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Fuzzy Parameters and Their Arithmetic Operations in Supply Chain Systems

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1. Introduction

We ask the question: what is the purpose of this chapter in the whole book? This chapter is a supplement to fuzzy supply chains. The whole book could itself be divided into two parts according to the assumption whether the supply chain is a deterministic or non-deterministic system. For non-deterministic supply chains, the uncertainty is the main topic to be considered and treated. From the history of mathematics and its applications, the considered uncertainty is the randomness treated by the probability theory. There are many important and successful contributions that consider the randomness in supply chain system analysis by probability theory (Beamon, 1998; Graves & Willems, 2000; Petrovic et al., 1999; Silver & Peterson, 1985). In 1965, L.A.Zadeh recognized another kind of uncertainty: Fuzziness (Zadeh, 1965). There are several works engaged on the research of fuzzy supply chains (Fortemps, 1997; Giachetti & Young, 1997; Giannoccaro et al., 2003; Petrovic et al., 1999; Wang & Shu, 2005). While this chapter is a supplement of fuzzy supply chains, the author is of the opinion that the parameters occurring in a fuzzy supply chain should be treated as fuzzy numbers. How to estimate the fuzzy parameters and how to define the arithmetic operations on the fuzzy parameters are the key points for fuzzy supply chain analysis. Existing arithmetic operations implemented in supply chain area are not satisfactory in some situations. For example, the uncertainty degree will extend rapidly when the product \times interval operation is applied. This rapid extension is not acceptable in many applications. To overcome this problem, the author of this chapter presented another set of arithmetic operations on fuzzy numbers (Alex, 2007). Since the new arithmetic operations on fuzzy numbers are different from the existing operations, the fuzzy supply chain analysis based on the new set of arithmetic operations is different from the fuzzy supply chain analysis introduced earlier. That is why the author has presented his modeling of fuzzy supply chains based on the earlier work here as a supplement to works on the fuzzy supply chains.

In Section 2, as a preliminary section, the structure and basic concepts of supply chains are described mathematically. The simple supply chains which are widely used in applications are defined clearly. Even though there have been a lot descriptions on supply chains, the author thinks that the pure mathematical description on the structure of supply chains here

Source: Supply Chain, Theory and Applications, Book edited by: Vedran Kordic, ISBN 978-3-902613-22-6, pp. 558, February 2008, I-Tech Education and Publishing, Vienna, Austria

is a special one and specifically needed in this and subsequent sections. In Section 3, the estimation of fuzzy parameters and the arithmetic operations on fuzzy parameters are introduced. In Section 4, based on the fuzzy parameter estimations and arithmetic operations, the fuzzy supply chain analysis will be built. The core of supply chain analysis is the determination of the order-up-to levels in all sites. By means of the possibility theory (Zadeh, 1978), a couple of real thresholds the optimistic and the pessimistic order-up-to levels is generated from the fuzzy order-up-to the level of site with respect to a certain fill rate r . There are no mathematical formulae to calculate the order-up-to levels for all sites in general supply chains, but this is an exception whenever a simple supply chain is stationary. In Section 5, the stationary simple supply chain and the stationary strategy are introduced and the optimistic and pessimistic order-up-to the levels at all sites of a stationary simple supply chain are calculated. An example of a stationary simple supply chain is given in Section 6. Conclusions are given in Section 7.

2. The basic descriptions of supply chains

A supply chain consists of many sites (also know as stages) and each site (stage) C_i provides/produces a certain kind of part/product p_j at a certain unit/factory. For simplicity, assume that different units provide different kinds of parts/products. Let $C = \{c_1, c_2, \dots, c_n\}$ be the set of all sites in a supply chain, and C^* be an extension of such that it includes the set of external suppliers denoted by Y and the set of end-customer centers denoted by Z :

$$C^* = Y \cup C \cup Z \quad (2.1)$$

We will simply treat an external supplier or an end-customer center also as a site. There is a relationship among the sites of C^* : If a site c_i uses materials/parts/products from a site c_j , then we say the site c_j supplies the site c_i and is denoted as $c_j \rightarrow c_i$. The site c_j is called an *up-site* of c_i , and c_i is called a *down-site* of c_i . The suppliers in Y have no up-sites and the customers in Z have no down-sites in C^* . The relation of supplying can be described in mathematics as a subset $S \subseteq C^* \times C^*$:

$$(c_j, c_i) \in S \text{ if and only if } c_j \rightarrow c_i. \quad (2.2)$$

If we do not consider the case of a site supplying itself, then the supplying relation S is anti-reflexive, i.e., for any $c_j \in C^*$, $c_j \rightarrow c_j$ is not possible. If we do not consider the case of two sites supplying each other, then S is anti-symmetric, i.e., for any $c_i, c_j \in C^*$, if $c_i \rightarrow c_j$, then $c_j \rightarrow c_i$ is not possible.

Definition 2.1 A *Supply chain* (C^*, S) is a set of sites C^* equipped with a supplying relation S , which is an anti-reflexive and anti-symmetric relation on C^* .

An anti-reflexive and anti-symmetric relation S ensures that there is no cycle occurring in the graph of a supply chain.

Set $S^1 = S$. For any $n > 1$, set

$$S^n = \{(c_k, c_i) \mid \exists c_j \in C^* \text{ such that } (c_k, c_j) \in S^{n-1}, (c_j, c_i) \in S\} \quad (2.3)$$

It is obvious that S^n will become an empty set when n is large enough. Let h be a number large enough such that S^h is empty. Set

$$S^* = S^1 \cup S^2 \cup \dots \cup S^h. \quad (2.4)$$

S^* denotes the enclosure of the supplying relation on S . S^* is the relation of "supplying directly or indirectly." It is obvious that S^* is still an anti-reflexive and anti-symmetric relation. It is also obvious that S^* is a transitive relation. i.e., if $(c_k, c_j) \in S^*$ and $(c_j, c_i) \in S^*$, then $(c_k, c_i) \in S^*$.

For any site $c_j \in C$, let D_j and U_j be the set of down-sites and up-sites of c_j , respectively. Suppose that $D_j^1 = D_j$. For any $n > 1$, set

$$D_j^n = \{c_i \mid \exists c_{i'} \in D_j^{n-1} \text{ such that } c_{i'} \rightarrow c_i\} \quad (2.5)$$

$$U_j^n = \{c_i \mid \exists c_{i'} \in U_j^{n-1} \text{ such that } c_i \rightarrow c_{i'}\} \quad (2.6)$$

The sites belonging to D_j^n and U_j^n are called the n -generation down-sites and up-sites of c_j , respectively. Clearly, any down-site is the 1-generation down-site, and any up-site is the 1-generation up-site. It is obvious that D_j^n or U_j^n may become an empty set when n is large enough. Set

$$D_j^* = \cup \{D_j^k \mid k = 1, 2, \dots, h\} \quad (2.7)$$

$$U_j^* = \cup \{U_j^k \mid k = 1, 2, \dots, h\}. \quad (2.8)$$

These are the enclosures of D_j and U_j , and are called the *down-stream* and *up-stream* of c_j , respectively.

Proposition 2.1 For any $c_j \in C$, the downstream D_j and the upstream U_j of c_j are disjoint.

Proof Assume D_j and U_j are joint, then there is at least a site called c_i belonging to both D_j and U_j simultaneously. This leads to $c_i \leftrightarrow c_j$, which is contradicted with the

requirement of the anti-symmetric of S^* . Thus, the assumption is not true, and it proves that D_j and U_j are disjoint.

Proposition 2.1 just ensures that the upstream and the downstream of a site are disjoint. Unfortunately, two different generations of up-sites (or down-sites) may be intersected:

For example, let c_1 be a site supplying sugar, c_2 be a site supplying the cake mix for cakes, and c_3 be the site supplying the birthday-cakes. We have that $c_1 \rightarrow c_2$, $c_2 \rightarrow c_3$, and $c_1 \rightarrow c_3$. Since c_1 is the up-site of c_2 and c_2 is the up-site of c_3 , so that c_1 is the 2-generation up-site of c_3 . But c_1 is also the first generation up-site of c_3 . So that $U_3^1 \cap U_3^2 \neq \emptyset$. Such situations may bring complexity to the research.

