Chapter

Financial Time Series Analysis via Backtesting Approach

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Abstract

This book chapter investigated the place of backtesting approach in financial time series analysis in choosing a reliable Generalized Auto-Regressive Conditional Heteroscedastic (GARCH) Model to analyze stock returns in Nigeria. To achieve this, The chapter used a secondary data that was collected from www.cashcraft.com under stock trend and analysis. Daily stock price was collected on Zenith bank stock price from October 21st 2004 to May 8th 2017. The chapter used nine different GARCH models (standard GARCH (sGARCH), Glosten-Jagannathan-Runkle GARCH (gjrGARCH), Exponential GARCH (Egarch), Integrated GARCH (iGARCH), Asymmetric Power Autoregressive Conditional Heteroskedasticity (ARCH) (apARCH), Threshold GARCH (TGARCH), Non-linear GARCH (NGARCH), Nonlinear (Asymmetric) GARCH (NAGARCH) and The Absolute Value GARCH (AVGARCH) with maximum lag of 2. Most the information criteria for the sGARCH model were not available due to lack of convergence. The lowest information criteria were associated with apARCH (2,2) with Student t-distribution followed by NGARCH(2,1) with skewed student t-distribution. The backtesting result of the apARCH (2,2) was not available while eGARCH(1,1) with Skewed student t-distribution, NGARCH(1,1), NGARCH(2,1), and TGARCH (2,1) failed the backtesting but eGARCH (1,1) with student t-distribution passed the backtesting approach. Therefore with the backtesting approach, eGARCH(1,1) with student distribution emerged the superior model for modeling Zenith Bank stock returns in Nigeria. This chapter recommended the backtesting approach to selecting reliable GARCH model.

Keywords: financial, time series, backtesting, GARCH, ARCH-LM

1. Introduction

Time series is a series of observation collected with respect to time. The time could be in minutes, hours, daily, weekly, monthly, yearly etc. Time series data can be seen and applied in all fields of endeavors such as engineering, geophysics, business, economics, finance, agriculture, medical sciences, social sciences, meteorology, quality control etc. [1] but this chapter focused on financial time series analysis.

In the field of time series analysis, Autoregressive Moving Average (ARMA) and Autoregressive Integrated Moving Average (ARIMA) models are popular and excellent for modeling and forecasting univariate time series data as proposed by Box and Jenkins in 1970 but many times these models failed in analyzing and forecasting financial time series [2], this is because the ARMA and ARIMA models are used to model conditional expectation of a process but in ARMA model, the conditional variance is constant. This means that ARMA model cannot capture process with time-varying conditional variance (volatility) which is mostly common with economic and financial time series data [3].

In economic and financial time series literatures, time-varying is more common than constant volatility, and accurate modeling of time volatility is of great importance in financial time series analysis by financial econometricians [4]. In practice, financial time series contains uncertainty, volatility, excess kurtosis, high standard deviation, high skewness and sometimes non normality [3]. Therefore, to model and capture properly the characteristics of financial time series models such as Auto-Regressive Conditional Heteroscedastic (ARCH), Generalized Auto-Regressive Conditional Heteroscedastic (GARCH), multivariate GARCH, Stochastic volatitlity (SV) and various variants of the models have been proposed to handle these characteristics of financial time series [5]. This chapter would focus on univariate GARCH models. In practice, the backtesting approach compliment the estimated GARCH model, in order to select a reliable GARCH model useful for real life application.

This book chapter aimed at obtaining reliable GARCH model via backtesting approach using daily Zenith bank Nigeria plc stock returns.

2. Empirical literature reviews of previous studies

Emenogu, et al. [3] modeled and forecasted the Guaranty Trust (GT) Bank daily stock returns from January 22,001 to May 82,017 data set collected from a secondary source. The ARMA-GARCH models, persistence, half-life and backtesting were used to analyzed the collected data using student t and skewed student t-distributions, and the analyses are carried out R environment using rugarch and performanceAnaytics Packages. The study revealed that using the lowest information criteria values only could be misleading rather we added the use of backtesing. The ARMA(1,1)-GARCH(1,1) models fitted exhibited high persistency in the daily stock returns while the days it takes for mean-reverting of the models is about 5 days, but unfortunately the models failed backtesting. The results further revealed ARMA(1,1)-eGARCH (2,2) model with student t distribution provides a suitable model for evaluating the GT bank stock returns among the competing models while it takes less than 30 days for the persistence volatility to return back to its average value of the stock returns. They recommended that researchers should adopt backtesting approach while fitting GARCH models while GT bank stocks investor should be assured that no matter the fluctuations in the stock market, the GT bank stock returns has the ability to returns to its mean price return.

Asemota and Ekejiuba [6] examined the volatility of banks equity weekly returns for six banks (coded B1 to B6) using GARCH models. Results reveal the presence of ARCH effect in B2 and B3 equity returns. In addition, the estimated models could not find evidence of leverage effect. On evaluating the estimated models using standard criteria, EGARCH (1, 1) and CGARCH (1, 1) model in Student’s t-distribution are adjudged the best volatility models for B2 and B3 respectively. The study recommends that in modeling stock market volatility, variants of GARCH models and alternative error distribution should be considered for robustness of results. The study also recommended for adequate regulatory effort by the CBN over commercial banks operations that will enhance efficiency of their stocks performance and reduce volatility aimed at boosting investors’ confidence in the banking sector.

Adigwe, et al. [7], examined the effect of stock market development on Nigeria's economic growth. The objective of the study was to determine if stock market development significantly impact on the country's economic growth. Secondary data were employed for the study covering 1985 to 2014. Ordinary Least Square (OLS) econometric technique was used for the time series analysis in which variations in economic growth was regressed on market capitalisation ratio to GDP, value of stock traded ratio to GDP, trade openness and inflation rate. The analysis revealed that stock market has the potentials of growth inducing, but has not contributed meaningfully to Nigerian economic growth, since only 26.5% of variations in economic growth were explained by the stock market development variables. Based on this, they suggested for an encouragement of more investors in the market, improvement in the settlement system and ensuring investors' confidence in the market.

