Chapter

Convective Heat and Mass Transfer of Two Fluids in a Vertical Channel

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Abstract

A mathematical model for convective heat and mass transfer of two immiscible fluids in a vertical channel of variable width with thermo-diffusion, diffusion-thermal effects is presented. The governing boundary layer equations generated for momentum, angular momentum, energy and species concentration are solved with appropriate boundary conditions using Galeriken finite element method. The effects of the pertinent parameters are studied in detail. Furthermore, the rate of heat transfer, mass transfer and shear stress near both walls is analyzed.

Keywords: vertical channel, immiscible fluids, finite element method, heat and mass transfer

1. Introduction

The developments are carried out in the field of fluid dynamics which was initiated by Euler [1] by proving his famous equations of fluid flow for ideal (inviscid) fluids. Fluid dynamics is a subset of the science that looks at the materials in motion. Hydrodynamics deals with the fluids which are in motion. Fluid dynamics comes under science of fluid mechanics along with the other subcategories as fluid statics, which corresponds to fluids at rest, while fluid dynamics includes fluids in motion. Fluid is defined as a matter in a gas or liquid state. Fluid dynamics is governed by the regulations of preservation of mass, energy and linear momentum. These laws state that the total amount in a closed model remains unchangeable and the energy and mass cannot be formed or demolished. They can deform but will not disappear. Another governing law is the continuum hypothesis which defines that they are uninterrupted and their characteristics fluctuate all over. The history of fluid dynamics can be found in Rouse and Ince [2] and Tokaty [3].

1.1 Micropolar fluid

The subject of micro-polar fluids attained higher degree by many researchers because when the fluid is with the suspended particles we cannot analyze the properties of fluid flow by regular Newtonian fluid characteristics. Generally this fluid is defined as non-Newtonian comprising of small firm cylindrical matters, polymer liquids, liquefied suspensions, animal blood and such related components. The existence of dust or smoke especially, gas are characterized as micro-polar fluids in fluid dynamics. Eringen [4, 5] had taken the initiation in describing the subject of micropolar fluids. In his theory he considered the local impacts emerging from the micro-structure and the inherent movement of the fluid elements. Peddieson and McNitt [6], Ariman et al. [7] addressed many investigations and applications of micropolar fluid mechanics, which are also described in the works of Lukaszewicz [8] and Eringen [9]. A.J. Chamkha et al. [10] analyzed the wholly established free convection of micropolar fluid in a upright passage.

1.2 Magnetohydrodynamics (MHD)

MHD is the science concerned with the motions of electro fluids and their interactions with magnetic fields. It is a vital branch and comparatively new in the field of fluid dynamics. When a conducting fluid is moving along a magnetic field, it results in induction of an electric field current which in turn produces the body forces. According to Faraday's principle, on passage of electric current in a magnetic area, it experiences a force making to direct it at right angles to the electric field. Similarly, if the conductor has electromagnetic forces of the same order as the hydrodynamical and inertial forces, these forces are taken in the equation of motion along with the other forces. The integration of Navier–Stokes relations of fluid dynamics and Maxwell's expressions of electromagnetism describe magneto hydrodynamics which are to be solved simultaneously. There are many scientific & technical applications in the literature: heating and flow control in metal structures, power production from 2-phase models or seeded high temperature gases, magnetic constraints of extreme temperature plasma and dynamo that develop magnetic field in environmental matters.

The concept of MHD flow of the boundary layer in a vertical channel is greatly considered in present metallurgical and metal processing fields. Most of the metallic materials are manufactured from the molten state. It is significant to determine the heat transfer in metals, which are electric conductors. Therefore, a controlled cooling system is required, so that, it can be regulated through an external magnetic field.

1.3 Convective heat and mass transference

Convection is the movement of molecules within the fluids. It belongs to the fundamental means of heat and mass transference that is carried out by ways of diffusion and random Brownian movement of distinct liquid elements. In our context, convection refers to the totality of advective and diffusive transfer. However, it is taken for only advective phenomena. A mechanism of transfer of heat occurring due to bulk motion of fluids is regarded as convective heat transfer. Emphasis is given to heat that is being passed and distributed.

Extensive research has been done over convective heat and mass transference of the fluid flow in vertical channels and other geometries. The existence of temperature and concentration differences or gradients lead to the convective heat and mass transfer and it is regarded as an area of study for broad examination because it is applied in several engineering issues, which are common in atmospheric buoyancy induced actions, liquid and semi-solid bodies and so on. There are quite a large number of application in the heat and mass transfer flows like, rocket nozzles, nuclear power plants, air craft and its re-entry in atmosphere, chemical and process instruments, mist formation and dispersal, temperature and humidity circulation over cultivation farms, plants destruction because of freezing, etc. Packham [11] considered the steady co-current motion of two immiscible viscous fluids in a parallel tube, the fluid interface being ripple-free and plane. Shail [12] considered the Hartmann flow of a conducting fluid in the pass way between two parallel insulating sheets of unbounded length, there exists a sheet of non-conductive fluid between the conductive fluid and the upper passage layer and given the conclusion that considerable increase could be attained in the conductive fluid velocity for appropriate proportions of the depth and viscosity of the two fluids. Beckermann et al. [13] conducted a numerical and experimental study to analyze the fluid motion and heat transfer in a upright rectangular cover which is occupied partly with a vertical layer of a fluid-saturated absorbent structure, where it is determined that the fluid quantity entering the fluid area to the absorbent layer is dependent on the Darcy & Rayleigh numbers. Lohrasbi and Sahai [14] researched in 2-phase MHD flow and heat transfer with the 1-phase conductive fluid.

1.4 Viscous dissipation

Viscous dissipation relates to the conversion of kinetic to internal energy (heating up the fluid) with respect to viscosity. It plays a significant part in normal convection in numerous units which hold huge deviations of gravitational force Gebhart [15]. Gebhart and Mollendorf [16] examined viscous dissipation in peripheral normal convection by taking in account of exponential deviation of wall temperature using resemblance relation. Fand and Brucker [17] stated that the impact of viscous dissipation is important in case of normal/natural convection in absorbent structure with respect to their investigational correlation for the heat transference in peripheral motions. Fand et al. [18] validated the comment for the Darcy method by experimental and analytical means when predicting the heat transfer coefficient from a parallel chamber implanted in an absorbent medium. Viscous dissipation performs as a heat source and heats the medium considerably. Nakayama and Pop [19] evaluated the influence of viscous dissipation on the Darcy's free convection towards an arbitrary shaped non-isothermal matter placed in a permeable medium. Murthy and Singh [20] observed viscous dissipation on non-Darcy normal convection from an erect flat sheet in a permeable medium saturated with Newtonian fluid. It is deduced that heat transfer decreases significantly with the presence of viscous dissipation effect. El-Amin [21] analyzed the impact of viscous dissipation and Joule heating on magneto fluid dynamics forced convection jointly on a non-isothermal straight container fixed in a fluid saturated permeable membrane. Bejan [22] defined that the calculations are limitedin examining the dissipation effect by means of a stable, 1-D energy relation, based on the analogical form with viscous dissipation effect. Pantokratoras [23] evaluated the viscous dissipation effects in a normal convection using a warmed straight plate. Seddeek [24] investigated viscous dissipation effect and thermophoresis on Darcy Forchheimer mixed convection in a fluid saturated permeable medium. Duwairi et al. [25] studied the effects of viscous dissipation and Joule heating employing an isothermal cone in a saturated porous medium. Various non-Newtonian fluids have high viscosity because the irreparable criterion owing to viscous dissipation sometimes becomes vital. Hence it motivates investigators to analyze the effects of viscous dissipation in a non-Newtonian fluid saturated permeable medium. Cortell [26] analyzed viscous dissipation effect and thermal boundary layer radiation on a nonlinear wide plate. Kairi and Murthy [27] analyzed the viscous dissipation impact over normal convection heat and mass transference from an upright cone in a non-Newtonian fluid saturated non-Darcy absorbent structure. Cortell [28] analyzed the influences of suction, viscous dissipation and thermal radiation over heat transfer of a power-law fluid past a boundless permeable sheet.

1.5 Diffusion effects

The diffusion effects namely thermal-diffusion (Soret) and diffusion-thermo (Dufour) are highly important in fluid mechanics. Soret is the transfer of mass formed by temperature gradients, i.e. species diversity evolving in a primary homogeneous blend directed to a thermal gradient. Diffusion-thermo effect is the heat transfer or the heat flux formed by concentration gradient. The problems concerned to heat and mass transference and density variations with temperature and concentration lead to integrated buoyancy convected force. The diffusion impacts influence the flow field in boundary membrane on an upright channel.

Chapman and Cowling [29] developed the diffusion-thermo and thermal-diffused heat and mass effects. Eckert and Drake [30] suggested that Dufour effect has widened magnitude and so this effect should not be ignored. Kafoussias and Williams [31] included the boundary layer flows with Soret and Dufour effects for the combined forced-normal convection problem. Anghel et al. [32] analyzed the Dufour and Soret effects of a free convection boundary layer on a vertical field inserted in a permeable membrane. Postelnicu [33] evaluated the effects of thermal-diffusion and diffusion-thermo on combined heat and mass transference in natural convection boundary layer flow in a Darcian porous media under transverse magnetic effect. Alam and Rahman [34] analyzed the effects of thermal-diffusion and diffusion-thermo on combined and free convection heat and mass transference flow past an erect permeable flat sheet inserted in a porous membrane with or without flexible suction. In many studies, Dufour and Soret effects are ignored based on a minor magnitude order than Fourier's and Fick's laws effect. The effect of thermal-diffusion and diffusion-thermo influences over the motion area in mixed convection boundary-layer on an upright surface kept in a permeable medium and on mixed convection flow past a vertical permeable even sheet with varying suction. Chamkha and Ben-Nakhi [35] studied the combined convection flow in the existence of thermal radiation with an erect porous layer embedded in an absorbent media considering thermal-diffusion and diffusion-thermo effects. El-Aziz [36] examined the Dufour and Soret effects on MHD heat and mass transference on a porous widening layer in the presence of thermal radiation in a combined manner. Maleque [37] considered only the diffusion-thermo effect on convective heat and mass transference past a rotating permeable disk, in where the thermal-diffusion effect is ignored. Anwar Beg et al. [38] described the thermal-diffusion and diffusion-thermo impacts by numerically studying the free convection MHD heat and mass transfer over a stretching layer with saturated permeable structure. Pal and Chatterjee [39] study shows combined convected magneto hydrodynamic heat and mass transference past a stretching plate considering Ohmic dissipated thermal-diffusion and diffusion-thermo impacts with micro-polar fluid. MHD flow of a pair of immiscible and conducting fluids within isothermal and insulated moving sheets under an applied electric and inclined magnetic effect and with an induced magnetic field has been investigated by Stamenkovic et al. [40].