Definition 2.2 A supply chain (C^*, S) is called a *simple supply chain* if for any site c_j in C ,

$$n \neq n' \Rightarrow (D^n \cap D^{n'} = \emptyset \text{ and } U^n \cap U^{n'} = \emptyset) \quad (2.9)$$

For a simple supply chain (C^*, S) , any site can be in at most one generation of upstream and at most one generation of downstream of another site.

Set

$$B = \{c \in C \mid \exists c^* \in Y \text{ such that } c^* \rightarrow c\}, \text{ or} \quad (2.10)$$

$$O = \{c \in C \mid \exists c^* \in Z \text{ such that } c \rightarrow c^*\}. \quad (2.11)$$

We call a site belonging to B the *boundary site* and a site belonging to O the *root site* of C . For a boundary site $c_b \in B$, U_b should contain at least an external supplier:

$U_b \cap Y \neq \emptyset$. If U_b does only contain external suppliers, i.e., $U_b \subseteq Y$, then c_b is called a *proper boundary site*. For a root site $c_0 \in O$, D_0 should contain at least a customer: $D_0 \cap Z \neq \emptyset$. If D_0 does only contain customers, i.e., $D_0 \subseteq Z$, then c_0 is called a *proper root site*.

We can specify some of the most important cases of simple supply chains as follows:

Case 1. Linear supply chains: A linear supply chain is a simple supply chain (C^*, S) , C^* contains one supplier-site and one root site c_0 , and each site in C has one 1-generation down-site and one 1-generation up-site.

It is obvious that the construction of a linear chain can be drawn as follows:

$$\text{supplier} \rightarrow c_b \rightarrow c_{h-2} \rightarrow \cdots \rightarrow c_1 \rightarrow c_0 \rightarrow \text{customer} \quad (2.12)$$

Case 2. Anti-tree supply chains: An anti-tree supply chain is a simple supply chain (C^*, S) , C^* contains at least two supplier sites and only one root site c_0 , each site in C has one 1-generation down-site but any number of 1-generation up-sites, and all sites are in

the upstream of the only one root site c_0 . An anti-tree chain represents a centralized supply chain.

It is obvious that all sites in C can be divided as different up-generations of c_0 . If $c_j \in U_0^n$, we say that the (generation) code of c_j for c_0 is n , and denoted as $\chi_j = \chi_{j0} = n$. Since the supply chain is simple so that for any site c_j in C with code n , there is one and only one linear chain connecting the site c_j and c_0 given by:

$$c_j \rightarrow c_{(n-1)} \rightarrow \cdots \rightarrow c_{(1)} \rightarrow c_0 \quad (2.13)$$

Case 3. Multiple anti-trees supply chains: A multiple anti-trees supply chain is a simple supply chain (C^*, S) , $C^* = C_1^* \cup C_2^* \cup \cdots \cup C_m^*$, and for $1 \leq k \leq m$, (C_k^*, S_k) are anti-tree supply chains, where $S_k = S \cap (C_k^* \times C_k^*)$, the constraint of S on C_k^* . Each root site $c_{0(k)}$ is a proper root site. A multiple anti-trees chain represents a decentralized supply chain.

Omitting the proof, we can say that a multiple anti-trees supply chain is a combination of several anti-tree supply chains. It is obvious that there are several supplier-sites and many proper root sites. Each site in C has no limit on the number of 1-generation down-sites and 1-generation up-sites, but each site should be in the upstream of at least one proper root site. It is obvious each site c_j in C has a code χ_{j0} for a root-site c_0 if $c_j \rightarrow c_0$, and has one and only one linear chain connecting c_j and c_0 . Case 2 is a generalization of case 1, and the case 3 is a generalization of case 2. In the rest of the chapter, we will limit our attention to case 2 of a simple supply chain.

For each site c_j in C , let $q_{ji}(t)$ be the order quantity of p_j -part/material from the down-site c_i , which is called the *order-away quantity* of c_j at time t . While $q_{kj}(t)$, the p_k -part/material quantity in up-site c_k ordered by c_j , is called the *order-in quantity* of c_j at time t .

The following *review period policy* is assumed here: For any site c_j in C , the time of ordering in the up-parts could not be arbitrary, but limited at $t_j, t_j + T_j, t_j + 2T_j, \cdots$. These timings are called the *review times*, and $T_j > 0$ is called the *review period* of c_j . To be simple, assume that $t_j = 0$ for any c_j in C .

For any site $c_j \in C$, suppose that $c_j \rightarrow c_i$. Set

$$\alpha_j(nT_j) = \sum \{q_{ji}(t) \mid \exists i \in C; (n-1)T_j \leq t < nT_j\}, \quad (2.14)$$

is the number of p_j -parts that has been ordered to be sent out to the down-site of c_j during the last period $(n-1)T_j \leq t < nT_j$ and is called the *passed away number* of p_j 's in the last period. Set

$$\bar{\alpha}_j(nT_j) = \left(\frac{1}{T_j}\right)(\alpha_j(nT_j)). \quad (2.15)$$

This is called the *order-away rate* of p_j at the time t . For a root-site c_0 , the passed-away number of p_0 -products is called the *demand number* at time t denoted as $d(nT_0) = \alpha_0(nT_0)$. Set

$$\bar{d}(t) = (1/T_0)(d(t)). \quad (2.16)$$

This is called the *demand rate* of p_0 at the time t .

Suppose that each p_i -product/part is produced by means of w_{ji} pieces of p_j -parts, we call w_{ji} the *equivalence* of a p_i -part for the p_j -part. For any site pair $(c_j, c_i) \in S$, there is an equivalence value w_{ji} , which reflects the production ingredient of down-products by means of the up-parts.

In case 2, for any site c_j in C with code $\chi_j = n$, there is one and only one linear chain connecting it to its root site c_0 as: $c_j \rightarrow c_{(n-1)} \rightarrow \cdots \rightarrow c_{(1)} \rightarrow c_0$. Set

$$w_j = w_{(j)(n-1)} w_{(n-1)(n-2)} \cdots w_{(1)(0)}. \quad (2.17)$$

This is called the *equivalence* of a product for the p_j -part. The production of each final product p_0 needs w_j pieces of p_j -parts to supply it.

The main problem in supply chain analysis is: How to set up the reasonable inventory levels in all sites of C ? Let $I_j = I_j(t)$ be the real inventory of p_j -parts of site c_j at time $t = nT_j$. This should be a negative number whenever it is in shortage at the time. We

do not want a site to be in the shortage, so we want that $I_j > 0$; While its value should not be too high since then there will be a high inventory maintenance cost; The goal of supply chain management is to minimize the supply chain inventory cost and to limit the possibility of shortage as much as possible.

The expected inventory level of the site c_j at the time $t = nT_j$ should be responsible not only for supplying the down-site of c_j during the next period $[nT_j, (n+1)T_j]$, but also

for a longer time until the birth of the next batch of p_j -parts produced from up-parts ordered in c_j at the next review time $t' = (n+1)T_j$. The length from $t = nT_j$ to the mentioned time can be denoted as

$$T_j^* = T_j + L_j. \quad (2.18)$$

This is called the *looking time* of c_j ; while L_j is called the *replenishment time* of c_j . The concrete expression of L_j is

$$L_j = M_j + G_j + P_j, \quad (2.19)$$

where

$$M_j = \max\{M_{kj} \mid k \in U_j\};$$

$$G_j = \max\{G_{kj} \mid c_k \in U_j\};$$

$$P_j = (\alpha_j(nT_j) \times T_j \times \tau_j \times (1 + \varphi_j \times \vartheta_j)) / C_j. \quad (2.20)$$

M_{kj} is the time of transferring the ordered p_k -parts from the site c_k to the site c_j at a review time $t = nT_j$, called the *material lead time* from c_k to c_j ; G_{kj} is the time of delaying of the transferring of the ordered p_k -parts owing to the shortage of p_k -parts, called the *delay time* of p_k -parts for c_j ; P_j is the time of transferring the p_k -parts into p_j -parts at the site c_j , called the *production time* of c_j , with the following parameters: τ_j the cycle time for p_j ; φ_j the estimated number of occurrences of downtime; ϑ_j , the duration of downtime on the production line for c_j ; C_j the production capacity, the working hours per day, allocated for c_j . Set

$$S_j = \alpha_j(nT_j) \times (T_j + L_j), \quad (2.21)$$

which stands for the reasonable inventory level of site c_j at time $t = nT_j$. S_j is called the order-up-to level of site c_j at time $t = nT_j$.

$$S_{kj}^* = w_{kj} \times (S_j - I_j), \quad (2.22)$$

which is the real order of p_k -parts from site c_j at time $t = nT_j$.

