{16\pi^2 T_C \Delta T}{7\zeta(3)} F \tag{1}$$

where

$$\begin{aligned} F = -trAA^+ + \nu trA^+AP + (trA^+A)^2 + trAA^+AA^+ + trAA^+A^*A^T - \\ -trAA^TA^*A^+ - (1/2)trAA^TtrA^*A^+, \\ \nu = 7\zeta(3)\mu^2H^2/4\pi^2T_C\Delta T \end{aligned} \tag{2}$$

The effective action F is identical in form with that arising in the case of three–dimensional (3D) superconducting system. The difference is connected with the fact that the matrix A with elements a_{ia} for the two–dimensional system is a 2×3 matrix instead of 3×3 matrix in the case of three–dimensional (3D) superconducting system. The matrix P is the projector on the third axis: $P = \delta_{i3}\delta_{j3}$

$$P = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The following equation for the condensate matrix A could be obtained by minimizing F:

$$\begin{aligned} -A + \nu AP + 2(trAA^+)A + 2AA^+A + 2A^*A^TA - 2AA^TA^* - \\ -(trAA^T)A^* = 0. \end{aligned} \tag{3}$$

There are several solutions of Eq. (3), corresponding to the different superfluid phases. Let us consider the following possibilities:

$$\begin{aligned} A_1 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, A_2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, A_3 = \frac{1}{4}\begin{pmatrix} 1 & i & 0 \\ i & -1 & 0 \end{pmatrix}, \\ A_5 = \frac{1}{\sqrt{3}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, A_6 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, A_7 = \frac{1}{2}\begin{pmatrix} 0 & \pm 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\ A_8 = \left(\frac{1-\nu}{3}\right)^{1/2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, A_9 = \left(\frac{1-\nu}{4}\right)^{1/2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \end{pmatrix}. \end{aligned} \tag{4}$$

The corresponding values of the effective action F are equal to:

$$\begin{aligned} F_1 = -\frac{1}{4}, F_2 = -\frac{1}{4}, F_3 = -\frac{1}{8}, F_4 = -\frac{1}{6}, F_5 = -\frac{1}{6}, \\ F_6 = -\frac{1}{4}, F_7 = -\frac{1}{4}, F_8 = -(1-\nu)^2/6, F_9 = -(1-\nu)^2/4. \end{aligned} \tag{5}$$

The quantity of the effective action F for the first eight phases does not depend on H. The minimum value of $F = -1/4$ is reached for phases A_1 and A_2 as well as for the phases with matrices A_6 and A_7 and A_9 (last state has minimum energy in zero magnetic field ($\nu = 0$)).

The first two phases have been discovered by Brusov and Popov [7, 8] in the films of superfluid ^{3}He. Authors [7, 8] have called them the a – and b –phases and have proved that the phases a– and b– are stabile relative to the small perturbations. Brusov and Popov [7, 8] have calculated the full collective mode spectrum for two these phases. Brusov et al. [9–14] have calculated the full collective mode spectrum for A_6 and A_7 states.

2.1.2 The collective mode spectrum

The full collective mode spectrum for each of these phases consists of 12 modes (the number of degrees of freedom). Among them we have found Goldstone modes as well as high frequency modes (with energy (frequency) which is proportional to energy of the gap in single–particle spectrum).

The results obtained by Brusov and Popov [7, 8] and Brusov et al. [9–14] are shown below for collective mode spectrum for different two–dimensional superconducting states under p–pairing.

The collective mode spectrum for a–phase with order parameter $\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$:

$$\begin{gathered} E^2 = \frac{c_F^2 k^2}{2}\left(1 - \frac{5c_F^2 k^2}{96\Delta^2}\right), \text{(3 modes)} \\ E^2 = 2\Delta^2 + c_F^2 k^2/2, \text{(6 modes)} \\ E^2 = 4\Delta^2 + (0.500 + i0.433)c_F^2 k^2. \text{(3 modes)} \end{gathered} \tag{6}$$