1.6 Two fluid flow

For many years, Scientists and Engineers have been showing interest in two phase flows, which arise in many industrial applications. The two-phase fluid flow phenomena are important in pipe flows, fluidized beds, sedimentation, gas purification, transport processes and shock waves. The study of dynamics of two phase fluid system is concerned with the motion of a liquid or gas containing immiscible, suspended stokesian solid particles. In the equations of motion of two phase fluid flows, which are the modified form of Navier-Stoke's equations, the presence of dust adds an extra force term which represents the interaction between the dust and the fluid particles. The modified form of Navier–Stokes equations coupled with Euler equations of motion for perfect fluids are used as the equations of motion of fluid phase and particle phase, respectively. Practical application of these flows may be found in heat exchanges utilizing liquid metal or liquid sodium coolants in the area of thermal instability in boiling heat transfer studies. Malashetty and Leela [41, 42] studied two-fluid flow and heat transference in a parallel fluid passage in both conductive phases. Such investigations are beneficial to understand the slag layer effects over heat transfer features of a coal-fired magneto hydrodynamic generator. Vajravelu et al. [43] dealt with the hydromagnetic unstable motion of two immiscible conducting fluids between two porous media of different porosity. Malashetty and Umavathi [44] studied 2-phase magnetohydrodynamic flow and heat transference in a sloped passage, where 1-phase is conductive and the transport characteristics of the fluids are assumed to be unvarying. Srinivasan et al. [45] theoretically studied the two immiscible fluid models in a permeable membrane by considering the impacts of non-Darcian boundary and inertia. Malashetty et al. [46] explored the complexities of completely established 2-fluid magneto hydrodynamic flow including and excluding applied electric field in an slant pass way and described the solution of energy and momentum equations, using perturbation method for smaller value product of Prandtl and Eckert number in completely progressed free convection 2-fluid MHD flow of a tilted passage.

2. Mathematical formulation

The two infinite plates are kept at $Y = -h_1$ and $Y = h_2$ initially as shown in **Figure 1** and the two sheets are isothermal with dissimilar temperatures T_1 and T_2 respectively. The distance $-h_1 \leq Y \leq 0$ represents region-1 and the distance $0 \leq Y \leq h_2$ represents region-2 where the first one occupies micropolar fluid and the other, viscous fluid. Here the buoyancy force determines the fluid flow.

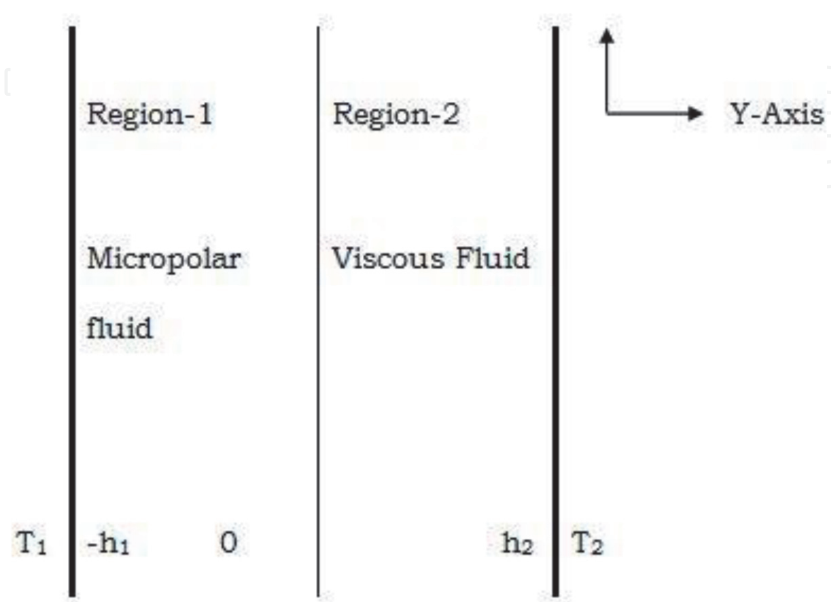

Figure 1.
Schematic diagram of the problem.

The governing conditions for the problem are developed based on the assumptions stated below:

1. The flow is assumed to be 1-D, steady, laminar, immiscible and incompressible.

2. The transport characteristics of the two fluids are presumed to be constant.

3. The fluid flow is fully developed.

4. The temperature and heat flows are continuous at the interface.

5. $T_1 > T_2, C_1 > C_2$.

3. Governing equations

The governing equations which are derived in Chapter-II under the above assumptions yields.

Region-1:

$$\frac{\mathrm{dU_1}}{\mathrm{dY}} = 0 \text{ [Law of conservation of mass]} \tag{1}$$

$$\rho_1 = \rho_0[1 - \beta_{1T}(T_1 - T_0) - \beta_{1C}(C_1 - C_0)] \text{ [Physicalstate]} \tag{2}$$

$$\frac{\mu_1 + K}{\rho_1}\frac{d^2U_1}{\mathrm{dY}^2} + \frac{K}{\rho_1}\frac{\mathrm{dn}}{\mathrm{dY}} + g\beta_{1T}(T_1 - T_0) + g\beta_{1C}(C_1 - C_0) - \frac{\sigma {B_0}^2 U_1}{\rho_1} = 0 \text{ [Momentum]} \tag{3}$$

$$\gamma\frac{d^2n}{\mathrm{dY}^2} - K\left[2n + \frac{\mathrm{dU_1}}{\mathrm{dY}}\right] = 0 \text{ [Conservation of Angular Momentum]} \tag{4}$$

where $\gamma = \left(\mu_1 + \frac{K}{2}\right)j$

$$\frac{k_1}{\rho_1 C_p}\frac{d^2T_1}{\mathrm{dY}^2} + \frac{1}{\rho_1 C_p}\left[\mu_1\left(\frac{\mathrm{dU_1}}{\mathrm{dY}}\right)^2 + \frac{\rho_1 D_1 K_{T1}}{C_{S1}}\frac{d^2C_1}{\mathrm{dY}^2}\right] = 0 \text{ [Energy]} \tag{5}$$

$$D_1\frac{d^2C_1}{\mathrm{dY}^2} + \frac{D_1 K_{T1}}{T_M}\frac{d^2T_1}{\mathrm{dY}^2} = 0 \text{ [Diffusion]} \tag{6}$$

Region-2:

$$\frac{\mathrm{dU_2}}{\mathrm{dY}} = 0 \text{ [Continuity]} \tag{7}$$

$$\rho_2 = \rho_0[1 - \beta_{2T}(T_2 - T_0) - \beta_{2C}(C_2 - C_0)] \text{ [State]} \tag{8}$$

$$\frac{\mu_2}{\rho_2}\frac{d^2U_2}{\mathrm{dY}^2} + g\beta_{2T}(T_2 - T_0) + g\beta_{2C}(C_2 - C_0) - \frac{\sigma {B_0}^2 U_2}{\rho_2} = 0 \text{ [Momentum]} \tag{9}$$

$$\frac{k_2}{\rho_2 C_p}\frac{d^2T_2}{\mathrm{dY}^2} + \frac{1}{\rho_2 C_p}\left[\mu_2\left(\frac{\mathrm{dU_2}}{\mathrm{dY}}\right)^2 + \frac{\rho_2 D_2 K_{T2}}{C_{S2}}\frac{d^2C_2}{\mathrm{dY}^2}\right] = 0 \text{ [Energy]} \tag{10}$$

$$D_2\frac{d^2C_2}{\mathrm{dY}^2} + \frac{D_2 K_{T2}}{T_M}\frac{d^2T_2}{\mathrm{dY}^2} = 0 \text{ [Diffusion]} \tag{11}$$

The above equation models (1) to (11) are solved by the following boundary and interface parameters.

$U_1 = 0$ at $Y = -h_1$, $U_2 = 0$ at $Y = h_2$,$U_1(0) = U_2(0)$,
$T = T_1$ at $Y = -h_1$, $T = T_2$ at $Y = h_2$,$T_1(0) = T_2(0)$,
$C = C_1$ at $Y = -h_1$, $C = C_2$ at $Y = h_2$,$C_1(0) = C_2(0)$,
$n = 0$ at $Y = -h_1$,$(\mu_1 + K)\frac{dU_1}{dY} + Kn = \mu_2 \frac{dU_2}{dY}$ at $Y = 0$,
$\frac{dn}{dY} = 0$ at $Y = 0$, $k_1 \frac{dT_1}{dY} = k_2 \frac{dT_2}{dY}$ at $Y = 0$,$D_1 \frac{dC_1}{dY} = D_2 \frac{dC_2}{dY}$ at $Y = 0$.

The following non dimensional variables form the equation systems (1) to (11) in to dimensionless form:

$y = \frac{Y}{h_1}$(region-1), $y = \frac{Y}{h_2}$ (region-2), $u_1 = \frac{U_1}{U_0}$, $u_2 = \frac{U_2}{U_0}$, $\theta_1 = \frac{T_1 - T_0}{\Delta T}$,
$\theta_2 = \frac{T_2 - T_0}{\Delta T}$, $N = \frac{h_1}{U_0} n$, $j = h^2$ (Characteristic length), $K' = \frac{K}{\mu_1}$, $c_1 = \frac{C_1 - C_0}{\Delta C}$, $c_2 = \frac{C_2 - C_0}{\Delta T}$, $\mathrm{Gr} = \frac{g\beta_{1T}\Delta T h_1{}^3}{\nu_1{}^2}$, $\mathrm{Gc} = \frac{g\beta_{1C}\Delta C h_1{}^3}{\nu_1{}^2}$, $R = \frac{U_0 h_1}{v_1}$, $Sr = \frac{D_1 K_{T1} \Delta T}{T_M \Delta C U_0 h_1}$, $Sc = \frac{v_1}{D_1}$, $Du = \frac{D_1 K_{T1} \Delta C}{C_P C_{S1} \nu_1 \Delta T}$,
$M = \frac{\sigma B_0{}^2 h_1{}^2}{\mu_1}$, $\Pr = \frac{\mu_1 Cp}{k_1}$, $Ec = \frac{U_0{}^2}{Cp\ \Delta T}$,
$C_S = \frac{C_{S1}}{C_{S2}}$, $K_T = \frac{K_{T1}}{K_{T2}}$, $D = \frac{D_1}{D_2}$, $h = \frac{h_1}{h_2}$, $m = \frac{\mu_1}{\mu_2}$, $\alpha = \frac{k_1}{k_2}$, $\rho = \frac{\rho_1}{\rho_2}$, $b_1 = \frac{\beta_{1T}}{\beta_{2T}}$, $b_2 = \frac{\beta_{1C}}{\beta_{2C}}$,$\nu = \frac{\nu_1}{\nu_2}$.