The main task in supply chain analysis is the determination of the order-up-to levels $\{S_j\}_{(j=1,\dots,n)}$ in all sites of the chain at a time t .

3. Fuzzy parameters and their estimation and arithmetic operations

Since this chapter is a supplement of fuzzy supply chain analysis, we avoid repeating the statements on what is fuzziness, what is the different between fuzziness and randomness, and so on. But it should be emphasized here again that fuzzy theory is good at imitating the subjective experience of human beings. When we face an unknown parameter with fuzziness in a supply chain, the natural way is representing it by a fuzzy number. There are two key points: First, how to estimate the parameters? i.e., how to get a fuzzy number to represent the estimation by experts for a parameter? Second, how to make reasonable arithmetic operations on the fuzzy parameters?

3.1 How to estimate a fuzzy parameter?

The fuzzy estimation reflects the subjective measurement about a real number by an expert (or a group of experts) who has knowledge and experience with respect to the estimated parameter. The process of subjective estimation has no general rules as guide; every case has its own approach. An expert pointing out the location of an expected number depends on his inference, which is based on the experience of grasping the main essential factors in the practical situation. Under some factor-configuration, the expert will make a choice. But when the factor-configuration has been changed, the expert will have another choice. To acquire an expert’s estimation into a fuzzy number, we could learn from psychological statistics. There are many methods that could be adopted. To be simple, the author shortens some of the methods and suggests by asking an expert the following questions:

Question 1: What is the real number in your mind, which is the most acceptable for you to represent a fuzzy parameter α ?

Let a real number a be the answer, then we say that the fuzzy parameter α has the estimation value a , denoted as $a = m(\alpha)$.

Question 2: What is the confidence on your estimation for α ? Please place the mark \times on a proper location in the real number line that represents the confidence interval $[0, 1]$. The expert points out a mark \times at the proper position in the interval $[0, 1]$ to represent the degree of his confidence on the estimation of the number in question 1. For example, according to the location of the mark shown in the Fig. 1, we can get a real number $\varphi = 0.75$, which is called as the *confidence degree* of the expert on his estimation.

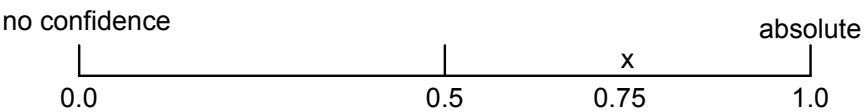


Figure 1. The confidence on the parameter estimation

If the confidence degree equals 1, then the expert must make sure that the estimation value a is true absolutely and there is no error in the estimation. If the confidence equals to 0, then the expert knows nothing about this estimation.

Suppose that there is a group of experts that make estimations of fuzzy parameters within a supply chain system. Each expert has a score $\rho \in [0,1]$ to represent his skill degree on subjective estimation. The closer the score value is to 1 the higher the authority. The score can be measured and adjusted by the success rate in practical situations. ρ is called the *authority index* of the expert. The product of the authority index ρ of an expert and the confidence degree φ of his estimation on a fuzzy parameter represents the subjective accuracy of this estimation, denoted as $\tau = \rho \times \varphi$. We call $\delta = 1 - \tau$ the *ambiguity degree* of the estimation. A fuzzy parameter α can be represented by a pair of two real numbers, its estimation value a and its ambiguity degree δ :

$$\alpha = a(1 \pm \delta), \quad (0 \leq \delta \leq 1). \quad (3.1)$$

The ambiguity degree of the parameter α could also be called the *estimation error* of the estimation in α , and denoted as $\delta = e(\alpha)$. The formula (3.1) looks like the representation of error in measurement theory. Yes, they are very similar. The only difference is: The error in measurement is caused by the impreciseness of instruments and observation; while the ambiguity is caused by the fuzziness in subjective estimation. In the error theory, there are two kinds of errors: absolute error and relative error. The ambiguity reflects the error in subjective estimation and it is not an absolute error, but a relative error. The relative error plays a more essential role. For examples, when we estimate that the height of the wall as 2 ± 0.2 units, the estimation value is $a = 2$ units and the absolute error is $a \times \delta = 0.2$; when we estimate that the length of the street is 2000 ± 200 units, the estimation value is $a = 2000$ units and the absolute error is $a \times \delta = 200$; when we estimate that the length of an insect is 0.002 ± 0.0002 units, the estimation value is $a = 0.002$ and the absolute error is $a \times \delta = 0.0002$. There are differences in the three examples, but the relative error is the same $\delta = 0.1$. The estimation errors are invariable on the changing of unit. It reflects the intrinsic quality of subjective estimation.

We represent the membership function of a fuzzy parameter estimation by a triangle fuzzy number taking its peak at the estimation value a and its radius as $r = |a| \times \delta$:

$$\mu_{\alpha}(x) = \begin{cases} 0 & \text{if } -\infty < x \leq a - r \\ 1 + \frac{x - a}{r} & \text{if } a - r < x \leq a \\ 1 - \frac{x - a}{r} & \text{if } a < x \leq a + r \\ 0 & \text{if } a + r < x < \infty \end{cases} \quad (3.2)$$

Since $0 \leq \delta \leq 1$, a fuzzy parameter is a special triangle fuzzy number whose radius is $r \leq |a|$.

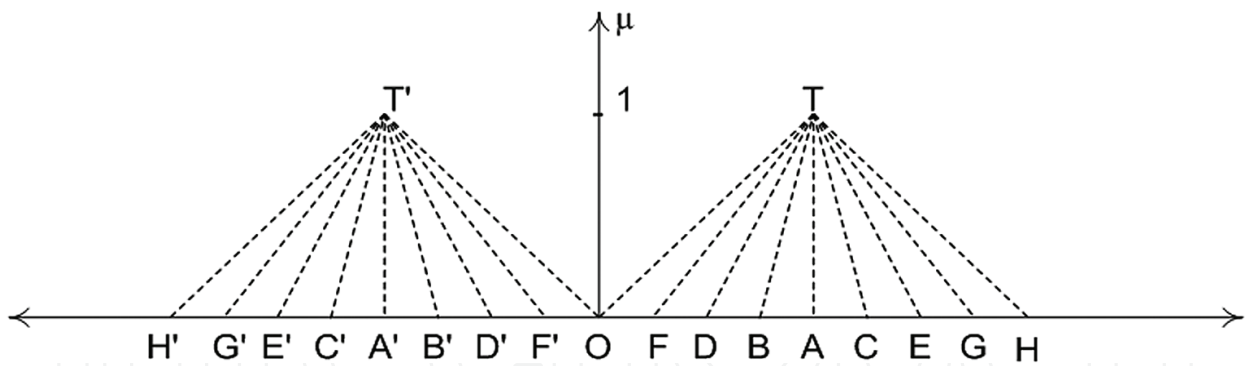


Figure 2. Example of 10 fuzzy parameters

In the Fig. 2, we can see a set of fuzzy parameters with estimation value $a = 100$ have membership functions shown as the broken lines ATA, BTC, DTE, FTG , and OTH with ambiguity $\delta = 0, 0.25, 0.5, 0.75$, and 1 , respectively; those fuzzy parameters with estimation value $a' = -100$ have membership functions shown as the broken lines $A'T'A', B'T'C', D'T'E', F'T'G'$, and $O'T'H'$ with ambiguity $\delta = 0, 0.25, 0.5, 0.75$, and 1 , respectively.

Definition 3.1 Given a positive real number $0 \leq \delta^* \leq 1$, we call V , the set of fuzzy parameters

$\alpha = a \pm r$ with $\frac{r}{|a|} \leq \delta^*$, the δ^* -systems of fuzzy parameters.

For example, suppose that V is a 0.05 -system of fuzzy parameters. The fuzzy parameter

$2 \pm 1 \notin V$ since $\frac{r}{|a|} = 0.5 > 0.05$. The fuzzy parameter $1 \pm 0.05 \in V$ since

$\frac{r}{|a|} = 0.05$.