Yaya, et al. [8] examined the application of nonlinear Smooth Transition- Generalized Autoregressive Conditional Heteroscedasticity (ST-GARCH) model of Hagerud on prices of banks' shares in Nigeria. The methodology was informed by the failure of the conventional GARCH model to capture the asymmetric properties of the banks' daily share prices. The asymmetry and non-linearity in the model dynamics make it useful for generating nonlinear conditional variance series. From the empirical analysis, we obtained the conditional volatility of each bank's share price return. The highest volatility persistence was observed in Bank 6, while Bank 12 had the least volatility. Evidently, about 25% of the investigated banks exhibited linear volatility behavior, while the remaining banks showed nonlinear volatility specifications. Given the level of risk associated with investment in stocks, investors and financial analysts could consider volatility modeling of bank share prices with variants of the ST-GARCH models. The impact of news is an important feature that relevant agencies could study so as to be guided while addressing underlying issues in the banking system.

Emenike and Aleke [9] studied the daily closing prices of the Nigerian stocks from January 1996 to December 2011 used the asymmetric GARCH variants. Their result showed strong evidence of asymmetric effects in the stock returns and therefore proposed EGARCH as performing better than other asymmetric rivals.

Arowolo [10] examined the forecasting properties of linear GARCH model for daily closing stocks prices of Zenith bank Plc in the Nigerian Stock Exchange. The Akaike and Bayesian Information Criteria (AIC and BIC) techniques were used to obtain the order of the GARCH (p,q) that best fit the Zenith Bank return series. The information criteria identified GARCH (1,2) as the appropriate model. His result further supported the claim that financial data are leptokurtic.

Emenike and Ani [11], examined the nature of volatility of stock returns in the Nigerian banking sector using GARCH models. Individual bank indices and the All-share Index of the Nigerian Stock Exchange were evaluated for evidence of volatility persistence, volatility asymmetry and fat tails using data from 3 January 2006 to 31 December 2012. Results obtained from GARCH models suggest that stock returns volatility of the Nigerian banking sector move in cluster and that volatility persistence is high for the sample period. The results also indicate that stock returns distribution of the banking sector is leptokurtic and that sign of the innovations have insignificant influence on the volatility of stock returns of the banks. Finally, the findings of this study show that the degree of volatility persistence is higher for the All Share Index than for most of the banks.

Abubakar and Gani [12] re-examined the long run relationship between financial development indicators and economic growth in Nigeria over the period 1970–2010. The study employed the Johansen and Juselius (1990) approach to cointegration and Vector Error Correction Modeling (VECM). Their findings revealed that in the long-run, liquid liabilities of commercial banks and trade openness exert significant positive influence on economic growth, conversely, credit to the private sector, interest rate spread and government expenditure exert significant negative influence. The findings implied that, credit to the private sector is marred by the identified problems and government borrowing and high interest rate are crowding out investment and growth. The policy implications are financial reforms in Nigeria should focus more on deepening the sector in terms of financial instruments so that firms can have alternatives to banks' credit which proved to be inefficient and detrimental to growth, moreover, government should inculcate fiscal discipline.

3. Model specification

This study focuses on the GARCH models that are robust for forecasting the volatility of financial time series data; so GARCH model and some of its extensions are presented in this section.

3.1 Autoregressive conditional heteroskedasticity (ARCH) family model

Every ARCH or GARCH family model requires two distinct specifications, namely: the mean and the variance Equations [13]. The mean equation for a conditional heteroskedasticity in a return series, y_t is given by

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (1)$$

where $\varepsilon_t = \varphi_t \sigma_t$.

The mean equation in Eq. (1) also applies to other GARCH family models. $E_{t-1}(.)$ is the expected value conditional on information available at time *t-1*, while ε_t is the error generated from the mean equation at time t and φ_t is the sequence of independent and identically distributed random variables with zero mean and unit variance.

The variance equation for an ARCH(p) model is given by

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \ \dots \ + \alpha_p a_{t-p}^2 \quad (2)$$

It can be seen in the equation that large values of the innovation of asset returns have bigger impact on the conditional variance because they are squared, which means that a large shock tends to follow another large shock and that is the same way the clusters of the volatility behave. So the ARCH(p) model becomes:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \ \dots \ + \alpha_p a_{t-p}^2 \quad (3)$$

Where $\varepsilon_t \sim N$ (0,1) *iid*, $\omega > 0$ and $\alpha_i \geq 0$ for $i > 0$. In practice, ε_t is assumed to follow the standard normal or a standardized student-*t* distribution or a generalized error distribution [14].

3.2 Asymmetric power ARCH

According to Rossi [15], the asymmetric power ARCH model proposed by [16] given below forms the basis for deriving the GARCH family models.

Given that:

$$\begin{aligned} r &= \mu + a_t, \\ \varepsilon_t &= \sigma_t \varepsilon_t, \\ \varepsilon_t &\sim N(0,1) \\ \sigma_t^\delta &= \omega + \sum_{i=1}^{p} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \end{aligned} \tag{4}$$

where

$$\begin{array}{ll} \omega > 0, & \delta \ge 0 \\ \alpha_i \ge 0 & i = 1, 2, \dots, p \\ -1 < \gamma_i < 1 & i = 1, 2, \dots, p \\ \beta_j > 0 & j = 1, 2, \dots, q \end{array}$$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative "shocks".

3.3 Standard GARCH(p, q) model

The mathematical model for the sGARCH(p,q) model is obtained from Eq. (4) by letting $\delta = 2$ and $\gamma_i = 0, i = 1, \dots, p$ to be:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{5}$$

Where $a_t = r_t - \mu_t$ (r_t is the continuously compounded log return series), and. $\varepsilon_t \sim N$ (0,1) *iid*, the parameter α_i is the ARCH parameter and β_j is the GARCH parameter, and $\omega > 0$, $\alpha_i \ge 0$, $\beta_j \ge 0$, and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$, [17].

The restriction on ARCH and GARCH parameters $\left(\alpha_i, \beta_j\right)$ suggests that the volatility (a_i) is finite and that the conditional standard deviation (σ_i) increases. It can be observed that if q = 0, then the model GARCH parameter (β_j) becomes extinct and what is left is an ARCH(p) model.

To expatiate on the properties of GARCH models, the following representation is necessary:

Let $\eta_t = a_t^2 - \sigma_t^2$ so that $\sigma_t^2 = a_t^2 - \eta_t$. By substituting $\sigma_{t-i}^2 = a_{t-i}^2 - \eta_{t-i}$, ($i$ = 0,…, q) into Eq. (3), the GARCH model can be rewritten as

$$a_t = \alpha_0 + \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) a_{t-i}^2 + \eta_t - \sum_{j=1}^{q} \beta_j \eta_{t-j}, \tag{6}$$

It can be seen that $\{\eta_t\}$ is a martingale difference series (i.e., $E(\eta_t)$ = 0 and. $\text{cov}\left(\eta_t, \eta_{t-j}\right) = 0$, for $j \ge 1$). However, $\{\eta_t\}$ in general is not an iid sequence.