The dimensionless forms of governing equations thus obtained are:

Region-1:

$$\frac{d^2N}{dy^2} - \frac{2K'}{2 + K'}\left(2N + \frac{du_1}{dy}\right) = 0 \tag{12}$$

$$(1 + K')\frac{d^2u_1}{dy^2} + K'\frac{dN}{dy} + \frac{Gr}{R}\theta_1 + \frac{Gc}{R}c_1 - Mu_1 = 0 \tag{13}$$

$$\frac{1}{\Pr R}\frac{d^2\theta_1}{dy^2} + \frac{Ec}{R}\left(\frac{du_1}{dy}\right)^2 + \frac{Du}{R}\frac{d^2c_1}{dy^2} = 0 \tag{14}$$

$$\frac{1}{ScR}\frac{d^2c_1}{dy^2} + Sr\frac{d^2\theta_1}{dy^2} = 0 \tag{15}$$

Region -2

$$\frac{d^2u_2}{dy^2} + \frac{m}{b_1\rho h^2}\frac{Gr}{R}\theta_2 + \frac{m}{b_2\rho h^2}\frac{Gc}{R}c_2 - \frac{mM}{h^2}u_2 = 0 \tag{16}$$

$$\frac{\rho h}{\alpha}\frac{1}{\Pr R}\frac{d^2\theta_2}{dy^2} + \frac{\rho h}{m}\frac{Ec}{R}\left(\frac{du_2}{dy}\right)^2 + \frac{c_s h}{DK_T}\frac{D_u}{R}\frac{d^2c_2}{dy^2} = 0 \tag{17}$$

$$\frac{h}{D}\left(\frac{1}{ScR}\right)\frac{d^2c_2}{dy^2} + \frac{h}{K_T D}Sr\frac{d^2\theta_2}{dy^2} = 0 \tag{18}$$

The dimensionless boundary and interface conditions thus formed are:

$$\begin{aligned}
&u_1 = 0 \text{ at } y = -1, u_2 = 0 \text{ at } y = 1, u_1(0) = u_2(0),\\
&\theta_1 = 1 \text{ at } y = -1, \theta_2 = 0 \text{ at } y = 1, \theta_1(0) = \theta_2(0),\\
&c_1 = 1 \text{ at } y = -1, c_2 = 0 \text{ at } y = 1, c_1(0) = c_2(0),\\
&N = 0 \text{ at } y = -1, \frac{du_1}{dy} + \frac{K'}{1 + K'}N = \frac{1}{mh(1 + K')}\frac{du_2}{dy} \text{ at } y = 0,\\
&\frac{dN}{dy} = 0 \text{ at } y = 0, \frac{d\theta_1}{dy} = \frac{1}{h\alpha}\frac{d\theta_2}{dy} \text{ at } y = 0, \frac{dc_1}{dy} = \frac{1}{hD}\frac{dc_2}{dy} \text{ at } y = 0.
\end{aligned} \tag{19}$$

4. Solution of the problem

The finite element method as described in Chapter-II is applied in solving the dimensionless coupled differential equations generated by the fluid flows. For the problem discussed here, it is considered that each region is classified into 100 linear elements and each element is 3 nodded.

The element equations associated with Eqs. (12) to (18) is as follows:

$$\int_{y_i}^{y_{i+1}} \left(\frac{d^2N}{\mathrm{dy}^2} \text{-} \frac{2K'}{2+K'} \left[2N + \frac{\mathrm{du}_1}{\mathrm{dy}} \right] \right) \eta_k \mathrm{dy} = 0 \tag{20}$$

$$\int_{y_i}^{y_{i+1}} \left((1+K') \frac{d^2u_1}{\mathrm{dy}^2} + K' \frac{\mathrm{dN}}{\mathrm{dy}} + \frac{\mathrm{Gr}}{R} \theta_1 + \frac{\mathrm{Gc}}{R} c_1 - \mathrm{Mu}_1 \right) \eta_k \mathrm{dy} = 0 \tag{21}$$

$$\int_{y_i}^{y_{i+1}} \left(\frac{1}{\mathrm{Pr}R} \frac{d^2\theta_1}{\mathrm{dy}^2} + \frac{\mathrm{Ec}}{R} \left(\frac{\mathrm{du}_1}{\mathrm{dy}} \right)^2 + \frac{\mathrm{Du}}{R} \frac{d^2c_1}{\mathrm{dy}^2} \right) \eta_k \mathrm{dy} = 0 \tag{22}$$

$$\int_{y_i}^{y_{i+1}} \left(\frac{1}{\mathrm{Sc}R} \frac{d^2c_1}{\mathrm{dy}^2} + \mathrm{Sr} \frac{d^2\theta_1}{\mathrm{dy}^2} \right) \eta_k \mathrm{dy} = 0 \tag{23}$$

$$\int_{y_i}^{y_{i+1}} \left(\frac{d^2u_2}{\mathrm{dy}^2} + \frac{m}{b_1\rho h^2} \frac{\mathrm{Gr}}{R} \theta_2 + \frac{m}{b_2\rho h^2} \frac{\mathrm{Gc}}{R} c_2 - \frac{\mathrm{mM}}{h^2} u_2 \right) \chi_k \mathrm{dy} = 0 \tag{24}$$

$$\int_{y_i}^{y_{i+1}} \left(\frac{\rho h}{\alpha} \frac{1}{\mathrm{Pr}R} \frac{d^2\theta_2}{\mathrm{dy}^2} + \frac{\rho h}{m} \frac{\mathrm{Ec}}{R} \left(\frac{\mathrm{du}_2}{\mathrm{dy}} \right)^2 + \frac{c_s h}{\mathrm{DK}_T} \frac{D_u}{R} \frac{d^2c_2}{\mathrm{dy}^2} \right) \chi_k \mathrm{dy} = 0 \tag{25}$$

$$\int_{y_i}^{y_{i+1}} \left(\frac{h}{D} \left(\frac{1}{\mathrm{Sc}R} \right) \frac{d^2c_2}{\mathrm{dy}^2} + \frac{h}{K_T D} \mathrm{Sr} \frac{d^2\theta_2}{\mathrm{dy}^2} \right) \chi_k \mathrm{dy} = 0 \tag{26}$$

Where η_k and χ_k denotes the shape functions of a typical element (y_i, y_{i+1}) in the region 1 and 2 correspondingly.

On integrating the above equations and by replacing the finite element Galerkin calculations,

$$u_1^i = \sum_{j=1}^{3} u_j^i \eta_j^i,\ c_1^i = \sum_{j=1}^{3} c_j^i \eta_j^i,\ N^i = \sum_{j=1}^{3} N_j^i \eta_j^i,\ \theta_1^i = \sum_{j=1}^{3} \theta_j^i \eta_j^i,$$

$$u_2^i = \sum_{j=1}^{3} u_j^i \chi_j^i,\ c_2^i = \sum_{j=1}^{3} c_j^i \chi_j^i,\ \theta_2^i = \sum_{j=1}^{3} \theta_j^i \chi_j^i.$$

From Eq. (20) we get

$$\int_{y_i}^{y_{i+1}} \frac{\mathrm{dN}^i}{\mathrm{dy}} \frac{d\eta_k}{\mathrm{dy}} \mathrm{dy} + \frac{2K'}{2+K'} \int_{y_i}^{y_{i+1}} \left[2N^i \eta_k \mathrm{dy}\text{-} \frac{d\eta_k}{\mathrm{dy}} u_1^i \right] \mathrm{dy} = \left[\eta_k \frac{\mathrm{dN}^i}{\mathrm{dy}} + \eta_k u_1^i \right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[a^i_{\mathrm{kj}}\right]\left[N^i_k\right] + \left[b^i_{\mathrm{kj}}\right]\left[u^i_k\right] = \left[Q_1{}^i_j\right] \tag{27}$$

where $a^i_{\mathrm{kj}} = \int\limits_{y_i}^{y_{i+1}} \frac{d\eta_k}{\mathrm{dy}}\frac{d\eta^i_j}{\mathrm{dy}}\,\mathrm{dy} + \frac{2K'}{2+K'}\int\limits_{y_i}^{y_{i+1}}\left[2\eta_k\eta^i_j\right]\mathrm{dy}$

$$b^i_{\mathrm{kj}} = -\frac{2K'}{2+K'}\int\limits_{y_i}^{y_{i+1}}\left[\frac{d\eta^i_j}{\mathrm{dy}}\eta_k\right]\mathrm{dy}$$

$$Q_1{}^i_j = \left[\eta_k\frac{\mathrm{dN}^i}{\mathrm{dy}} + \eta_k u^i_1\right]_{y_i}^{y_{i+1}}$$