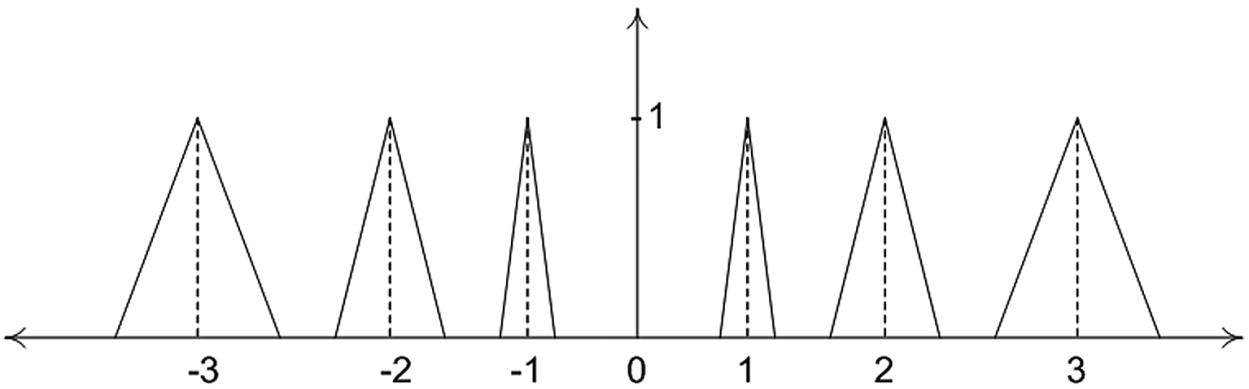


Figure 3. The δ^* -system of fuzzy parameters

In the Fig. 3, the radius of the fuzzy parameter $-3 \pm 3\delta^*$ is $3\delta^*$, the radius of the fuzzy parameter $-2 \pm 2\delta^*$ is $2\delta^*$, and the radius of the fuzzy parameter $-1 \pm \delta^*$ is δ^* . The radius of fuzzy parameter $1 \pm \delta^*$ is δ^* ; the radius of fuzzy parameter $2 \pm 2\delta^*$ is $2\delta^*$; and the radius of fuzzy parameter $3 \pm 3\delta^*$ is $3\delta^*$. As we see from figure 3, the

estimation values closer to zero, the narrower the membership function width; the estimation value farther away from zero, the wider the membership function width. However, the ambiguities of the fuzzy parameters in a δ^* -system are all restricted by δ^* . A δ^* -system includes not only those fuzzy parameters whose ambiguities are equal to δ^* , but all fuzzy parameters whose ambiguities are less than δ^* . The δ^* -systems are not disjoint but expanded when the parameter δ^* is increasing: δ_1^* -system $V_1 \subseteq \delta_2^*$ -system V_2 ($\delta_1 \leq \delta_2$).

Proposition 3.1 Suppose that V is a δ^* -system of fuzzy parameters, where $0 \leq \delta^* \leq 1$. For any non-zero fuzzy parameter $\alpha = a \pm r \in V$, the support of α does not contain zero as an inner point. i.e., $0 \notin (a - r, a + r)$.

Proof Assume that $0 \in (a - r, a + r)$. If $a > 0$, then $1 - \frac{r}{a} < 0 < 1 + \frac{r}{a}$. Then

$1 - \delta^* \leq 1 - \frac{r}{a} < 0$, i.e., $\delta^* > 1$. This is a contradiction to the requirement of $\delta^* \leq 1$.

Suppose that $a < 0$, then $1 - \frac{r}{a} > 0 > 1 + \frac{r}{a}$. Since $\delta^* \geq \left| \frac{r}{a} \right| = -\frac{r}{a}$,

$0 > 1 + \frac{r}{a} \geq 1 - \delta^*$, i.e., $\delta^* > 1$. This is a contradiction with the requirement of $\delta^* \leq 1$.

According to the reduction to absurdity, the assumption is not true. So $0 \notin (a - r, a + r)$.

Using Proposition 3.1, we can say that a fuzzy parameter α is positive if the estimation value of α is positive, and α is negative if the estimation value of α is negative.

Proposition 3.1 constrains the fuzzy parameters in our δ -system in pure sign, i.e., the support of any fuzzy parameter does not contain zero. This is not a real constraint in practical but reflects such a faith in the thinking process: Human beings like to do fuzzy estimation on "how much" but not fuzzy on the main direction to do it. For example, suppose we are telling somebody: "To go to the post office, turn left and go about 150 meters". It may be acceptable if the distance is not estimated precisely; the distance is not exactly 150 meters, instead it is 164 meters. But it is not acceptable if the direction to turn left is wrong. A δ -system is free in use if we put the zero point in such a place from where the directions toward West and East are distinguished.

It is worth noting that the ambiguity δ of a fuzzy parameter α could be larger than zero whenever its estimation value $a = 0$. In this case, $\alpha = a \pm |0| \times \delta = a \pm 0$. Indeed, for a fuzzy parameter with estimation value zero, it can have arbitrary ambiguity δ .

However, we can make an assumption that for a fuzzy parameter with zero estimation value, we rewrite its ambiguity as zero no matter how large its ambiguity is.

The fuzzy parameters we defined here indeed are triangle fuzzy numbers with a little constraint. The reason for making a different name for them is not to emphasize the constraint, but to emphasize the different definitions of arithmetic operations on them.

3.2 Arithmetic operations of fuzzy parameters

The existing arithmetic operations of fuzzy numbers are based on the extension principle of set mappings and in accordance with the operations of interval numbers are:

$$[a, b] + [c, d] = [a + c, b + d] \quad (1)$$

$$[a, b] - [c, d] = [a - d, b - c] \quad (2)$$

$$[a, b] \times [c, d] = [\min\{ac, ad, bc, bd\}, \max\{ac, ad, bc, bd\}] \quad (3)$$

$$\frac{[a, b]}{[c, d]} = \left[\min\left\{\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right\}, \max\left\{\frac{a}{c}, \frac{a}{d}, \frac{b}{c}, \frac{b}{d}\right\} \right] \quad (4)$$

The operation product \times in equations (3) has the problem that the range of the interval may increase rapidly. For example, consider two interval numbers $I = [-2, 3]$ and $I' = [100, 200]$. According to equation (3) the product of I and I' is $I \times I' = [-400, 600]$. The range of interval I is 5, the range of interval I' is 100. But the range of the interval $I \times I'$ is 1000. This rapid expansion of the range of the interval $I \times I'$ is not acceptable. The radius of fuzzy numbers will extend rapidly when performing the operations of product and division.

In the search for new fuzzy arithmetic calculus where the uncertainty involved in the evaluation of the underlying operation does not increase excessively, there has been some works done in fuzzy set theory. D. Dubois and H. Prade (Dubois & Prade, 1978; Dubois & Prade, 1988) have employed the t-norm to extend the operation of membership degrees for defining the Cartesian product of fuzzy subsets and then generalized Zadeh's extension principle to t-extension principle. Their work has made an order among different t-norms using an inequality according to its effectiveness of restraining the increasing of uncertainty involved in the evaluations across calculations. The more the t-norm is to the left of the inequality the better the arithmetic operation. The minimum t-norm T_m , which corresponds to the existing operations related to equations (1) through (4), sits on the right-extreme end of the inequality. People then look toward the left of the inequality to search for a t-norm to get more reasonable fuzzy calculations along the t-norm ordering. This is a direction guiding our research. Especially, people focus attention on the t-norm T_w , which sits on the left-extreme end of the t-norm ordering inequality. Many worthy works have been published recently along this direction (Hong, 2001; Mares & Mesiar, 2002) Mula et al., 2006).

