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Oseen's Flow Past Axially Symmetric Bodies in Magneto Hydrodynamics

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Abstract

In the present technical note, drag on axially symmetric body for conducting fluid in the presence of a uniform magnetic field is considered under the no-slip condition along with the matching condition ($\rho^2 U^2 = H_0^2 \mu^3 \sigma$) involving Hartmans number and Reynolds number to define this drag as Oseen's resistance or Oseen's correction to Stokes drag is presented. Oseen's resistance on sphere, spheroid, flat circular disk (broadside) are found as an application under the specified condition. These expressions of Oseen's drag are seems to be new in magneto-hydrodynamics. Author claims that by this idea, the results of Oseen's drag on axially symmetric bodies in low Reynolds number hydrodynamics can be utilized for finding the Oseen's drag in magneto hydrodynamics just by replacing Reynolds number by Hartmann number under the proposed condition.

Keywords: stokes drag, Oseen's resistance, conducting fluid, magnetic field, Hartman number, Reynolds number

1. Introduction

There are many fluids like plasmas, liquid metals, salt water, and electrolytes etc. lies under the class of magneto hydrodynamics and attracted the attention of mechanical engineers, scientists and chemists for a longer period of time. The main significant quantity of magneto hydrodynamic fluid past an axially symmetric particle or object is the drag experienced by the stationary body or moving through the fluid.

It was **George Gabriel** Stokes [1] who gave the idea of Stokes drag on sphere by solving the Navier–Stokes equation combining with continuity equation under no-slip boundary condition by neglecting the convective inertia terms in the vicinity of spherical body. The then, this idea is known as Stokes law. This Stokes law or Stokes approximation is valid only in the vicinity of the body which breaks down at distance far away from the body. This breaks down of Stokes solution at far distance from the body is known as **Whitehead's paradox** [2]. It was **Oseen** [3], who pointed out the origin of Whitehead's paradox and suggest a scheme for its resolution (see [4]). In this scheme, Oseen has corrected the drag on the sphere, called Oseen's correction to Stokes drag, namely.

$$D = D_s [1 + (3/8) R], \quad (1)$$

where D_s is the classical Stokes drag and 'R' is the Reynolds number.

Chester [5] studied the effect of magnetic field on Stokes flow in a conducting fluid and modified the classical Stokes drag solution by magnetic field, which is uniform at infinity and is in the direction of flow of the fluid, given as

$$D = D_s \left\{ 1 + \frac{3}{8} M + \frac{7}{960} M^2 - \frac{43}{7680} M^3 + O(M^4) \right\}, \quad (2)$$

Where D_s is the classical Stokes drag and 'M' is the Hartmann number. He also proved that when the magnetic Reynolds number R_m , is small the magnetic field is essentially independent of the fluid motion. **Ludford** [6] discussed the effect of an aligned magnetic field on Oseen flow of a conducting fluid. **Payne and Pell** [7] have tackled the Stokes flow problem for a class of axially symmetric bodies and found the general expression of Stokes drag on axially symmetric bodies in terms of stream function. **Imai** [8] has discussed the flow of conducting fluid past bodies of various shapes. **Gotoh** [9] has discussed the magneto hydrodynamic flow past a sphere and calculated the drag on sphere. **Chang, I-Dee** [10] studied the problem of Stokes flow of a conducting fluid past an axially symmetric body in the presence of a uniform magnetic field and gave the formula of drag on axially symmetric body placed in the conducting fluid under the effect of uniform magnetic field. He utilizes the perturbation technique given by **Proudman and Pearson**[11]. In his Ph. D. thesis at Harvard University, **Blerkom** [12] studied the magneto-hydrodynamic flow of a viscous fluid past a sphere.