From Eq. (21) we get

$$\int\limits_{y_i}^{y_{i+1}}(1+K')\frac{d\eta_k}{\mathrm{dy}}\frac{\mathrm{du}^i_1}{\mathrm{dy}}\,\mathrm{dy} + \int\limits_{y_i}^{y_{i+1}}K'\frac{d\eta_k}{\mathrm{dy}}N^i\mathrm{dy}\text{-}\frac{\mathrm{Gr}}{R}\int\limits_{y_i}^{y_{i+1}}\eta_k\theta^i_1\mathrm{dy}\text{-}$$
$$\frac{\mathrm{Gc}}{R}\int\limits_{y_i}^{y_{i+1}}\eta_k c^i_1\,\mathrm{dy} + M\int\limits_{y_i}^{y_{i+1}}\eta_k u^i_1\,\mathrm{dy} = \left[\text{-}(1+K')\eta_k\frac{\mathrm{du}^i_1}{\mathrm{dy}} - K'\eta_k N^i\right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[c^i_{\mathrm{kj}}\right]\left[u^i_k\right] + \left[d^i_{\mathrm{kj}}\right]\left[N^i_k\right] + \left[e^i_{\mathrm{kj}}\right]\left[\theta^i_k\right] + \left[f^i_{\mathrm{kj}}\right]\left[c^i_k\right] = \left[Q_2{}^i_j\right] \tag{28}$$

where $c^i_{\mathrm{kj}} = \int\limits_{y_i}^{y_{i+1}}(1+K')\frac{d\eta^i_j}{\mathrm{dy}}\frac{d\eta_k}{\mathrm{dy}}\,\mathrm{dy} + M\int\limits_{y_i}^{y_{i+1}}\eta^i_j\eta_k\mathrm{dy},\ d^i_{\mathrm{kj}} = K'\int\limits_{y_i}^{y_{i+1}}\left[\frac{d\eta^i_j}{\mathrm{dy}}\eta_k\right]\mathrm{dy}$

$$e^i_{\mathrm{kj}} = -\frac{\mathrm{Gr}}{R}\int\limits_{y_i}^{y_{i+1}}\left[\eta^i_j\eta_k\right]\mathrm{dy},\ f^i_{\mathrm{kj}} = -\frac{\mathrm{Gc}}{R}\int\limits_{y_i}^{y_{i+1}}\left[\eta^i_j\eta_k\right]\mathrm{dy}$$

$$Q_2{}^i_j = \left[\text{-}(1+K')\eta_k\frac{\mathrm{du}^i_1}{\mathrm{dy}} - K'\eta_k N^i\right]_{y_i}^{y_{i+1}}$$

From Eq. (22) we get

$$\frac{1}{\mathrm{Pr}R}\int\limits_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{d\theta^i_1}{\mathrm{dy}}\mathrm{dy} - \frac{\mathrm{Ec}}{R}\int\limits_{y_i}^{y_{i+1}}\eta_k\left(\frac{\mathrm{du}^i_1}{\mathrm{dy}}\right)^2\mathrm{dy}\text{-}\frac{\mathrm{Du}}{R}\int\limits_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{\mathrm{dc}^i_1}{\mathrm{dy}}\mathrm{dy}$$
$$= \left[\frac{1}{\mathrm{Pr}R}\eta_k\frac{d\theta^i_1}{\mathrm{dy}} - \frac{\mathrm{Du}}{R}\eta_k\frac{\mathrm{dc}^i_1}{\mathrm{dy}}\right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[g^i_{\mathrm{kj}}\right]\left[\theta^i_k\right] + \left[u^i_k\right]^T\left[h^i_{\mathrm{kj}}\right]\left[u^i_k\right] + \left[m^i_{\mathrm{kj}}\right]\left[c^i_k\right] = \left[Q_3{}^i_j\right] \tag{29}$$

where

$$g_{\mathrm{kj}}^{i}=\frac{1}{\mathrm{Pr}\,R}\int_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{d\eta_j^i}{\mathrm{dy}}\mathrm{dy},\,h_{\mathrm{kj}}^{i}$$

$$=\frac{-\mathrm{Ec}}{R}\int_{y_i}^{y_{i+1}}\begin{bmatrix}\left(\frac{d\eta_1^i}{\mathrm{dy}}\right)^2 & \left(\frac{d\eta_1^i}{\mathrm{dy}}\right)\left(\frac{d\eta_2^i}{\mathrm{dy}}\right) & \left(\frac{d\eta_1^i}{\mathrm{dy}}\right)\left(\frac{d\eta_3^i}{\mathrm{dy}}\right)\\ \left(\frac{d\eta_2^i}{\mathrm{dy}}\right)\left(\frac{d\eta_1^i}{\mathrm{dy}}\right) & \left(\frac{d\eta_2^i}{\mathrm{dy}}\right)^2 & \left(\frac{d\eta_2^i}{\mathrm{dy}}\right)\left(\frac{d\eta_3^i}{\mathrm{dy}}\right)\\ \left(\frac{d\eta_3^i}{\mathrm{dy}}\right)\left(\frac{d\eta_1^i}{\mathrm{dy}}\right) & \left(\frac{d\eta_3^i}{\mathrm{dy}}\right)\left(\frac{d\eta_2^i}{\mathrm{dy}}\right) & \left(\frac{d\eta_3^i}{\mathrm{dy}}\right)^2\end{bmatrix}[\eta_k]\mathrm{dy}.$$

$$m_{\mathrm{kj}}^{i}=\frac{-\mathrm{Du}}{R}\int_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{d\eta_j^i}{\mathrm{dy}}\mathrm{dy},\,Q_{3\,j}^{\;i}=\left[\frac{1}{\mathrm{Pr}\,R}\eta_k\frac{d\theta_1^i}{\mathrm{dy}}-\frac{\mathrm{Du}}{R}\eta_k\frac{\mathrm{dc}_1^i}{\mathrm{dy}}\right]_{y_i}^{y_{i+1}}.$$

From Eq. (23) we get

$$\frac{1}{\mathrm{Sc}\,R}\int_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{\mathrm{dc}_1^i}{\mathrm{dy}}\mathrm{dy}+\mathrm{Sr}\int_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{d\theta_1^i}{\mathrm{dy}}\mathrm{dy}=\left[\frac{1}{\mathrm{Sc}\,R}\eta_k\frac{\mathrm{dc}_1^i}{\mathrm{dy}}+\mathrm{Sr}\,\eta_k\frac{d\theta_1^i}{\mathrm{dy}}\right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[n_{\mathrm{kj}}^{i}\right]\left[c_k^i\right]+\left[p_{\mathrm{kj}}^{i}\right]\left[\theta_k^i\right]=\left[Q_{4\,j}^{\;i}\right] \tag{30}$$

where $n_{\mathrm{kj}}^{i}=\frac{1}{\mathrm{Sc}\,R}\int_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{d\eta_j^i}{\mathrm{dy}}\mathrm{dy},\,p_{\mathrm{kj}}^{i}=\mathrm{Sr}\int_{y_i}^{y_{i+1}}\frac{d\eta_k}{\mathrm{dy}}\frac{d\eta_j^i}{\mathrm{dy}}\mathrm{dy}$

$$Q_{4\,j}^{\;i}=\left[\frac{1}{\mathrm{Sc}\,R}\eta_k\frac{\mathrm{dc}_1^i}{\mathrm{dy}}+\mathrm{Sr}\,\eta_k\frac{d\theta_1^i}{\mathrm{dy}}\right]_{y_i}^{y_{i+1}}$$

From Eq. (24) we get

$$\int_{y_i}^{y_{i+1}}\frac{d\chi_k}{\mathrm{dy}}\frac{\mathrm{du}_2^i}{\mathrm{dy}}\mathrm{dy}\text{-}\frac{m}{b_1\rho h^2}\frac{\mathrm{Gr}}{R}\int_{y_i}^{y_{i+1}}\chi_k\theta_2^i\mathrm{dy}\text{-}\frac{m}{b_2\rho h^2}\frac{\mathrm{Gc}}{R}\int_{y_i}^{y_{i+1}}\chi_k c_2^i\,\mathrm{dy}$$

$$+\frac{\mathrm{mM}}{h^2}\int_{y_i}^{y_{i+1}}\chi_k u_2^i\,\mathrm{dy}=\left[\text{-}\chi_k\frac{\mathrm{du}_2^{\;i}}{\mathrm{dy}}\right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[C_{\mathrm{kj}}^{i}\right]\left[u_k^i\right]+\left[D_{\mathrm{kj}}^{i}\right]\left[\theta_k^i\right]+\left[E_{\mathrm{kj}}^{i}\right]\left[c_k^i\right]=\left[Q_{5\,j}^{\;i}\right] \tag{31}$$

where $C^i_{\text{kj}} = \int\limits_{y_i}^{y_{i+1}} \frac{d\chi^i_j}{\text{dy}} \frac{d\chi_k}{\text{dy}} \text{dy} + \frac{\text{mM}}{h^2} \int\limits_{y_i}^{y_{i+1}} \chi^i_j \chi_k \text{dy}$, $D^i_{\text{kj}} = -\frac{m}{b_1 \rho h^2} \frac{\text{Gr}}{R} \int\limits_{y_i}^{y_{i+1}} \left[\chi^i_j \chi_k\right] \text{dy}$,

$E^i_{\text{kj}} = -\frac{m}{b_2 \rho h^2} \frac{\text{Gc}}{R} \int\limits_{y_i}^{y_{i+1}} \left[\chi^i_j \chi_k\right] \text{dy}$, $Q_{5}{}^i_j = \left[-\chi_k \frac{\text{du}^i_2}{\text{dy}}\right]_{y_i}^{y_{i+1}}$.