The extension principle is a prudent principle in mathematics to define set-operations. It considers all possible; no omission! That is why it causes the extension rapidly. Based on the extension principle, any definition of the operation \times for fuzzy numbers could not avoid the decreasing of uncertainty, even using the t-norm T_w . The operations of random variables are indeed defined according to a kind of extension principle, which can carry probabilities. Existing arithmetic operations for fuzzy numbers and the operations for

random variables are all constructed in an objective approach. However, experts' estimation is a subjective approach. It is a decisive principle: Don't care about omissions, but do aim at the essential point; neglect the unimportant points even though they are possible to occur; only concentrate on the most important location. The width (radius) of the membership function of a fuzzy parameter does not reflect on any relevant objective distribution, but only the subjective accuracy. The arithmetic operations of fuzzy parameters keep the operations on the estimated values of the fuzzy parameters. As ordinary real numbers, they keep ordinary arithmetic operations. The additional consideration here is the operations of their estimation errors. When two fuzzy parameters α_1 and α_2 have the same estimation error δ , then the same estimation error δ is applied to $\alpha_1 \pm \alpha_2$ or $\alpha_1 \times \alpha_2$, or $\alpha_1 \div \alpha_2$; If they have different estimation errors, then the estimation error of $\alpha_1 \pm \alpha_2$ or $\alpha_1 \times \alpha_2$, or $\alpha_1 \div \alpha_2$ must be between the two original estimation errors. Hence the following definition:

Definition 3.2 Let $\alpha_i = a_i \pm |a_i| \times \delta_i$, ($i = 1, 2$). The arithmetic operations of fuzzy parameters are defined as:

$$m(\alpha_1 + \alpha_2) = a_1 + a_2, e(\alpha_1 + \alpha_2) = \delta_1 * \delta_2; \quad (3.3)$$

$$m(\alpha_1 - \alpha_2) = a_1 - a_2, e(\alpha_1 - \alpha_2) = \delta_1 * \delta_2; \quad (3.4)$$

$$m(\alpha_1 \times \alpha_2) = a_1 \times a_2, e(\alpha_1 \times \alpha_2) = \delta_1 * \delta_2; \quad (3.5)$$

$$m(\alpha_1 \div \alpha_2) = a_1 \div a_2, e(\alpha_1 \div \alpha_2) = \delta_1 * \delta_2. \quad (3.6)$$

Here

$$\min\{\delta_1, \delta_2\} \leq \delta_1 * \delta_2 \leq \max\{\delta_1, \delta_2\}. \quad (3.7)$$

For simplicity, we define $\delta_1 * \delta_2 = \max\{\delta_1, \delta_2\}$ in this work. The inequalities in (3.7) could be called the *estimation-error-limitation principle*. This effectively prevents the rapid extension of uncertainty when the arithmetic operations of fuzzy parameters are taken into consideration.

It is not difficult to see that the new arithmetic operation definitions on fuzzy parameters and the ordinary arithmetic operation definitions of fuzzy numbers are coincident for the operations $+$ and $-$ whenever $\delta_1 = \delta_2$. Of course, they are not coincident on the \times and \div operations.

4. The application of the new arithmetic operations in supply chains

We observe that the value of $q_{ji}(t)$, the order-away quantity of C_j at time t , is not known yet. If it is not deterministic, then uncertainties occur when we take estimation on this value.

As mentioned earlier, there are two kinds of uncertainties: randomness and fuzziness. If there are enough data representing the past values and the conditions related to those data are continuing onto the present, then the value $q_{ji}(t)$ could be treated as a random variable; otherwise, if there is not enough statistical data available, or if the conditions have been changed, it could be better treated as a fuzzy parameter and be estimated as mentioned in Section 3. As a supplement to the existing treatments of uncertainty in supply chain analysis, this chapter presents a new phase of the uncertainties treatment when we encounter randomness and fuzziness simultaneously. Accordingly, no matter whether the parameter is a random variable or a fuzzy number, it is always treated as a fuzzy parameter:

$$q_{ji}(t) = m(q_{ji}(t))(1 \pm \delta(q_{ji}(t)));$$

$$m(q_{ji}(t)) = E(q_{ji}(t)), \delta(q_{ji}(t)) = \sigma(q_{ji}(t)) \text{ whenever } q_{ji}(t) \text{ is a random variable} \quad (4.1)$$

Here $E(q_{ji}(t))$ is the mathematical expectation of $q_{ji}(t)$ and $\sigma(q_{ji}(t))$ is the root-mean-square error of $q_{ji}(t)$. Why does the author treat a random variable as a fuzzy parameter? Because the core modeling for fuzzy supply chain analysis is imitating the experts' experiences. Experts responsible for fuzzy supply chain analysis apply their skill at two stages: 1. Estimating value of each involved parameter; 2. Choosing of arithmetic operations on fuzzy parameters according to the estimation-error-limitation principle. In the first stage, if the estimated value is the mathematical expectation of a random variable, then the expert could rely on the objective methods in probability theory and get the resulting value. That is fine! It could save expert's time to do subjective estimation. Whenever the fuzzy estimations have been input into the second stage, the root-mean-square error has been transferred into the estimation error in the fuzzy parameters' operations. This will not involve operations on the probability distributions of random variable. Apart from taking the operations \pm on independent variable, there may not be any need to do rigid probabilistic operations on random variables' in the practical applications.

Similarly, the *order-away quantity* of C_j at time t , $a_j(t)$, is also a fuzzy parameter no matter it is a random variable or a fuzzy number.

$$\alpha_j(t) = m(\alpha_j(t))(1 \pm e(\alpha_j(t))). \quad (4.2)$$

Here

$$m(\alpha_j(t)) = \sum \{m(q_{ji}(t')) \mid \exists i \in C, t \leq t' < t + T\} \quad (4.3)$$

$$e(\alpha_j(t)) = \max \{e(\alpha_j(t')) \mid \exists i \in C \text{ and } t \leq t' \leq t + T\} \quad (4.4)$$

The *order-away rate* of p_j at the time t , $\bar{\alpha}_j(t)$, is also a fuzzy parameter no matter it is a random variable or a fuzzy number.

$$\bar{\alpha}_j(t) = m(\bar{\alpha}_j(t))(1 \pm e(\bar{\alpha}_j(t))), \quad (4.5)$$

where

$$m(\bar{\alpha}_j(t)) = \frac{1}{T_j} (m(\alpha_j(t))), \quad (4.6)$$

$$e(\bar{\alpha}_j(t)) = e(\alpha_j(t)). \quad (4.7)$$

For a root-site C_0 , the demand number at time t is treated as a fuzzy parameter $d(t) = m(d(t))(1 \pm e(d(t)))$. The demand rate of p_0 at the time t , treated as a fuzzy parameter

$$\bar{d}(t) = m(\bar{d}(t))(1 \pm e(\bar{d}(t))). \quad (4.8)$$

The material lead time from C_k to C_j , M_{kj} , is also to be treated as a fuzzy parameter no matter it is a random variable or a fuzzy number.

$$M_{kj} = m(M_{kj})(1 \pm e(M_{kj}))$$

$$m(M_{kj}(t)) = E(M_{kj}(t)), e(M_{kj}(t)) = \sigma(M_{kj}(t)) \text{ whenever } M_{kj}(t) \text{ is a random variable} \quad (4.9)$$

$$M_j = m(M_j)(1 \pm e(M_j)); \quad (4.10)$$

Here

$$m(M_j) = \max \{m(M_{kj}) \mid k \in U_j\} \quad (4.11)$$

$$e(M_j) = \max \{e(M_{kj}) \mid k \in U_j\} \quad (4.12)$$

The delay time of p_k -parts for C_j , G_{kj} , is also to be treated as a fuzzy parameter:

$$G_{kj} = c(G_{kj})(1 \pm \delta(G_{kj}));$$

$$m(D_{kj}(t)) = E(G_{kj}(t)), e(G_{kj}(t)) = \sigma(G_{kj}(t)) \text{ whenever } G_{kj}(t) \text{ is a random variable} \quad (4.13)$$

$$G_j = m(G_j)(1 \pm e(G_j)) \quad (4.14)$$

Where

$$m(G_j) = \max \{m(G_{kj}) \mid k \in U_j\} \quad (4.15)$$

$$e(G_j) = \max \{e(G_{kj}) \mid k \in U_j\} \quad (4.16)$$

Although it is possible along the same line as above, we omit writing the fuzzy parameters such as the deterministic quantities: the cycle time τ_j , the estimated number of occurrences of downtime φ_j , the duration of downtime on the production line \mathcal{G}_j . While the production capacity C_j is a deterministic number, it could be also treated as a fuzzy parameter provided we take $m(C_j) = C_j$ and $e(C_j) = 0$. The replenishment time of c_j , L_j , is also treated as a fuzzy parameter

$$L_j = m(L_j)(1 \pm e(L_j)), \quad (4.17)$$

where

$$m(L_j) = m(M_j) + m(G_j) + P_j. \quad (4.18)$$

The order-up-to level for site c_j at time t , $S_j(t)$ is a fuzzy parameter, which is the product of $\alpha_j(nT_j)$ and $(T_j + L_j)$ is:

$$S_j(t) = \alpha_j(nT_j) \times (T_j + L_j) = m(S_j(t))(1 \pm e(S_j(t))), \quad (4.19)$$

where

$$\begin{aligned} m(S_j(t)) &= m(\alpha_j(nT_j)) \times (T_j + m(L_j)), \\ e(S_j(t)) &= \max\{e(\alpha_j(nT_j)), e(L_j)\}. \end{aligned} \quad (4.20)$$

