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Chapter

Architectural Design Canons from Middle Ages and Before: An Inspiration for Modern Sustainable Construction

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Abstract

The role of geometry and arithmetic in ancient building is common knowledge, but it has seldom been proved by measured drawing. This chapter looks for the remote origins of design criteria and ancient canons, and their application in representative antique and medieval architecture. Architectural design had to reflect the universal cosmic Order and Harmony and the ancient and medieval architect-designer had to rely on the same intangible instruments, i.e. the geometry and the arithmetic's, created by the Divine Geometer. The geometry of forms and the numbers of quantities and dimensions served as a mayor instrument for developing coherent modulation in the design and the structure of the building and his environment. They also served as a symbol and an allegorical sign to convey intangible messages from the commissioner. Metric analysis reveals this evident design practices and their probable semantic content. This is illustrated in the analysis of six cases: the Cheops pyramid at Memphis, the Pantheon at Rome, the Charlemagne's Palace Chapel at Aachen, the Our Lady's Cathedral at Chartres, the S. Francis Basilica at Assisi and the Castel del Monte at Andria. This historic examples should inspire modern creative design and modern sustainable construction.

Keywords: metric analysis, modulation, medieval design, design arithmetic and geometry, design history, ad quadratum, proportion, golden mean, sustainable construction, Cheops pyramid, Pantheon Rome, Carolingian Chapel Aachen, cathedral at Chartres, S. Francis Basilica, Castel del Monte Andria

1. Introduction

The study of architectural history is and always has been one of the fundamentals in each architect's education; up to the 1920's circa also the design classes were embedded by the replica of the Greek temple orders and the imitations taken from ancient reference books such as Vitruvius, Palladio, Viollet-le-duc and many other. This changed after the 1920's in favor of a more multidisciplinary and technical orientated training, promoted by "Ornament and Crime" (A. Loos, 1908), "Form follows function" (R. Sullivan, 1900 ca.) and other design maxims. It caused a proportionate impoverishment of building history knowledge and of traditional design methods. One said the architect-artist's mind had to be liberated from all historic ballast, and

should be able to create within his free individual creativity. This is a discussable principle with potentially quite negative consequences as architecture is not only a question of artistic creativity or aesthetical harmony, nor a pure functional or technical discipline: "Architectura … nascitur et fabrica et ratiocinatione …" ("Architecture is born by craftsmanship and balanced rationality", Vitruvius, I°sec. b.C.) in ([1], I,1). Architecture (with capital) needs both approaches and apart from the Vitruvian "utilitas, venustas et firmitas [1]", Architecture always had an existential and universal dimension dealing with bringing sense and structure in the surrounding space, including physical communication with meanings and messages' to the observer [2].

The often poor knowledge on historic design criteria nowadays, inevitably leads in many cases to a considerable loss of 'sense' and a different type of 'meaning and message' in contemporary projects. Many heritage buildings get their conservation status because of tangible cultural and historic characteristics, and in many cases it is completed with a large intangible content expressed through symbolisms and allegories. Unfortunately, very often this symbolism and allegories get lost today as man is not familiar any more with the ancient allegorical languages. Also the other way around, modern design rarely uses those so called 'old-fashioned' allegorical indications in such speechless but most effective communication between designer and observer. Medieval buildings are particularly representative for the presence of this mostly forgotten intangible communication content, expressed through the symbolism of form, number, proportion, material or color. Based on the analysis of some representative medieval buildings, this chapter illustrates and tries to detect such design indicators to inspire the contemporary designer, not suggesting a flat imitation but a personal modern interpretation and use of the very same ancient design indicators. The two mayor instruments to all kind of allegorical allusions in medieval design are the geometry of the architectural form and the arithmetic's within the different quantities and dimensions.

This book aims contributing in sustainable construction. The easy re-use or reconversion without great structural change or loss of architectural identity is part of all sustainability and certainly one of the most crucial assignments today. Recent experiences on the reconversion of existing fabric or the recuperation of ancient abandoned structures, mostly for evident economic reasons, have proved abundantly that reconversion or recuperation is much easier and less invasive with ancient well-modulated traditional buildings as it is the case with some contemporary building or probably shall be with one of the super eye-catching designed ones, created by great archistars as e.g. the Bilbao Guggenheim Museum or the Baku Heydar Aliyer Center. Certainly, those superlatives are strong signs of digital design and technical knowledge, but their quality remains one-sided, limited to never seen forms and materials. They do not show great flexibility nor long lasting esthetic pleasure; any probable later intervention, as proof of sustainable (re-)use, risks to damage considerably their actual identity. Society needs avant-garde, but this has to be applied with cure and caution. Contemporary design should reconsider the historic canons, take profit of the three thousand years' experience, evaluate and integrate the old principles for harmony and sustainable use in the modern design algorithms to guarantee qualitative architecture and long lasting construction.

2. Ancestral origins of numbers and geometries in West-European architectural design

2.1 The ancient Greek law on measured and figured numbers (Pythagoras and the Pythagoreans)

Unlike what many people thinks, architectural design is not so much a question of spontaneous creativity but much more of theoretical and technical knowledge.

"Ars sine scientia nihil est" according the well know exclamation of the French master builder Jean Mignot consulting the Milan Cathedral builders in 1400 ca. The theoretical "scientia" at Mignot's time was little and approximate, the artistic drive on the contrary was all the stronger. The success of so many ancient buildings, in particular the audacious finely jointed gothic structures were the result of practical experience during about 3000 years of building since the first Mesopotamian temples. Those ancient buildings were always expression not only of specific needs but also of the then living spiritual concepts about society, religion and esthetics. As in many other disciplines, also architecture and design owe a lot to ancient Greek philosophers from the early 10th century BC (and the Egyptians before them) as founders of West-European culture. Within the larger context of the Mediterranean Basin they developed a world view, not precisely as told in Genesis, but quite similar, i.e. created by a Supreme Divinity who organized and structured the initial chaos using calculated and measured geometric forms. This cosmos of a well ordered celestial and terrestrial creation by the Divine Geometer was the example that man had to follow in structuring his own small local chaos of space. All architectural project implies structure of space, and for that reason, all architectural design must be based on calculation, arithmetic and geometry. This idea was further developed, especially by the Christian scholastics (ca. 9th–13th century) and became an existential obligation for all architectural projects. This explains the permanent presence of numbers and geometries in architectural design for more than 3000 years. In this prospective, one could consider Plato, Aristotle, Pythagoras and Euclid (and the unnamed Egyptian and Mesopotamian priests) as the founders of European design principles.

To study the cosmos's structure, Greek philosophers developed arithmetic number systems and geometric procedures to explain the phenomena of life and nature [3, p. 7]. Numbers are abstract concepts related with the quantity of things, but in relation with real sets or groups they become a tangible reality which, in ancient metaphysic thinking, got very often an intangible connotation or symbolic value, variable in history. In architecture, this number symbolisms got related with the physical quantity of distinct built elements or with the measured quantity of length, width, height or volume. Also the procedure to establish a single number, i.e. the type of calculus by simple arithmetic using the four basic operations (addition, detraction, multiplication and dividing) or more complicate ones (square or cubic root), and the position of each single number as part of a sequence (arithmetic, harmonic or geometric progressions) got specific symbolic meaning and became associated with human or natural phenomena or events. In particular the 'harmonic progression', i.e. where each number of a simple sequence stays in 'harmonious' proportion to the previous and the following number, were popular and looked for[1]. The numbers, visible in real quantities (e.g. number of piers, of bays, of rooms, of corners, of stair-steps, ecc.) were a fundamental part of each building design; the same numbers served as the metaphoric indicator par excellence for expressing intangible values or messages such as power, devotion, glory, utility, science, beauty, harmony and other.

This is not the place to enter in detail about the number systems and the numeral calculus, nor about the wide range of symbolic values in Greek and/or medieval

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[1] The neo-platonians distinguished ten types of 'harmonic' sequences (including the progression in XIIIth century called after Fibonacci) as those of at least three consecutive numbers of which the proportion between the central number and the previous and the following number 'sounded' particularly harmonious and rhythmical, conform with the essentials of the Pythagorean harmonic canon – the numbers indicating the respective length of each string in playing the Greek lyre, e.g. 1–2-3; 2–4-5; 3–5-8; ecc. [4], p. 43] [5], I,16.

numbering. This chapter only stresses their presence and application since ancestral times, and their fundamental role in the genesis of all pre-industrial building projects. Understanding the ancient metaphors, hidden behind the physical quantities and dimensions in the building, is not so easy as the correct lecture and interpretation of the dimensions presumes the often missing knowledge about the metric unit (yard, foot, cubit?), about the eventual modulus (fixed group or set of units) and about the measuring and building conventions at the time and the place of the design. On top of this uncertainty, the modern observer is seldom familiar enough with the ancient design canons and number or figure symbolisms. The Pythagoreans (IVth-IIIth century BC) knew many types of numbers: real or rational ones, integers, fractions, even and uneven ones, primes, perfect numbers[2], as well as irrational and complex ones (roots, unlimited ratio's such as π (= circonference : diameter of cercle = 3,14 ...) or φ (golden mean = 1,618 ...), and numbers with virtual connotations (sacral, male, female[3]) and still other types.

Number 'one' is seen as the most important number, being the origin of everything, not only in arithmetic calculation but also in the natural world and the cosmos (also the justification for monotheism; although many cults worshiped a Divine Threesome in one Union, i.e. the Holy Trinity in Christianity). Number 'two', first and only even prime, represents dualism, the base of philosophy and all science; number 'three' means the female and number 'four' the male element in the 3–4-5 triangle. Number 'four' also refers to all groups of four elements in nature: the basic elements of everything (earth, water, fire, air); four cardinal directions, four seasons and, in Christian context, e.g. the four evangelists). The sum of these first four initial numbers 1 + 2 + 3 + 4 gives the number 10 (the sequence called "tetractys"), creating the sacral number 'ten', representing the universal order. Because of this special property, 'ten' got a special 'mystic' value and the Pythagoreans cultivated a particular preference for decades and pentades in arithmetic calculus and their homonymous polygons in geometry. The tetractys sequence generated the concept of calculated harmony in a eight-divided music-scale (from second to octave, the double of four tone intervals[4]), and the ancient eight-divided foot unit as well as the modern decimal measuring system. Also Vitruvius, explaining and defending the use of anthropomorphic dimensions, presented the number 'ten' as a sacral and most 'beautiful' number ([1], III,275)[5]. The theory and philosophy on the use and allegorical value of numbers in ancient times is large and filled with unexpected results, but their decisive role in pre-industrial design and sometime also in post-industrial projects, is evident.

The most curious invention from ancient Greece, without any doubt, regards the concept of 'figured numbers'. This means that the number (except number 'one')

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[2] A 'perfect' number is each positive integer which is the sum of all his divisors except itself (e.g. 6, 28, 496, 8128; there are only this four perfect numbers under one million).

[3] The gender-ification of some numbers goes back to Plutarch (1st century a.C) who tells in his book *Isis and Osiris* that the vertical side of each Pythagorean orthogonal triangle (i.e. of which the length of all three sides sign a integer number, e.g. 3–4-5; 5–12-13; or 7–24-25, ...) is considered a male element and the horizontal base is the female element; the hypotenuse was seen as the product (the child) of the union of both cathedes.