A GARCH model can be regarded as an application of the ARMA idea to the squared series a_t^2. Using the unconditional mean of an ARMA model, results in this

$$\mathrm{E}\left(a_t^2\right) = \frac{\alpha_0}{1 - \sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i)}$$

provided that the denominator of the prior fraction is positive [14].

When p = 1 and q = 1, we have GARCH(1, 1) model given by:

$$\begin{aligned} a_t &= \sigma_t \varepsilon_t, \\ \sigma_t^2 &= \omega + \alpha_1 a_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \end{aligned} \tag{7}$$

3.4 GJR-GARCH(p, q) model

The Glosten-Jagannathan-Runkle GARCH (GJRGARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$.

When $\delta = 2$ and $0 \le \gamma_i < 1$,

$$\begin{aligned} \sigma_t^2 &= \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i}\right)^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \\ &= \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-1}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i}\right) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{8}$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i^2 (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \end{cases}$$

i.e.;

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \alpha_i \left\{(1+\gamma_i)^2 - (1-\gamma_i)^2\right\} S_i^- \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2 + \sum_{i=1}^{p} 4\alpha_i \gamma_i S_i^- \varepsilon_{t-i}^2$$

where $S_i^- = \begin{cases} 1 \text{ if } \varepsilon_{t-i} < 0 \\ 0 \text{ if } \varepsilon_{t-i} \ge 0 \end{cases}$,

Now define

$$\alpha_i^* = \alpha_i (1-\gamma_i)^2 \text{ and } \gamma_i^* = 4\alpha_i \gamma_i,$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i^* S_i^- \varepsilon_{t-1}^2 \tag{9}$$

Which is the GJRGARCH model [15].

But when $-1 \le \gamma_i < 0$,

Then recall Eq. (8)

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$= \omega + \sum_{i=1}^{p} \alpha_i \left(|\varepsilon_{t-i}|^2 + \gamma_i^2 \varepsilon_{t-1}^2 - 2\gamma_i |\varepsilon_{t-i}| \varepsilon_{t-i} \right) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i^2 (1-\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \\ \omega + \sum_{i=1}^{p} \alpha_i (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \end{cases}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \left\{ (1-\gamma_i)^2 - (1+\gamma_i)^2 \right\} S_i^+ \varepsilon_{t-i}^2$$

$$= \omega + \sum_{i=1}^{p} \alpha_i (1+\gamma_i)^2 \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i \{ 1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i \} S_i^+ \varepsilon_{t-i}^2$$

Where

$$S_i^+ = \begin{cases} 1 \text{ if } \varepsilon_{t-i} > 0 \\ 0 \text{ if } \varepsilon_{t-i} \le 0 \end{cases}$$

also define

$$\alpha_i^* = \alpha_i (1+\gamma_i)^2 \text{ and } \gamma_i^* = -4\alpha_i \gamma_i,$$

then

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i^* \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i^* S_i^+ \varepsilon_{t-1}^2 \tag{10}$$

which allows positive shocks to have a stronger effect on volatility than negative shocks [15]. But when $p = q = 1$, the GJRGARCH(1,1) model will be written as

$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma S_i \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{11}$$

3.5 IGARCH(1, 1) model

The integrated GARCH (IGARCH) models are unit- root GARCH models. The IGARCH (1, 1) model is specified in Grek [18] as

$$a_t = \sigma_t \varepsilon_t; \sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1-\beta_1) a_{t-1}^2 \tag{12}$$

Where $\varepsilon_t \sim \mathrm{N}(0, 1)$ *iid*, and $0 < \beta_1 < 1$, Ali (2013) used α_i to denote $1 - \beta_i$.

The model is also an exponential smoothing model for the $\{a_t^2\}$ series. To see this, rewrite the model as.

$$\begin{aligned} \sigma_t^2 &= (1-\beta_1) a_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \\ &= (1-\beta_1) a_{t-1}^2 + \beta_1 \left[(1-\beta) a_{t-2}^2 + \beta_1 \sigma_{t-2}^2 \right] \\ &= (1-\beta_1) a_{t-1}^2 + (1-\beta_1) \beta_1 a_{t-2}^2 + \beta_1^2 \sigma_{t-2}^2. \end{aligned} \tag{13}$$

By repeated substitutions, we have

$$\sigma_t^2 = (1-\beta_1)\left(a_{t-1}^2 + \beta_1 a_{t-2}^2 + \beta_1^2 a_{t-3}^3 + \cdots\right), \tag{14}$$

which is the well-known exponential smoothing formation with β_1 being the discounting factor [14].

3.6 TGARCH(p, q) model

The Threshold GARCH model is another model used to handle leverage effects, and a TGARCH(p, q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t=i})a_{t-i}^2 + \sum_{j=1}^{q}\beta_j \sigma_{t-j}^2, \tag{15}$$

where N_{t-i} is an indicator for negative a_{t-i}, that is,

$$N_{t-i} = \begin{cases} 1 & \text{if } a_{t-i} < 0, \\ 0 & \text{if } a_{t-i} \geq 0, \end{cases}.$$

and α_i, γ_i, and β_j are nonnegative parameters satisfying conditions similar to those of GARCH models, [14]. When $p = 1, q = 1$, the TGARCH(1, 1) model becomes:

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1})a_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{16}$$

3.7 NGARCH(p, q) model

The Nonlinear Generalized Autoregressive Conditional Heteroskedasticity (NGARCH) Model has been presented variously in literature by the following scholars [19–21]. The following model can be shown to represent all the presentations:

$$h_t = \omega + \sum_{i=1}^{q}\alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q}\gamma_i \varepsilon_{t-i} + \sum_{j=1}^{p}\beta_j h_{t-j} \tag{17}$$

Where h_t is the conditional variance, and ω, β and α satisfy $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$. Which can also be written as

$$\sigma_t = \omega + \sum_{i=1}^{q}\alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q}\gamma_i \varepsilon_{t-i} + \sum_{j=1}^{p}\beta_j \sigma_{t-j} \tag{18}$$