From Eq. (25) we get

$$\frac{\rho h}{\alpha} \frac{1}{\text{Pr} R} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{d\theta^i_2}{\text{dy}} \text{dy} - \frac{\rho h}{m} \frac{\text{Ec}}{R} \int\limits_{y_i}^{y_{i+1}} \chi_k \left(\frac{\text{du}^i_2}{\text{dy}}\right)^2 \text{dy-} \frac{c_s h}{\text{DK}_T} \frac{\text{Du}}{R} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{\text{dc}^i_2}{\text{dy}} \text{dy} = \left[\frac{\rho h}{\alpha} \frac{1}{\text{Pr} R} \chi_k \frac{d\theta^i_2}{\text{dy}} - \frac{c_s h}{\text{DK}_T} \frac{\text{Du}}{R} \chi_k \frac{\text{dc}^i_2}{\text{dy}}\right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[F^i_{\text{kj}}\right] \left[\theta^i_k\right] + \left[u^i_k\right]^T \left[G^i_{\text{kj}}\right] \left[u^i_k\right] + \left[H^i_{\text{kj}}\right] \left[c^i_k\right] = \left[Q_{6}{}^i_j\right] \tag{32}$$

where $F^i_{\text{kj}} = \frac{\rho h}{\alpha} \frac{1}{\text{Pr} R} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{d\chi^i_j}{\text{dy}} \text{dy}$,

$$G^i_{\text{kj}} = -\frac{\rho h}{m} \frac{\text{Ec}}{R} \int\limits_{y_i}^{y_{i+1}} \begin{bmatrix} \left(\frac{d\chi^i_1}{\text{dy}}\right)^2 & \left(\frac{d\chi^i_1}{\text{dy}}\right)\left(\frac{d\chi^i_2}{\text{dy}}\right) & \left(\frac{d\chi^i_1}{\text{dy}}\right)\left(\frac{d\chi^i_3}{\text{dy}}\right) \\ \left(\frac{d\chi^i_2}{\text{dy}}\right)\left(\frac{d\chi^i_1}{\text{dy}}\right) & \left(\frac{d\chi^i_2}{\text{dy}}\right)^2 & \left(\frac{d\chi^i_2}{\text{dy}}\right)\left(\frac{d\chi^i_3}{\text{dy}}\right) \\ \left(\frac{d\chi^i_3}{\text{dy}}\right)\left(\frac{d\chi^i_1}{\text{dy}}\right) & \left(\frac{d\chi^i_3}{\text{dy}}\right)\left(\frac{d\chi^i_2}{\text{dy}}\right) & \left(\frac{d\chi^i_3}{\text{dy}}\right)^2 \end{bmatrix} [\chi_k] \text{dy}$$

$H^i_{\text{kj}} = -\frac{c_s h}{\text{DK}_T} \frac{\text{Du}}{R} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{d\chi^i_j}{\text{dy}} \text{dy}$, $Q_{6}{}^i_j = \left[\frac{\rho h}{\alpha} \frac{1}{\text{Pr} R} \chi_k \frac{d\theta^i_2}{\text{dy}} - \frac{c_s h}{\text{DK}_T} \frac{\text{Du}}{R} \chi_k \frac{\text{dc}^i_2}{\text{dy}}\right]_{y_i}^{y_{i+1}}$.

From Eq. (26) we get

$$\frac{h}{D} \frac{1}{\text{Sc} R} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{\text{dc}^i_2}{\text{dy}} \text{dy} + \frac{h}{K_T D} \text{Sr} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{d\theta^i_2}{\text{dy}} \text{dy} = \left[\frac{h}{D} \frac{1}{\text{Sc} R} \chi_k \frac{\text{dc}^i_2}{\text{dy}} + \frac{h}{K_T D} \text{Sr} \chi_k \frac{d\theta^i_2}{\text{dy}}\right]_{y_i}^{y_{i+1}}$$

The stiffness matrix equation corresponding to the above is

$$\left[L^i_{\text{kj}}\right] \left[c^i_k\right] + \left[M^i_{\text{kj}}\right] \left[\theta^i_k\right] = \left[Q_{7}{}^i_j\right] \tag{33}$$

where $L^i_{\text{kj}} = \frac{h}{D} \frac{1}{\text{Sc} R} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{d\chi^i_j}{\text{dy}} \text{dy}$, $M^i_{\text{kj}} = \frac{h}{K_T D} \text{Sr} \int\limits_{y_i}^{y_{i+1}} \frac{d\chi_k}{\text{dy}} \frac{d\chi^i_j}{\text{dy}} \text{dy}$

$$Q_{7}{}^i_j = \left[\frac{h}{D} \frac{1}{\text{Sc} R} \chi_k \frac{\text{dc}^i_2}{\text{dy}} + \frac{h}{K_T D} \text{Sr} \chi_k \frac{d\theta^i_2}{\text{dy}}\right]_{y_i}^{y_{i+1}}$$

The Langrange's interpolation polynomials are used as the shape functions at each of the nodes are considered as follows:

$$\eta_i^1 = \frac{\left(y - \left(\frac{2i-101}{100}\right)\right)\left(y - \left(\frac{2i-100}{100}\right)\right)}{\left(\left(\frac{2i-102}{100}\right) - \left(\frac{2i-101}{100}\right)\right)\left(\left(\frac{2i-102}{100}\right) - \left(\frac{2i-100}{100}\right)\right)},$$
$$\eta_i^2 = \frac{\left(y - \left(\frac{2i-102}{100}\right)\right)\left(y - \left(\frac{2i-100}{100}\right)\right)}{\left(\left(\frac{2i-101}{100}\right) - \left(\frac{2i-102}{100}\right)\right)\left(\left(\frac{2i-101}{100}\right) - \left(\frac{2i-100}{100}\right)\right)},$$
$$\eta_i^3 = \frac{\left(y - \left(\frac{2i-101}{100}\right)\right)\left(y - \left(\frac{2i-102}{100}\right)\right)}{\left(\left(\frac{2i-100}{100}\right) - \left(\frac{2i-102}{100}\right)\right)\left(\left(\frac{2i-100}{100}\right) - \left(\frac{2i-101}{100}\right)\right)}.$$

and similarly for $\chi_i^1, \chi_i^2, \chi_i^3$.

The shear stress values, heat (Nusselt number) and mass transfer rate (Sherwood number) are calculated at both walls as per the following relations:

$$\mathrm{St}_1 = \left[\frac{\partial u_1}{\partial y}\right]_{y=-1}, \mathrm{St}_2 = \left[\frac{\partial u_2}{\partial y}\right]_{y=1}, \mathrm{Nu}_1 = \left[\frac{\partial \theta_1}{\partial y}\right]_{y=-1}, \mathrm{Nu}_2 = \left[\frac{\partial \theta_2}{\partial y}\right]_{y=1}, \mathrm{Sh}_1 = \left[\frac{\partial c_1}{\partial y}\right]_{y=-1}, \mathrm{Sh}_2 = \left[\frac{\partial c_2}{\partial y}\right]_{y=1}.$$

5. Results and discussion

The numerical solution of the system of equations is analyzed for several values of the governing factors and its corresponding graphical representations are resulted. Thermal Grashof (Gr), Molecular Grashof (Gc) and Reynolds numbers (R), Magnetic field (M) and Material parameters (K′) and Dufour (Du), Schmidt (Sc), Soret (Sr) and Eckert numbers (Ec) are fixed as Gr = 5, Gc = 5, R = 3, M = 3,

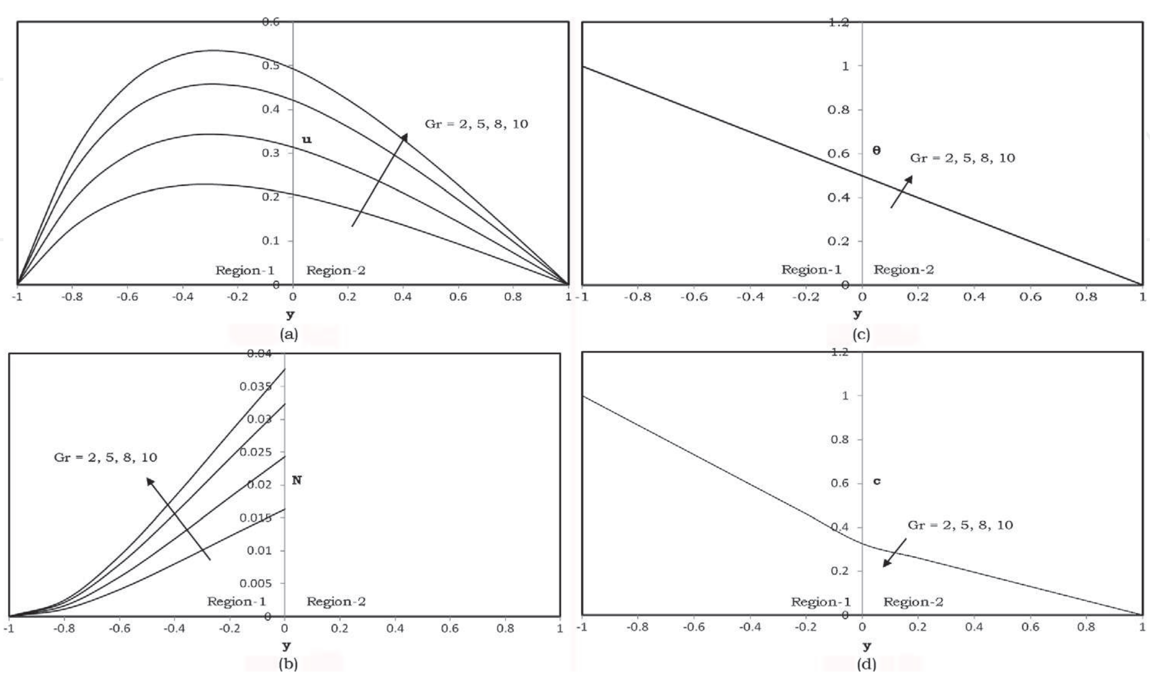

Figure 2.
(a) Represents behavior of u. (b) Represents behavior of N for Gr. (c) Represents behavior of θ. (d) Represents behavior of c for Gr.

$K' = 0.1$, $Du = 0.08$, $Sr. = 0.1$, $Sc = 0.66$, $Sr. = 0.001$ for all the profiles excepting the varying parameter.