Brenner [13] calculated the Oseen resistance of a particle of arbitrary shape in terms of classical Stokes drag and Reynolds number 'R'. **Chester** [14] investigated the validity of the Oseen equations, for incompressible, viscous flow past a body, as an approximation to the Navier–Stokes equations. He determined the drag correctly to the first order in the Reynolds number, though the detailed velocity field is not correct to this order. Moreover, this force can be deduced simply from knowledge of the force on the body according to Stokes's approximation. He also analyzed the generalization of drag including the magneto-hydrodynamic effects when the fluid is conducting and the flow takes place in the presence of a magnetic field. **Kanwal** [15] has obtained the drag on solid bodies moving through the viscous and electrically conducting fluids. **Mathon and Ranger** [16] tackled the problem of magneto-hydrodynamic streaming flow past a sphere at low Hartmann numbers. **Bansal and Kumari** [17] have studied the MHD slow motion past a sphere and calculated the drag on sphere in both Stokes and Oseen's limits. **Datta and Srivastava** [18] proved a new form of Stokes drag on axially symmetric bodies based on geometric variables. **Venkatalaxmi et al.** [19] have obtained a general solution of Oseen equations based on the suggestions given by **Lamb** [20]. The Oseenlet is used for application purposes in their work. **Srivastava and srivastava** [21] calculated the Oseen's drag on axially symmetric bodies with the use of DS-conjecture given by **Datta and Srivastava** [18]. **Sellier and Aydin** [22] provided the fundamental free-space solutions for a steady axi-symmetric MHD viscous flow. **Ghosh et al.** [23] studied the effect of penetration of magnetic field on full magneto hydrodynamic flow past a circular cylinder. **Ibrahim and Tulu** [24] discussed the MHD boundary layer flow past a wedge with heat transfer and viscous effects of Nano fluid embedded in porous media. **Reza and Rajasekhar** [25] tackled the problem of shear flow over a rotating plate in the presence of magnetic field.

For in depth information regarding the classical Stokes drag and Oseen's drag on axi-symmetric bodies in relativistic fluid mechanics and magneto hydrodynamics,

the books of Oseen [4] (in German Language), Happel and Brenner [26], Childress [27], Ferraro and Plumpton [28], Milne-Thompson [29], Cabannes [30], Mirela and Pop [31], Kim and Karrila [32] are referred by author.

2. Formulation of problem

We consider the equation of low Reynolds number flow of an incompressible conducting fluid past an axi-symmetric body in a magnetic field which is uniform at infinity. **Chester** [5] proved that when the magnetic Reynolds number R_m is small the magnetic field is essentially independent of fluid motion. For the case where the body and the fluid have nearly the same permeability, a uniform magnetic field will result i.e. $\mathbf{H}' = H_0 \mathbf{i}$ = magnetic field at infinity. This indicates from the symmetry that there is no electric field, since for all such flows the electric currents form closed circuits. The governing equations and the no-slip boundary conditions for the present problem now becomes [10]

$$-\nabla p + \nabla^2 \mathbf{v} - M^2 [\mathbf{v} - (\mathbf{v} \cdot \mathbf{i}) \mathbf{i}] = \mathbf{0}, \quad (3)$$

$$\nabla \cdot \mathbf{v} = 0 \quad (4)$$

$$\mathbf{v} \rightarrow \mathbf{i} \text{ as } r \rightarrow \infty (r^2 = x^2 + y^2 + z^2), \quad (5)$$

$$\mathbf{v} = \mathbf{0} \text{ at the body.} \quad (6)$$

In Eqs. (3–5), all entities are non-dimensional and their abbreviations are as follows;

U = free-stream velocity,

a = characteristic length of body,

$$v = \frac{v'}{U}, p = \frac{a(p' - p'_\infty)}{\rho \nu U}, x = \frac{x'}{a}, \text{ etc.,}$$

$$Re = \frac{Ua}{\nu} = \text{Reynolds number,}$$

$$R_m = Ua\mu\sigma = \text{magnetic Reynolds number,}$$

$$M = \mu H_0 a \left(\frac{\sigma}{\rho \nu} \right)^{\frac{1}{2}} = \text{Hartmann number,}$$

\mathbf{i} = unit vector along x-direction.

Other symbols have their usual meanings in electro-hydrodynamics and magneto-hydrodynamics. Primed entities are in physical units (as per [5, 10]).