The collective mode spectrum for b–phase with order parameter $\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$:

$$\begin{gathered} E^2 = \frac{c_F^2 k^2}{2}\left(1 - \frac{5c_F^2 k^2}{48\Delta^2}\right), \text{(2 modes)} \\ E^2 = \frac{3c_F^2 k^2}{4}\left(1 - \frac{c_F^2 k^2}{72\Delta^2}\right), \text{(1 mode)} \\ E^2 = \frac{c_F^2 k^2}{4}\left(1 - \frac{c_F^2 k^2}{48\Delta^2}\right), \text{(1 mode)} \\ E^2 = 2\Delta^2 + c_F^2 k^2/2, \text{(4 modes)} \\ E^2 = 4\Delta^2 + (0.500 - i0.433)c_F^2 k^2, \text{(2 mode)} \\ E^2 = 4\Delta^2 + (0.152 - i0.218)c_F^2 k^2, \text{(1 mode)} \\ E^2 = 4\Delta^2 + (0.849 - i0.216)c_F^2 k^2. \text{(1 mode)} \end{gathered} \tag{7}$$

It is seen that in a– and b–phases the so–called two–dimensional (2D) sound with velocity $v_2 = c_F/\sqrt{2}$ exists. Note that dispersion coefficient of 2D–sound in b–phase is twice higher than in a–phase. We should remind that in bulk systems the three–dimensional sound with velocity $v_3 = c_F/\sqrt{3}$ is well known). After Brusov et al. [7, 8] this result has been reproduced by a number of authors (Nagai [15], Tewordt [16] etc.).

The collective mode spectrum for the phase with order parameter $\frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & i \end{pmatrix}$:

$$\begin{aligned} E^2 &= 0, (3\ \text{modes}); \\ E^2 &= 2\Delta^2, (6\ \text{modes}); \\ E^2 &= 4\Delta^2. (3\ \text{modes}) \end{aligned} \tag{8}$$

The collective mode spectrum for two phases with order parameters $\frac{1}{2}\begin{pmatrix} 0 & \pm 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$:

$$\begin{aligned} E^2 &= 0, (4\ \text{modes}); \\ E^2 &= 2\Delta^2, (4\ \text{modes}); \\ E^2 &= 4\Delta^2. (4\ \text{modes}) \end{aligned} \tag{9}$$

The collective mode spectrum for the phase with order parameter $\frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$:

$$\begin{aligned} E^2 &= 0, (4\ \text{modes}); \\ E^2 &= 2\Delta^2, (4\ \text{modes}); \\ E^2 &= 4\Delta^2. (4\ \text{modes}) \end{aligned} \tag{10}$$

3. Two–dimensional d–Wave superconductivity

3.1 2D–model of d–pairing in CuO_2 planes of HTSC

The existence of CuO_2 planes — the common structural factor of HTSC — suggests we consider two–dimensional (2D) models. For two–dimensional (2D) quantum antiferromagnet (AF) it was shown that only the *d*–channel provides an attractive interaction between fermions. The *d*– pairing arises also in symmetry classifications of CuO_2 planes HTSC. In Sr_2RuO_4 where the p–pairing appears to have been precluded by recent NMR experiments, the two–component d–wave order parameters, namely {d_{xz},d_{yz}} and even with admixture of g–wave {$d_{x2\ -\ y2}$,$g_{xy\ (x2\ -\ y2)}$}, are now the prime candidates for the order parameter of the quasi–two–dimensional Sr_2RuO_4.

The two–dimensional (2D) model of *d*– pairing in the CuO_2 planes of HTSC has been developed by Brusov and Brusova (BB) [9, 10, 13] and Brusov, Brusova and Brusov (BBB) [14] using a path integration technique. The hydrodynamic action functional, obtained by path integration over "fast" and "slow" Fermi–fields, has been used under construction of this model. This hydrodynamic action functional determines all properties of the CuO_2 planes and, in particular, the spectrum of collective excitations.

To develop the model of d–pairing in the two–dimensional (2D)–case we modify the three–dimensional (3D) considered by us in Ref. 1.