[4] The tetractys property has generated the 'Pythagorean Canon' in music theory, comparable with the geometric proportion (i.e. the length of the Greek lyre strings) and the acoustic harmony in sound proportion or symphonic composition [4], pl.XXXV

[5] Also Vitruvius connected the use of numbers with the origins of the cosmos created by God. 'Ten' was considered the most 'perfect and sacral' number as man, created after God's image, had ten fingers to serve the Lord [1], 1,II,27.

should not be seen as a single independed entity, but as a set or distribution, or as a part within a progression, and can be represented in space (linear, superficial or volumetric). The abstract number indicates the ratio between a certain quantity and the unit or dimension of that quantity on which it is relying (in this case on two-dimensional figures or surfaces or three-dimensional volumes). The philosophical background of the concept is more complex and relies on Plato's theory on the proportions of volumes in the dialog Thééthète and presumes the alchemic mixture of arithmetic and geometry [4], p. 45. The concept of 'figured numbers' is particularly useful and explanatory in case of irrational numbers such as root $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ or the real quantities π = 3,14 ... or φ = 1,618 ... as this are infinite ratios. For example, when it is impossible to write the result of $\sqrt{2}$ = 1,41421... as a complete and absolutely correct cipher as the result is infinite, the same quantity can perfectly and correctly be indicated and represented in space as the finite length of the diagonal of a square with the side equal unity. In such context, $\sqrt{2}$ is called a 'figured number' as it is associated with the finite length of this line. This is quite important concerning the 'measurability' of the building and admits the integration of root-proportions (most popular in medieval design where $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ appear frequently) in the design without creating the feeling of approximation or ambiguity (although the tracing of infinite ratios did not create any practical problem at the building site as all dimensions were traced using compass and not with measuring rod). All numbers indicate an abstract quantity which had to be measurable and made tangible in space by length, height or volume. This double character (arithmetic and geometric) of the 'figured' number or ratio is the reason and the instrument at the same time for the presence of the polygons in plan and elevation of medieval buildings; together with the number symbolisms they expressed different kind of allegories or hidden messages, being understood only by the initiated members[6]. For the modern observer, it is always an intriguing challenge to discover the hidden symbolisms in ancient buildings.

The combination of architecture and number philosophy has nothing to do with "numerology", being a predominant esoteric discipline of fortune-telling and kabalistic or astrologic reading of phenomena about man and nature. It does not apply the scientific and rational 'theory of numbers' as intended by Greek philosophers, although even they did not use always the most objective logic, as e.g. by naming male and female numbers, inherited from Egypt. Part of this "numerology" is the practice of the old Hebraic 'gematric'-modus (i.e. giving a numerical value to each letter of the alphabet, making it possible to convert letter-words into a mathematical value), used sometimes in the design of mayor buildings but forbidden in church design by the ecclesiastic authorities.

Finally, the rather primitive measuring instruments and the long lasting construction programs, forced ancient building practice to use preferably integer quotes and simple fractions (half, quarter, third), to facilitate tracing and execution on the building site. This explains the preference for integer numbers in the design of plan and elevation of a building. One also has to consider metric rounding after theoretic calculation and the difference between theory and practice to facilitate execution. Such condition on top of the normal building tolerances, on top of the physical degradation and deformation of historic buildings, ask for benevolent interpretation margins.

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[6] One should remember that architectural projects were created by highly educated officials at the service of King or Bishop, and in this quality they were very well aware of the current philosophical and technical achievements of that time.

2.2 Euclid's geometry, the basic instrument in structuring and meditating on space

The geometry is indispensable in structuring any chaotic space. Similar with the procedures used by the Divine Geometer, also man had to create order and harmony by using appropriate geometric figures and proportions. Euclid of Alexandria (ca. 325–265 b.C.) wrote the first systematic manual on this matter, and from that period, a large gamma of regular and irregular geometric forms was developed. The numeric quantity and form of the angles and sides of the figures, in combination with other geometric properties as size, symmetry, congruency, similarity or opposition, they got special symbolic meaning in their architectural application. The most evident figures used in architectural design are the different types of lines (strait, bowed, dotted, alternated), the regular bi-dimensional figures (square, circle, triangle, polygons), and their tri-dimensional derivate. Plato's description in Timaeus on the symbolic content of the regular polyhedra found many applications: the tetrahedron (fire), cube (earth), octahedron (air), dodecahedron (heaven with 12 constellations), and icosahedron (water) [3].

The wohltemperierte amalgamation of geometry and numbering were the necessary conditions for all harmonious architecture; they were the real determinants and driving forces in the design process, and the real generators of all architectural styles. Everything must be calculated, measured and proportionate, as the Holy Bible's verse "Omnia in mensura et numero et pondere disposuisti" (Thou hast ordered all things in measure, number and weight – Book of Wisdom 11:21). This also explains the frequent presence of specific proportions such as the 'Golden Mean' or 'Divina Proportione φ' (= 1,618) not because it was seen as a particular beautiful proportion, but because it was a unique and exclusive value obtained through division of another number (or length of a line) by the 'extreme and mean ratio' or 'mean proportional' and could be seen as a squared figure or 'figured' quantity . The success of this 'golden mean' was also connected with the quite simple procedure to draw it with the compass. It got a particular semantic content as being the irrational, infinite quantity, related with the 'figured' pentagonal number 'five' in the proportion between the diagonal and the side of the pentagon ($\varphi = \frac{1}{2}\left(1 + \sqrt{5}\right)$.

As said, the most frequent geometries were the circle, the square and the triangle, as this were the most easy figures to draw up with simple instruments as wooden rod and cord, compass and plumb, but also because of their specific semantics generated since ancestral times. Before Columbus (ca. 1492), the image people had about the structure of the cosmos was that of a flat and square earth (with Jerusalem in the center) and a celestial half globe. It seems logic that the square and the circle, representing the earth and the heaven, were the first geometric figures used in architectural creations. Plato's vision on the origin of the cosmos and the 'elementary triangle' as the fundament of all matter, together with his exaltation of mathematics and geometry at the expense of artistic creativity, contributed considerably in the use of different kind of triangles and the five polyhedra. Christian philosophers extended the ancient semantics with biblical or religious connotations as e.g. the circle became the representation of human society with God in the center and the people staying on the circle line, equidistant from the center and meaning that everyone is equally considered and protected by God. The square represented the walled Terrestrial Paradise or the walled terrestrial and celestial city of Jerusalem.

Apart from the circle, the triangle (equilateral, isosceles, rectangular, proportioned) and the four-angle polygon (square, rectangle, parallelogram, trapezium and rhomb) are the most frequent figures in architectural design, because of their

large semantic spectrum and the easy designing technique. For that reason, they are the most popular geometries in architecture. The design 'ad quadratum' i.e. using different squarely formats connected, turned around, divided or superposed, was very popular in all kind of utility-building, 'ad circulum' was frequently used in centralized buildings (e.g. sepulchral monuments, baptisteries); 'ad triangulum' was most appropriate for the design of the building elevation and applied in many church buildings. Also in the panoptic of triangle-types, all had his specific symbolic meaning related with their arithmetic and geometric properties, e.g. the equilateral or 'perfect' triangle (symbol for the divine Trinity: three gods equipotential, united in one figure), the rectangular Pythagorean triangle with figured numbers on each side and, with female base and male height; the isosceles triangle symbolizes Christ: divine and human at the same time. Also triangles with specific ratios were used, as e.g. the 'Egyptian' triangle (as it signs the profile of the Cheops pyramid) is a isosceles triangle with the height equal to 0,625 (= 5/8) of the base (the most beautiful triangle according to Plutarch). Similar semantic discours got connected with all mentioned polygons (pentagon, hexagon, octagon and other).

3. Vitruvius and 1.500 years of modulation through numbers and geometries

As numbers and geometries are the mayor determinants in free architectural design (i.e. without conditions from the commissioner, from materials, or from topography), the definition of the right number and the proper geometry will determine the quality and the legibility of the final product. The correct selection and the proper combination of both determinants within the context of a given assignment, signs the art and the discipline of good design. Design is a research activity, similar at all other scientific research, and this chapter has not to enter in research methodologies or procedures, but intends to look after those tangible criteria used by earlier generations. Moreover, every designer, working on a specific commission, sitting in front of an empty piece of paper (or a white computer display) knows very well the process of trial and error, characteristic for all design processes.

This was not different in ancient times; one has to go back to the Roman legionary-architect Marcus Vitruvius Pollio (ca.81–15 b.C.) to read about procedures and criteria in architectural design. According Vitruvius, the decision on 'what number to choose?' is given in the proportions of the human body. The numbers of a good design should respect the metric relations between different parts of the body, to be multiplied according the necessity of the project. The use of anthropomorphic proportions and the human body as guiding principle in architectural design was an ancestral tradition adopted from Mesopotamia and Egypt and further used in all West-European cultures, up to the Modulor of Le Corbusier dd.1930 ca. (**Figure 1a-e**).

The same Vitruvius gives indications about the geometry in the architectural project, not directly by speaking about geometric figures, but by explaining the disposition and distribution of each individual quantity. He puts 'ordinatio et quantitas' (in Greek: taxis and posotys) as the first of five conditions, what confirms what was said before on the importance of the 'number'. The second design determinant, the geometry, is included in the 'ordinatio' and 'dispositio' (in Greek: diathesin) meaning the appropriate attention on the three design aspects and image-interpretation, i.e. iconographic, orthographic and scenographic criteria. Furthermore, Vitruvius explains the need for 'eurythmia' (general visual harmony), 'symmetria et analoghia' (harmony and similarity between elements by using a
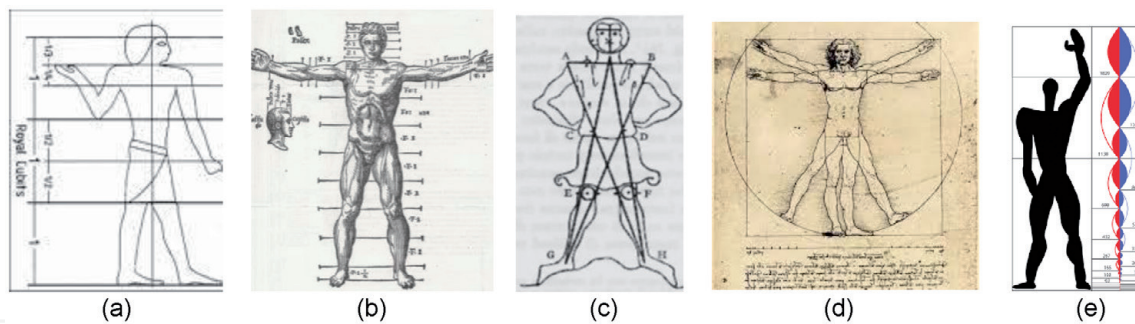
Figure 1.
The proportions of the human body as reference for harmonic design. (a) Ancient Egypt (ca.2000 b.C.), (b) Vitruvius (1st century B.C.; illustration from the Italian edition by F. de Franceschi & J. Criegher in Venice dd. 1567), (c) Villard de Honnecourt (ca. 1235), (d) Leonardo da Vinci, the 'Vitruvian man' (ca.1490), (e) Le Corbusier, the 'Modulor' (ca. 1935).

common 'modulus' for each part of the building, and 'convenientia et distributio (greek: oikonomia)' i.e. equilibrated administration of the available resources and space and, during the execution of the project, a proper division of the costs by calculation ([1], I,1,2). This topics are all well-known fundamentals of architectural theory, and consequently also the fundamentals in architectural design.