Following the perturbation method given by **Proudman and Pearson** [11], Chang [10] has solved the above equations under the no-slip boundary conditions and obtained the drag on axially symmetric body in terms of classical Stokes drag D_s and Hartmann number as

$$D = D_s \left(1 + \frac{D_s}{16\pi\mu a U} M \right) + O(M^2), \quad (7)$$

where D_s is the Stokes drag for flow without magnetic field.

Now, in the section-4, we prove that the solution of drag given in Eq. (7) is Oseen's drag or Oseen's correction to Stokes drag by utilizing the idea of Oseen's

resistance given by **Brenner** [13] for axially symmetric body under some specific constraints on dimensionless parameters responsible for the two flow configurations. The matching condition for Hartmann number and Reynolds number is provided as well.

3. Oseen's equations and Oseen's drag

Let us consider the axially symmetric arbitrary body of characteristic length 'a' placed along principal axis (x-axis, say) in a uniform stream U of viscous [4, 13] fluid of density ρ and kinematic viscosity ν . When particle Reynolds number Ua/ν is low, the steady motion of incompressible fluid around this axially symmetric body is governed by Stokes equations [26],

$$\mathbf{0} = -\left(\frac{1}{\rho}\right)\text{grad } p + \nu \nabla^2 \mathbf{u}, \text{div } \mathbf{u} = 0, \quad (8)$$

subject to the no-slip boundary condition. It was **Oseen** in 1910, who pointed out the origin of Whitehead's paradox and suggest a scheme for its resolution (see [4]). In order to rectify the difficulty, Oseen proposed that uniformly valid solutions of the problem of steady streaming flow past a body at small particle Reynolds numbers could be obtained by solving the linear equations

$$(\mathbf{U} \cdot \text{grad}) \mathbf{u} = -\left(\frac{1}{\rho}\right)\text{grad } p + \nu \nabla^2 \mathbf{u}, \text{div } \mathbf{u} = 0, \quad (9)$$

known as Oseen's equation. **Oseen** [4] obtained an approximated solution of his equations for flow past a sphere, from which he obtained the Stokes drag formula [**Happel and Brenner** [26] p. 44, (Eqs. 2-8)] under the no-slip conditions (Eqs. 5, 6) as

$$D = 6 \pi \mu a U \left[1 + \frac{3}{8} \text{Re} + O(\text{Re}^2) \right], \quad (10)$$

where $\text{Re} = \rho Ua/\mu$ is bodies Reynolds number.

Based on Oseen's above idea and **Chang's**[10] expression of drag in terms of Hartmann number 'M', Brenner gave the expression of Oseen drag on axially symmetric body moving with equal velocity U and identical orientation through the unbounded fluid in terms of Reynolds number 'Re' as

$$D = D_s \left[1 + \frac{D_s}{16\pi\mu Ua} \text{Re} \right] + O(\text{Re}^2), \quad (11)$$

where 'a' is any characteristic particle or body dimension and $\text{Re} = \rho Ua/\mu$ is the particle Reynolds number.

4. Matching condition for Hartmann number and Reynolds number

In the expression of drag (Eq. 7) given by **Chang**[10], the Hartmann number 'M' is treated as small. Similarly, in the expression of drag (Eq. 11) given by **Brenner** [13], the Reynolds number 'Re' is also treated as small. Now, we can define the drag D (Eq. 7) as Oseen's correction to classical Stokes drag D_s on axially

symmetric body having characteristic length 'a' placed under uniform stream velocity U parallel to the principal axis(x-axis, say) when the two small dimensionless parameters M and Re matches to be equal i.e. $M = Re$ provides

$$\frac{\rho U a}{\mu} = \mu H_0 a \left(\frac{\sigma}{\mu} \right)^{1/2},$$

$$\text{or } \rho^2 U^2 = H_0^2 \mu^3 \sigma. \quad (12)$$

Under this condition, drag on axially symmetric body in the presence of a uniform magnetic field described by **Chang**[10] is defined as Oseen's drag or Oseen's correction to Stokes drag in magneto-hydrodynamics. In the next section, we find the Oseen's drag on sphere and spheroid in terms of Hartmann number 'M' as an application which is the main task of interest for mechanical engineers.