3.8 The exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model

The EGARCH model was proposed by Nelson [22] to overcome some weaknesses of the GARCH model in handling financial time series pointed out by [23], In particular, to allow for asymmetric effects between positive and negative asset returns, he considered the weighted innovation

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E(|\varepsilon_t|)], \tag{19}$$

where θ and γ are real constants. Both ε_t and $|\varepsilon_t| - E(|\varepsilon_t|)$ are zero-mean iid sequences with continuous distributions. Therefore, $E[g(\varepsilon_t)] = 0$. The asymmetry of $g(\varepsilon_t)$ can easily be seen by rewriting it as

$$g(\varepsilon_t) = \begin{cases} (\theta+\gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) & \text{if } \varepsilon_t \geq 0, \\ (\theta-\gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) & \text{if } \varepsilon_t < 0. \end{cases} \tag{20}$$

An EGARCH(m, s) model, according to Dhamija and Bhalla [24] can be written as

$$a_t = \sigma_t\varepsilon_t, \ \ln\left(\sigma_t^2\right) = \omega + \sum_{i=1}^{s} \alpha_i \frac{|a_{t-i}| + \theta_i a_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j \ln\left(\sigma_{t-i}^2\right), \tag{21}$$

Which specifically results in EGARCH (1, 1) being written as

$$\begin{aligned} a_t &= \sigma_t\varepsilon_t \\ \ln\left(\sigma_t^2\right) &= \omega + \alpha\left([|a_{t-1}| - E(|a_{t-1}|)]\right) + \theta a_{t-1} + \beta \ln\left(\sigma_{t-1}^2\right) \end{aligned} \tag{22}$$

where $|a_{t-1}| - E(|a_{t-1}|)$ are iid and have mean zero. When the EGARCH model has a Gaussian distribution of error term, then $E(|\varepsilon_t|) = \sqrt{2/\pi}$, which gives:

$$\ln\left(\sigma_t^2\right) = \omega + \alpha\left(\left[|a_{t-1}| - \sqrt{2/\pi}\right]\right) + \theta a_{t-1} + \beta \ln\left(\sigma_{t-1}^2\right) \tag{23}$$

3.9 The absolute value GARCH (AVGARCH)

An asymmetric GARCH (AGARCH), according to Ali [25] is simply

$$a_t = \sigma_t\varepsilon_t; \sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i |\varepsilon_{t-i} - b|^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{24}$$

While the absolute value generalized autoregressive conditional heteroskedasticity (AVGARCH) model is specified as:

$$a_t = \sigma_t\varepsilon_t; \sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i} + b| - c\ (\varepsilon_{t-i} + b))^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{25}$$

3.10 Nonlinear (asymmetric) GARCH, or N(a)GARCH or NAGARCH

NAGARCH plays key role in option pricing with stochastic volatility because, as we shall see later on, NAGARCH allows you to derive closed-form expressions for European option prices in spite of the rich volatility dynamics. Because a NAGARCH may be written as

$$\sigma_{t+1}^2 = \omega + \alpha\sigma_t^2(z_t - \delta)^2 + \beta\sigma_t^2 \tag{26}$$

And if $z_t \sim IIDN(0,1)$, z_t is independent of σ_t^2 as σ_t^2 is only a function of an infinite number of past squared returns, it is possible to easily derive the long run, unconditional variance under NGARCH and the assumption of stationarity:

$$\begin{aligned} E\left[\sigma_{t+1}^2\right] &= \overline{\sigma}^2 = \omega + \alpha E\left[\sigma_t^2 (z_t - \delta)^2\right] + \beta E\left[\sigma_t^2\right] \\ &= \omega + \alpha E\left[\sigma_t^2\right] E\left(z_t^2 + \delta^2 - 2\delta z_t\right] + \beta E\left[\sigma_t^2\right] \\ &= \omega + \alpha \overline{\sigma}^2 \left(1 + \delta^2\right) + \beta \overline{\sigma}^2 \end{aligned} \tag{27}$$

Where $\overline{\sigma}^2 = E\left[\sigma_t^2\right]$ and $E\left[\sigma_t^2\right] = E\left[\sigma_{t+1}^2\right]$ because of stationary. Therefore

$$\overline{\sigma}^2\left[1 - \alpha\left(1 + \delta^2\right) + \beta\right] = \omega \Rightarrow \overline{\sigma}^2 = \frac{\omega}{1 - \alpha\left(1 + \delta^2\right) + \beta} \tag{28}$$

Which exists and positive if and only if $\alpha\left(1 + \delta^2\right) + \beta < 1$. This has two implications:

i. The persistence index of a NAGARCH(1,1) is $\alpha\left(1 + \delta^2\right) + \beta$ and not simply $\alpha + \beta$;

ii. a NAGARCH(1,1) model is stationary if and only if $\alpha\left(1 + \delta^2\right) + \beta < 1$.

See details in [22].

3.11 Persistence

The low or high persistency in volatility exhibited by financial time series can be determined by the GARCH coefficients of a stationary GARCH model. The persistence of a GARCH model can be calculated as the sum of GARCH (β_1) and ARCH (α_1) coefficients that is $\alpha + \beta_1$. In most financial time series, it is very close to one (1) [26, 27]. Persistence could take the following conditions:

If $\alpha + \beta_1 < 1$: The model ensures positive conditional variance as well as stationary.

If $\alpha + \beta_1 = 1$: we have an exponential decay model, then the half-life becomes infinite. Meaning the model is strictly stationary.

If $\alpha + \beta_1 > 1$: The GARCH model is said to be non-stationary, meaning that the volatility ultimately detonates toward the infinitude [27]. In addition, the model shows that the conditional variance is unstable, unpredicted and the process is non-stationary [28].

3.12 Half-life volatility

Half-life volatility measures the mean reverting speed (average time) of a stock price or returns. The mathematical expression of half-life volatility is given as

$$Half - Life = \frac{\ln(0.5)}{\ln(\alpha_1 + \beta_2)}$$

It can be noted that the value of $\alpha + \beta_1$ influences the mean reverting speed [27], which means that if the value of $\alpha + \beta_1$ is closer to one (1), then the volatility shocks of the half-life will be longer.