As Vitruvius expresses general and universal principles for correct and good architecture, it's evident to find clear applications of his maxims in the design of all medieval buildings, even when there is no written testimony about the spread of his "Ten books of Architecture" in medieval times. The first evident reference to his manual is the drawing of Leonardo's 'Vitruvian Man' (dd. 1490 ca.) as the new standard for artistic proportion and design. This means that the input of Vitruvius's maxims never disappeared, not even after quasi 1500 years.

The one and only authentic written source on medieval design are the 65 pages the Picardian masterbuilder Villard de Honnecourt drew during his travels in Flanders and the Nord of France in the beginning of XIII° century. This sketchbook reveals some of the design techniques of his time, with particular attention for the proportion of building elements and sculptural decoration. Villard does not mention many numbers but shows in his drawings all geometric auxiliary lines and frames to guarantee the correct "euritmia" and "eumetria" in the project. He introduces the aid of a geometric pattern in the design of figures, building plans and elevations, and uses quasi exclusively square or triangular grids. His ideal plan for the "glize desquarie" (squared church) for the Cistercian convent is of particular interest as it represents the model of many West-European medieval churches. Basically, the plan adopts the three naves Latin basilica type with enlarged choir section, flat-ended apse and transept. It is easy to recognize the Vitruvian proportions of a double square module in the central nave axes (longitudinal sequence 6:2:4; transept 4:2:4) and a transversal section in the sequence 1:2:1, with the addition of co-modulated single square lateral naves in both directions. Villard also adds three alternatives for richer gothic choir and apse projects ([6], Taf.28) (**Figure 2a-c**).

In the metric analysis of the St. Francis Church in Assisi [7], we also found the presence of a double grid design i.e. the superposition of a first principal square grid defining the sequence of open spaces for practical use, and a secondary in-between-grid for structural elements (walls and columns) (largely preceding the SAR-design method [7] presented as much innovative in the 1960's).

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[7] SAR = Stichting Architecten Research, Netherlands (1964–1990) on the initiative of prof. J. Habraken (T.U.Delft), was working on new housing concepts with large intervention of the future private inhabitants. In stead of the traditional design frames following the axis of the loadbearing structures, they used a doubled design frame with separated spaces for effective utility and for structural elements.
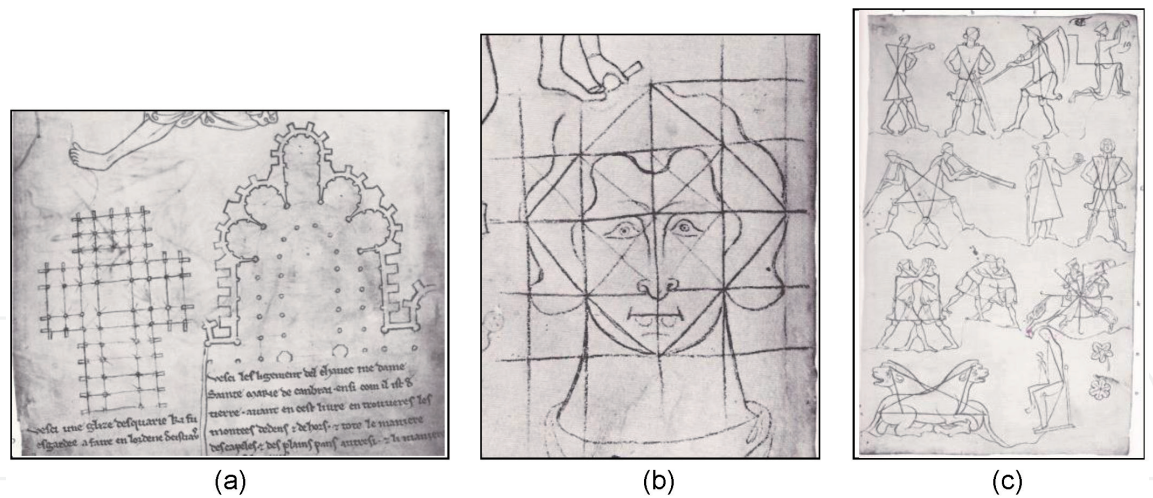
(a)   (b)   (c)

Figure 2.
Sketches from VILLARD DE HONNECOURT, before 1235 [6]; (a) linear modulated plan of the 'squared church' and ground floor of a large choir with deambulation and radiating chapels; (b) design of a men's head on a squared modulated grid, (c) group of figures, using geometric patterns.

As far as we could check, nor Vitruvius nor Villard use the word *modulus* in their texts, but it is obvious that the 'modulus' must be the key in any design project. This is what Vitruvius intends by combining "ordinatio et quantitas", and what he makes explicit in his definition on "symmetria … est ex ipsius operis membris conveniens consensus, ex partibusque separatis ad universae figurae speciem ratae partis responses, …" (symmetry means bringing convenient consensus between all parts of the project, in the separate elements as well as by applying universal forms and figures….) ([6], I, 20–21). He continues by insisting on the use of the dimensions and the proportions within the perfect human body (mutual ratio's between head, chest, arm, palm, foot, finger ecc…) as reference for all design. Villard does not say it by so many words, but presupposes the criterion of mathematical harmonious proportions in the totality of the project as well as in the mutual relation between separated components. He shows it through his graphic analysis and fragmentation of the design steps in his figures (without giving numeric quotes). Also for Villard, symmetry and analogy seem the main criteria.

The choice of the right modulus is the first indicator for the appropriate "ratiocinatione" (= the result of a rational decision-making process) by a most polyvalent-educated architect. In consult with his commissioner, he has to decide about the architectural typology of the building, determined by geometry and size, considering the symbolic capacity of both criteria. In medieval church building, the width of the choir indicated the module for the whole building; the numeric length of the modulus was most symbolic and took normally a number from the Holy Bible or some event related with the purpose of the project. In civil building, the length of the modulus could be of any kind related with the commissioner or the function of the building, as can be seen in the examples below.

Our 'youngest' example dates from ca. 1250 a.D. (Castel del Monte, Andria), but evidently, the modulated design practice did not stop after that period; quite on the contrary! The renaissance architects rediscovered and re-interpreted Vitruvius; and in one way or another, up to the end of the XIXth century all architectural design, working with traditional materials and traditional structures, took profit of the old master's procedures, and 'translated' them in their own contemporary language. But the analysis of the design praxis in the Modern Times is a topic for another study.

4. Selected cases of ancient modulated design

4.1 GIZA (Egypt) Great Pyramid of Cheops, dd. ca. 2570 BC, monumental tomb

Long before the Pythagorean philosophers, the design of this monumental tomb expresses a stupefacient simplicity and coherency in geometry and dimension, fully compatible with the Egyptian vision on the society and the cosmos, witnessing the exceptional culture and knowledge of some 5.000 years ago (**Figure 3a-c**).

The symbolic geometry is obvious: the combination of a square ground floor, symbolizing the flat plane of the earth, and the upwards rising triangular flanks directed versus the celestial globe, with the mummy of the pharaoh and his wife, waiting for rebirth, at the center of the monument. The sides of the square are closely aligned to the four cardinal points. According to some researchers, inside passages and corridors are orientated versus astrologic constellations at the time of building. The King's and Queen's tomb chambers are located in the geometric gravity center of the construction, on emplacement and distances of interior corridors related to the golden mean proportion φ ([9], I, pl.XLIV).

The physical environment of the pyramid, the access way, the sphinx statue and the position of other monumental tombs of pharaoh related people or animals show a well-considered geometric and measured design.

The pyramid's external dimensions witness an equally exceptional design with a selection of most allegorical and harmonious numbering, combining length of sides, diameter, vertical and sloping heights, including also particular prime numbers and the irrational ratio's π and φ (**Figure 3b,c**):

- groundfloor: square (440 x 440) Egyptian Royal Cubits (Erc) of ca.0,524 m = 230,56 m x 230,56 m;

- height (original): 280 Erc = 146,70 m;

- base angle ca. 51,575°, top angle: ca. 76,85°, length slope 220 Erc/cos 51,575° = 354Erc = 185,48 m

- diameter groundfloor ca. 622,25 Erc = 326 m

- ratio (height: side) = 280/440 Erc = 0,636 = ~ 5: 8 (= ~ 0,618 = golden mean) giving origin at the so called 'egyptian triangle', according Plutarch, the most beautiful triangle as it is derived from the equilateral triangle, see **Figure 3b**)
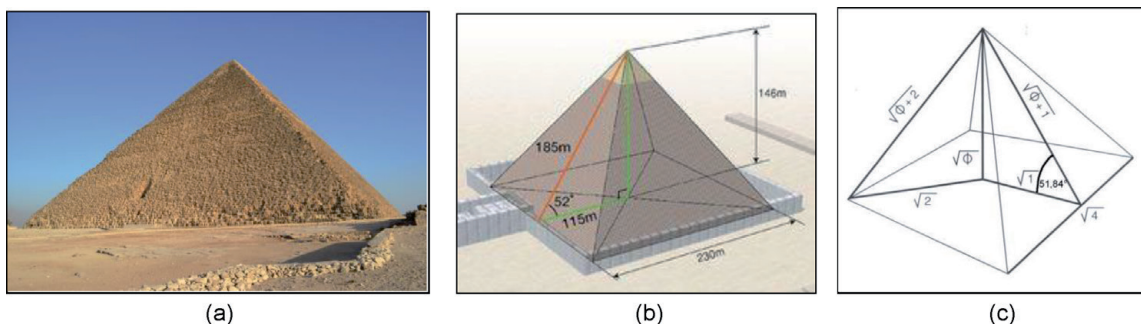

(a)   (b)   (c)

Figure 3.
Pyramid of pharaoh Cheops at Giza a. general view [8], photo: Giza-legacy.Ch; b + c: Geometries (drawings: Pinterest).

- the ratio (height: ½ side) or the ratio between both cathedes of half of the isosceles profile signs = Erc (280/220) = 1,272727...= $\sqrt{\varphi}$ which results as double irrational by $\sqrt{}$ and φ, and with the exceptional sequence of a repeated number 27 (=3x3x3) after comma, also being easy to draw as it equals the ratio (14:11). This property illustrates the value of number three, being the fundamental (according Plato) of all terrestrial matter.

- ratio (perimeter: height) = 1760/280 Erc = 6,2857 equates to 2π to an accuracy of better than 0.05 percent (corresponding to the approximation of π as 22:7).

- other surprising arithmetic ratios and golden- mean- related dimensions are signed in **Figure 3c**.

The integer dimensions (440 and 280 Erc) as well as the irrational ratio's 5:8 and $2\pi = 2(44:14)$; (note the numeric resemblances between 44 ↔ 440 and 2(14) ↔ 280) also include (unknown) allegorical messages. [8]; [Wikipedia].