5. Flow past sphere

We consider the sphere generated due to the revolution of circle of radius 'a' about axis of symmetry. The Oseen's drag on sphere of radius 'a' placed under conducting fluid of uniform velocity U and uniform magnetic field H_0 is given by (7) as

$$D = D_s \left(1 + \frac{D_s}{16\pi\mu a U} M \right) + O(M^2),$$

but for sphere, the classical Stokes drag $D_s = 6\pi\mu U a$, then, we have

$$D = D_s \left(1 + \frac{3}{8} M \right) + O(M^2), \quad (13)$$

which is in confirmation with Oseen's drag (Eq. 10) on sphere given by **Oseen** [4] and **Chester** [5] under the aforesaid condition(Eq. 10).

6. Flow past spheroid

6.1 Prolate spheroid

We consider the prolate spheroid generated by revolution of ellipse having semi-major axis length 'a' and semi-minor axis length 'b' about axis of symmetry. Stokes drag on prolate spheroid placed in uniform axial flow, with velocity U, parallel to axis of symmetry (x-axis) is given as (by utilizing DS conjecture given in [18])

$$D_s = \frac{16 \pi \mu U a e^3}{[-2e + (1 + e^2) \ln \frac{1+e}{1-e}]}. \quad (14)$$

Now, the Oseen's correction as well as the solution of Oseen's equation (Eq. 9) may be obtained for same prolate spheroid by substituting the value of Stokes drag (Eq. 14) in Brenner's formula (Eq. 11) under the matching condition (Eq. 10) as

$$\begin{aligned}\frac{D}{D_s} &= 1 + \frac{16 \pi \mu U a e^3}{16 \pi \mu U a [-2e + (1 + e^2) \ln \frac{1+e}{1-e}]} M + O(M^2), \\ &= 1 + \frac{e^3}{[-2e + (1 + e^2) \ln \frac{1+e}{1-e}]} M + O(M^2),\end{aligned}\quad (15)$$

$$= 1 + \frac{3}{8} \left[1 - \frac{2}{5} e^2 - \frac{17}{175} e^4 \dots \right] M + O(M^2), \quad (16)$$

where $M = \mu H_0 a \left(\frac{\sigma}{\rho \nu} \right)^{\frac{1}{2}}$ is Hartmann number and $R = \left(\frac{\rho U a}{\mu} \right)$ is the Reynolds number. The same solution may be re-written, when we take particle Reynolds number $R = \left(\frac{\rho U b}{\mu} \right)$, by using $b/a = (1-e^2)^{1/2}$, as

$$= 1 + \frac{e^3}{\sqrt{1-e^2} [-2e + (1 + e^2) \ln \frac{1+e}{1-e}]} M + O(M^2), \quad (17)$$

$$= 1 + \frac{3}{8} \left[1 + \frac{1}{10} e^2 + \frac{109}{1400} e^4 \dots \right] M + O(M^2). \quad (18)$$

Equations (Eq. 16) and (Eq. 18) immediately reduces to the case of sphere (given in Eq. 13) in the limiting case as $e \rightarrow 0$. On the other hand, the closed form expressions (Eq. 15) and (Eq. 17) due to Oseen for prolate spheroid appears to be new for magneto hydrodynamics as no such type of expressions are available in the literature for comparison.

6.2 Oblate spheroid

We consider the oblate spheroid generated by revolution of ellipse having semi-major axis length 'b' and semi-minor axis length 'a' about axis of symmetry. Stokes drag on oblate spheroid placed in uniform axial flow, with velocity U, parallel to axis of symmetry (x-axis) is given as (by utilizing DS conjecture given in [18])

$$D_s = \frac{8 \pi \mu U a e^3}{[e\sqrt{1-e^2} - (1-2e^2) \sin^{-1}e]}. \quad (19)$$