The profiles of all the governing parameters are depicted from **Figures 2–10**. The flow in micropolar region is found to be more than the flow in viscous region. The variations of linear momentum and angular momentum are clear for each and every governing parameter. The variation of temperature and diffusion are very narrow except for the parameters R, Du, Sr., and Sc. The temperature and diffusion are uniform across the channel and are found to be significant at the mid region of

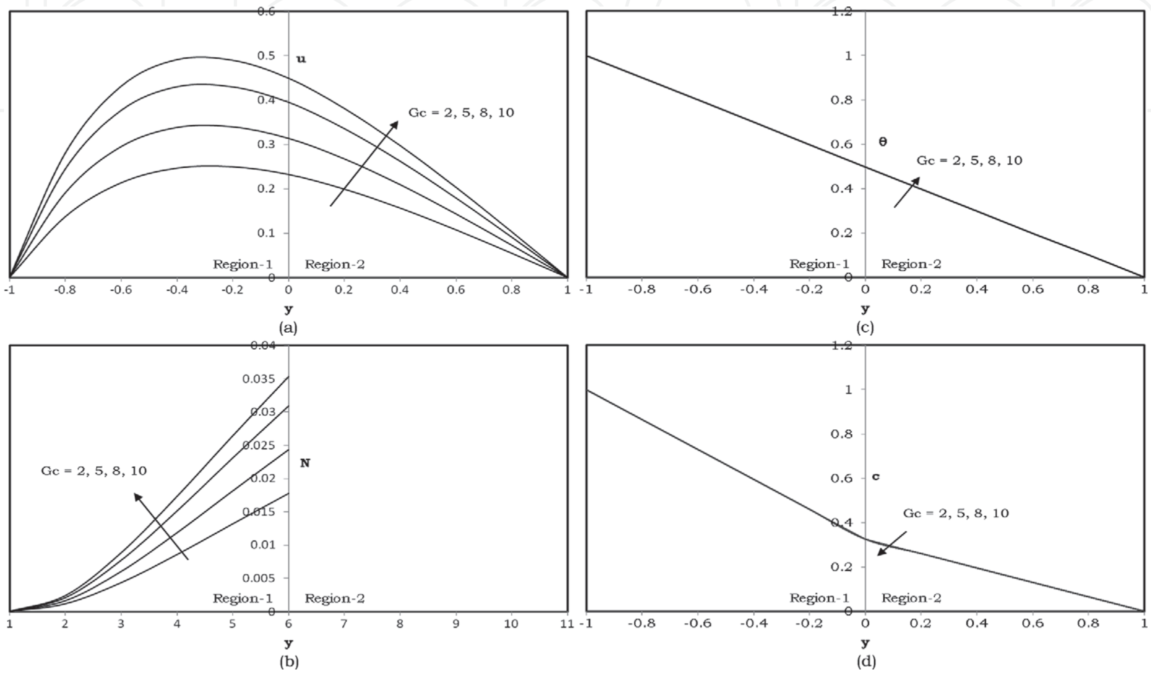

Figure 3.
(a) Represents behavior of u. (b) Represents behavior of N for Gc. (c) Represents behavior of θ. (d) Represents behavior of c for Gc.

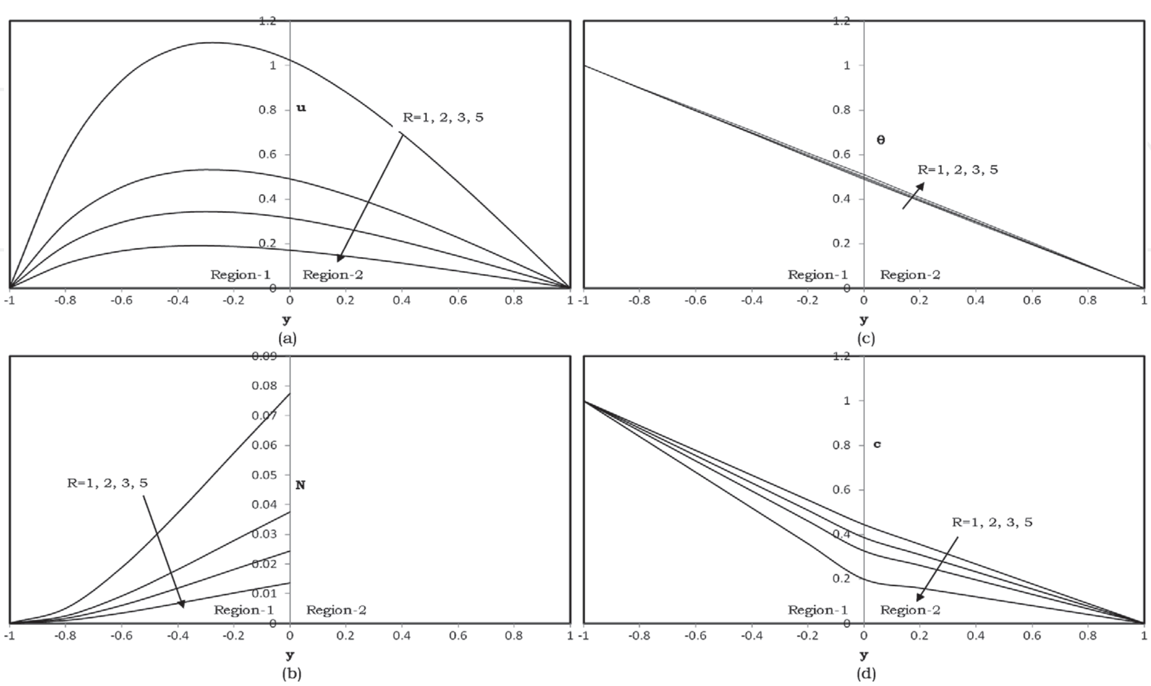

Figure 4.
(a) Represents behavior of u. (b) Represents behavior of N for R. (c) Represents behavior of θ. (d) Represents behavior of c for R.

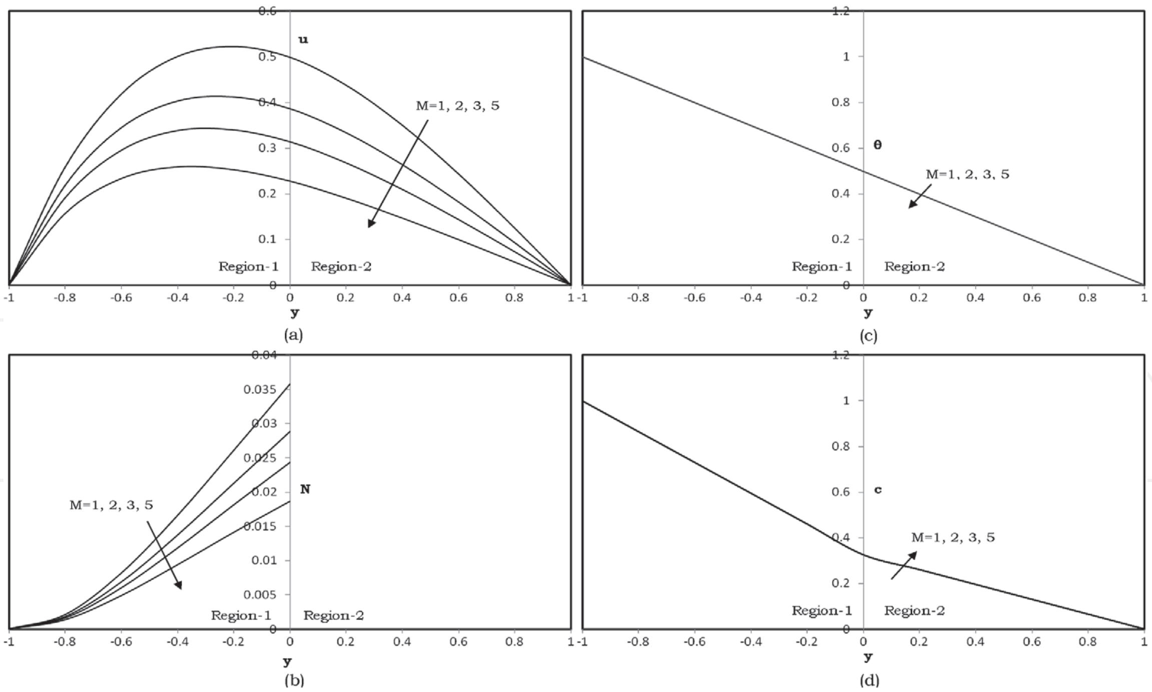

Figure 5.
(a) Represents behavior of u. (b) Represents behavior of N for M. (c) Represents behavior of θ. (d) Represents behavior of c for M.

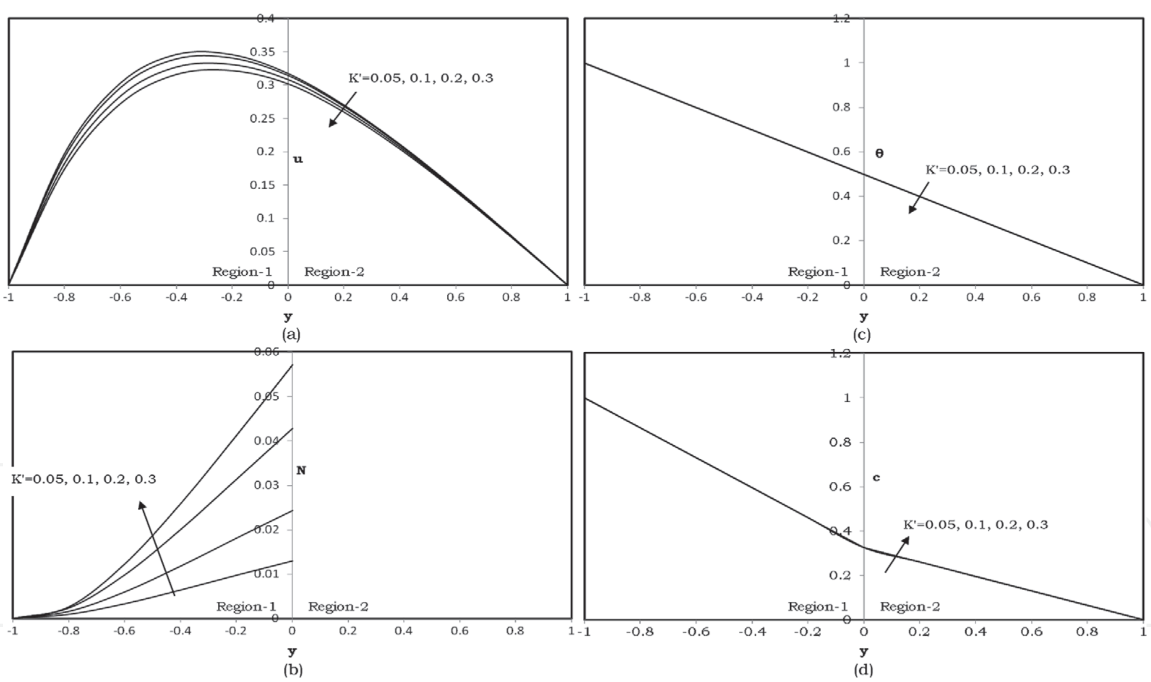

Figure 6.
(a) Represents behavior of u. (b) Represents behavior of N for K′. (c) Represents behavior of θ. (d) Concentration profiles for K′.

the channel. The diffusion is slightly effected at the interface due to two fluids. Hence the two fluid flow model has much importance in the real time systems. All our results are compared with earlier studies and they are validated.