4.2 ROMA, pantheon - founded ca. 23 b.C.; present structure dd. ca. 125 a.C. - temple

This ancient Roman temple, dedicated to all God's, is a most interesting monument for many reasons, a.o. for his unusual design showing a multilayered intangible content(**Figure 4a-d**). The building was founded as a rectangular temple about
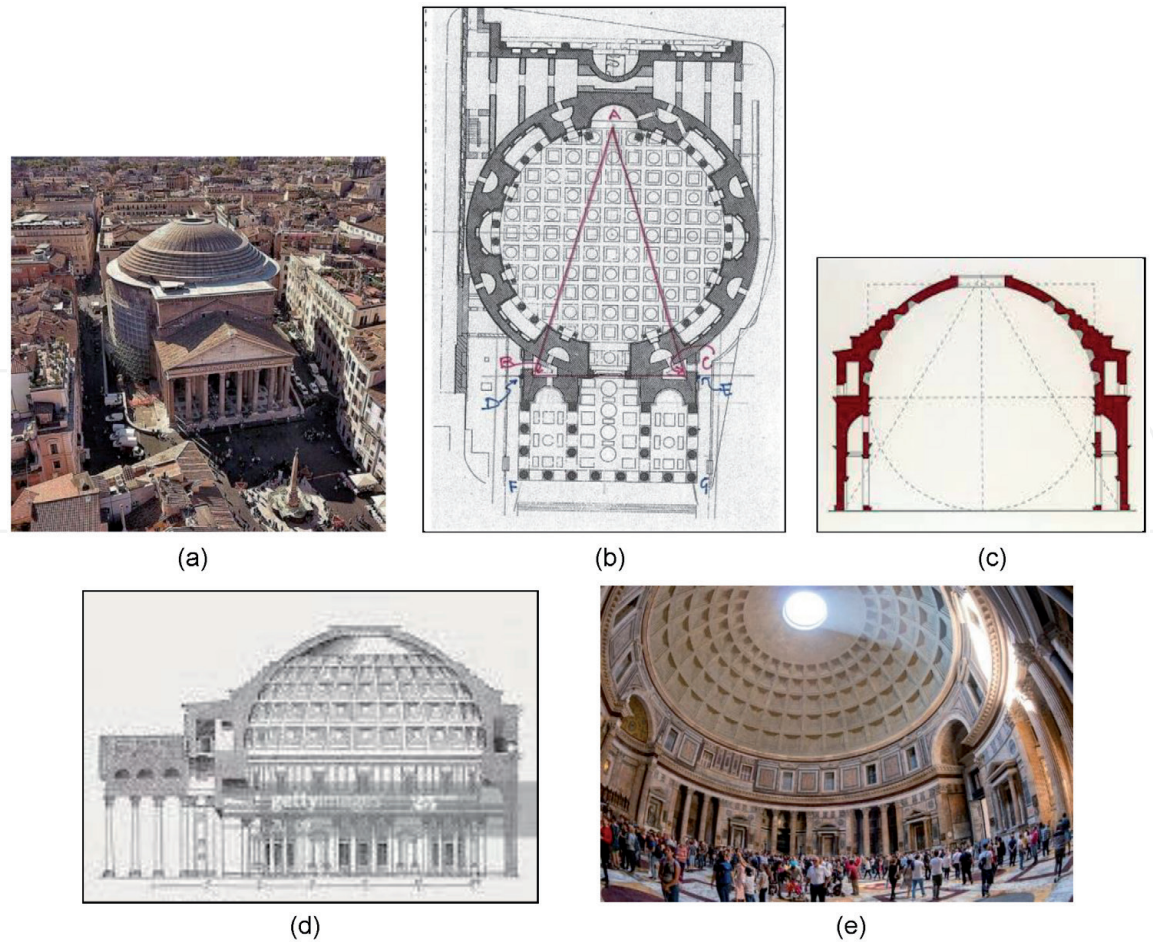

(a) (b) (c)

(d) (e)

Figure 4.
Roma, pantheon: (a) aerial view (photo Pinterest); (b) ground floor of rotunda and so-called portico (Wikimedia & author); (c) transversal section (Pinterest); (d) longitudinal section (Getty images); (e) general view rotunda inside (Wikimedia).

27 b.C. by Marco Vipsanio Agrippa (ca. 63–12 b.C.), rebuilt two times, in his present form by the Emperor Hadrian in ca.125 a.C.. The monument shows two distinct parts: the mayor part or 'Rotunda' with cupola, and a squared classic temple, today serving as entrance area. The overall image shows two buildings, with two separated spatial identities, two formats and two functions.

The Rotunda represents the allegoric bricked envelop of an impressive regular globe. The horizontal diametrical plane of the globe divides the interior in two equally high volumes: the upper part shows a half-sphere dome, structured with cassettes and at the top an open oculus, the only entrance of natural light; the lower half is a cylindrical volume, for his part divided, according the golden mean ratio, in a lower section including a sequence of niches, apses and columns, and a upper dome-tambour section elaborated with squared cornice patterns. The cylindri-cal walls of the lower half of the interior are richly decorated by different kind of materials and forms. The Rotunda is the evident metaphor for the cosmos with the dome as the celestial half-round and the cylindrical lower space representing the terrestrial world, everything dominated by the central oculus representing the Supreme Divinity, generating life and dynamism through the zenithal light entering from the oculus .

The second part, the so-called portico, minor but nevertheless substantial part of the design, seems conform with one of the traditional Greek-temple-inspired models described in Vitruvius manual ([1], I, III, 284). However, the design does not follow this models too much; on the contrary, it is much closer with the most ancient tripartite Etruscan-Italian temple as described below. This deviant design from the conventional temple-pattern, seems an explicit demonstration of the own native Etruscan and Italian temple-building origins, different from those imported by the first Greek colonists in the 3th and 2th century b.C. It is as if Agrippa Vipsanio, commissioner of the first temple and son in law of the first roman emperor Antony-Augustus (with divine status), or one of his successors involved in the reconstruction of the temple during the 1st and 2d century a.C., liked to stress the native identity of the italic people, denying the fact of roman sacral architecture being indebted to the Greek (although this clearly appears from the first temples e.g. in Sicily and Paestum, or from the great basilica's built on the Fori Romani). The squared portico part has a net floor area of circa half that of the Rotunda, which is much too large for being only 'portico' (or 'pronaos' as it is wrongly called in some literature). The approximate ground floor surface ratio 1:2, and the change from the rectangular versus the circular format, rather seems a conscious combination of old and new, an indication on the start of a new era for Rome, reminding the remote origins of the first Sannite (origin of the temple's founder Agrippa Vip-sanio?), Tuscan or other settlers in Central Italy, and Rome's passage as the capital of a new Mediterranean and European empire. The creation of the square antichambre-like temple before entering the incomparable grandeur of the Rotunda reinforces the expressivity of this last one. Both united entities are a rare example of architectural design as a political statement, materializing history and social order.

• The geometry of the double monument

The circular Rotunda: as easily deductible from (**Figure 4a-e**), the geometry of the Rotunda is a most regular compilation of circles, squares and even a equilateral triangle (transversal section) in bi-dimensional and tridimensional edition, all of them having their specific semantic content. The rhythmic alternation of the tripar-tite 'negative' savings and 'positive' porches on the lower cylindrical ring, and the polychrome with the changing incidence of natural light from the oculus at the top,

creates a great sense of wealth and dynamism. The frequency of the number eight in the design, by quantity (e.g. 8 mayor interior savings and 8 minor exterior ones in the perimeter wall) and by dimension (e.g. the overall inside diameter of 146 roman feet – see below), as well as the ancestral semantic of this number (being the first cubic number as $8 = 2^3$) do believe that eight (and his composing factors 2–3-4) is the numeric modulus for the design of the complete building (to be completed with the number 5 as we'll see further). The modulus should be found also in the net width of the opening of niches and apses, but the necessary metric information to prove such assumption, was not available. Apart from the 8 mayor niches/apses, the interior parietal composition includes 8 jutting out porches; both artifacts create a imaginary cylindrical space-filling web of 8 + 8 = 16 isosceles spiked volumes, pointing to the central vertical axis connecting nadir and zenith of the as imaginary interior sphere.

The squared entrance-temple is designed according the archetype of the three-cellae open Tuscan temple surrounded by columns and divided by two intermediate rows of three columns. The net inside width of the three cellae-areas is traced according the golden mean ratio. The two lateral cells have a small apse at the end for some simulacrum, the wider middle area covers the common central longitudinal axis with the double entrance door of the Rotunda, and is orientated versus the head apse of this Rotunda with the statue or the seat of the emperor. The entrance-temple is surrounded by eight columns at the front and three columns at each side with a short in- between piece of wall connecting with the Rotunda. The temple front looks similar with the Vitruvian models but other research is needed to identify all metric differences. After conversion of the Pantheon ensemble in a Christian church in 7th century, specially this entrance part and the in-between connecting structure suffered various amputations of materials and additions with demolition afterwards of crowning towers, but the present image should approach the authentic one.

- The arithmetic at the double monument

The choice of the numbers, i.e. the dimensions and the quantities of the decorative elements (niches, apses, columns, porches, cornices, marble paneling), as well as their spatial distribution are additional indicators for the intangible messages. This chapter cannot enter into the detailed design aspects of this elements, but one can safely conclude that the Pantheon ensemble is probably the very first example of a fully integrated Gesamtkunstwerk based on the ancestral symbolic geometry and numbering; this last by using the very basic integer quantities 1–2–3-4-5-(1,618= φ), and their derives 6–8-10 and other. We illustrate this by the only three beginning and most decisive choices of the design process.

- Three decisive choices at the start of the design process

1. The very first step concerns the choice on the architectural typology and form, based on the functional requirements and the symbolic content, mentioned before. This resulted in the option to create an innovative cosmic sphere imitation in a format which was never done before; to be connected with a reminder of the presumed architectural origins of Rome, i.e. the Tuscan temple. The innovation is proved by the technical audacity and capacity to build a dome structure signing the incredible span of ca. 43,40 m or 146 rf, which is the largest span ever in building from antiquity up to mid XIX° century! The 146rf dimension is certainly not arbitrary; the radius of 73rf = 1 + 8 + (8x8) might refer to the number 8 = 2x2x2 as the probable numeric modulus for the entire

composition. Unfortunately, we do not know what symbolic content Agrippa or Hadrian connected with the number 8 modulus.

It's interesting to notice also the open oculus diameter of ca. 8,35 m = ~ 28rf which signs ca. 28:146 = ~ 1:5 ratio to the central diametric plane of the sphere, and a reference to the semantic of number five, the golden mean ratio's (including $\sqrt{5}$) and the pentagonal symmetries in other parts of the building.

2. The second decision concerns the likewise exceptional thickness of the external wall of ca. 5,90 m = ~ 20,00 rf (=2x2x5). Considering the presence of the deep niche savings, the minimum thickness of the external bricked shell is reduced at ca. 2,36 m ~ 8,00 rf (including the todays disappeared external marble wall cladding), which seems comparable with other antique buildings. This quote is not the result of any structural calculation (although technical experience must have been involved), but of an exclusive geometric property. Indeed, the external diameter signs ca. 55,20 m or 186,50 rf, and his ratio to the inside 146rf diameter equals 186/146 = 1,274 = ~ 1,2727 = $\sqrt{1,618}$ or the square root of the golden mean proportion (with connotation of sequence after comma 27 = 3x3x3 similar with Cheops ratio). This means that the length of both diameters (external versus internal) are part of a Diophantic triangle progression[8], referring once again to the golden mean proportion φ = 1,618 ([10], Ù 49). Such arithmetic property only occurs in exceptional cases, and is highly appreciated as the irrational square root as well as the irrational φ value results a double infinity reference, most symbolic for the cosmos, for the divine status of the emperor Augustus and for the future of the Roman empire [9].