The main distinctions between 3D and 2D cases are as follows:

a. The orbital moment $\vec{l}\left(\left|\vec{l}\right| = 2\right)$ should be perpendicular to the plane and can have only two projections on the $\hat{z}$– axis: ± 2 instead of the three–dimensional (3D) case where the orbital moment can have five projections on the $\hat{z}$– axis: ± 2; ± 1;0. Because the d– pairing is a singlet the total spin of the pair is equal zero, so in the case of the two–dimensional (2D) d– pairing one has $1 \times 2 \times 2 = 4$ degrees of freedom. Thus the superconductive state in this case can be described by complex symmetric traceless 2×2 matrices $c_{ia}(p)$, which have the same number of degrees of freedom $(2 \times 2 \times 2 - 2 - 2 = 4)$. This number is equal to the number of the CM in each phase. Note that in the three–dimensional (3D) case this number is equal to 10, as well as the number of the collective modes in each phase.

b. The pairing potential t is given by:

$$t = v\left(\hat{k}, \hat{k}'\right) = \sum_{m=-2,\,2} g_m Y_{2m}\left(\hat{k}\right) Y^*_{2m}\left(\hat{k}'\right) \tag{11}$$

We consider the case of circular symmetry $g_2 = g_{-2} = g$, which is describes by one coupling constant g. Note, that less symmetric cases require both constants g_2 and g_{-2}. We consider the circularly symmetric case where:

$$v\left(\hat{k}, \hat{k}'\right) = g\left[Y_{2-2}\left(\hat{k}\right) Y^*_{2-2}\left(\hat{k}'\right) + Y_{22}\left(\hat{k}\right) Y^*_{22}\left(\hat{k}'\right)\right] \tag{12}$$

c. $\mathbf{x}$ will be a 2D–vector and square “volume” will be $S = L^2$ (instead of $V = L^3$ as in 3D case).

Account these distinctions between the two–dimensional (2D) and the three–dimensional (3D) cases we will describe our Fermi–system by the anticommuting functions $\chi_s(\mathbf{x}, \tau)$, $\overline{\chi}_s(\mathbf{x}, \tau)$, defined in the square volume $S = L^2$ and antiperiodic in “time” τ with period $\beta = T^{-1}$.

After path integrating over slow and fast Fermi–fields (which is a very similar to 3D one) one gets the effective action functional S_{eff}, which takes (formally) the same form as in 3D case.

The number of degrees of freedom in the case of two–dimensional (2D) d–pairing is equal to 4. By the other words, one has two complex canonical variables. It is easy to see from non–diagonal elements of $\hat{M}$ matrix that the following canonical variables should be chosen:

$$c_1 = c_{11} - c_{22}, c_2 = c_{12} + c_{21}. \tag{13}$$

One has for the conjugate variables:

$$c_1^+ = c_{11}^+ - c_{22}^+, c_2^+ = c_{12}^+ + c_{21}^+. \tag{14}$$

Below we transform the effective action functional S_{eff} to these new variables. One has:

$$S_{\text{eff}} = (2g)^{-1} \sum_{p,j} c_j^+(p) c_j(p) + \frac{1}{2} \ln\, \det \frac{\hat{M}\left(c_j^+, c_j\right)}{\hat{M}\left(c_j^{+(0)}, c_j^{(0)}\right)} \tag{15}$$

where

$$\begin{aligned} M_{11} &= Z^{-1}[i\omega - \xi + \mu(\mathbf{H}\boldsymbol{\sigma})]\delta_{p_1p_2} \\ M_{22} &= Z^{-1}[-i\omega + \xi + \mu(\mathbf{H}\boldsymbol{\sigma})]\delta_{p_1p_2} \\ M_{12} = M_{21}^{+} &= \sigma_0\alpha(\beta S)^{-1/2}(c_1\cos 2\phi + c_2\sin 2\phi). \end{aligned} \tag{16}$$

The effective functional S_{eff} determines all properties of considering model system – superconducting CuO_2 planes. It determines, in particular, the full collective–mode spectrum, consisting of four collective–modes in each phase.

3.2 The collective mode spectrum

Two SC states arise in the symmetry classification of CuO_2 planes with OP which are proportional to $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ respectively. In the former phase the gap is proportional to $Y_{22} + Y_{2-2} \sim \sin^2\theta|\cos 2\phi| \sim |\cos 2\phi|$ while in the later one is proportional to $-i(Y_{22} - Y_{2-2}) \sim \sin^2\theta|\sin 2\phi| \sim |\sin 2\phi|$. For 2D case we put $\theta = \pi/2$ and $\sin\theta = 1$.