3.13 Backtesting

Financial risk model evaluation or backtesting is an important part of the internal model's approach to market risk management as put out by Basle Committee on

Banking Supervision [29]. Backtesting is a statistical procedure where actual profits and losses are systematically compared to corresponding VaR estimates [30]. This book chapter adopted Backstesting techniques of [29]; The test was implemented in R using rugarch package and this test considered both the unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances (see details in [31, 32].

The unconditional (Kupiec) test also refer to as POF-test (Proportion of failure) with its null hypothesis given as

$$H_0 : p = \hat{p} = \frac{y}{T}$$

Here y is the number of exceptions and T is the number of observations and k is the confidence level. The test is given as

$$LR_{POF} = -2\ln\left(\frac{(1-p)^{T-y}p^y}{\left[1-\left(\frac{y}{p}\right)^{T-y}\left(\frac{y}{T}\right)^y\right.}\right).$$

Under the null hypothesis that the model is correct and LR_{POF} is asymptotically chi-squared (χ^2) distributed with degree of freedom as one (1). If the value of the LR_{POF} statistic is greater than the critical value (or p-value$<$0.01 for 1% level of significant or p-value$<$0.05 for 5% level of significant) the null hypothesis is rejected and the model then is inaccurate.

The Christoffersen's Interval Forecast Test combined the independence statistic with the Kupiec's POF test to obtained the joint test [30, 31]. This test examined the properties of a good VaR model, the correct failure rate and independence of exceptions, that is condition coverage (cc). the conditional coverage (cc) is given as

$$LR_{cc} = LR_{POF} + LR_{ind}$$

Where

$$LR_{ind} = \sum_{i=2}^{n}\left[-2\ln\left(\frac{p(1-p)^{u_i-1}}{\left(\frac{1}{u_i}\right)\left(1-\frac{1}{u_i}\right)^{u_i-1}}\right)\right] - 2\ln\left(\frac{p(1-p)^{u-1}}{\left(\frac{1}{u}\right)\left(1-\frac{1}{u}\right)^{u-1}}\right)$$

Where u_i is the time between exceptions I and i-1 while u is the sum of u_i.

If the value of the LR_{cc} statistic is greater than the critical value (or p-value$<$0.01 for 1% level of significant or p-value$<$0.05 for 5% level of significant) the null hypothesis is rejected and that leads to the rejection of the model.

3.14 Distributions of GARCH models

In this study we employed two innovations namely student t and skewed student t distributions they can account for excess kurtosis and non-normality in financial returns [28, 33].

The student t-distribution is given as

$$f(y) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{y^2}{v}\right)^{-\frac{(v+1)}{2}};\ -\infty < y < \infty$$

The Skewed student t-distribution is given as

$$f(y;\mu,\sigma,v,\lambda)=\begin{cases} bc\left(1+\frac{1}{v-2}\left(\frac{b\left(\frac{y-\mu}{\sigma}\right)+a}{1-\lambda}\right)^{2}\right)^{-\frac{v+1}{2}}, if\ y<-\dfrac{a}{b} \\ bc\left(1+\frac{1}{v-2}\left(\frac{b\left(\frac{y-\mu}{\sigma}\right)+a}{1+\lambda}\right)^{2}\right)^{-\frac{v+1}{2}}, if\ y\geq-\dfrac{a}{b} \end{cases}$$

Where v is the shape parameter with $2 < v < \infty$ and λ is the skewness parameter with $-1 < \lambda < 1$. The constants a, b and c are given as

$$a=4\lambda c\left(\frac{v-2}{v-1}\right); b=1+3(\lambda)^{2}-a^{2}; c=\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)\Gamma\left(\frac{v}{2}\right)}}$$

Where μ and σ are the mean and the standard deviation of the skewed student t distribution respectively.

4. Method of data collection

The data used in this study is a secondary data that was collected from www.cashcraft.com under stock trend and analysis. Daily stock price was collected on Zenith bank stock price from October 21st 2004 to May 8th 2017.

The returns was calculated using the formula below

$$R_t = \ln P_t - \ln P_{t-1} \tag{29}$$

Where R_t is stock returns; P_t is the present stock price; P_{t-1} is the previous stock price and ln is the natural logarithm transformation. Then total observation becomes 3070.

5. Results and discussion

The section presented the results emanating from the analysis and discussions of results.

Figure 1 below presented the plot of the log of Zenith Bank returns which is the first step in financial time series analysis. The plot revealed some spikes at the early part of the return series while later the series returns became stable.

Figure 2 below presented the plot of the cleansed log of Zenith Bank returns, this is necessary to remove any possible outlier that may be presents in the return series.

The **Table 1** below presented the descriptive statistics of the zenith bank return series. The **Table 1** revealed a maximum return as 0.338000 while minimum return as −0.405850. The average return as 0.000114 which signifies a gain in the stock for the period under study. The series is negatively skewed with high value of kurtosis. The return series is not normally distributed and the return series is stationary with presence of ARCH effects in the return series, these are typical characteristics of a financial return series [34, 35].

Table 2 below presents the selection criteria values for daily zenith Bank stock returns based on the student and skewed student t-distributions. The log returns of

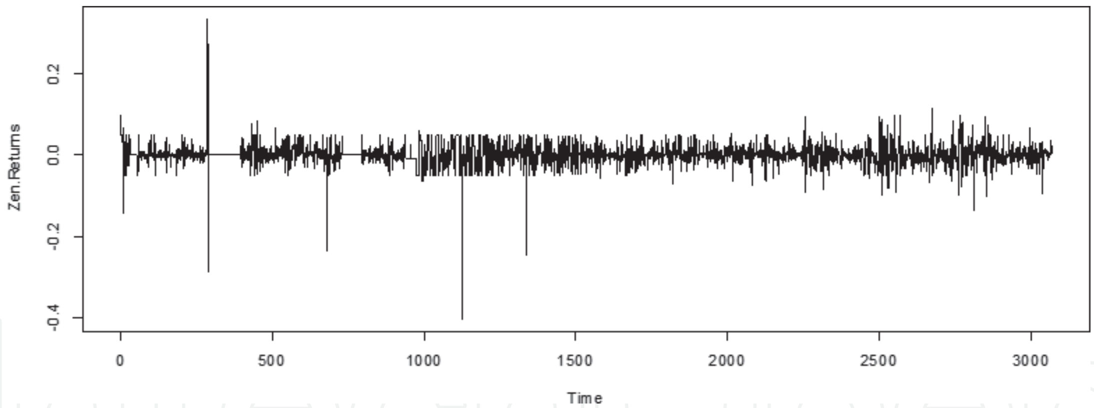

Figure 1.
The time plot of the log of zenith Bank returns.