3. The third decision regards the dimensions of the entrance-temple, signing a double squared ground floor of ca. (31x17,5)m = ~(104x59)rf or a normal length to width ratio of approximate 2:1. Archeological excavations at the end of 19th century have proved the presence of more steps and a normal stylobate space to get on the columned entrance area; this means that the squaring could have been slightly different from today. More important however is the geometric connection between the Rotunda and this open temple structure. After searching and calculating possibilities, the author found out that the net frontal width BC (free passage between the side walls) of the portico (ca. 31,0 m) is given by the base of the 'golden' or 'sublime' triangle ABC inscribed in the Rotunda, with the top A in the center of the head-apse and the base along the inner line of the double (formerly bronze) entrance door of the Rotunda. (**Figure 4b**). The 'golden triangle' is found in the spikes of regular pentagons and decagons, and is a isosceles triangle such that the ratio of the hypotenuse to base is equal to the golden mean. The calculated hypotenuse signs 48,45 m (diameter from apse to entrance door) ÷ cos. 18° (half top angle of 36°) = 50,946 and 50,946 ÷ 31,0 (measured width of the portico's front) = 1,643 or, within normal tolerances, identic to the golden mean ratio 1,618.

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[8] Called after the Greek mathematician Diophantes of Alexandria (ca. III° century AD). The property of a Diophantic triangle means that the length of his sides can be expressed as part of a geometric sequence with a reason (i.e. the constant proportion between two successive numbers of the sequence) that equals 1,272 or $\sqrt{\varphi}$ = $\sqrt{1,618}$.

[9] Many of the antique geometric and arithmetic symbols have been adopted by early Christianity; the first Church Fathers and Scholastics christianized the ancestral pagan semantics, converting and adapting them into biblical and christian concepts.

The next design step draws a double square, sided ½ 31 m, at the left and the right of the central longitudinal axis to create the overall entrance-temple area DEFG. Further on, each of both composing squares get divided according golden mean ratio with the width of the lateral bay (with end-apse) as the ratios mayor and half of the central area as the ratios minor. The joining of both minors results in the central area along the common longitudinal axis of the ensemble. By this procedure, the entrance temple and the Rotunda get physical (through geometry and numbering) and spiritual (through various semantic) most intimately connected.

We notice, once again, the application of the golden mean ratio. The frequency of this ratio in so many design procedures indicates his particularly powerful allegoric meaning as indicator of cosmic harmony in life and society and of rebirth and infinity of man.

4.2.1 Conclusion

This concise analysis discovered several unexpected qualities of the Pantheon ensemble. They are not simple architectural 'curiosities' but existential part of the building's identity. The multilayered image of this 2000 year's old ensemble shows the resilience of simple but conscious design and his timeless capacity for tangible and intangible communication.

4.3 AACHEN (Germany), Carolingian Imperial chapel, start 795 a.D., dedication 803 a.D.; (extended with gothic choir and several side chapels from ca. 1350 onwards)

The chapel of the imperial palace of Charlemagne is another example of the impact of the geometry and the arithmetic in the dialog between the building and his observer. The commissioner is the first West- European emperor Charlemagne (742-814 a.D.) after the fall of Rome. For the chapel of his palace, he chooses the model of what he might have seen on his conquests (e.g. S. Vitale at Ravenna dated ca. 530 a.D.) and what linked him with his illustrious predecessors. He adopts, for the first time applied on this scale in the northern-of-the-Alps countries, similar innovative design which includes a lot of Christian and imperial symbolisms and allegories. We refer to the architectural history books for all details – e.g. see [11]. Our limited notes mark the most evident design characteristics employed as tangible instruments in the communication of intangible contents .

• The geometry (unit of 1 cubit = 0,4281 m [12])

The plan: although the external image of the Carolingian building looks almost circular, the fundamental plan concept is of a squared design, i.e. a central octagon, which is the result of two superposed identic squares of which one is rotated over 45°. The central octagonal area is surrounded by a ring of eight squared chapels connected by eight triangular interspaces, generating the hexadecagonal external envelop. The central area is connected with the ring area through eight arched passages, covering quasi the full length of the octagon's side (**Figure 5b and e**).

The vertical section shows two concentric volumes; the central octagonal higher one, open up to the top, and the surrounding hexadecagonal ring of two levels: the ground floor spanned by cross ribbed vaults and a upper gallery similar to the Romanesque matroneum, looking into the central octagonal space through a double
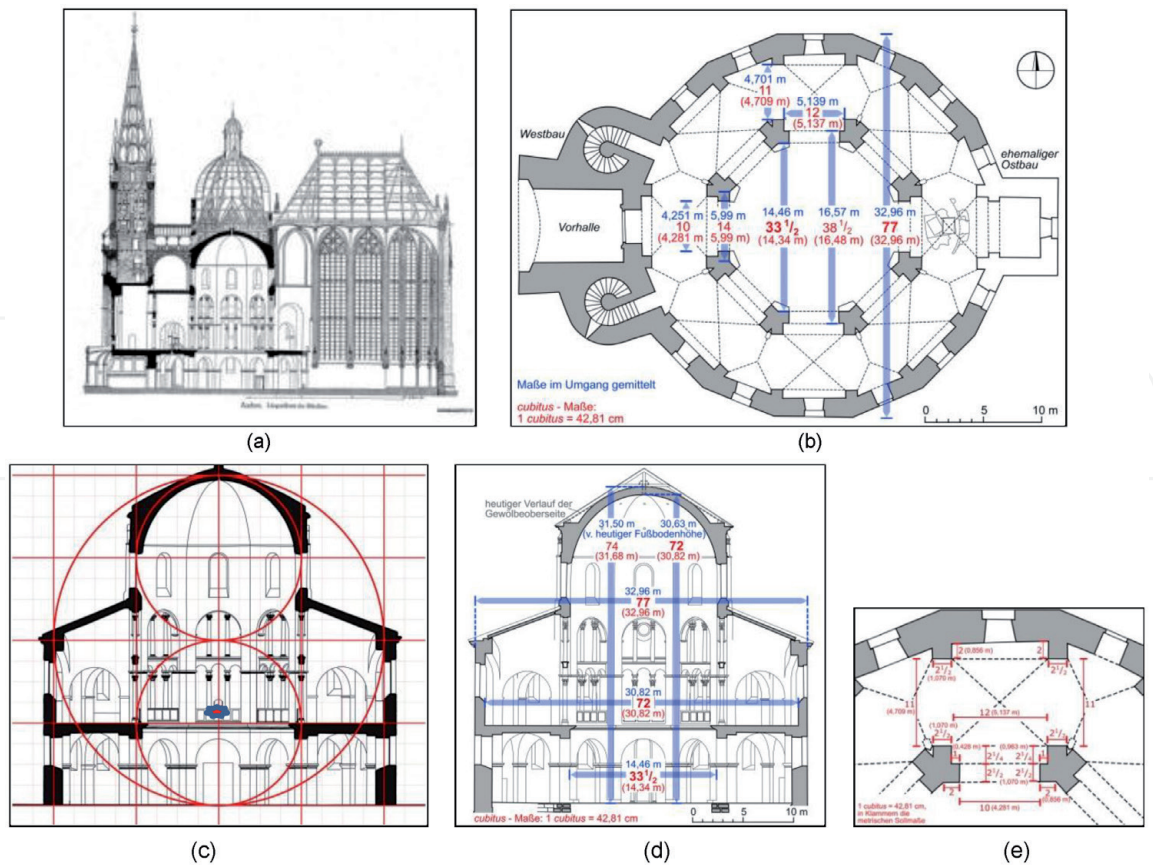
Figure 5.
(a) Aachen, our Lady's cathedral, longitudinal section with the carolingian chapel at the west half and the gothic choir (14th -15th century) at the east half of the ensemble. The tower above the west entrance and the raised dome above the octoganal Centre date from 18th & 19th century (drawing Kunstdenkmäler Rheinland, 1916). (b) Aachen, our Lady's cathedral, ground floor of the end-8th century Kaiserkapelle (drawing from [12]). (c) squared proportions of different sections of the building and indication (star) of the emperor's chair at the center of the lower circle (d): Arithmetic modulation and ratio's in the octagon; (e): Mutual proportions in the squared ring-chapels / drawings from [12].

superposed tripartite arched opening, divided by two columns. With the outside windows at the top of the octagon, the central area gives the impression of a four-level structure, where the physical evidence of the surrounding ring-volume counts only two floors (**Figure 5c**).

The center of the chapel forms a regular octagonal prism; the overall circumscribing volume marks a regular virtual cube, with the sides equal to the hexadecagonal's diameter, inside of ca. 30,82 m = 72 cubits, and outside of 32,96 m = 77 cubits. The prism's inside height from floor to the top also signs 72 cubits. The outside top-cornice of the hexadecagonal ring signs the precise horizontal middle plane (height 36 cubits) of the chapel, separating virtually the ground floor with the upper gallery of the imaginary 'terrestrial world' from the single globe on the top, inscribed in the dome and tambour volume, the imaginary space with the presence of the one God. Some parallel can be made with the domed upper half (with oculus) of the Roman Pantheon, apart from the octagonal perimeter versus the roman circular one. The sequence of spaces in the lower surrounding ring volume applies a most regular and symmetric geometry with congruent cubic volumes alternated by triangular spikes on both levels of the ring space (**Figure 5c**).

The amalgamation of the three most fundamental geometries: the square (part of the octagon), the circle (inside and outside virtual volumes, plus the staircase towers flanking the entrance), and the triangle (the eight spikes inside the hexadecagonal ring-space) generates an exquisite allegoric ensemble, open to all kind of profane and religious interpretations. In addition to the predominant

centralizing plan-concept, the entrance extension with two massive towers in the west generates a additional east–west longitudinal axis (different from the Ravenna example[10]) with the emperor's marble coronation armchair at the center of the upper squared floor section, looking east towards the central golden altar. Although the setting of the armchair might have been different at Charles's time, it signs the beginning of a later developed Romanesque tradition to build a special imperial room on top of the west-entrance of more double oriented churches, as one can still admire in some large Meuse and Rhine churches (e.g. St. Servaes at Maastricht, Netherlands)[11] .

The hexadecagonal chapel was a integrated part of a squared parceled design covering the complete imperial palace site, including an open forecourt to the chapel, two small and one larger basilica's for the emperor's activities as well as several residential buildings. On top of the sophisticated chapel geometry and the squared environmental design, there should have been involved also astronomic considerations related with the orientation of the single buildings and, concerning the chapel, with the incidence of natural light, the original emplacement of the altar, the location of the emperor's chair, and other particularities, but this seems not enough studied yet.