Brusov and Brusova [9, 10] and Brusov, Brusova and Brusov [14] have calculated the collective– mode spectrum for both of these states. In the first approximation the collective excitations spectrum is determined by the quadratic part of S_{eff}, obtained by the shift $c_j(p) \to c_j^{(0)} + c_j(p)$ in S_{eff}. Here $c_j^{(0)}$ are the condensate values of the canonical Bose–fields $c_j(p)$.

The collective mode spectrum for the phases with order parameters $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The spectra in both phases turns out to be identical. Brusov and Brusova [9, 10] found two high frequency modes in each phase with following energies (frequencies):

$$\begin{aligned} E_1 &= \Delta_0(1.42 - i0.65), \\ E_2 &= \Delta_0(1.74 - i0.41). \end{aligned} \tag{17}$$

Note that the energies of both modes turn out to be complex. This results from the d–pairing, or in other words, via the disappearance of a gap in the chosen directions. In this case the Bose–excitations decay into fermions. This leads to a damping of the collective modes. The value of imaginary part of energy is 23% for the second mode and 46% for first one. Thus both modes should be regarded as resonances and the second mode is better defined than the first.

The other two modes are Goldstone or low–energy modes (with energy $\leq 0.1\Delta_0$).

4. Lattice symmetry and collective mode spectrum

The made calculations of the collective mode spectrum are not completely self–-consistent, because Brusov, Brusova and Brusov [14] working within spherical symmetry approximation, use the order parameters obtained with taking the lattice symmetry into account. The taking the lattice symmetry into account, as we mentioned above, requires a few coupling constants using instead of one. The number of collective excitations (collective modes) in superconducting state, which is equal to number of degrees of freedom, will change too (note, that in case of spherical symmetry it is equal to 10). In case of the simple irreducible representation (IR) the number of collective modes is equal to twice number of irreducible representation dimensionality. For orthorhombic (OR) symmetry and singlet pairing all irreducible representations are one dimensional (1D), so in each superconducting state there are two modes corresponding to phase and amplitude variations. Amplitude mode is high frequency with $E\approx 2\Delta$, where Δ is the gap in a single particle spectrum.

Among irreducible representations of tetragonal (TG) symmetry there are 1D as 2D (remind that we consider the singlet pairing). Thus in addition to the superconducting states, with two collective modes of conventional superconductors there are states which have four collective modes, none of which are Goldstone. We would like to mention, that for cylindrical Fermi–surface (D_∞) among collective modes there is Goldstone mode in $(1, 0)$ and $(1, 1)$ states but there is not Goldstone mode in $(1, i)$ state.

Because it looks like that there is a mixture of different irreducible representations (corresponding, for example, to s– and d–wave states or to two different d–wave states: $d_{x^2-y^2}$ and d_{xy}; or d_{xz} and d_{yz}) it will be interesting to investigate the collective mode spectrum in this case for different admixture values of s–wave state (d_{xy}– state). Considered by Brusov et al. particular case of $d_{x^2-y^2} + id_{xy}$ state [1] shows that such consideration leads to very interesting results. One more possibility is connected with the recent experiments in Sr_2RuO_4 where the p–pairing appears to have been precluded by recent NMR experiments, the two–component d–wave order parameters, namely $\{d_{xz}, d_{yz}\}$ and even with admixture of g–wave $\{d_{x2-y2}$, $g_{xy(x2-y2)}\}$, are now the prime candidates for the order parameter of the quasi–two–dimensional Sr_2RuO_4. So, it will be interesting to study the collective mode spectrum in such states.

5. Conclusions

We consider all superconducting states, arising in symmetry classification of p-wave and d-wave 2D–superconductors, and calculate the full collective modes spectrum for each of these states.

The collective mode spectrum could manifest itself in microwave impedance technique, in ultrasound experiments, ultrasound velocity measurements and others. They allow determine the type of pairing and the symmetry of order parameter in HTSC and HFSC.

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