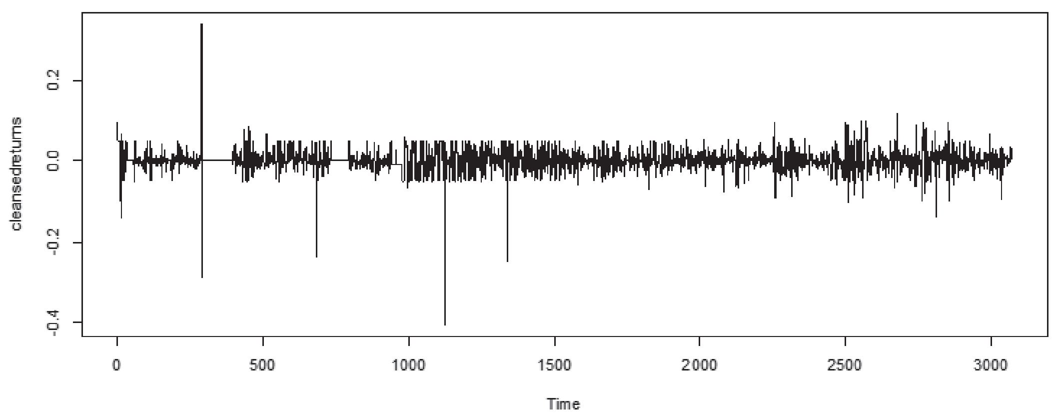

Figure 2.
The time plot of the removal of possible outliers in the log of zenith Bank returns.

Statistic	Value
Mean	0.000114
Median	0.000000
Maximum	0.338000
Minimum	−0.405850
Std. Dev.	0.027600
Skewness	−1.267452
Kurtosis	33.76662
Jarque-Bera	121905.9 (p = 0.00000)
Number of Observation	3070
Unit root testing	
ADF	−47.11172 (p = 0.0000)
DF-GLS	−1.842682
PP	−46.52078 (p = 0.0000)
ARCH test	
	Chi-squared = 123.05, df = 12, p-value <2.2e-16

Table 1.
Descriptive statistics and unit root testing of zenith Bank stock returns.

Model	Information criteria	Std t innovation	Skewed stdt innovation
sGARCH (1,1)	Akaike Bayes Shibata Hannan-Quinn	NA	NA
sGARCH (2,1)	Akaike Bayes Shibata Hannan-Quinn	NA	NA
sGARCH(2,2)	Akaike Bayes Shibata Hannan-Quinn	NA	−5.3667 −5.3530 −5.3667 −5.3618
gjrGARCH(1,1)	Akaike Bayes Shibata Hannan-Quinn	−5.5110 −5.5012 −5.5110 −5.5075	NA
gjrGARCH(2,1)	Akaike Bayes Shibata Hannan-Quinn	−5.5493 −5.5356 −5.5493 −5.5444	NA
gjrGARCH(2,2)	Akaike Bayes Shibata Hannan-Quinn	NA	NA
eGARCH (1,1)	Akaike Bayes Shibata Hannan-Quinn	−5.0584 −5.0485 −5.0584 −5.0548	−5.0587 −5.0469 −5.0587 −5.0545
eGARCH (2,1)	Akaike Bayes Shibata Hannan-Quinn	−5.0853 −5.0716 −5.0853 −5.0804	−5.0859 −5.0702 −5.0859 −5.0802
eGARCH (2,2)	Akaike Bayes Shibata Hannan-Quinn	−5.0196 −5.0039 −5.0196 −5.0140	NA
iGARCH (1,1)	Akaike Bayes Shibata Hannan-Quinn	−5.1474 −5.1415 −5.1474 −5.1453	−5.1498 −5.1420 −5.1498 −5.1470
iGARCH (2,1)	Akaike Bayes Shibata Hannan-Quinn	−5.1527 −5.1449 −5.1527 −5.1499	−5.1526 −5.1428 −5.1527 −5.1491
iGARCH (2,2)	Akaike Bayes Shibata Hannan-Quinn	−5.1496 −5.1397 −5.1496 −5.1460	−5.1547 −5.1429 −5.1547 −5.1505
TGARCH(1,1)	Akaike Bayes Shibata Hannan-Quinn	−5.8914 −5.8815 −5.8914 −5.8878	−5.8920 −5.8803 −5.8921 −5.8878
TGARCH(2,1)	Akaike Bayes Shibata Hannan-Quinn	−5.9253 −5.9115 −5.9253 −5.9203	−5.8819 −5.8662 −5.8819 −5.8763

Model	Information criteria	Std t innovation	Skewed stdt innovation
TGARCH(2,2)	Akaike	−5.8908	−5.8752
	Bayes	−5.8751	−5.8575
	Shibata	−5.8908	−5.8752
	Hannan-Quinn	−5.8851	−5.8688
NGARCH(1,1)	Akaike	−15.563	−13.191
	Bayes	−15.553	−13.179
	Shibata	−15.563	−13.191
	Hannan-Quinn	−15.559	−13.187
NGARCH(2,1)	Akaike	−14.470	−16.419
	Bayes	−14.458	−16.405
	Shibata	−14.470	−16.419
	Hannan-Quinn	−14.466	−16.414
NGARCH(2,2)	Akaike	−9.5866	−11.248
	Bayes	−9.5729	−11.232
	Shibata	−9.5866	−11.248
	Hannan-Quinn	−9.5817	−11.242
apARCH(1,1)	Akaike	−7.8258	NA
	Bayes	−7.8140	
	Shibata	−7.8258	
	Hannan-Quinn	−7.8216	
apARCH(2,1)	Akaike	−8.1226	−8.7718
	Bayes	−8.1069	−8.7541
	Shibata	−8.1226	−8.7718
	Hannan-Quinn	−8.1170	−8.7654
apARCH(2,2)	Akaike	−16.904	9.4341
	Bayes	−16.886	9.4538
	Shibata	−16.904	9.4341
	Hannan-Quinn	−16.897	9.4412
NAGARCH(1,1)	Akaike	−5.1428	−5.1402
	Bayes	−5.1330	−5.1285
	Shibata	−5.1428	−5.1403
	Hannan-Quinn	−5.1393	−5.1360
NAGARCH(2,1)	Akaike	−5.1296	−5.1343
	Bayes	−5.1158	−5.1186
	Shibata	−5.1296	−5.1343
	Hannan-Quinn	−5.1246	−5.1286
NAGARCH(2,2)	Akaike	−5.1221	−5.0439
	Bayes	−5.1063	−5.0262
	Shibata	−5.1221	−5.0439
	Hannan-Quinn	−5.1164	−5.0375
AVGARCH(1,1)	Akaike	−5.8467	−5.6004
	Bayes	−5.8349	−5.5866
	Shibata	−5.8467	−5.6004
	Hannan-Quinn	−5.8425	−5.5954
AVGARCH(2,1)	Akaike	−5.6197	−5.9524
	Bayes	−5.6020	−5.9327
	Shibata	−5.6197	−5.9524
	Hannan-Quinn	−5.6134	−5.9453
AVGARCH(2,2)	Akaike	−5.4227	−5.8644
	Bayes	−5.4031	−5.8428
	Shibata	−5.4228	−5.8644
	Hannan-Quinn	−5.4157	−5.8567