- A rich arithmetic reading (**Figure 5b-e**)

The dimensions and quantities in the Imperial Chapel reveal the intended communication by specific dimensions. It starts with the choice for the octagonal plan, derived from the square, and including the semantics of the number four, eight, sixteen, and the multiplication factor two, appearing frequently in the building. The single entrance with the two flanking towers maybe symbolizes the double nature of his commissioner as worldly and religious sovereign, or also refers to the two columns crowning the entrance of King Solomon's temple (1Kings,7,15–22) indicating imperial dynamism and power. The overall design might have been guided by Alcuinus of York (732–804), chief philosopher and theologian at Charlemagne's court, as all mayor dimensions, after conversion into the then local cubitus (= ~ yard of ca.41,81 cm), result symbolic for biblical quotes. In this perspective, the design modulation does not appear that much a physical condition for related quotes, but a mental and metaphysical set of numbers taken from the Holy

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[10] It's interesting to compare the *Aachen* design with the older S.Vitale at *Ravenna* dd.525–547. This last one applies the octagonal plan for the central area as well as for the surrounding ring-volume. The passage between the central and the perimeter ring space in Ravenna is made through tripartite bowed niches and the exterior wall has a different windows distribution, which result in a more open and better illuminated interior space. There is a double entrance, passing through a full-width narthex, deviating from the central symmetry axis. The overall design includes a rich allegoric content by geometry and arithmetic.

[11] One has to realize the different character of Charlemagne's palace chapel, deviating substantially from the current typologies for private chapels in aristocratic palaces or castles of the early Middle Ages. Normally, such chapels were built as two strictly separated entities: a upper volume at the exclusive use by the noble men, and a lower volume (without any direct connection with the upper one) for the use by the lower social classes, or in many cases also as memorial chapel for the family tombs of the seigniors. Such physical separation was done for evident security reasons, but also as confirmation of social hierarchy. Charlemagne introduced here a different typology: on one hand a octagonal sanctuary in his double identity exclusively at his personal service with double floor level and double horizontal areas i.e. the central octagon and the surrounding chapels, all in one volume, visually connected and also physically by two stairs; and on the other hand two traditional rectangular small basilica's, built at the outside left and right of the hexadecagon, both at the service of the palace staff.. .

Bible and the ancestral arithmetic symbolisms passed down from the Pythagorean philosophers. Nevertheless, there is great logic and uniformity in the geometric figures as well as in the choice of integer and simple numbering for the design of length, width and height[12] .

This logic is found in the harmonic multiples of small (metric) quantities, and simple mathematical sequences (e.g. the arched openings connecting the octagon and the hexadecagon sign 10–12-14 cub. (**Figure 5e**). As said, there is preference for prime numbers and biblical numbers, many of them borrowed from the Apocalypse and the there description of the celestial Jerusalem (Apo.21, 9). This Jerusalem is a squared city (the octagon signs the superposition of the terrestrial and the celestial Jerusalem), and was equally long, large and high, with a wall of 144 (=12x12) cub. Also the virtual cube of the chapel volume is equally high and large (inside 72 cub., outside 77 cub.[13]); the longitudinal side of the squares in the ring-volume signs 12 cub. and the transversal opening signs 11 cub.. The ratio 12:11 has his own semantic with 12 referring to the 12 apostles of Christ or to his presumed 33 year's life time on earth (to read as 12 = 2x(3 + 3), meaning the double nature of Christ (divine and human) living during 33 years; and the 11-quote indicates men's imperfection while Christ's perfection is signed by 12; to be seen also as 11=1/3 x 33 of Christ's lifetime and the metaphysical passage (12=4x3) from a squared central space into the triangular inserted spaces of the hexadecagonal ring.

The passages from the octagonal area into the surrounding area sign an opening of 10 cub.(with 10 being the 'perfect' quantity). In most medieval churches, the architectural modulus is indicated by the passage opening between apse and choir; this brings us to conclude that also in this case, the metric modulus has been ten cubits (= ~ 4,18 m), eventually in combination with a secondary 12 cub. Quote as a recall at the left and right of the passage opening (**Figure 5e**). The octagon's diameter of 33 ½ cub. is another reference to Christ's life time on earth (**Figure 5d**); the diagonal of the hexadecagonal and the altimetry design, signing the same quote of cub. 72 = 2x3x12 = 3(2x3x4); the external diameters of the octagon (38 ½ cub.) and the hexadecagonal (77 cub.) sign the evident proportion 1:2. Also the number 77 should not be fortuitous because, according the calculations by S. Augustine of Hippo (354–430), (based on the Old Testament history data), the number of generations from Adam to Christ counts 77! [12]. The lecture of this building's dimensions seems a florilège of biblical quotes, a very typical phenomenon in medieval church design .

4.4 CHARTRES (France), Our Lady's Cathedral : start rebuilding 1194 (after fire damage), dedication 1260

The design of this French gothic key monument followed the then current design criteria: a ground floor ad quadratum according the église carrée of Villard (**Figure 2a**), and an elevation ad triangulum. Although the available metric documentation [13–17]; [18], p. 84, is not uniform, the evidence of a coherent modulated design is undeniable, including the inevitable smaller or bigger differences

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[12] The analysis of the measured drawings from [12] proves once again that ancient modulation normally refers to the free passable space in between structural elements as walls and piers. Quite rarely also the thicknesses of this elements are included in the modulated design or are part of the allegoric dimensions (except from the Diophantic geometry as it is the case for the Pantheon's exterior wall).

[13] The outside height of the dome shell actually signs only 74 cub. as the roof structure as well as the ground floor quote has been changed in a later period; the initial height should have been also 77 cub. [12].
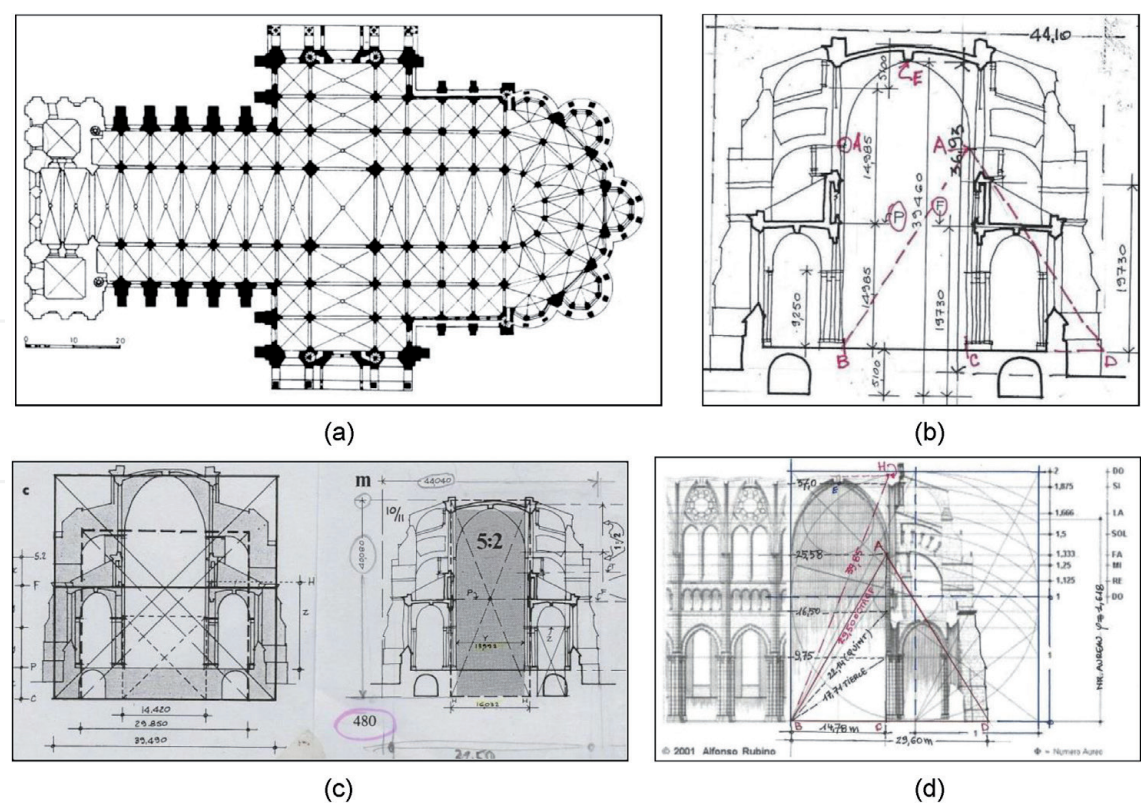
Figure 6.
Chartres, Our Lady's Cathedral. (a): Plan [13]. (b) Transversal section [16] and author. (c) Ratio's transversal section (drawing from [16]). (d) Hypothetical altimetry design according musical canon [Wikicommons + A. Rubino + author].

and deviations, caused by ca. 100 years construction activity by different master-building teams (**Figure 6a**). Similar to what we'll see in Assisi, also in Chartres there is the very first question about what might have been the metric standard applied on the building site because of the confusion in the historic terminology and the uncertainty on the real length unit in practice at that time: or the French "coudée" or cubit of 0,5236 m, or the ancient "pied du Roy" of 0,3236 m or maybe still another local standard .

As said, the metric modulus of the medieval church was given by the net passage opening between the apse and the choir or the choir and the nave, in this case signing **14,78 m** (and not the 16,40 m distance between the center-to-center columns axis's, as kept on in most publications)! Our reference publications propose different 'fabricated' measure units, quite close to the "pied du Roy", as e.g. 0,369 m in [14] or 0,333 m in [18], partly inspired by the Christian semantics associated with number three and his multiples. However, none of those are coherent nor with any historic evidence. By reasoning back to front, comparing with other medieval churches, and inspired by the case of the St. Francis Church in Assisi, it seems probable and plausible that the Chartres's bishop in charge, out of respect to the Roman Pope, absolute head of the Christian church, and maybe also out of piety tradition, imposed to practice the ancient roman foot standard of **0,296 m**, applied in Rome and in many Christian church buildings. This same standard was called in France also "pied de Cluny" for the evident reason that the Cluny convent and the Cistercian monks, since late 10th century, were the most active church builders in Europe. Converting on this base, we obtain a most sense full and most Christian symbolic modulus of 14,78 m: 0,296 = **50rf** (roman

foot). The acceptability of this "pied de Rome" unit is confirmed in the converting of some representative dimensions of the building (always considering the only free passable spaces).

However, for want of a recent complete measured drawing, we only note following data (**Figure 6a**):

- Architectural modulus: 14,78 m → 50rf = 2x5x5

- Transversal width of choir-bay (estimated) 6,25 m →21rf = 3x7

- The overall design grid in the choir and the nave signs a sequence of oblong rectangles and squares with slightly variable dimensions, apparently depending on the section of the columns and alternative ratio's in the tracing of a virtual geometric grid. The grid indicates the net squared areas of the quadrangles, separated by the construction strips, corresponding with the width of the vault-arches. The quadrangles can include different semantics such as 'golden rectangles' (φ related) or 'dynamic rectangles' ($\sqrt{}$ related).

  According our scaled drawings, the nave and choir rectangles sign theoretically (50x21)rf, the aisles (25x21)rf and the crossing (50x42)rf. All quadrangles are semantically considered equivalent with regular squares.

- Global inside length (from the entrance porte royal to the closing wall of the apse): 130,20 m → = 440 rf or 400rf of church passage and 40 rf of entrance zone between the towers;

- Global inside width of nave and two aisles: 32,40 m → = 109,46 rf or 100rf of passable church area and 9,46 rf for two rows of pillars construction zone;

- Global height inside the choir (from first outside stair- threshold of the entrance, i.e. the public space area, to the underside of the vault-keystone: 36,93 m → =125 rf = (5x5x5)rf (**Figure 6b**)

- Global height from crypt-floor to nave vault: 39,46 m → 133,33 rf (to read as separated ciphers, with double reference to the 33 years Christ lived on earth)

- As shown in (**Figure 6b**) there is no doubt about the elevation design ad triangulum with the modulus BC equal the half-side of the equilateral triangle with top A at quote 25,58 m (= 29,50x1/2 $\sqrt{3}$) , indicating also the impost of the nave-vaults arches and the bottom of the first outside flying buttress.