Figure 2(a)–(d) illustrate the effect of Grashof numbers on velocity, angular velocity, temperature and diffusion. As Gr increases the velocity and angular velocity increases substantially. The buoyancy enhances the flow in both regions i.e. thermal buoyancy force dominates the viscous force in both regions of the channel and it is found to be more in micropolar region. The lowest velocity corresponds to

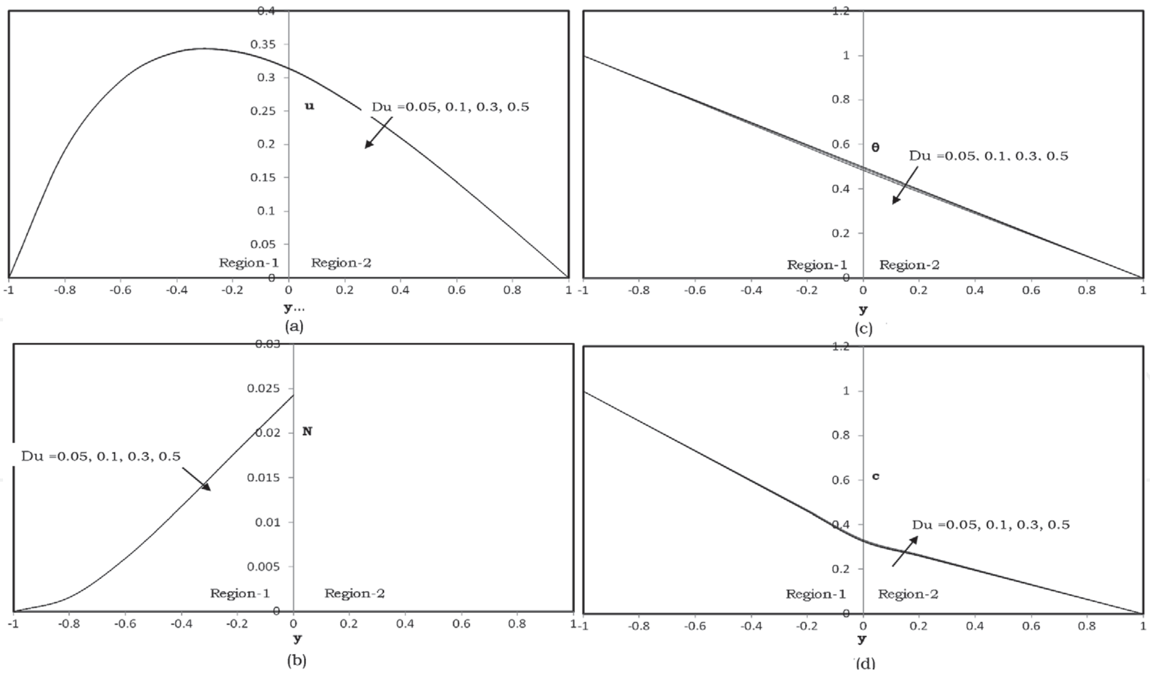

Figure 7.
(a) Represents behavior of u. (b) Represents behavior of N for Du. (c) Represents behavior of θ. (d) Represents behavior of c for Du.

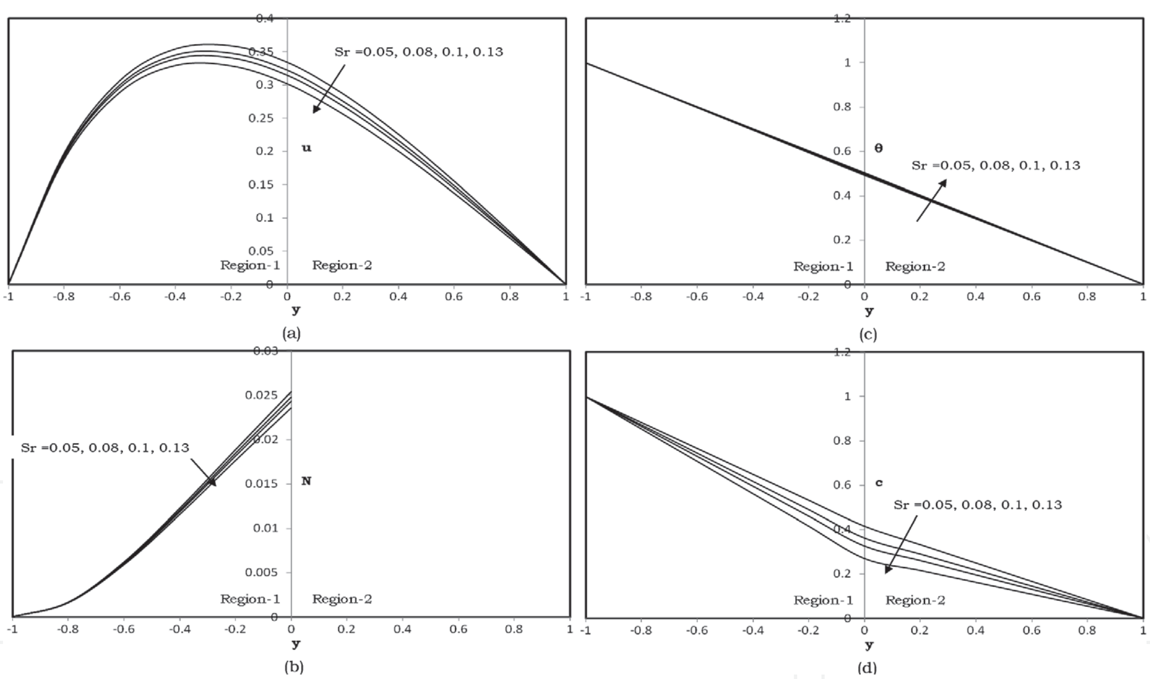

Figure 8.
(a) Represents behavior of u. (b) Represents behavior of N for Sr. (c) Represents behavior of θ. (d) Represents behavior of c for Sr.

Gr = 2. Higher Gr values boost up the flow in both regions. As Gr increases the minute enhancement of temperature and diffusion are observed. Similar observations are noticed with all the variations of Gc which are plotted in **Figure 3(a)–(d)**.

Figure 4(a)–(d) describe the Reynolds number (R) impact on velocity, angular velocity, temperature and diffusion. The reduction of velocity is found with increase of Reynolds number due to domination of inertial force on viscous force in both regions of the channel and found more drastic in viscous region. Also reduces the micro rotation with increase of Reynolds number. The effect of inertial forces enhances the temperature and reduction of the diffusion is shown with increase of R.

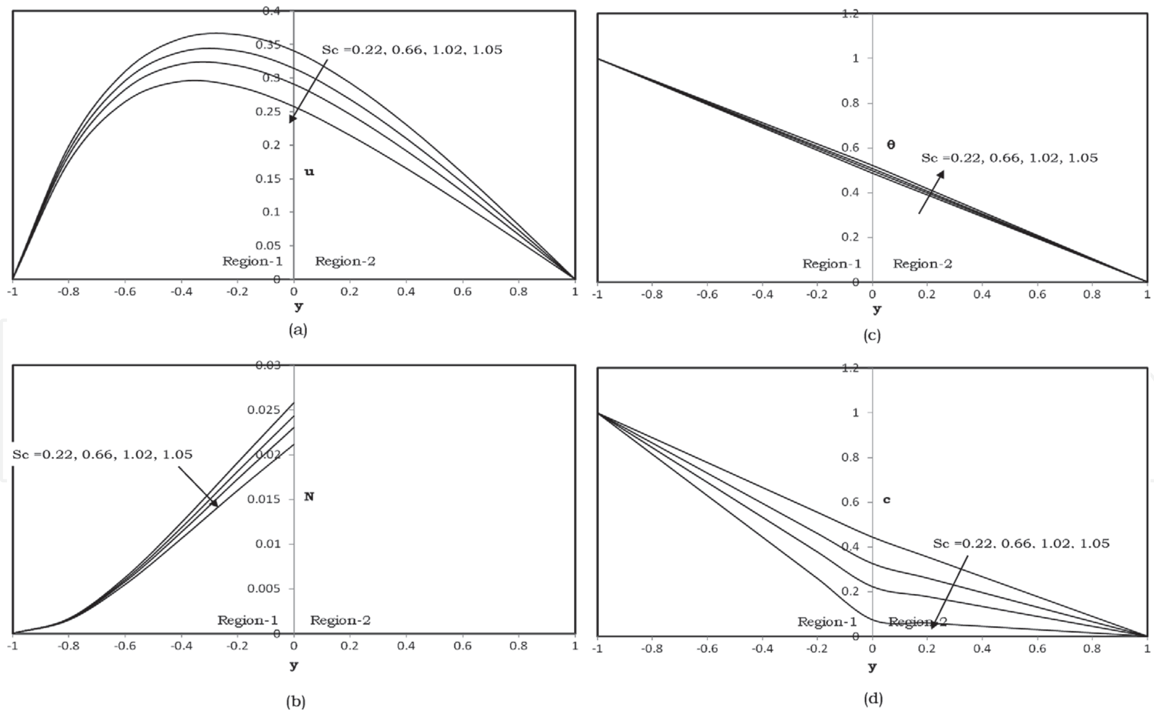

Figure 9.
(a) Represents behavior of u. (b) Represents behavior of N for Sc. (c) Represents behavior of θ. (d) Represents behavior of c for Sc.