Now, the Oseen's correction as well as the solution of Oseen's equation (Eq. 19) may be obtained for same oblate spheroid by substituting the value of Stokes drag (6.6) in Brenner's formula (Eq. 11) under the matching condition (Eq. 10) as

$$\begin{aligned}\frac{D}{D_s} &= 1 + \frac{8 \pi \mu U a e^3}{16 \pi \mu U a [e\sqrt{1-e^2} - (1-2e^2) \sin^{-1}e]} M + O(M^2), \\ &= 1 + \frac{e^3}{2[e\sqrt{1-e^2} - (1-2e^2) \sin^{-1}e]} M + O(M^2),\end{aligned}\quad (20)$$

$$= 1 + \frac{3}{8} \left[1 - \frac{1}{10} e^2 - \frac{31}{1400} e^4 \dots \right] M + O(M^2), \quad (21)$$

where $M = \mu H_0 a \left(\frac{\sigma}{\rho \nu} \right)^{\frac{1}{2}}$ is Hartmann number and $R = \left(\frac{\rho U a}{\mu} \right)$ is the Reynolds number. The same solution may be re-written, when we take particle Reynolds number $R = \left(\frac{\rho U b}{\mu} \right)$, by using $b/a = (1-e^2)^{1/2}$, as

$$= 1 + \frac{e^3}{2\sqrt{1-e^2} \left[e\sqrt{1-e^2} - (1-2e^2) \sin^{-1} e \right]} M + O(M^2), \quad (22)$$

$$= 1 + \frac{3}{8} \left[1 + \frac{2}{5} e^2 + \frac{61}{200} e^4 \dots \right] M + O(M^2). \quad (23)$$

Equations (Eq. 21) and (Eq. 23) immediately reduces to the case of sphere (given in Eq. 13) in the limiting case as $e \rightarrow 0$. On the other hand, the closed form expressions (Eq. 20) and (Eq. 22) due to Oseen for oblate spheroid appears to be new as no such type of expressions are available in the literature for comparison.

7. Flat circular disk (broadside on)

Lamb [20] provided the Stokes drag on flat circular disk of radius 'a' placed broadside on facing towards the uniform stream of velocity U as.

$$D_s = 16\mu Ua. \quad (24)$$

Now, under the matching conditions (Eq. 10), the Oseen's drag on circular disk placed under the effect of magnetic field is given by Chang's rule (Eq. 7) in terms of Hartmann number as

$$D = D_s \left(1 + \frac{D_s}{16\pi\mu a U} M \right) + O(M^2)$$

or

$$\begin{aligned} D &= 16\mu a U \left(1 + \frac{16\mu a U}{16\pi\mu a U} M \right) + O(M^2) \\ &= 16\mu a U \left(1 + \frac{M}{\pi} \right) + O(M^2), \end{aligned} \quad (25)$$

where $M = \mu H_0 a \left(\frac{\sigma}{\rho \nu} \right)^{\frac{1}{2}}$ is Hartmann number and $R = \left(\frac{\rho U a}{\mu} \right)$ is the Reynolds number. This drag immediately reduces to the classical one as $D=D_s$ defined by Lamb [20]. This Oseen's drag (Eq. 25) may also be reduced directly from oblate result (Eq. 20) by taking $e \rightarrow 1$ or $b \rightarrow 0$.

8. Conclusion

The problem of Oseen flow of an incompressible conducting fluid past axially symmetric body in the presence of a uniform magnetic field is tackled. The matching conditions are obtained by equating the small dimensionless Hartmann

number and Reynolds number ensuring the **Chang's**[10] solution of drag to be Oseen's drag on same body in terms of Hartmann number and classical Stokes drag. Under no-slip boundary conditions, the closed form expressions are calculated to obtain the Oseen's drag on spheroid (prolate and oblate) and flat circular disk in terms of classical Stokes drag and Hartmann number. These expressions are further extended to the form containing powers of eccentricity 'e'. All forms reduce into the classical Oseen's drag on sphere of radius 'a' given by **Oseen's** [4] and **Chester** [5, 14]. These expressions of Oseen's drag seem to be new in magneto-hydrodynamics. Following the same idea, the Oseen's drag may be calculated in terms of Hartmann number for other body configurations like deformed sphere, cycloidal body of revolution, egg-shaped body, Cassini oval, hypocycloidal body etc.

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