- The top E signs the intersection of the two vault curves 'at the fifth point', at the height from outside threshold of 37 m in a ratio versus the width of the nave (= modulus) of 14,78 m:37 m = 1: 2,5. The modulus signs half of the length of the diagonal AB (=side equilateral triangle).

- The external transversal section of the nave(including the crypt areas) describes a quasi-squared plane (height 40 m x large 44 m) = 10:11, which signs the already noticed semantics. The capitals of the triforium corridor (quote 19,73 m) sign the geometric middle of the full inside elevation; the overall transversal section on the nave forms a 5:2 rectangle (**Figure 6b and c**).

In this limited list, we see more references to number five and his multiples, including the number 100 and 400 as reference to the exclusive hecatompedon quantity (i.e. 100 = 10x10 what means a more than perfect number) and a supplementary indicator for the 50rf as present modulus.

To conclude, we have to mention the interesting hypothesis on the relation between architectural geometry canons and the music harmony canons, which seems to find confirmation in the elevation of Chartres's nave (**Figure 6d**). Just as good architecture should bring order in a chaotic space, also harmonic music is as a cosmos imposed upon chaos [19], 251ff. Considering the side-length of the equilateral triangle (= double modulus) as an octave interval, one connects some significant altimetry quotes or virtual lengths from the opposite floor-bottom, with the tierce (lower capitals of central piers), the quart (keystone vault aisles), the quint (passageway triforium), sext or sept (lower threshold nave windows), first octave (impost vault-arch nave), second octave (~ keystone vault-arch nave). This needs further study before being confirmed.

4.5 ASSISI, S. Francis Basilica; start 1228 - dedication 1253 (extended in later decades)

The S. Francis Basilica at Assisi was built to worship the sepulcher of Francis of Assisi (1182–1226), founder of the mendicant Order of the Minorite Fathers. The work started March 1228, but after two years, mid 1230, the project mission was extended with a second assignment i.e. becoming the representative mother church of the new religious Order. Integrating this new function in the same, still under construction, sepulchral and pilgrimage church, did not seem possible and for want of space on the same site, it was decided to build a fully separated second church on top of the first one, keeping the same external wall's perimeter and a similar inside spatial distribution. Such audacious project got realized after reinforcing the already built exterior walls and after inserting a new type of cross-rib vaulting for the lower church. This phased realization resulted in the actually superposed double-church, characterized not only by two different functions (a Lower devotional Church for S. Francis's tomb and a Upper Church for the Fathers Convent's services), but also by a double architectural and artistic identity: a first late-Romanesque Umbrian Lower Church and a second early-Gothic European Upper Church.

The design of the Lower Church (excluding later extensions) adopted the traditional middle sized Umbrian single-nave and single level church model. The geometry and the arithmetic's on dimensions and quantities of the Lower Church integrated the ancient Pythagorean traditions, modifying them according Christian semantics with tangible imitation of some iconic Christian churches of that time i.e. the S. Peters Basilica of Rome and, as it was built in full crusaders period, also the S. Sepulcher of Jerusalem, and an allegoric record of the since long demolished biblical Temple of King Solomon on Mount Moriah in Jerusalem. This multifaceted mission was realized in a multilayered design, building a quite old-fashioned Romanesque Lower Church and an innovative Upper Church, this last one in the new transalpine (French) gothic style and structure, the very first application of this new architecture in Italy. It's interesting to notice the remarkable coordination in the design of dimensions, forms and structures within two architecturally and structurally so different buildings. Under this prospective, the S. Francis double church is a interesting example of design resilience and flexibility avant la lettre [7, 20, 21].

- The geometry

Notwithstanding the two-phased origin of the S. Francis Basilica[14], the geometry and the arithmetic follow the ancestral symbolisms of figures and numbers, independent from the clearly different stylistic character of each church and the references in orientation, distribution and architectural forms to the three mentioned iconic Christian churches .The plan-geometry of the S. Francis Basilica is a evident ad quadratum design, persisting in the early XIII° century four bays Romanesque Lower church, continued in the mid XIII° century Upper Church and the XIV°-XVII° side chapels. The modulus is given by the width (11,84 m = 40 roman feet of 0,296 m) of the passage in between the apse and the crossing. As a one-nave church, the initial three nave-bays sign a perfect (40x40)rf square, separated from each other by a narrow strip for the transversal arches. The crossing signs (43x43) rf in order to realize an inside area with hecatompedon overall length from the XIII°century entrance door to the end of the apse. The bell-tower adopts the same modulation, applied however on the outside perimeter. The design of the later additions and modifications (the superposed second church, the east entrance-transept of the lower church and seven of the twelve lateral chapels) followed similar squared design. The lateral chapels however applied a reduced modulus of 23 or 24 roman feet (**Figure 7b**). The geometry of the two transept sections (added in a second campaign) are two 'golden rectangles', applying the same 40 rf modulus in longitudinal direction and defining the transversal width according the golden mean proportion (40 rf x 0,618 = 24,72 rf = ~ 7,32 m). The overall analysis of the modulation permitted the author to discover some revisions and changes in the design, realized in the course of building, and to identify the probable chronology of each section of this medieval project [7].

The vertical section of the Lower Church signs a surprising ad circulum design, i.e. each bay includes a regular sphere with a diameter equal to the 40 rf modulus. The nadir of each sphere does not coincide with the pavement level, what should be the normal design, but with the quote of the Saint's sarcophagus, below the pavement. The sarcophagus is located at the base of a virtual sphere, inscribed in the crossing of the nave and the transept. The same sarcophagus marks also the starting point for an imaginary vertical axis, passing the middle of the mayor altar, rising to the zenith, as to indicate that the remains of S. Francis, a unique relic treasure with exceptional thaumaturgy capacities, is the best go-between to resolve men's problems. To stress this capacity, the difference between the extrados and the intrados perimeter of the circular transversal ribs delimitating the crossing, have been calculated according to a 1/3 reduced width of the diophantic proportion (similar to the Pantheon's enveloping wall) indicating the double irrational and golden mean reference of $\sqrt{\varphi}$ (**Figure 7d and f** [20, 21]. Both churches also include a symbolic parcours on rebirth, indicated by the geometric path from the Lower Church virtually ascending through both apses and the Upper Church [23], p. 117.

As the initial design provided with two identic churches, both ground floors are similar apart from the nave's interior width, which is little larger in the Upper Church as the exterior wall thickness of the Upper Church is only half of those of the Lower Church. However, the opening between apse and crossing also in the Upper Church keeps strict on the architectural modulus of 40 rf, notwithstanding

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[14] The denomination as 'Basilica' is rather misleading as the church is not conform with the historic basilica building type; it is a exceptional honorary title, awarded in 1756 in occasion of the 500th years anniversary of the dedication of the double church, to give the S. Francis church the same ecclesiastic privileges as the ancient Christian Basilicas in Rome.
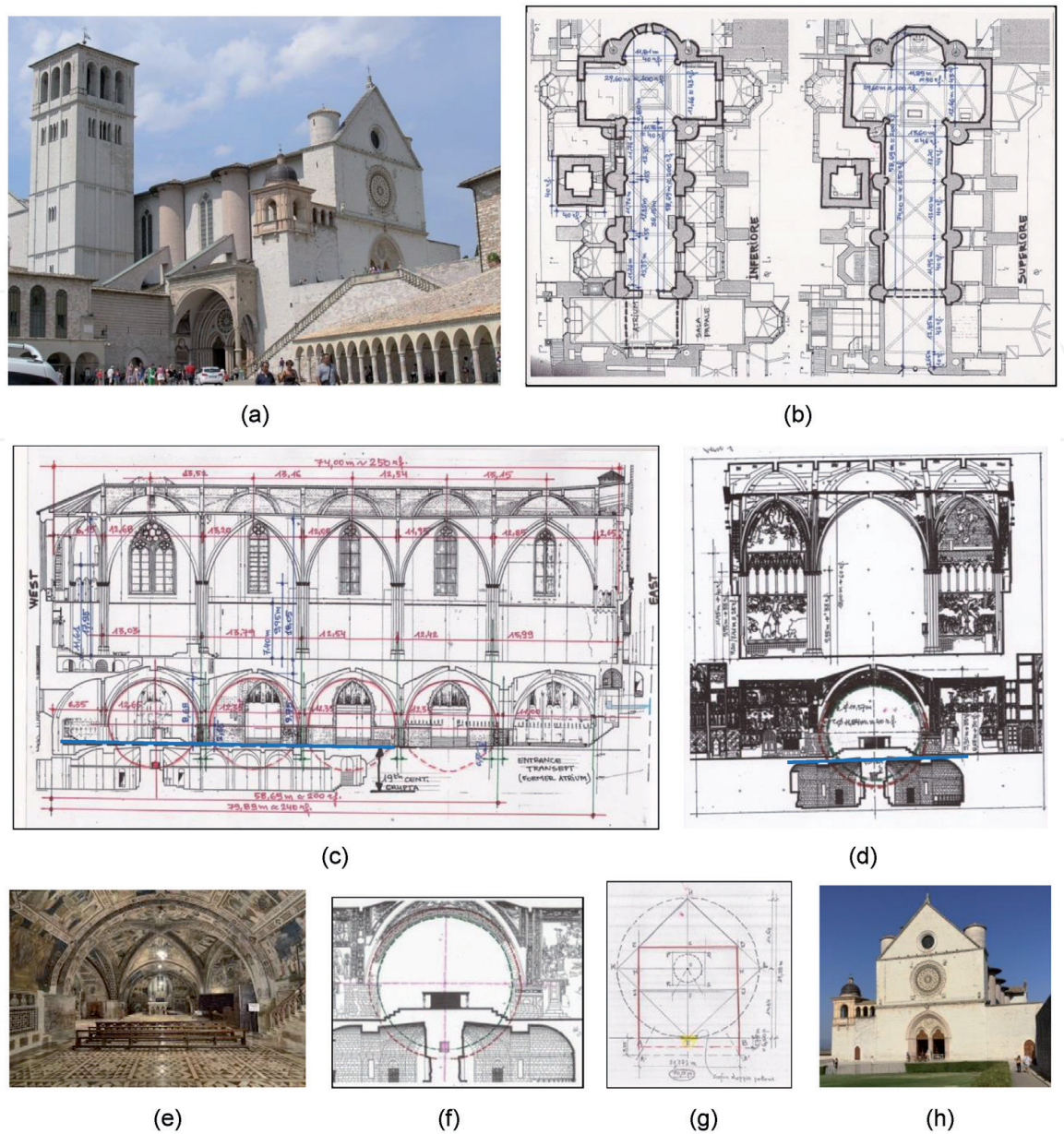
Figure 7.
Double St. Francis basilica, Assisi. (a) General view from south-east [Wikidata]. (b) Squared ground floors lower basilica (left) and upper basilica (right); the black delineation indicates the mid-thirteenth initial three-bays church, the gray volumes the later additions [22] & author. Francis basilica, Assisi. (c) longitudinal section over both churches, extended in XIX° century with crypt. (d) transversal section over the transept (drawings [22] and author). S. Francis basilica, Assisi. (e) Lower church, view on the main altar, versus west [Wikimedia]. (f) Transversal section over main altar lower church and XIX° century crypt with location of sarcophagus in the nadir of reduced diophantic sphere (FOART measuring & author). (g, h) Geometric design of the east façade upper church (drawing and photo: Author).

the different geometry of the apse (circular in the Lower Church and decagonal in the Upper Church). The stylistic differences in ornamental design (gothic versus romanesque) is also very evident as this emerges e.g. in the massive trilobate wall-piers of the Lower Church in contrast with the five-lobate clustered piers in the Upper Church. We also noticed several metric design irregularities, e.g. the vertical axis' of the Lower and Upper Church wall-piers are not well centralized, nor the length of the bays and squared plan-grid of the Upper Church are very regular. This are obvious indications for the separate design of both churches, spread over more years and more chief-master builders; maybe also connected with the medieval concept on metric tolerance's and the not so perfect measuring instruments. However, the visual impact of this metric irregularities is negligible as they get disguised by the full polychromic decoration of all walls and vaults.