Note: *NA-Not Available.*

Table 2.
GARCH models and their performance on the log returns of daily log zenith Bank returns.

Model	Student t distribution					Skewed Student t distribution				
eGARCH (1,1)	Robust Standard Errors:					Robust Standard Errors:				
		Estimate	Std. Error	t value	Pr(>|t|)		Estimate	Std. Error	t value	Pr(>|t|)
	omega	−0.35726	0.021440	–16.6632	0.00000	omega	−0.35394	0.020239	–17.4879	0.00000
	alpha1	-0.10324	0.065840	-1.5681	0.11687	alpha1	-0.10597	0.065963	-1.6065	0.10817
	beta1	0.94958	0.001992	476.7729	0.00000	beta1	0.95011	0.001801	527.6448	0.00000
	gamma1	0.53417	0.080367	6.6467	0.00000	gamma1	0.53018	0.079261	6.6891	0.00000
	shape	2.64439	0.347058	7.6194	0.00000	skew	1.02439	0.012253	83.6013	0.00000
						shape	2.64681	0.345965	7.6505	0.00000
	Weighted Ljung-Box Test on Standardized Residuals					Weighted Ljung-Box Test on Standardized Residuals				
	------------------------------------					------------------------------------				
				statistic	p-value				statistic	p-value
	Lag [1]			19.23	1.160e-05	Lag [1]			18.99	1.314e-05
	Lag[2*(p+q)+(p+q)-1] [2]			19.28	6.872e-06	Lag[2*(p+q)+(p+q)-1] [2]			19.04	7.944e-06
	Lag[4*(p+q)+(p+q)-1] [5]			19.95	1.999e-05	Lag[4*(p+q)+(p+q)-1] [5]			19.71	2.335e-05
	d.o.f=0					d.o.f=0				
	H0: No serial correlation					H0: No serial correlation				
	Weighted ARCH LM Tests					Weighted ARCH LM Tests				
	------------------------------------					------------------------------------				
		Statistic	Shape	Scale	P-Value		Statistic	Shape	Scale	P-Value
	ARCH Lag [3]	0.02173	0.500	2.000	0.8828	ARCH Lag [3]	0.02100	0.500	2.000	0.8848
	ARCH Lag [5]	0.04570	1.440	1.667	0.9957	ARCH Lag [5]	0.04417	1.440	1.667	0.9959
	ARCH Lag [7]	0.06498	2.315	1.543	0.9998	ARCH Lag [7]	0.06276	2.315	1.543	0.9998

Table 3.
Parameter estimates and ARCH LM tests of the GARCH models.

the daily stock price of Zenith Bank returns were modeled with nine different GARCH models (sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH) with maximum lag of 2. Most the information criteria for the sGARCH model were not available because the model fails to converge. The lowest information criteria were associated with apARCH (2,2) with Student t-distribution followed by NGARCH(2,1) with skewed student t distribution. The caution here is that GARCH model should not be selected only based on information criteria only but the significance value of the coefficients, goodness-of-test fit and backtesting should be considered also [3]. The estimated GARCH models for the zenith bank stock with nine different GARCH models (sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH) shows that most of the coefficients of the fitted GARCH models were

Models	Std		Skewed std	
	Persistence	**Half-life volatility**	**Persistence**	**Half-life volatility**
sGARCH (1,1)	NA	NA	NA	NA
sGARCH (2,1)	NA	NA	NA	NA
sGARCH(2,2)	NA	NA	0.9783281	31.63581
gjrGARCH(1,1)	0.9945289	126.3447	NA	NA
gjrGARCH(2,1)	0.9939226	113.7067	NA	NA
gjrGARCH(2,2)	NA	NA	NA	NA
eGARCH (1,1)	0.9495821	13.39848	0.9501065	13.543
eGARCH (2,1)	0.9799226	34.17597	0.9802555	34.75809
eGARCH (2,2)	0.9775862	30.57714	NA	NA
iGARCH (1,1)	1	infinity	1	infinity
iGARCH (2,1)	1	infinity	1	infinity
iGARCH (2,2)	1	infinity	1	Infinity
TGARCH(1,1)	0.9463794	12.57713	0.9587135	16.43969
TGARCH(2,1)	0.9529079	14.36961	0.9506704	13.70184
TGARCH(2,2)	0.9315479	9.775345	0.9470317	12.73636
NGARCH(1,1)	0.9925847	93.1287	0.9732531	25.56687
NGARCH(2,1)	0.984207	43.54208	0.9888705	61.93282
NGARCH(2,2)	0.9704479	23.10679	0.9971636	244.0279
apARCH(1,1)	0.9759139	28.42987	NA	NA
apARCH(2,1)	0.9829391	40.28021	0.9853317	46.90736
apARCH(2,2)	0.9869038	52.58005	0.9513766	13.90596
NAGARCH(1,1)	0.9933088	103.2444	0.9950269	139.0335
NAGARCH(2,1)	0.9910378	76.99442	0.9942849	120.9365
NAGARCH(2,2)	0.9974602	272.5659	0.9978423	320.8989
AVGARCH(1,1)	0.9579476	16.13387	0.9315018	9.768526
AVGARCH(2,1)	0.9311321	9.714181	0.9513755	13.90564
AVGARCH(2,2)	0.9635552	18.6704	0.9633697	18.57406

Table 4.
Persistence and half-life volatility of the GARCH models of daily log zenith Bank stock returns.