We note that shortage may occur whenever

$$I_j(t) < S_j(nT_j), \quad (4.21)$$

where $nT_j \leq t < nT_j + L_j(t)$. It implies that the shortage interval could be roughly written as $(-\infty, S_j)$. But since S_j is a fuzzy number, the interval is not a crisp interval. To conveniently control the inventory, we need to pick out two thresholds from S_j : two real numbers S_j^o and S_j^p called the *optimistic* and the *pessimistic* order-up-to levels of site c_j , respectively. They are determined by the following equations:

$$\Pi^+(S_j^o) = r = N(S_j^p), \quad (4.22)$$

where r is a given fill rate, A typical value is $r = 0.95$; and

$$\Pi^+(x) = \max\{\mu_{S_j}(u) \mid u \leq x\}, \quad N(x) = 1 - \max\{\mu_{S_j}(u) \mid x > u\}. \quad (4.23)$$

These are called the left possibility function and the necessary function of fuzzy variable S_j respectively. We have a formula to get the two real order-up-to levels from the fuzzy order-up-to level:

4.1 Proposition

$$\begin{aligned} S_j^o &= (1 - e(S_j) + r \times e(S_j)) \times m(S_j); \\ S_j^p &= (1 + r \times e(S_j)) \times m(S_j). \end{aligned} \quad (4.24)$$

Proof The increasing part of the left possibility function is in accordance with the left-wing of the membership function of the fuzzy parameter S_j . It is obvious that

$$\Pi^+(S_j^o) = \mu_{S_j}(S_j^o) = (S_j^o - m(S_j) + e(S_j) \times m(S_j)) / (m(S_j) \times e(S_j)).$$

From (4.22) we get

$$(S_j^o - m(S_j) + e(S_j) \times m(S_j)) / m(S_j) \times e(S_j) = r$$

$$S_j^o = r \times m(S_j) \times e(S_j) + m(S_j) - e(S_j) \times m(S_j) = (1 - e(S_j) + r \times e(S_j)) \times m(S_j)$$

This is the optimistic threshold. Similarly, we can get the pessimistic threshold.

A key task of fuzzy supply chain analysis is the determination of the optimistic and the pessimistic order-up-to levels of all sites in the supply chain.

5. Stationary strategy

The roles of a supply chain are transferring raw materials as parts-flow, flowing down along the supply chain network, and the quantities of the flow are determined by information-flow flowing up inversely. There are no mathematical formulae to calculate the order-up-to levels for all sites in general supply chains. However, there could be the possibility for special simple supply chains, which are stationary supply chains defined as follows:

Definition 5.1 Suppose that (C, S) is a simple supply chain. When $\bar{d}(t) \equiv \bar{d}(a \leq t \leq b)$

where \bar{d} is a fuzzy parameter, we say that the simple supply chain is *stationary* on the interval $[a, b]$.

Just as the stationary random process has a stationary distribution, a stationary supply chain has a stationary possibility distribution with a constant demand rate \bar{d} . Of course, the real demand from the customers is still a variable. No matter how complex the supply chain system is, we can think of it as a network of water flow. In the water flow network, we will have a stationary flow whenever the input equals the output at every node. To maintain a stationary flow in a supply chain network, the best way is for any site to know how many units passed away during the last period; how many units should be ordered back in the review time. Here comes the stationary strategy.

Stationary Strategy: For a simple supply chain with any site $c_j \in C$, and any $n > 0$, the number of passed away quantity $\alpha_j(nT_j)$ of c_j is known at the time $t = nT_j$, without special note, the default order-in quantities of c_j may be given by the following formula:

$$q_{kj}(nT_j) = w_{kj} \times \alpha_j(nT_j) \quad (k \in U_j). \quad (5.1)$$

Stationary strategy aims to lead the parts-flow within the supply chain network achieving the equilibrium between output and input at every site. Even though the equilibrium is not synchronous but with a time-delay, the supply chain network will keep constant inventory for each site after a while.

Proposition 5.1 Suppose that a simple supply chain is stationary: $\bar{d}(t) \equiv \bar{d}$. Under the stationary strategy, the passed-away number for each site c_j in C is also stationary which is given by:

$$\alpha_j(nT_j) \equiv \alpha_j = w_j \times \bar{d} \quad (n = 1, 2, \dots). \quad (5.2)$$

Proof. We use the principle of mathematical induction for the code $n = \chi(c_j)$.

Assume that the Proposition is true for any c_j with code $n = \chi(c_j) = 1$. Indeed, $\chi(c_j) = 1$ implies that $c_j \rightarrow c_0$. It is obvious that (5.2) is true for the base case.

Suppose that (5.2) is true for n , we are going to prove that it is true for $n + 1$. Suppose that $c_j \rightarrow c_i$ and $\chi(c_j) = n$, then $\alpha_j(t) = w_i \times \bar{d}$. We have that

$$\alpha_j(mT_j) = w_{ji} \times \alpha_i(mT_i) = w_{ji} \times w_i \times \bar{d} = w_j \times \bar{d}$$

So (5.2) is true.

According to (4.20), we have

$$\begin{aligned} m(S_j(t)) &= w_j \times m(\bar{d}) \times (T_j + m(L_j)) \\ e(S_j(t)) &= \max\{e(\bar{d}), e(L_j)\} \end{aligned} \quad (5.3)$$

Since the chain is in stationary, we can write $m(L_j)$ in detail according to (4.1)-(4.18) and get

$$\begin{aligned} m(S_j) &= w_j \times m(\bar{d}) \\ &\times (T_j + \max\{c(M_{kj}) \mid k \in U_j\} + (m(\bar{d}) \times w_j \times T_j \times c(\tau_j) \times (1 + c(\varphi_j) \times c(\mathcal{G}_j)))) / C_j) \\ e(S_j) &= \max\{\max\{e(M_{kj}) \mid k \in U_j\}, e(\bar{d}), e(\tau_j), e(\varphi_j), e(\mathcal{G}_j)\}. \end{aligned} \quad (5.4)$$

According to (4.24), we get

$$S_j^o = (1 - e(S_j) + r \times e(S_j)) \times w_j \times m(\bar{d}) \times (T_j + \max\{\dots\} + \dots)$$

$$S_j^p = (1 + r \times e(S_j)) \times w_j \times m(\bar{d}) \times (T_j + \max\{\dots\} + \dots) \quad (5.5)$$

The likely situations of a simple supply chain system are that: 1. Supply chain is stationary; 2. the inventory in each site is keeping its order-up-to level. In this situation, the simple chain is in the optimal situation and the parts flow is stationary with the minimum inventory cost and fulfills the target fill rate on the final products at the root.

Definition 5.2: A simple supply chain is called *optimal* if it is in the stationary situation and the inventory number equals to the order-up-to level in all the sites of the chain.

When a simple supply chain is stationary but the inventory number is not equal to the order-up-to level in each site, then we can take the following strategy to push the supply chain to attain an optimal situation:

Optimal strategy: For a simple stationary supply chain at the review time $t = nT_j$ on the site c_j ,

1. If $I_j(t) \leq S_j^o + w_k \times m(\bar{d}) \times T_j$, then take $q_{kj} = w_k \times m(\bar{d}) \times T_j + (S_j^o - I_j(t))$
2. If $I_j(t) > S_j^p + w_k \times \bar{d} \times T_j$, then take $q_{kj} = 0$.

(5.6)

Here $I_j(t)$ is the inventory of c_j at review time t . We can see that the optimal strategy (5.6) is the same as the stationary strategy (5.1) whenever $I_j(t) = S_j$. It means that whenever the inventory equals the order-up-to level, the optimal strategy automatically returns to the stationary strategy to keep the inventory at the order-up-to level successively. The optimal situation could be conserved until the demand rate \bar{d} is changed.