The vertical geometry and the wall's elevation of the Upper Church is very different from the Lower Church. As said, the Upper Church design expresses the introduction of the gothic design canons, inspired by the gothic examples in the French and English Normandy and the Scholastic church fathers. The design implies a predominant ad triangulum geometry and the application of the different types of pointed arches as indicated by Villard de Honnecourt [6].

- The arithmetic numbering (conversion of metric unit: Roman foot = 0,296 m)

The numeric quantities in both churches reflect a large collection out of the mentioned sources from pagan antiquity, the Holy Bible, the Christian Scholastics, and the practical need for simple and rounded numbers. It starts with the choice of the horizontal as well as the vertical modulus in both churches, equal **40 rf** (a number appearing more than 100 times in the Holy Bible and also the modulus of King Solomon's Temple); the overal inside length (from apse to entrance) of the sepulchral church signs 200 rf (=5x40), the inside width of the transept signs 100 rf, i.e. a single and a double hecatompedon (the length of the mayor antique Greek temple); the inside length of the extended Upper Church signs 250 rf; the free height in the Lower Church from the pavement to the top of the transversal arch signs 30 rf and 33 rf to the vaults keystone signs; the piers' impost in the Upper Church signs also 33 rf; the height of the vaults keystone signs 60 rf or a 1:2 proportion regarding the Lower Church height. In the Lower Church one finds the omnipresence of number three in different combinations, in the Upper Church, one finds similar arithmetic combinations of number five in dimensions (e.g. length 250 rf = 2x5x5x5) and in structural elaboration (e.g. five-lobate wall-piers). The number five cult and the ad quadratum design is largely visible in the Upper Church east façade, which could serve as the tangible synthesis of a rich and most interesting intangible program (**Figure 7g-h**).

4.6 ANDRIA (Italy), Castle del Monte, imperial castle, built 1241-1250

Maybe the best known and most enigmatic medieval building with regard to architectural design is the castle built mid XIII°century near the site of a former small Our Lady's convent in Andria (Puglia, Italy), commissioned by Frederic II (1194–1250), the then Holy Roman Emperor. It illustrates the amalgamation of European (Romanesque), classic antiquity (Roman) and Islamic design traditions, clearly combined with Christian semantics about life, rebirth, cosmic structure and the role of Jerusalem as religious and geographic center of the flat terrestrial world. Indeed, the octagon was since Babylonian times the main symbol for rebirth and eternal life (cfr. The supreme goddess Isthar was imaged by a octagonal star); the Pythagoreans loved the number eight for being the first prime 'regenerating' three times (8 = 2x2x2), and also the Christians used the octagonal plan for the baptistery, as this is the building where men get spiritual rebirth. In metaphysics, the number eight was the sum of three and five, two numbers with particular semantic meaning, and the octagon was associated with the person of Christ, as He was part of a tripartite God but also human being, using the five human senses (3 + 5 = 8), reborn the third day after death.

In the case of Castle del Monte, we propose a more tangible explanation for the triple octagonal design (i.e. the inner courtyard, the outside perimeter and eight octagonal edge-towers). As emerges from the plan and the geometry, the octagonal design seems a quasi-imitation, of the ca. 600 years earlier built Islamic octagonal 'Dome of the Rock' built on top of Mount Moriah in Jerusalem. This 'Dome of the Rock' as well as the adjacently located smaller 'Dome of the Chain' were at
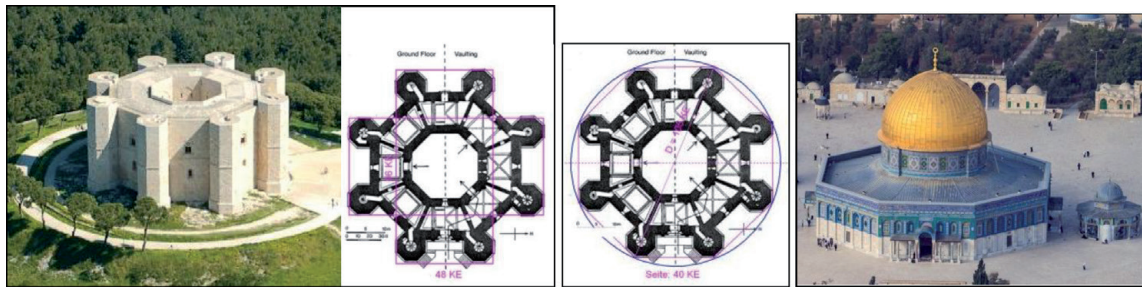
Figure 8.
*Andria, Castel del Monte: (a) General aerial view. (b, c) squared and circle geometry of the ground floor
(photo, drawings: Pinterest). (d) Jeruzalem, 'Dome of the Rock' and the adjacent, at east, small 'Dome
of the Chain', both with octogonal plan, located in the center of the Temple Mount platform at Jeruzalem
(Wikipedia).*

Frederick's time converted into a Christian 'Templum Dominum', considered the geographic center of the world and also the most holy place for all three monotheist religions (Islam, Judaic and Christian). The 'bare stone platform', the center of the Dome of the Rock, was covered with the most extraordinary intangible content as it should have been the place where Abram sacrificed (Gen. 22), where Jacob dreamed (Gen. 28,11), where Solomon's Temple had his main altar (1Kings 6; 2 Cron.), and from where Mohammed ascended to heaven. Notwithstanding the Muslim origin and property of this Dome, it seems quite plausible that Frederick II, also crowned King of Jerusalem since 1229, was inspired by this unique symbolic content, and that he ordered the construction of an octagonal 'center of the world'- interpretation in his South Italian territories. Although the origin as a pagan (= Islamic) building, it was seen as inalienable part of the Christian heritage, and the imitation of similar octagonal building, substituting Solomon's Temple, was the best way to prove this towards his subjects. On top of this, it could be seen as a sign of obligingness from an illuminated and open minded Emperor towards the Muslim society (who had many Arabic scientists at his court) (**Figure 8d**).

There also might have been a second motive in play. Political history learns about the quasi permanent conflict between Frederick II and the then Pope Gregory IX. This last one had started in 1228 the before cited Sepulchral Basilica for S. Francis in Assisi, including also some small residence at the papal service, mainly for devotional reasons but also to consolidate papal political power in Central Italy. As the Castle del Monte was built in the same years (i.e. before 1240–1249) as the S. Francis Basilica at Assisi (built 1228–1253)), this Castle del Monte might also have been meant as a imperial answer in virtual confrontation with the papal project in Assisi. This hypothesis of ours can explain the evident link in the design with the biblical structure in Jerusalem and changes the Castle from a hunting refuge, as presented in literature, into a political and religious statement, expressed by this most unusual triple octagonal castle design. (One should note that the actual rectangular Al-aqsa mosque as well as the 'Dome of the Rock'- shrine has been damaged and rebuilt several times, but the mayor geometry of the 'Dome' has been preserved).

4.6.1 Geometry and numbers

Many hypothesis on the semantics of this exceptional format have been proposed, but still today, there are more questions than answers [24]. In most literature, it is presented as a 'hunting' castle, or part of a greater military defense chain of castles by Frederick II, but none of this hypothesis make sense in this totally isolated location on a ca. 500 m high top. As said, it seems more a political sign against Pope Gregory IX (building the S. Francis basilica on top of the Assisi's Collis

Inferiore), and a confirmation of the emperor's personal contribution as the leader of the 6th crusade, bringing the Jerusalem holy places under Christian control. The plausible intention of goodwill towards the Muslim world, the probable and significant astrologic input in the design of the castle, and the most probable sacral, social and political message gives this monument a complex and multilayered content, as intriguing as his design mixes the geometric and the arithmetic canons of Europe and the Mediterranean Basin.

The design draws two concentric octagons, with an open courtyard at the center and eight small octagonal towers at each corner of the external perimeter. Based on a possible (but not sure) metric unit of the 'imperial foot' of 0,52 m, the side of the overall circumscribed octagon signs (again) 40 feet and the ca.54,60 m diameter of the circle circumscription is to convert into the number 54,60: 0,52 = 105 feet, which indicates the particular arithmetic sequence of 105 = 1x3x5x7 i.e. the product of the first uneven prime numbers. The plan design is strictly squared in all particularities (**Figure 8b, c**). Each of the two floors is divided in eight trapezium-form rooms, covered by a central cross-rib square vault and two triangular side vaults. The parallel with the 'Dome of the Rock '-temple is evident as also this one is designed as a double concentric octagon signing the same overall diameter of 105 feet or ca. 55 m, what supports the hypothesis about a conscious imitation in Andria. A further connection with Islamic architecture is given by the slim-line corner-towers reminding the typology of the 4th century square towers alongside the byzantine defense wall round Constantinople, although the octagonal towers at the Andria-castle do not seem too much for defense as well as for semantic reasons.

However, any further comment on a probable metric modulation or other design parallels are hindered by sufficiently controllable historic and metric information about the initial composition (this last aspect hampered by the lack of net interior dimensions as all inside decorative marble wall-cladding has disappeared, what makes authentic reliable measuring impossible). Further research is needed.

5. Conclusion

The above mentioned small selection of architectural and structural characteristics illustrate the impact and extraordinary image building capacity of the ancient design procedures and the role of semantics and symbolisms. The Vitruvian maxims on a.o. ordinatio, analogia, symmetria, euritmia [1], as well as the Pythagorean semantics on quantity, harmonic numbering and arithmetic sequences were the guiding criteria in the historic design; this same criteria became universal quality indicators by which also modern designers can be inspired in the choice and the definition about the form and the geometry of their project. Recent E.U. research programs [25] have proved in several occasions the possible input of heritage building analysis by digital modeling; the application of similar techniques and algorithms might be reversible and be introduced in creative new design.

Sustainable architectural construction means not only an economic or utilitarian driven concern but implies a at least as important social and humanitarian assignment. The quality of contemporary projects, and the long-term guarantee for visual and functional quality in the built environment will certainly be enriched by a design which is not looking for eye-catching artistic effects, as this last ones change with the wind, but by a comprehensive design based on the universal human scaled modulation of all times, with a simply legible semantic communication.

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