Model	Distributions	Alpha	Expected Exceed	Actual VaR Exceed	Unconditional Coverage (Kupiec) H_0: Correct Exceedances	Conditional Coverage (Christoffersen) H_0: Correct Exceedances and independence of Failure
eGARCH (1,1)	Student t	1%	10.7	10	LR.uc Statistic: 0.047 LR.uc Critical: 6.635 LR.uc p-value: 0.828 Reject Null: NO	LR.cc Statistic: 0.236 LR.cc Critical: 9.21 LR.cc p-value: 0.889 Reject Null: NO
		5%	53.5	67	LR.uc Statistic: 3.332 LR.uc Critical: 3.841 LR.uc p-value: 0.068 Reject Null: NO	LR.cc Statistic: 3.497 LR.cc Critical: 5.991 LR.cc p-value: 0.174 Reject Null: NO
	Skewed student t	1%	10.7	10	LR.uc Statistic: 0.047 LR.uc Critical: 6.635 LR.uc p-value: 0.828 Reject Null: NO	LR.cc Statistic: 0.236 LR.cc Critical: 9.21 LR.cc p-value: 0.889 Reject Null: NO
		5%	53.5	74	LR.uc Statistic: 7.425 LR.uc Critical: 3.841 LR.uc p-value: 0.006 Reject Null: YES	LR.cc Statistic: 7.428 LR.cc Critical: 5.991 LR.cc p-value: 0.024 Reject Null: YES
NGARCH (1,1)	Student t	1%	10.7	76	LR.uc Statistic: 171.505 LR.uc Critical: 6.635 LR.uc p-value: 0 Reject Null: YES	LR.cc Statistic: 175.258 LR.cc Critical: 9.21 LR.cc p-value: 0 Reject Null: YES
		5%	53.5	135	LR.uc Statistic: 93.627 LR.uc Critical: 3.841 LR.uc p-value: 0 Reject Null: YES	LR.cc Statistic: 101.753 LR.cc Critical: 5.991 LR.cc p-value: 0 Reject Null: YES
NGARCH (2,1)	Skewed student t	1%	10.7	74	LR.uc Statistic: 163.466 LR.uc Critical: 6.635 LR.uc p-value: 0 Reject Null: YES	LR.cc Statistic: 171.614 LR.cc Critical: 9.21 LR.cc p-value: 0 Reject Null: YES
		5%	53.5	141	LR.uc Statistic: 106.038 LR.uc Critical: 3.841 LR.uc p-value: 0 Reject Null: YES	LR.cc Statistic: 111.739 LR.cc Critical: 5.991 LR.cc p-value: 0 Reject Null: YES
apARCH (2,2)	Student t	1%	NA	NA	NA	NA
		5%	NA	NA	NA	NA
TGARCH (2,1)	Student t	1%	10.7	31	LR.uc Statistic: 25.744 LR.uc Critical: 6.635 LR.uc p-value: 0 Reject Null: YES	LR.cc Statistic: 25.755 LR.cc Critical: 9.21 LR.cc p-value: 0 Reject Null: YES
		5%	53.5	92	LR.uc Statistic: 24.225 LR.uc Critical: 3.841 LR.uc p-value: 0 Reject Null: YES	LR.cc Statistic: 24.823 LR.cc Critical: 5.991 LR.cc p-value: 0 Reject Null: YES

Note: *uc.LRstat: the unconditional coverage test likelihood-ratio statistic; uc.critical: the unconditional coverage test critical value; uc.LRp: the unconditional coverage test p-value; cc.LRstat: the conditional coverage test likelihood-ratio statistic; cc. critical: the conditional coverage test critical value; cc.LRp: the conditional coverage test p-value; NA: not available.*

Table 5.
Backtesting of the GARCH models: GARCH roll forecast (backtest length: 1070) for the log daily zenith Bank stock returns.

not significant at 5% level except for eGARCH (1,1) model that provided significant coefficients in most cases. In the overall, most of the estimated GARCH models revealed absence of serial correlation in the error terms and absence of ARCH effects in the residuals. Because of limited space, we presented only the result of eGARCH (1,1) model in **Table 3** above.

Persistence of GARCH model measure whether the estimated GARCH model is stable or not as shown in **Table 4** above. In financial time series literature it should be less than 1 [3, 36]. Most of the models are stable except for iGARCH model. The half-life measure how long it will take for mean-reversion of the stock returns. The result revealed an average of 10 days for mean-reversion to take place.

The **Table 5** above presented the backtesting test of some selected GARCH model. The backtesting result of the apARCH (2,2) was not available while eGARCH(1,1) with Skewed student t-distribution, NGARCH(1,1), NGARCH(2,1), and TGARCH (2,1) failed the backtesting but eGARCH (1,1) with student t-distribution passed the backtesting approach which is supported by the results in **Table 5** above. Therefore with the backtesting approach, eGARCH(1,1) with student t-distribution emerged the superior model for modeling Zenith Bank stock returns in Nigeria [30, 31]. This chapter recommended the backtesting approach to selecting reliable GARCH model for estimating stock returns in Nigeria.

6. Conclusions

This book chapter investigated the place of backtesting approach in financial time series analysis in choosing a reliable GARCH Model for analyzing stock returns. To achieve this, The chapter used a secondary data that was collected from www.cashcraft.com under stock trend and analysis. Daily stock price was collected on Zenith bank stock price from October 21st 2004 to May 8th 2017. The chapter used nine different GARCH models (sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH) with maximum lag of 2. Most the information criteria for the sGARCH model were not available because the model could to converged. The lowest information criteria were associated with apARCH (2,2) with Student t distribution followed by NGARCH(2,1) with skewed student t distribution. The caution here is that GARCH model should not be selected only based on information criteria only but the significance value of the coefficients, goodness-of-test fit and backtesting should be considered also [3].

The backtesting result of the apARCH (2,2) was not available while eGARCH (1,1) with Skewed student t distribution, NGARCH(1,1), NGARCH(2,1), and TGARCH (2,1) failed the backtesting but eGARCH (1,1) with student t distribution passed the backtesting approach. Therefore with the backtesting approach, eGARCH(1,1) with student distribution emerged the superior model for modeling Zenith Bank stock returns in Nigeria [30, 31]. This chapter recommended the backtesting approach to selecting reliable GARCH model.

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Conflict of interest

The Author declares no conflict of interest.

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