6. Example

To apply the theory described above to a problem, an example (Wang and Shu, 2005) is adapted in this section. Assume that a supply chain contains one distribution center, the root-site c_0 and six production facilities: c_0 has one up-site c_1 ; c_1 has three up-sites c_2 , c_3 , and c_4 ; c_2 has an up-site c_5 ; and c_5 has one up-site c_6 . The site c_1 has also two external suppliers s_1 and s_2 . The sites c_3 , c_4 and c_6 are proper boundary sites: four external suppliers s_4 , s_5 , s_6 , and s_7 , supply the site c_3 , s_3 supplies the site c_6 , and s_8 supplies the site c_4 . So that the supply chain for the problem consists of $C = \{c_0, c_1, c_2, c_3, c_4, c_5, c_6\}$ and $C^* = C \cup \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8\}$. The graphical representation of the supply chain is shown in Fig. 5.

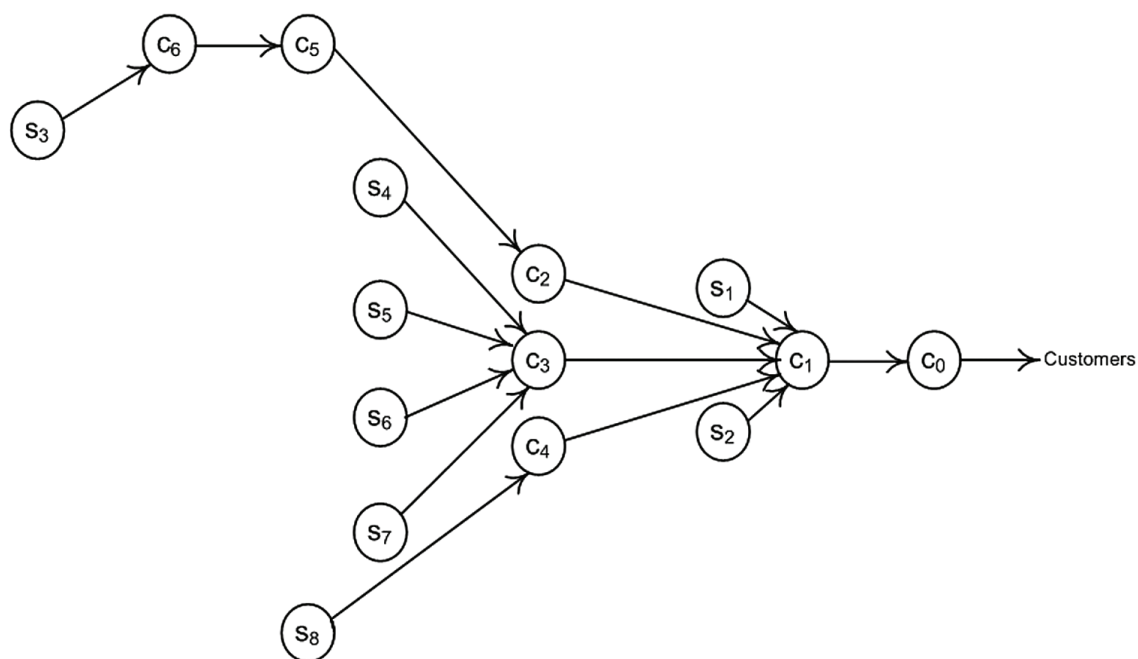


Figure 5. Example of a simple supply chain network

Assume that the *equivalence* of a product for p_j -parts is $w_j = 1, (j = 1, 2, \dots, 6)$. The supply chain is simple and is assumed stationary and the daily customer demand for the finished product at the root-site c_0 is the fuzzy number $\bar{d} = 200(1 \pm 0.5)$.

Assume that the review periods (days) are given as: $T_0 = 2, T_1 = 3, T_2 = 4, T_3 = 3, T_4 = 3, T_5 = 4, T_6 = 5$. Let the production cycle times (10^{-2} times/hr) be given as the following with estimations:

$$m(\tau_1) = 4.2, \quad m(\tau_2) = 2.0, \quad m(\tau_3) = 3.0, \quad m(\tau_4) = 3.3, \quad m(\tau_5) = 3.2, \\ m(\tau_6) = 2.8$$

and degree of ambiguities:

$$e(\tau_1) = 0.3, \quad e(\tau_2) = 0.25, \quad e(\tau_3) = 0.3, \quad e(\tau_4) = 0.2, \quad e(\tau_5) = 0.1, \quad e(\tau_6) = 0.3.$$

Let the downtime frequencies (10^{-3} times/hr) be given as the following with estimations:

$$m(\varphi_1) = 1.3, \quad m(\varphi_2) = 1.4, \quad m(\varphi_3) = 1.3, \quad m(\varphi_4) = 1.5, \quad m(\varphi_5) = 1.7, \\ m(\varphi_6) = 1.5$$

and degree of ambiguities:

$$e(\vartheta_1) = 0.15, \quad e(\vartheta_2) = 0.14, \quad e(\vartheta_3) = 0.12, \quad e(\vartheta_4) = 0.2, \quad e(\vartheta_5) = 0.13, \\ e(\vartheta_6) = 0.12.$$

Let the downtime (hr/time) be given as the following with estimations:

$$m(\vartheta_1) = 2.0, \quad m(\vartheta_2) = 2.2, \quad m(\vartheta_3) = 2.3, \quad m(\vartheta_4) = 1.9, \quad m(\vartheta_5) = 3.0, \\ m(\vartheta_6) = 2.5,$$

and the degree of ambiguities

$$e(\mathcal{G}_1) = 0.15, \quad e(\mathcal{G}_2) = 0.14, \quad e(\mathcal{G}_3) = 0.12, \quad e(\mathcal{G}_4) = 0.2, \quad e(\mathcal{G}_5) = 0.13, \\ e(\tau_6) = 0.12$$

Let the capacities (hr/day) be given as:

$$C_1 = 16, C_2 = 16, C_3 = 40, C_4 = 32, C_5 = 40, C_6 = 40.$$

Let the transition time (days) be given as the following with estimations:

$$m(M_0) = 2.0, \quad m(M_{10}) = 1.0, \quad m(M_{21}) = 3.0, \quad m(M_{31}) = 2.0, \quad m(M_{41}) = 3.0, \\ m(M_{52}) = 2.0, \quad m(M_{65}) = 3.0,$$

and the degree of ambiguities:

$$e(M_0) = 0.5, \quad e(M_{10}) = 0.5, \quad e(M_{21}) = 3.0, \quad e(M_{31}) = 0.25, \quad e(M_{41}) = 0.17, \\ e(M_{52}) = 0.25, \quad e(M_{65}) = 0.17.$$

Let the transition time (days) for the external suppliers be given as the following with estimations:

$$m(M_{s_{1,1}}) = 4.0, \quad m(M_{s_{2,1}}) = 5.0, \quad m(M_{s_{3,6}}) = 4.0, \quad m(M_{s_{4,3}}) = 3.0, \\ m(M_{s_{5,3}}) = 5.0, \quad m(M_{s_{6,3}}) = 4.0, \quad m(M_{s_{7,3}}) = 2.0, \quad m(M_{s_{8,4}}) = 3.0,$$

and the degree of ambiguities:

$$e(M_{s_{1,1}}) = 0.25, \quad e(M_{s_{2,1}}) = 0.2, \quad e(M_{s_{3,6}}) = 0.25, \quad e(M_{s_{4,3}}) = 0.17, \\ e(M_{s_{5,3}}) = 0.2, \\ e(M_{s_{6,3}}) = 0.25, \quad e(M_{s_{7,3}}) = 0.25, \quad e(M_{s_{8,4}}) = 0.17.$$