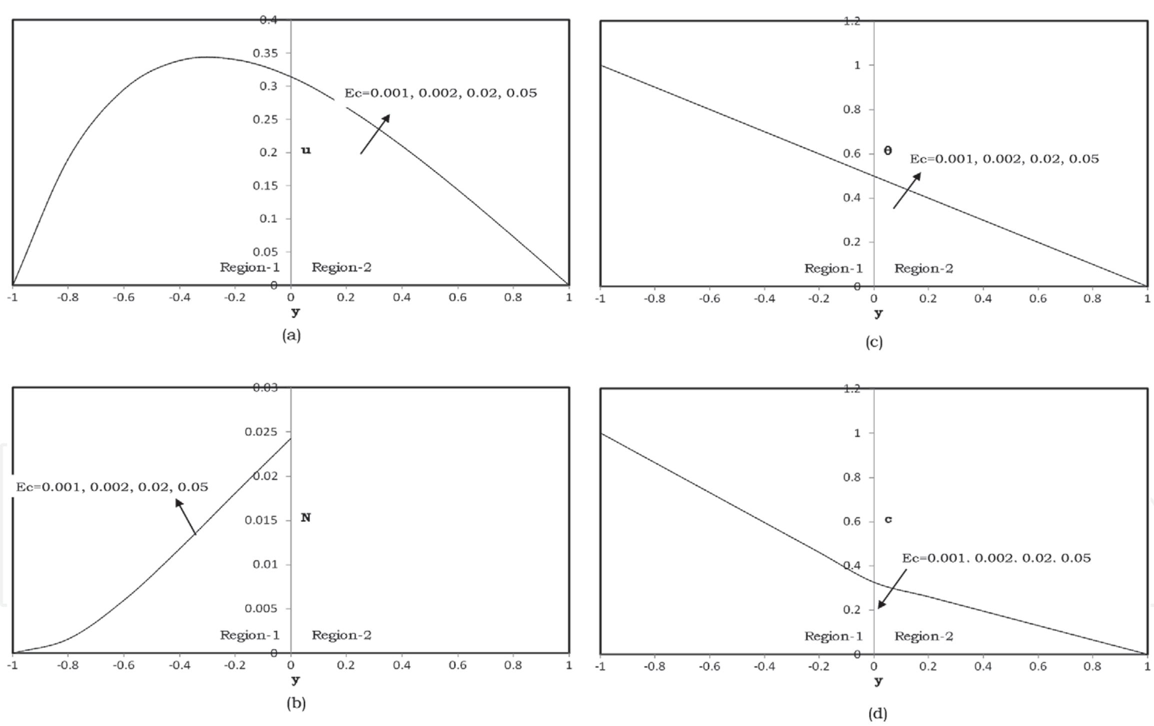

Figure 10.
(a) Represents behavior of u. (b) Represents behavior of N for Ec. (c) Represents behavior of θ. (d) Represents behavior of c for Ec.

Figure 5(a)–(d) describe the magnetic field (M) effect on velocity, angular velocity, temperature and diffusion. They portray that there could be seen reduction in velocity and angular velocity as M increases. It shows that magnetic field has a tendency to retard fluid velocity and angular velocity due to the formation of resistive Lorentz force, where when magnetic effect is applied to the fluid, it tends to retard the fluid motion. The magnetic field parametric impact on temperature and diffusion is minute.

Figure 6(a)–(d) explain the effect of material parameter (K′) on velocity, angular velocity, temperature and diffusion. The effect of this parameter is very significant in both velocity and angular velocity, as K′ increases the velocity decreases significantly and this is reversed with respect to angular velocity. Minute effect is observed for both temperature and diffusion.

Figure 7(a)–(d) explain the effect of Dufour number (Du) on velocity, angular velocity, temperature and diffusion. **Figure 7(a)** depict the fact that as Du increases, i.e. when molecular diffusivity increases, the velocity is reduced and leads to reduction of micro rotation also from **Figure 7(b)** it clearly indicates the influence of the concentration gradients to the thermal energy flux in the flow. **Figure 7(c)** specifies that the temperature reduces with increase of molecular diffusivity over the thermal diffusivity. It is clear that the diffusion profiles increase with increase of dufour number as observed in **Figure 7(d).**

Figure 8(a)–(d) relates the Soret number (Sr) impact on velocity, angular velocity, temperature and diffusion. **Figure 8(a)** shows that as Sr. increases i.e. thermal diffusivity increases the decrease in velocity is found and micro rotation also decreases with increase of Sr. from **Figure 8(b)**. Soret number states the impact of temperature gradients stimulating considerable mass diffusion effects. Here, as Soret number increases it leads to rise in temperature and shows the decay in the fluid concentration from **Figure 8(c)** and **(d)**.

Figure (a)–(d) specify the Schmidt number (Sc) effect on velocity, angular velocity, temperature and diffusion. **Figure 9(a)** and **(b)** show that as Sc increases, velocity and angular velocity decreases significantly. From **Figure 9(c)** and **(d)** it is found that the temperature increases with increase of Sc and fluid concentration reduces with increase in Schmidt number.

Figure 10(a)–(d) define the effect of Eckert number (Ec) on velocity, angular velocity, temperature and diffusion. From all the Figures it is concluded that the enthalpy is not having much influence over the flow for small variation of enthalpy. **Figure 10(a)** and **(b)** show that the velocity and angular velocity increase when Eckert number increases. So it is observed that momentum and angular momentum are inversely proportional to enthalpy. From **Figure 10(c)** the temperature increases with increase of kinetic energy. The kinetic energy reduces the concentration of the fluid as shown from **Figure 10(d)**.

Table 1 shows the Shear stress and Nusselt and Sherwood numbers values with all the effects of all governing functions. From this table, it is observed that the absolute Shear stress enhances with increase in Gr on both the boundaries $y = -1$ and $y = 1$ because of buoyancy forces and similar nature is observed for Gc also. For increase of Reynolds number, magnetic field and Material parameter and Dufour, Soret and Schmidt numbers, the stress reduces on both the boundaries. This case is reversed for dissipation effect. The Nusselt number i.e. rate of heat transfer decreases on the boundary at $y = -1$ and increases on the other boundary $y = 1$ for the parameters Gr, Gc, R, Sr., Sc and Ec. The rise in convection is leading to reduction of heat transfer rate on the plate bounding the region -1, the reverse effect is observed for boundary of the region-2. Drastic heat transfer rate is observed for the variations of the Reynolds Number. The increase in the Reynolds number decreases the heat transfer rate on the left plate and enhances on the right plate. For the other parameters M, K′, Du the effect is reversal. The Sherwood number i.e. rate of mass transfer increase on the boundary at $y = -1$ and decrease at the boundary $y = 1$ for the parameters Gr, Gc, R, Sr., Sc, Ec. This is because the rise in convection and inertial forces leading to enhance the concentration. For the other parameters M, K′, Du the effect is reversal i.e. mass transfer increases at the left boundary and decreases at the right.

Gr	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
2	*−0.830728*	*0.238773*	*0.501802*	*0.498151*	*0.674487*	*0.325508*
5	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
8	*−1.59544*	*0.497188*	*0.501571*	*0.49826*	*0.674547*	*0.325425*
10	*−1.85037*	*0.583344*	*0.50146*	*0.498312*	*0.674576*	*0.325385*
Gc	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
2	*−0.867571*	*0.276385*	*0.50179*	*0.498161*	*0.674491*	*0.3255*
5	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
8	*−1.55858*	*0.459553*	*0.501593*	*0.498243*	*0.674541*	*0.325438*
10	*−1.78891*	*0.520603*	*0.501504*	*0.498279*	*0.674564*	*0.32541*
R	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
1	*−3.75411*	*1.2211*	*0.51129*	*0.487749*	*0.556265*	*0.443686*
2	*−1.84876*	*0.581703*	*0.507141*	*0.492627*	*0.614305*	*0.385672*
3	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
5	*−0.703233*	*0.195659*	*0.489827*	*0.51014*	*0.80178*	*0.198213*
M	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
1	*−1.56381*	*0.632044*	*0.50152*	*0.498322*	*0.674563*	*0.325375*
2	*−1.35498*	*0.468803*	*0.50164*	*0.498238*	*0.67453*	*0.325441*
3	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
5	*−1.02772*	*0.252261*	*0.501772*	*0.498159*	*0.674494*	*0.325503*
K′	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
0.05	*−1.25054*	*0.369786*	*0.501697*	*0.498198*	*0.674514*	*0.325472*
0.1	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
0.2	*−1.1461*	*0.364131*	*0.501719*	*0.498193*	*0.674509*	*0.325476*
0.3	*−1.08773*	*0.360146*	*0.501732*	*0.49819*	*0.674506*	*0.325478*
Du	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
0.08	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
0.1	*−1.213*	*0.36789*	*0.502207*	*0.497695*	*0.674236*	*0.325748*
0.3	*−1.21205*	*0.366921*	*0.50812*	*0.491804*	*0.670989*	*0.328973*
0.5	*−1.21073*	*0.365583*	*0.516295*	*0.483705*	*0.666499*	*0.333392*
Sr	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
0.05	*−1.24193*	*0.397433*	*0.510133*	*0.489762*	*0.584941*	*0.415051*
0.08	*−1.22481*	*0.379948*	*0.505131*	*0.494768*	*0.638104*	*0.361884*
0.1	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
0.13	*−1.19499*	*0.349512*	*0.496423*	*0.503481*	*0.730638*	*0.269345*
Sc	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***
0.22	*−1.25121*	*0.406909*	*0.512844*	*0.487049*	*0.556131*	*0.443864*
0.66	*−1.21308*	*0.367972*	*0.501705*	*0.498196*	*0.674512*	*0.325473*
1.02	*−1.17975*	*0.333945*	*0.491969*	*0.507938*	*0.777967*	*0.222014*
1.5	*−1.13199*	*0.285191*	*0.478018*	*0.521897*	*0.926194*	*0.073786*
Ec	***St-I***	***St-II***	***Nu-I***	***Nu-II***	***Sh-I***	***Sh-II***

Gr	St-I	St-II	Nu-I	Nu-II	Sh-I	Sh-II
0.001	−1.21308	0.367972	0.501705	0.498196	0.674512	0.325473
0.002	−1.21309	0.367979	0.501527	0.498276	0.674559	0.325413
0.02	−1.21331	0.368093	0.49832	0.499703	0.675391	0.324325
0.05	−1.21367	0.368285	0.492968	0.502086	0.676779	0.322509

Table 1.
Shear stress, Nusselt number, Sherwood numbers.

6. Conclusions

- Significant effect of Soret number on momentum, diffusion are observed
- Diffusion effects are reducing the momentum and are more pronounced in concentration.
- For enhancement of inertial force the momentum and diffusion reduces to a higher extent in case of micropolar region than viscous region.
- The diffusion parameters are reducing the magnitude of the shear stress on both the boundaries.
- For dufour heat transfer rate effect is enhancing on the hot plate and reducing on the cold plate but reverse effect is observed for rate on mass transfer. Exactly opposite is observed for Soret number.
- Reduction of heat transference rate on hot plate and enhancement on the cold plate is observed due to viscous dissipation.

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