According to (5.4), we get that

$$m(S_1) = m(\bar{d}) \times (T_1 + \max\{m(M_{21}), m(M_{31}), m(M_{41}), m(M_{s_{1,1}}), m(M_{s_{2,1}})\} + \\ + m(\bar{d}) \times T_1 \times m(\tau_1) \times (1 + m(\varphi_1) \times m(\mathcal{G}_1)) / C_1) \\ = 200 \times (3 + \max\{3.0, 2.0, 3.0, 4.0, 5.0\} + 200 \times 3 \times 0.042 \times (1 + 0.0013 \times 2) / 16) = 1916 \\ e(S_1) = \max\{\max\{e(M_{21}), e(M_{31}), e(M_{41}), e(M_{s_{1,1}}), e(M_{s_{2,1}})\}, e(\bar{d}), e(\tau_1), e(\varphi_1), e(\mathcal{G}_1)\} \\ = 0.5.$$

$$m(S_2) = w_2 \times m(\bar{d}) \times (T_2 + m(M_{52}) + m(\bar{d}) \times w_2 \times T_2 \times m(\tau_2) \times (1 + m(\varphi_2) \times m(\mathcal{G}_2)) / C_2) \\ = 200 \times (4 + 2.0 + 200 \times 4 \times 0.02 \times (1 + 0.0014 \times 2.2) / 16) = 1401;$$

$$e(S_2) = \max\{\max\{e(M_{52})\}, e(\bar{d}), e(\tau_2), e(\varphi_2), e(\mathcal{G}_2)\} = 0.5.$$

$$m(S_3) = m(\bar{d}) \times (T_3 + \max\{m(M_{s_{4,3}}), m(M_{s_{5,3}}), m(M_{s_{6,3}}), m(M_{s_{7,3}})\} + \\ + m(\bar{d}) \times T_3 \times m(\tau_3) \times (1 + m(\varphi_3) \times m(\mathcal{G}_3)) / C_3)$$

$$= 200 \times (3 + \max\{3.0, 5.0, 4.0, 2.0\}) + 200 \times 3 \times 0.03 \times (1 + 0.0013 \times 2.3) / 40 = 1690$$

$$e(S_3) = \max\{\max\{e(M_{s_{4,3}}), e(M_{s_{5,3}}), e(M_{s_{6,3}}), e(M_{s_{7,3}})\}, e(\bar{d}), e(\tau_3), e(\varphi_3), e(\vartheta_3)\} \\ = 0.5$$

$$m(S_4) = m(\bar{d}) \times (T_4 + \max\{m(M_{s_{8,4}})\}) + m(\bar{d}) \times T_4 \times m(\tau_4) \times (1 + m(\varphi_4) \times m(\vartheta_4)) / C_4 \\ = 200 \times (3 + 3.0 + 200 \times 3 \times 0.033 \times (1 + 0.0015 \times 1.9) / 32) = 1324;$$

$$e(S_4) = \max\{\max\{e(M_{s_{8,4}})\}, e(\bar{d}), e(\tau_4), e(\varphi_4), e(\vartheta_4)\} = 0.5.$$

$$m(S_5) = m(\bar{d}) \times (T_5 + \max\{m(M_{s_{6,5}})\}) + m(\bar{d}) \times T_5 \times m(\tau_5) \times (1 + m(\varphi_5) \times m(\vartheta_5)) / C_5 \\ = 200 \times (4 + 3.0 + 200 \times 4 \times 0.032 \times (1 + 0.0017 \times 3) / 40) = 1529;$$

$$e(S_5) = \max\{\max\{e(M_{s_{6,5}})\}, e(\bar{d}), e(\tau_5), e(\varphi_5), e(\vartheta_5)\} = 0.5.$$

$$m(S_6) = m(\bar{d}) \times (T_6 + \max\{m(M_{s_{3,6}})\}) + m(\bar{d}) \times T_6 \times m(\tau_6) \times (1 + m(\varphi_6) \times m(\vartheta_6)) / C_6 \\ = 200 \times (5 + 4.0 + 200 \times 5 \times 0.028 \times (1 + 0.0015 \times 2.5) / 40) = 1941;$$

$$e(S_6) = \max\{\max\{e(M_{s_{3,6}})\}, e(\bar{d}), e(\tau_6), e(\varphi_6), e(\vartheta_6)\} = 0.5.$$

Since the root-site c_0 is a non-production site, we have that

$$m(S_0) = m(\bar{d}) \times (T_0 + m(M_{10})) = 600;$$

$$e(S_0) = e(M_{10}) = 0.5.$$

According to (4.24), the optimal and the pessimistic order-up-to levels for the pre-specified rate $r = 0.95$ at the sites c_j , $j = 1, 2, \dots, 6$, are given as:

$$S_1^o = (1 - e(S_1) + r \times e(S_1)) \times m(S_1) = 1,868;$$

$$S_1^p = (1 + r \times e(S_1)) \times m(S_1) = 2,826.$$

$$S_2^o = (1 - e(S_2) + r \times e(S_2)) \times m(S_2) = 1,366;$$

$$S_2^p = (1 + r \times e(S_2)) \times m(S_2) = 2,066.$$

$$S_3^o = (1 - e(S_3) + r \times e(S_3)) \times m(S_3) = 1,648;$$

$$S_3^p = (1 + r \times e(S_3)) \times m(S_3) = 2,493.$$

$$S_4^o = (1 - e(S_4) + r \times e(S_4)) \times m(S_4) = 1,291;$$

$$S_4^p = (1 + r \times e(S_4)) \times m(S_4) = 1,953.$$

$$S_5^o = (1 - e(S_5) + r \times e(S_5)) \times m(S_5) = 1,491;$$

$$S_5^p = (1 + r \times e(S_5)) \times m(S_5) = 2,255.$$

$$S_6^o = (1 - e(S_6) + r \times e(S_6)) \times m(S_6) = 1,892;$$

$$S_6^p = (1 + r \times e(S_6)) \times m(S_6) = 2,863.$$

At the root site C_0 , the optimal and the pessimistic order-up-to levels at C_0 are

$$S_0^o = (1 - e(S_0) + r \times e(S_0)) \times m(S_0) = 585;$$

$$S_0^p = (1 + r \times e(S_0)) \times m(S_0) = 885.$$

Thus the order-up-to levels in all sites of supply chain can be easily calculated.

7. Conclusion

As a supplement on fuzzy supply chain analysis, this chapter presents modeling for supply chain problems. In particular it answers question such as the following to the readers:

1. How to estimate parameters with fuzziness in supply chains? How to imitate experts' experiences as an estimation process? How to change our used subjective approach to be an acceptable subjective way?
2. How to define the arithmetic operations for fuzzy parameters? How to abandon the prudent principle of classical mathematics and accept the decisive principle in subjective estimation? What is the direction to prevent the uncertainty-increasing during performing arithmetic operations on fuzzy parameters?
3. How to treat fuzzy parameters when the randomness and fuzziness occur simultaneously?
4. How to simplify the complex analysis of supply chain? What is a simple chain? What is a stationary supply chain? How to get some formulae to calculate the order-up-to levels in a stationary simple chain? How to extend the advantages of pure mathematical analysis to the general cases?

From the answers to these questions presented in this chapter, the reader will find out new aspects and new considerations. It will be helpful to reflect by asking this question again: Where is the purpose of this chapter in the book? Yes, it is a supplement of fuzzy supply chain analysis. But, in some sense, it is also a supplement of non-deterministic supply chain analysis. In some other sense, it is also a supplement of the pure mathematical analysis on supply chains.

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Supply Chain

Edited by Vedran Kordic

ISBN 978-3-902613-22-6

Hard cover, 568 pages

Publisher I-Tech Education and Publishing

Published online 01, February, 2008

Published in print edition February, 2008

Traditionally supply chain management has meant factories, assembly lines, warehouses, transportation vehicles, and time sheets. Modern supply chain management is a highly complex, multidimensional problem set with virtually endless number of variables for optimization. An Internet enabled supply chain may have just-in-time delivery, precise inventory visibility, and up-to-the-minute distribution-tracking capabilities. Technology advances have enabled supply chains to become strategic weapons that can help avoid disasters, lower costs, and make money. From internal enterprise processes to external business transactions with suppliers, transporters, channels and end-users marks the wide range of challenges researchers have to handle. The aim of this book is at revealing and illustrating this diversity in terms of scientific and theoretical fundamentals, prevailing concepts as well as current practical applications.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Rajan Alex (2008). Fuzzy Parameters and Their Arithmetic Operations in Supply Chain Systems, Supply Chain, Vedran Kordic (Ed.), ISBN: 978-3-902613-22-6, InTech, Available from:
http://www.intechopen.com/books/supply_chain/fuzzy_parameters_and_their_arithmetic_operations_in_supply_chain_systems

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